2 MIP model

Constant value M
N_bins: the number of bins given
N_items: the number of items given
W_j, H_j, C_j: the width, height, and cost of bin j, respectively
w_i, h_i: the width, height of item i, respectively

2.1 Decision Variables

* $X_{ij} = 1$: item i packed in bin j

$$\Rightarrow \sum_{i=1}^{\text{N\_items}} X_{ij} \geq 1 \Longleftrightarrow Z_j = 1\text{: bin j has been used}$$

⇒ To MIP:

$$\begin{cases} Z_j \leq \sum_{i=1}^{\text{N\_items}} X_{ij} * \text{M} \\ Z_j * \text{M} \geq \sum_{i=1}^{\text{N\_items}} X_{ij} \end{cases}$$

* $R_i = 1$: item i rotated 90 degree

Item's Coordinate:

l_i, r_i, b_i, t_i: left, right, bottom and top coordinates of item i
* if item i not rotated: $R_i = 0$

$$\Rightarrow \begin{cases} r_i = l_i + w_i \\ t_i = b_i + h_i \end{cases}$$

* if item i rotated: $R_i = 1$

$$\Rightarrow \begin{cases} r_i = l_i + h_i \\ t_i = b_i + w_i \end{cases}$$

⇒ To MIP:

$$\begin{cases} r_i = l_i + w_i * (1 - R_i) + h_i * R_i \\ t_i = b_i + h_i * (1 - R_i) + w_i * R_i \end{cases}$$

2.2 Constraints

2.2.1 Each item has to be placed in exactly 1 bin:

$$\sum_{j=1}^{\text{N\_bins}} X_{ij} = 1, \quad \forall i \in \{1, 2, ..., \text{N\_items}\}$$

2.2.2 No two items overlap:

if $X_{i_1 j} = X_{i_2 j} = 1$

$$r_{i_1} \leq l_{i_2} \text{ or } r_{i_2} \leq l_{i_1} \text{ or } t_{i_1} \leq b_{i_2} \text{ or } t_{i_2} \leq b_{i_1}$$

⇒ To MIP: add new variable e

$$\begin{cases} e \geq X_{i_1 j} + X_{i_2 j} \text{ - } 1 \\ e \leq X_{i_1 j} \\ e \leq X_{i_2 j} \\ r_{i_1} \leq l_{i_2} + \text{M} * (1 \text{ - } (r_{i_1} \leq l_{i_2})) \\ r_{i_2} \leq l_{i_1} + \text{M} * (1 \text{ - } (r_{i_2} \leq l_{i_1})) \\ t_{i_1} \leq b_{i_2} + \text{M} * (1 \text{ - } (t_{i_1} \leq b_{i_2})) \\ t_{i_2} \leq b_{i_1} + \text{M} * (1 \text{ - } (t_{i_2} \leq b_{i_1})) \\ (r_{i_1} \leq l_{i_2}) + (r_{i_2} \leq l_{i_1}) + (t_{i_1} \leq b_{i_2}) + (t_{i_2} \leq b_{i_1}) + (1 \text{ - e}) * \text{M} \geq 1 \\ (r_{i_1} \leq l_{i_2}) + (r_{i_2} \leq l_{i_1}) + (t_{i_1} \leq b_{i_2}) + (t_{i_2} \leq b_{i_1}) \leq e * M \end{cases}$$

2.2.3 Items cannot exceed the bin:

if $X_{ij} = 1$

$$\Rightarrow \begin{cases} w_i \leq r_i \leq W_j \\ h_i \leq t_i \leq H_j \end{cases}$$

⇒ To MIP:

$$\begin{cases} r_i \leq (1 - X_{ij}) * \text{M} + W_j \\ t_i \leq (1 - X_{ij}) * \text{M} + H_j \end{cases}$$

2.3 Objective Function

$$\textbf{minimize} \sum_{j=1}^{\text{N\_bins}} Z_j * C_j$$