An Effective Algorithm of Splitting Rent

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Problem.

Suppose a group of n suit-mates are renting a large house composed of n different bedrooms together. Different people have different preference order of the rooms and we want to achieve the following goals:

- Each person will be assigned a unique room, and preferably on their preferences.
- The total sum of payments must add up to the total rent of the whole house.

The problem might seems not so hard, maybe we can just have a bidding process so the person willing to pay the most can get the room. However, this process is not really reasonable, and the distribution of the room fees will not be deterministic based on the different order of biding the rooms, and it is simply hard to ensure that the sum of the rent would add up, *e.g.*, the sum of the fees for the first $n-1$ rooms already exceeded the rent of the house, or the sum of the fees for the first $n-1$ rooms is very low, and the last room is not preferable at all.

Almost surely, there would appear fractions between the suit-mates, which is not what we want. This problem would especially be trickier when there are certain people being significantly richer, or when most people do not want to pay a lot for the rent. We will be providing a **good solution** to propose an effective algorithm of splitting the rent among the suit-mates.

Algorithm.

Given the n suit-mates and n rooms, each resident propose a nonnegative price to each room such that the sum of the prices add up to the rent of the house.
Note: Throughout the algorithm, there could be situations of draws. In this case, either do rock-paper-scissors, or just deal it with the men's way – have a real fight.

- The person who is willing to pay the most out of all rooms will get that room, whereas the rent of that room would be the average of all proposed prices from all n people for that room.

Then, we have reduced the problem into $n-1$ people arranging $n-1$ rooms, while the sum of the prices shall add up to the remaining unpaid rent of the house.

- We can keep doing this procedure until there is 1 people left and 1 room left, who would get the room and pay the remaining unpaid rent after the $n-1$ paid rents.

For simplicity, we will be providing you with an example run. Suppose we have A, B, and J, three people aiming to split a house with rooms 1, 2, and 3. The total house rent is \$1000.

Suppose the first round of rent would get the following result:

	1	2	3
A	250	400	350
B	300	500	200
J	300	450	250

Now, B would get room 2 with the rent \$450. The remaining people A and J gets to distribute rooms 1 and 3 with a total of \$550.

Suppose the second round of rent would get the following results:

	1	3
A	200	350
J	300	250

Thus, A would get room 3 with the rent \$300. With J left, J would get room q with rent \$250.

In this example, we may observe that each person gets a room with a price that is relatively lower than the highest they would want to pay, and the person with higher desire for the better room did get the room they prefer the most as long as they want to pay more. ♣

One can consider this as a simple example with **Game Theory**. As a side note, as one of the model resembling economics, this works the best with the following assumptions:

- Each person is **rational**. But technically, if a person really wants to pay **a lot** for a room, they are the only person to be worse off.
- Each person should have the same **amount of information**. Either everyone knows each other's preference list or no one knows the preference list of one another. The complete transparency or complete opacity makes the model works out the best. Technically, some people have more information than the others, they might find some way out to *collude*.

Hope that this document could help you and your bros when splitting out rents, and sustain your bro-ship. Good luck!