3) a)

$K_1 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $K_2 = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$ $K_3 = (7, 2)$

$x K_1 \bmod 11$

$x K_1 = [x_1 \ x_2] \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = [x_1 + 3x_2 \quad x_2]$

$(x K_1) K_2 \bmod 11$

$(x K_1) K_2 = [x_1 + 3X_2 \quad x_2] \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = [2X_1 + 7X_2 \qquad x_1 + 3x_2]$

Affine

$y_x = 7(x) + 2$

$y_1 = 7(2x_1 + 7x_2) + 2 = 14x_1 + 49x_2 + 2$

$y_2 = 7(x_1 + 3x_2) + 2 = 7x_1 + 21x_2 + 2$

$\Rightarrow y_1 \bmod 11 = 14x_1 + 49x_2 + 2 = \underline{3x_1 + 5x_2 + 2 \ \ m11}$

$y_2 \bmod 11 = 7x_1 + 21x_2 + 2 = 7x_1 + 10x_2 + 2 \ \ m11$

encryption rule $\boxed{y = (y_1, y_2) = (3x_1 + 5x_2 + 2, \ 7x_1 + 10x_2 + 2)}$

b)

inverse affine

$7^{-1} m11 = 8 \Rightarrow x = 8(y-2) \ m11$

inv K2

$det(K_2) = 2(0) - 1(1) = -1$

$adj(K_2) = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

$K_2^{-1} = det(K_2)\, adj(K_2) = \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} m11 = \begin{bmatrix} 0 & 10 \\ 10 & 2 \end{bmatrix}$

$K_2^{-1} = \begin{bmatrix} 0 & 10 \\ 10 & 2 \end{bmatrix}$

inv K1

$det(K_1) = 1 - 0 = 1$

$adj(K_1) = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}$

$K_1^{-1} = det(K_1)\, adj(K_1) = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} m11 = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$

$K_1^{-1} = \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$

put it together

$y = (y_1, y_2) \Rightarrow x' = (x_1', x_2')$

$x_1' = 8(y_1 - 2) \ m11$

$x_2' = 8(y_2 - 2) \ m11$

$x' = (8(y_1 - 2) m11 \quad 8(y_2 - 2) m11)$

$x'' = x' K_2^{-1} = x' \begin{bmatrix} 0 & 10 \\ 10 & 2 \end{bmatrix} m11$

$x = x'' K_1^{-1} = x'' \begin{bmatrix} 1 & 8 \\ 0 & 1 \end{bmatrix}$

decryption rule $\boxed{x = [(8(y_1 - 2) m11) K_2^{-1}) K_1^{-1} m11, \ (8(y_2 - 2) m11) K_2^{-1}) K_1^{-1} m11]}$