小熊樂園资格考试：数学和逻辑
Bear Resort Qualification Examination: Mathematics and Logics

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个人信息 / Personal Information

姓名 / Name: _____   小熊樂園编号 / Bear Resort ID: _____

诚信声明: 我承诺在考试中没有接受外部协助，遵守考试规定，且我的答案能最佳反应我的实力。

Integrity Statement: I certify that I have completed the exam without external help, abode the exam instructions, and my answers reflect my capabilities at the best extent.

签字 / Signature: _____   日期 / Date: _____

单项选择题。请选择下列选项中最适合的答案，答对得 5 分，不答得 2 分，答错不得分。

Multiple Choice Questions. Please select the most suitable choice. 5 points for the correct answer, 2 point for no answer, and no points for incorrect answer.

1. "$x \in \mathbb{R}$" 是 "$x = 1$ 是 $x^3 = 1$ 的唯一解'' 的什么条件?

   "$x \in \mathbb{R}$" is which kind of condition for "$x = 1$ being the unique solution to $x^3 = 1$?"

   ○ 充分且必要条件 / Sufficient and necessary condition.

   ○ 充分但不必要条件 / Sufficient but not necessary condition.

   ○ 不充分但必要条件 / Not sufficient but necessary condition.

   ○ 不充分不必要条件 / Not sufficient nor necessary condition.

2. 假设 "如果小小熊的体重超过了 220 磅, 小熊会要求他减肥." 下列哪个选项是假设的等价命题?

   Suppose that "if little bear's weight exceeds 220lbs, beary will demand him to lose weight." Which of the following is the equivalent statement?

   ○ 如果小小熊的体重没有超过 220 磅, 小熊不会要求他减肥.
   If little bear's weight does exceed 220lbs, beary will not demand him to lose weight.

   ○ 如果小熊要求小小熊减肥, 那么小小熊的体重超过了 220 磅.
   If beary asks little bear to lose weight, then the weight of little bear exceeds 220lbs.

   ○ 如果小熊没有要求小小熊减肥, 那么小小熊的体重定没超过 220 磅.
   If beary does not ask little bear to lose weight, then the weight of little bear must not exceed 220lbs.

   ○ 如果小小熊的体重没有超过 220 磅, 小熊可能要求他减肥.
   If little bear's weight does exceed 220lbs, beary might demand him to lose weight.

3. 已知 "尺规作图无法三等分任意角." 下列说法正确有几个?

   Given "it is impossible to trisect an arbitrary angle using only a straightedge and compass." How many of the following statements are correct?

   > • 给定任意一个角, 尺规作图做出其三等分角是不可能的.
   > Given any angle, it is impossible to trisect it using only a straightedge and compass.
   >
   > • 尺规作图无法六等分任意角.
   > It is impossible to divide an arbitrary angle into six equal parts using only a straightedge and compass.
   >
   > • 给定任意一个角, 尺规作图做出其六等分角是不可能的.
   > Given any angle, it is impossible to divide it into six equal parts using only a straightedge and compass.

   ○ 0.

   ○ 1.

   ○ 2.

   ○ 3.

4. 小小熊是一头很冲动、暴躁的熊. 对于任意两头其他不一样的熊, 他总会更想揍两头熊里的一头. 下列说法正确的有几个?

Little bear is an emotional and fiery bear. For any two other distinct bears, he always wants to beat one of them more. How many of the following statements are correct?

- 对于任意一群熊, 小小熊总能找到自己最想揍的一头.

  For any group of bears, little bear can always find one that he wants to beat the most.

- 对于任意一群熊, 小小熊每次从中随机挑选两头没被他打过的熊并打他更想打的一头, 最后一头被打的熊一定是确定的.

  For any group of bears, little bear randomly selects two bears that he has not beaten yet, and beat up the one that he wants to beat more, the last bear to be beaten is deterministic.

- 对于任意一群熊, 如果小小熊总能找到自己最想揍的一头, 那么他一定能找到自己最不想揍的一头.

  For any group of bears, if little bear can always find one that he wants to beat the most, then he can always find one that he wants to beat the least.

○ 0.

○ 1.

○ 2.

○ 3.

5. 已知质数是因子仅为 1 或者自己的不小于 2 的整数, 下列说法错误的是?

Given a prime numbers are integers no less than 2 whose only factors are 1 and itself, which of the following is incorrect?

○ 59 是质数. / 59 is a prime number.

○ 有无限多的质数. / There are infinitely many prime numbers.

○ 假设 $p_1, \cdots, p_n$ 为前 n 个质数, 那么 $p_1 \times \cdots \times p_n + 1$ 一定是质数. / Suppose $p_1, \cdots, p_n$ are the first n prime numbers, then $p_1 \times \cdots \times p_n + 1$ must be a prime number.

○ 令 n 为正整数, $2^n + 1$ 不一定是质数. / Let n be a positive integer, $2^n + 1$ is not necessarily prime.

6. 已知度量空间 (计作 d, 在集合 S 内) 必须满足下列要求:

1. 对任意两点 $x, y \in S$, 满足 $d(x, y) \geq 0$, 且 $d(x, y) = 0$ 当且仅当 $x = y$.

2. 对任意两点 $x, y \in S$, 满足 $d(x, y) = d(y, x)$.

3. 对任意三点 $x, y, z \in S$, 满足 $d(x, z) \leq d(x, y) + d(y, z)$.

It is known that the metric space (denoted d, in set S), must satisfies the following:

1. For any two points $x, y \in S$, they satisfy $d(x, y) \geq 0$ and $d(x, y) = 0$ if and only if $x = y$.

2. For any two points $x, y \in S$, they satisfy $d(x, y) = d(y, x)$.

3. For any three points $x, y, z \in S$, they satisfy $d(x, z) \leq d(x, y) + d(y, z)$.

下列在 $\mathbb{R}^n$ 的度量提议里, 形成度量空间的是?

In the following proposed metric of $\mathbb{R}^n$, which of them are metric space?

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1. 对 $x = (x_1, \cdots, x_n)$ 和 $y = (y_1, \cdots, y_n)$, 令 / For $x = (x_1, \cdots, x_n)$ and $y = (y_1, \cdots, y_n)$, let:
$$d(x, y) = \sum_{i=1}^{n} (x_i - y_i).$$

2. 对 $x = (x_1, \cdots, x_n)$ 和 $y = (y_1, \cdots, y_n)$, 令 / For $x = (x_1, \cdots, x_n)$ and $y = (y_1, \cdots, y_n)$, let:
$$d(x, y) = \sum_{i=1}^{n} (x_i - y_i)^2.$$

3. 对 x 和 y, 令 / For x and y, let:
$$d(x, y) = \begin{cases} 0, & x = y, \\ 1, & x \neq y. \end{cases}$$

○ 仅 2. / Only 2.

○ 1 和 2. / 1 and 2.

○ 2 和 3. / 2 and 3.

○ 全部都是. / All of them.

7. 已知代数基本定理, 即 "对于任意非常数的复系数多项式, 至少有一个复数根." 下列哪个选项不正确的?

   Given the fundamental theorem of algebra, that is "for any non-constant complex-coefficient polynomial, it has at least one complex root." Which of the following is not correct?

   ○ 对于任意非常数的实系数多项式, 至少有一个实数根.

   For any non-constant real-coefficient polynomial, it has at least one real root.

   ○ 对于任意非常数的 n 阶复系数多项式, 至少有 n 个复数根.

   For any nonconstant degree n non-constant complex-coefficient polynomial, it has at least n complex root.

   ○ 对于任意非常数的 n 阶复系数多项式, 都可以写作 n 项 $x - a$ 的乘积.

   For any nonconstant degree n non-constant complex-coefficient polynomial, it can be written as n products of some of $x - a$.

   ○ 对于任意非零常数多项式, 一定没有复数根.

   For any nonzero constant polynomial, it must not have a complex root.

8. 原本有 3^n 颗等重的苹果 (假设 n 是正整数), 但一颗被小小熊在中间掏了一个无法观测的小洞. 已知该苹果略轻于其他 $3^n - 1$ 颗苹果. 在最坏情况下, 最少需要几次天平测量可以找到这颗苹果?

   There were 3^n apples of the same weight (suppose n is positive integer), but little bear ate an unobservable hole in one of it. It is known that this apple is a little lighter than the other $3^n - 1$ apples. In the worst case, at least how many times of use of a balance can distinguish this apple?

   ○ n.

   ○ $3n$.

○ $3^n - 1$.

○ 3^n.

9. 小熊樂園往往会在星期四举办若干集体活动, 每个集体活动都有相应的开始时间和结束时间. 樂園方希望举办尽可能多的活动, 下列贪婪算法中正确的是?

The Bear Resort often holds various events on Thursdays, and every event has its start time and end time. The resort wishes to hold as many events as possible, so which of the following greedy algorithm is correct?

○ 由较短的活动选取 / Select from the shorter events.

○ 由最早开始的活动选取 / Select from the earliest starting events.

○ 由最晚开始的活动选取 / Select from the latest starting events.

○ 由与其他活动重复最少的活动选取 / Select from the event that has least overlap with the other events.

10. 下列在命题中有多少个是 "$x = 3$" 的必要但不充分命题?

How many of the following statements are the necessary but not sufficient condition for "$x = 3$"?

- x 是 $x^2 = 9$ 的解. / x is the solution to $x^2 = 9$.
- x 是 $x^2 = 9$ 的正数解. / x is the positive solution to $x^2 = 9$.
- x 是奇质数. / x is an odd prime number.
- x 是最小的奇质数. / x is the smallest odd prime number.

○ 0.

○ 1.

○ 2.

○ 3.

11. 假设 x, y 为非负实数, 下列在命题中等价关系正确的有几项?

Suppose x and y are nonnegative real numbers, how many of the following equivalent statements are correct?

- $x \leq y$ 等价于对于所有 $\varepsilon > 0$, $x < y + \varepsilon$.
  $x \leq y$ is equivalently that for all $\varepsilon > 0$, $x < y + \varepsilon$.
- $x = 0$ 等价于对于所有正整数 C, $x < \frac{1}{C}$.
  $x = 0$ is equivalently that for all positive real integer C, $x < \frac{1}{C}$.
- $x = y$ 等价于对于 $x \leq y$ 且 $x \geq y$.
  $x = y$ is equivalently that $x \geq y$ and $x \leq y$.

○ 0.

○ 1.

○ 2.

○ 3.

12. 等价关系是数学结构中任意两个元素的关系. 对于任意元素 a, b, c, 等价关系需要满足:

- 自反性: $a \sim a$.

- 对称性: $a \sim b$ 当且仅当 $b \sim a$.

- 传递性: $a \sim b$ 且 $b \sim c$ 代表 $a \sim c$.

下列等价关系的提议中是等价关系的有几个?

Equivalence relation is a basic binary relationship between two elements in a mathematical structure. For any elements a, b, c, the equivalence relation must satisfy that:

- Reflexivity: $a \sim a$.

- Symmetry: $a \sim b$ if and only if $b \sim a$.

- Transitivity: $a \sim b$ and $b \sim c$ imply $a \sim c$.

How many of the following proposed equivalence relationship is equivalence relationship.

- 在所有整数中, $a \sim b$ 当且仅当 $a - b$ 是 12 的倍数.
  For all integers, $a \sim b$ if and only if $a - b$ is a multiple of 12.

- 在所有实数中, 对于任意给定方程 φ, $a \sim b$ 当且仅当 $\varphi(a - b) = 0$.
  For all real numbers, for any given function φ, $a \sim b$ if and only if $\varphi(a - b) = 0$.

- 在所有实数中, 存在某个线性方程 φ 使得 $a \sim b$ 当且仅当 $\varphi(a - b) = 0$.
  For all real numbers, there exists some linear function φ such that $a \sim b$ if and only if $\varphi(a - b) = 0$.

○ 0.

○ 1.

○ 2.

○ 3.

13. 下列命题中最多的等价命题个数是?

What is this maximum number of equivalent statements below?

- 对于任意若干非空集合, 可以构造一个新集合包含每个非空集合中的一个元素.
  Give a collection of nonempty sets, it is possible to construct a new set by choosing one element from each set.

- 每个集合中都存在某种排序下的最小元素.
  Each set has some least element under some ordering.

- 每个集合中都存在某种排序下的最大元素.
  Each set has some largest element under some ordering.

- 在任意子集排序下, 排列存在上界.
  For any ordering of the subsets, there exists an upper bound.

○ 1.

○ 2.

○ 3.

○ 4.

14. 考虑 24 小时制时间精确到分钟, 每刻的时间可以写作一个二维数组 $(h, m) \in \{0, 1, \cdots, 12\} \times \{0, 1, \cdots, 59\}$. 假设每过一分钟和时间进制一致, 且 $(23, 59)$ 至 $(0, 0)$ 不考虑时期变更, 该时间制形成了一个循环. 下列选项正确的是?

Consider the 24-hour time accurate up to minutes, the time at each moment can be written as a 2-dimensional tuple $(h, m) \in \{0, 1, \cdots, 12\} \times \{0, 1, \cdots, 59\}$. Suppose the elapse of each minute is the same as how time works, and we neglect the date difference from $(23, 59)$ to $(0, 0)$, the time system forms a cycle. Which of the following statement is correct?

○ 对于任意两个时间 (h_1, m_1) 和 (h_2, m_2), $(h_1 + h_2, m_1 + m_2)$ 仍遵循这个时间标准, 且表达了两个时段的和.

For any two times (h_1, m_1) and (h_2, m_2), $(h_1 + h_2, m_1 + m_2)$ still abide this standard, and expresses the sum of the two time spans.

○ 任意 (h, m) 可以被等价地转换为 $60h + m$ 并取除以 1439 的余数.

Any (h, m) can be equivalently transformed into $60h + m$ and keep the remainder after dividing 1439.

○ 假设 φ 是可以转换该时间模式的一个方程, 满足 $\varphi((0, 0)) = (0, 0)$ 且对于任意两个时间 (h_1, m_1) 和 (h_2, m_2) 满足 $\varphi((h_1, m_1) + (h_2, m_2)) = \varphi((h_1, m_1)) + \varphi((h_2, m_2))$, 且 + 满足时间制的加法, 那么 (h, m) 以至于 $\varphi((h, m)) = (0, 0)$ 的时间数量一定是 1439 的除数.

Suppose φ is a function that can transform between this time system, which satisfies that $\varphi((0, 0)) = (0, 0)$ and for any two times (h_1, m_1) and (h_2, m_2) satisfying that $\varphi((h_1, m_1) + (h_2, m_2)) = \varphi((h_1, m_1)) + \varphi((h_2, m_2))$, whereas + satisfies the ordinary time addition, then (h, m) such that $\varphi((h, m)) = (0, 0)$ must have the number of elements as a divisor of 1439.

○ 存在一个时间系统的乘法在满足分配律的情况下为任意时间寻找一个时间以至于两个时间的乘积都是同一时间.

There exists a time system multiplication that satisfies the distribution rule, such that for each time, we can find another time so that the product of those two times is a constant time.

15. 零幂算子是一个线性算子 S, 存在正整数 N 以至于 $S^N(x) = 0$ 对于任何输入 x. 其中 S^N 指 $S(S(\cdots S(x)))$ 的算子运算了 N 次. 下列说法正确的是?

A nilpotent operator is a linear operator S in which there exists a positive integer N such that $S^N(x) = 0$ for all x. In particular, S^N means $S(S(\cdots S(x)))$, where the operator is applied N times. Which of the following statement is correct.

○ 如果 S 是一个线性算子并满足: 对于任意 x 都存在正整数 N 以至于 $S^N(x) = 0$, 那么 S 是零幂算子.

If S is a linear operator satisfying that: for any x, there exists a positive integer N such that $S^N(x) = 0$, then S is a nilpotent operator.

○ 存在一个线性算子 K 以至于 $K\big(x + S(x)\big) = x$.
There exists a linear operator K such that $K\big(x + S(x)\big) = x$.

○ 两个零幂算子的和也是零幂算子, 且算子的归零系数是两个算子归零系数的最大公约数.
The sum of two nilpotent operator is nilpotent, and the order of the sum is the greatest common divisor of the order of the respective nilpotent operators.

○ 存在一个线性算子 K 以至于 $K\big(S(x)\big) = x$.
There exists a linear operator K such that $K\big(1 + S(x)\big) = x$.

16. 假设一个实系数多项式, 如果该多项式存在一个非实数根, 计作 $a + b\mathrm{i}$, 那么其共轭复数, 计作 $a - b\mathrm{i}$, 也将是该多项式的一个根. 请问下列条件中能组成 "多项式至少有一个实数根" 的充分条件有几个?
Suppose you have a polynomial of real coefficient, if the polynomial as a non-real root, say $a + b\mathrm{i}$, then its complex conjugate $a - b\mathrm{i}$ will also be a root to the polynomial. How many of the following conditions can be the sufficient condition for "polynomial has at least one real root?"

> • 多项式的系数都为实数, 并且阶数是 23.
> The polynomial has real coefficents, and the degree of the polynomial is 23.
>
> • 多项式的系数都为有理数, 并且阶数是 12.
> The polynomial has rational coefficents, and the degree of the polynomial is 12.
>
> • 多项式的系数都为整数, 并且阶数是 35.
> The polynomial has integer coefficents, and the degree of the polynomial is 35.
>
> • $x = 1$ 是多项式的一个根.
> $x = 1$ is a root of the polynomial.

○ 0.

○ 1.

○ 2.

○ 3.

17. 假设 $f(x)$ 连续, 已知傅立叶级数可以将 $f(x)$ 在区间 $(0, 2\pi)$ 写作:

$$f(x) \sim c + \sum_{n=1}^{\infty} \big(a_n \sin(nx) + b_n \cos(nx)\big).$$

下列描述正确的是?

Suppose $f(x)$ is continuous, it is known that Fourier series is can write the function $f(x)$ on interval $(0, 2\pi)$ as:

$$f(x) \sim c + \sum_{n=1}^{\infty} \big(a_n \sin(nx) + b_n \cos(nx)\big).$$

Which of the following description is correct?

○ 如果 $f(x)$ 是偶函数 (即 $f(x) = f(-x)$), 那么 $f(x)$ 在 $(-2\pi, 2\pi)$ 可以仅用 $\sin(nx)$ 表示.
If $f(x)$ is a even function (*i.e.*, $f(x) = f(-x)$), then $f(x)$ can be expressed with only

$\sin(nx)$ terms on $(-2\pi, 2\pi)$.

○ 如果 $f(x)$ 是偶函数 (即 $f(x) = f(-x)$), 那么 $f(x)$ 在 $(-2\pi, 2\pi)$ 可以仅用 $\cos(nx)$ 表示.
If $f(x)$ is a even function (*i.e.*, $f(x) = f(-x)$), then $f(x)$ can be expressed with only $\cos(nx)$ terms on $(-2\pi, 2\pi)$.

○ 如果 $f(x)$ 是奇函数 (即 $f(x) = -f(-x)$), 那么 $f(x)$ 在 $(-2\pi, 2\pi)$ 可以仅用 $\sin(nx)$ 表示.
If $f(x)$ is a odd function (*i.e.*, $f(x) = f(-x)$), then $f(x)$ can be expressed with only $\sin(nx)$ terms on $(-2\pi, 2\pi)$.

○ 如果 $f(x)$ 是奇函数 (即 $f(x) = -f(-x)$), 那么 $f(x)$ 在 $(-2\pi, 2\pi)$ 可以仅用 $\cos(nx)$ 表示.
If $f(x)$ is a odd function (*i.e.*, $f(x) = f(-x)$), then $f(x)$ can be expressed with only $\cos(nx)$ terms on $(-2\pi, 2\pi)$.

18. 已知在十进制中, 所有能被 3 或 9 整除的数的数位和也能被 3 或 9 整除. (例: 522 可以被 3 和 9 整除, 因为数位和是 $5 + 2 + 2 = 9$). 下列进制系统中也有同样属性的有几个?
Note that in decimal numbers, all the numbers that is divisible by 3 or 9 must have the digit sum divisible by 3 or 9. (*E.g.*: 522 is divisible by 3 and 9, since the digit sum is $5 + 2 + 2 = 9$). For the following given numerical system, how many of the following properties are true?

> • 在八进制中, 1 或 7 的整除.
> For octal (base-8) numbers, all numbers divisible by 1 or 7.
>
> • 在十七进制中, 2 或 16 的整除.
> For base-17 numbers, all numbers divisible by 2 or 16.
>
> • 在十二进制中, 3 或 11 的整除.
> For base-12 numbers, all numbers divisible by 3 or 11.
>
> • 在十五进制中, 14 的整除.
> For base-15 numbers, all numbers divisible by 14.

○ 1.

○ 2.

○ 3.

○ 4.

19. 已知下列关于 $f : \mathbb{R} \to \mathbb{R}$ 三个条件, 请将它们从最强至最弱排序.
Given the following three conditions about $f : \mathbb{R} \to \mathbb{R}$, order them from the strongest to the weakest.

1. 存在 $C > 0$, 以至于对于任意 $x, y \in \mathbb{R}$, $|f(x) - f(y)| \le C|x - y|$.

   There exists some $C > 0$, such that for any $x, y \in \mathbb{R}$, $|f(x) - f(y)| \le |x - y|$.

2. 对于任意 $\varepsilon > 0$ 和 $x \in \mathbb{R}$ 都存在 $\delta > 0$ 以至所有满足 $|x - y| < \delta$ 的 $y \in \mathbb{R}$ 都满足 $|f(x) - f(y)| < \varepsilon$.

   For any $\varepsilon > 0$ and $x \in \mathbb{R}$, there exists some $\delta > 0$ such that all $y \in \mathbb{R}$ in which $|x - y| < \delta$ satisfies that $|f(x) - f(y)| < \varepsilon$.

3. $f(x + y) = f(x) + f(y)$ 对于任意 $x, y \in \mathbb{R}$.

   $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$.

- ○ $1 \Longrightarrow 2 \Longrightarrow 3$.
- ○ $1 \Longrightarrow 3 \Longrightarrow 2$.
- ○ $3 \Longrightarrow 2 \Longrightarrow 1$.
- ○ $3 \Longrightarrow 1 \Longrightarrow 2$.

20. 定义一个方程 $f : \mathbb{R} \to \mathbb{R}$ 为单射, 即 $f(x) = f(y)$ 当且仅当 $x = y$. 下列条件中是 $f : \mathbb{R} \to \mathbb{R}$ 为单射的必要但不充分条件是?

    Define a function $f : \mathbb{R} \to \mathbb{R}$ to be injective as $f(x) = f(y)$ if and only if $x = y$. Which of the following condition is the necessary but not sufficient condition for f being injective?

    - ○ 存在一个方程 $g : \mathbb{R} \to \mathbb{R}$ 以至于对于任何 $x \in \mathbb{R}$, $g(f(x)) = x$.
      There exists a function $g : \mathbb{R} \to \mathbb{R}$ such that for any $x \in \mathbb{R}$, $g(f(x)) = x$.

    - ○ 存在一个方程 $g : \mathbb{R} \to \mathbb{R}$ 以至于对于任何 $x \in \mathbb{R}$, $f(g(x)) = x$.
      There exists a function $g : \mathbb{R} \to \mathbb{R}$ such that for any $x \in \mathbb{R}$, $f(g(x)) = x$.

    - ○ $f(x) = 0$ 最多有一个解.
      $f(x) = 0$ has at most one solution.

    - ○ $f(x) = x$ 最多有一个解.
      $f(x) = x$ has at most one solution.

21. 小小熊发明了一种坚固的玩具球. 他现在有 3 个一样坚固的样品, 且他希望在 120 层楼中 (1 层至 120 层, 共 120 层) 测试该球最高不会摔碎的楼层. 注意: 球摔碎后无法再次测试. 他至少需要多少次测试?

    Little bear invented a rigid toy ball. He now has 3 samples that are as rigid, and he wants to test on a 120-level floor (level 1 to 120, a total of 120 levels) of the highest floor such that the ball would not break. Note: The ball cannot be used for testing once it is broken. How many tests does he have to conduct at least?

    - ○ 4 次 / 4 times.
    - ○ 7 次 / 7 times.
    - ○ 8 次 / 8 times.
    - ○ 12 次 / 12 times.

22. 假设平面上有 n 个顶点, 任意两个顶点之间可能有一条连线. 以下条件中不是 "任意两点之间存在任意条连线组成的连接" 的充分条件的是?

    Suppose there are n vertices, and there could be one connection line between any of the two vertices. Which of the following is not the sufficient condition for "there exists a connection composed of some connection lines between any two points"?

    ○ 平面上一共有 $n^2 - 3n + 2$ 条连线.
    The plane has a total of $n^2 - 3n + 2$ connection lines.

    ○ 平面上一共有 $n - 1$ 条连线且不形成循环.
    The plane has a total of $n - 1$ connection lines and they do not form any cycle.

    ○ 若移除任意一点及其连线, 任意剩余的两点之间仍存在任意条连线组成的连接.
    If we remove any point and its associated connection lines, any two remaining points has a connection composed of some connection lines between any two points.

    ○ 若移除添加一点及 2 条该点的连线, 任意两点之间仍存在任意条连线组成的连接.
    If we add a point and two connection lines associated to the new point, any two points has a connection composed of some connection lines between any two points.

23. 如果下列选项中只有一个是真话, 那么这个选项是?

    If only one of the choices below is true, what is the choice?

    ○ 这个选项是唯一的真话 / This choice is the only truth.

    ○ 下面的选项里没有真话 / The choices below has no truth.

    ○ 上面的选项里没有真话 / The choices above has no truth.

    ○ 第三个选项不是真话 / The third choice is not truth.

24. 定义 $\top, \perp$ 作为两个元素, 其中 $\top^n = \square$, $\perp^2 = \square$, 且 $\top\perp\top = \perp$, 其中 $\square$ 代表单位元, 即在任何序列中的 $\square$ 可被直接移除. 在不包含 $\square$ 的情况下, $\top$ 和 $\perp$ 排列组合能生成多少种独特的非空序列?

    Define $\top, \perp$ as two elements, such that $\top^n = \square$, $\perp^2 = \square$, and $\top\perp\top = \perp$, where $\square$ is the identity element, that is, you can remove $\square$ in any sequence. When excluding $\square$, how many nonempty, unique sequences can the permutation of $\top$ and $\perp$ generate?

    ○ $n + 1$.

    ○ $2n - 1$.

    ○ $2^n - 1$.

    ○ 无限多 / Infinitely many.

这是本场考试的结尾。
This is the end of this Exam.

此页留白是有意的。
不要在本页书写。

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