1 CP model

$X_{ij} = 1$: item i in bin j
$Z_j = 1$: bin j has been used
$R_i = 1$: item i rotated 90 degree
l_i, r_i, b_i, t_i: left, right, bottom and top coordinates of item i

1.1 Constraints

1. Each item has to be placed in exactly 1 bin:

$$\sum_{j=1}^{m} X_{ij} = 1 \text{ for i in n}$$

2. No two items overlap:
if $X_{i_1 j} = X_{i_2 j} = 1$

$$r_{i_1} \leq l_{i_2} \text{ or } r_{i_2} \leq l_{i_1} \text{ or } t_{i_1} \leq b_{i_2} \text{ or } t_{i_2} \leq b_{i_1}$$

3. If item i rotated:
if $R_i = 0$

$$\Rightarrow \begin{cases} \mathrm{r}_i = l_i + w_i \\ \mathrm{t}_i = b_i + h_i \end{cases}$$

else $R_i = 1$

$$\Rightarrow \begin{cases} \mathrm{r}_i = l_i + h_i \\ \mathrm{t}_i = b_i + w_i \end{cases}$$

4. Items cannot exceed the bin:
if $X_{ij} = 1$

$$\Rightarrow \begin{cases} \mathrm{w}_i \leq r_i \leq \mathrm{W}_j \\ \mathrm{h}_i \leq t_i \leq \mathrm{H}_j \end{cases}$$

1.2 Another way to approach

Keep constraints: 1, 2, 4
3. If item i rotated:
if $R_i = 0$

$$\Rightarrow \begin{cases} \mathrm{w}_i = w_i \\ \mathrm{h}_i = h_i \end{cases}$$

else $R_i = 1$

$$\Rightarrow \begin{cases} \mathrm{w}_i = h_i \\ \mathrm{h}_i = w_i \end{cases}$$

I will show how to implement this constraint in CP solver in the solver file.