1 CP model

 $X_{ij} = 1$: item i in bin j

 $Z_j = 1$: bin j has been used

 $R_i = 1$: item i rotated 90 degree

 $l_i, r_i.b_i, t_i$: left, right, bottom and top coordinates of item i

1.1 Constraints

1. Each item has to be placed in 1 bin:

$$\sum_{i=1}^{m} X_{ij} = 1 \text{ for i in n}$$

2. No two items overlap:

if
$$X_{i_1j} = X_{i_2j} = 1$$

$$r_{i_1} \le l_{i_2}$$
 or $r_{i_2} \le l_{i_1}$ or $t_{i_1} \le b_{i_1}$ or $t_{i_2} \le b_{i_1}$

3.Items cannot exceed the bin:

if $X_{ij} = 1$

$$\Rightarrow \begin{cases} \mathbf{w}_i \le r_i \le \mathbf{W}_j \\ \mathbf{h}_i \le t_i \le \mathbf{H}_j \end{cases}$$

4. If item i rotated:

if $R_i = 0$

$$\Rightarrow \begin{cases} \mathbf{r}_i = l_i + w_i \\ \mathbf{t}_i = b_i + h_i \end{cases}$$

else $R_i = 1$

$$\Rightarrow \begin{cases} \mathbf{r}_i = l_i + h_i \\ \mathbf{t}_i = b_i + w_i \end{cases}$$

1.2 Another way to approach

Keep constrains: 1, 2, 3

4. If item i rotated:

if $R_i = 0$

$$\Rightarrow \begin{cases} \mathbf{w}_i = w_i \\ \mathbf{h}_i = h_i \end{cases}$$

else $R_i = 1$

$$\Rightarrow \begin{cases} \mathbf{w}_i = j_i \\ \mathbf{h}_i = w_i \end{cases}$$

I will show how to implement this constrain in CP solver in the solver file