GEOPHYSICAL RESEARCH LETTERS, VOL. 25, NO. 2, PAGES 207-210, JANUARY 15, 1998

Global and regional surface-wave inversions: A spherical-spline parameterization

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Abstract. A surface-wave phase-velocity inversion scheme based upon a compactly supported cubic B-spline basis on a triangular grid of knots with approximately equal inter-knot spacing is implemented to perform global, regional and variable resolution inversions. Numerical results show that the inverted phase-velocity maps based upon this method are similar to those obtained using a global spherical-harmonic representation. Nonlinear inversions incorporating ray refraction are feasible based upon the spherical-spline model parameterization and the resulting maps are only slightly different from those based upon the great-circle approximation. A multiresolution analysis is introduced to obtain finer resolution in regions with good ray coverage and lower resolution in regions with poor coverage.

Introduction

In the past decades seismologists have made rapid progress in determining the laterally varying structure of the mantle using high-quality digital seismic data [*Masters et al.*, 1982; *Woodhouse and Dziewonski*, 1984; *Su et al.*, 1994; *Ekström et al.*, 1997]. At present, global Earth models are commonly parameterized in terms of spherical harmonics. Such an expansion, however, necessitates a significant increase in computational time with increasing angular degree and is impractical in studies restricted to a portion of the globe. To remedy these limitations, a parameterization method that partitions the Earth's surface into a set of equal-area blocks and assigns each block a constant velocity has been employed in some studies [*Zhang and Tanimoto*, 1992; *Fukao et al.*, 1992]. Such a block parameterization can also be applied to regional studies; however, because of discontinuities from one block to another, it cannot be used to perform dynamic or kinematic ray tracing, which is essential to the surface-wave JWKB method [*Tromp and Dahlen*, 1992].

We introduce a spherical-spline inversion method that parameterizes the laterally variable phase velocity of a Love or Rayleigh surface wave in terms of cubic B-splines centered upon a discrete number of geographical knots [*Wang and Dahlen*, 1995]. This model parameterization allows us to perform nonlinear inversions based upon the surface-wave JWKB method. In addition, it enables us to design a variable grid whose spacing is determined by the ray coverage. *Fukao et al.* [1992] first introduced such a variable resolution scheme in the context of body-wave tomography based upon constant velocity blocks.

Paper number 97GL03634.
0094-8534/98/97GL-03634$05.00

Method

To represent a function on the surface of the sphere, we define a set of cubic B-spline basis functions in terms of the geodesic distance measured from a number of nearly evenly-distributed knots, as shown in Figure 1. The resulting model representation has continuous zeroth, first and second derivatives everywhere on the surface of the sphere. A triangular tessellation technique is used to configure the knots upon the Earth's surface; this method guarantees that the geodesic distance between neighboring grid points exhibits less than 8 percent variation, and allows arbitrary grid density. A detailed discussion of the spherical-spline parameterization can be found in *Wang and Dahlen* [1995].

In this study we consider the inversion of surface-wave phase anomaly data based upon spherical splines. The phase anomaly $\delta\psi_j$ due to lateral heterogeneity along the jth path can be expressed as

$$\delta\psi_j = k_0 \int_j \epsilon \, \mathrm{d}\Delta,$$

where k_0 is the wavenumber on a spherical Earth model, $\epsilon = \delta k/k_0$ is the fractional wavenumber perturbation, and Δ measures the angular distance either along the great-circle or the true ray path. Most current global imaging studies are based upon the great-circle approximation by virtue of its linearization of the inverse problem and computational efficiency. This approximation, however, neglects effects due to the refraction of the surface-wave ray path from the great circle. Synthetic studies have shown that the true ray path can deviate from the unperturbed great-circle by more than 3° even for 150 s R1 and G1 waves. We introduce a nonlinear inversion scheme for phase anomalies based upon surface-wave JWKB theory which accounts for effects due to lateral heterogeneity [*Tromp and Dahlen*, 1992]. We seek a standard damped least-squares solution obtained by minimizing the weighted difference between the observed and the predicted phase anomaly in addition to gradients in the model [*Ekström et al.*, 1997].

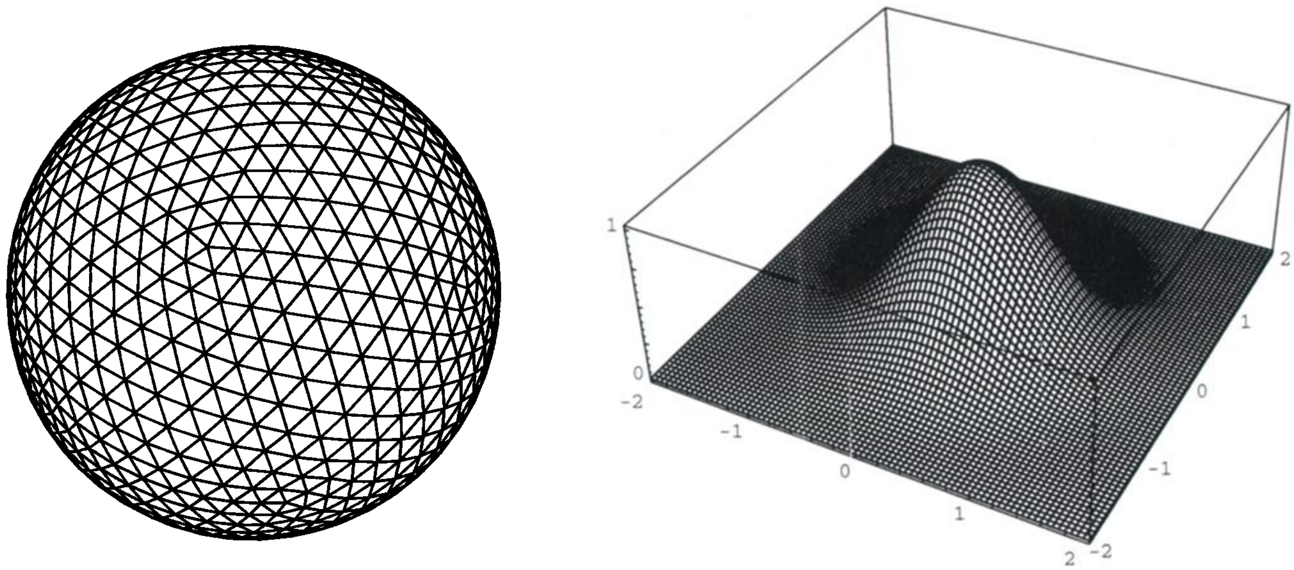

Figure 1. *Left*: Tessellated spherical triangles generated by a 9-fold subdivision of an icosahedron [*Wang and Dahlen*, 1995]. *Right*: Normalized cubic surface B-spline basis function.

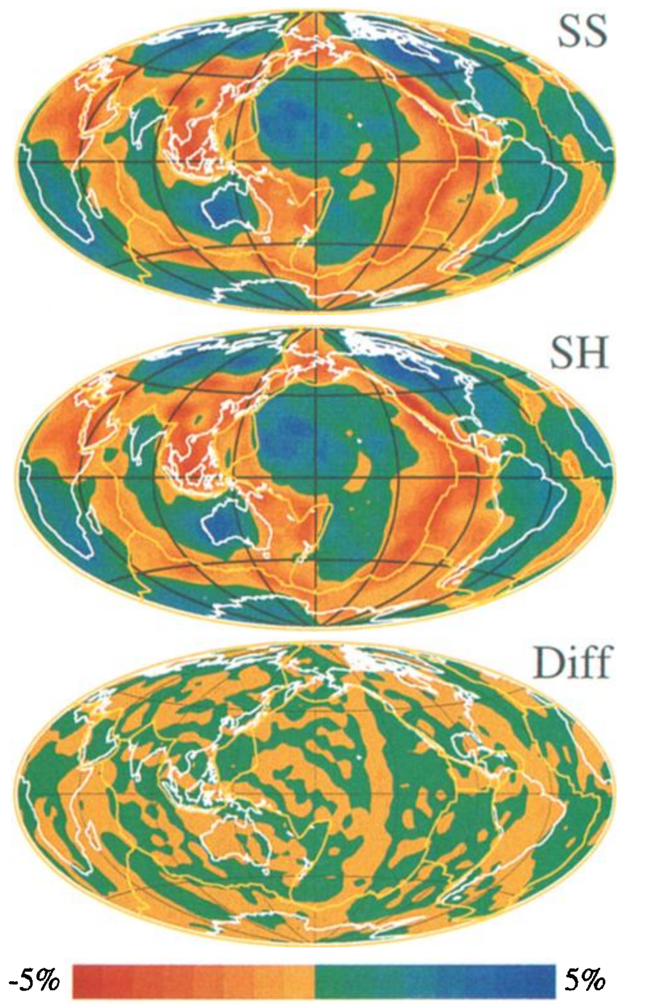

Figure 2. Fractional phase-velocity perturbation maps for 60 s fundamental Rayleigh waves obtained using a spherical-spline model with 1442 parameters (*Top*) and a spherical-harmonic representation with 1369 coefficients (*Middle*). The difference between these two maps is shown at the bottom.

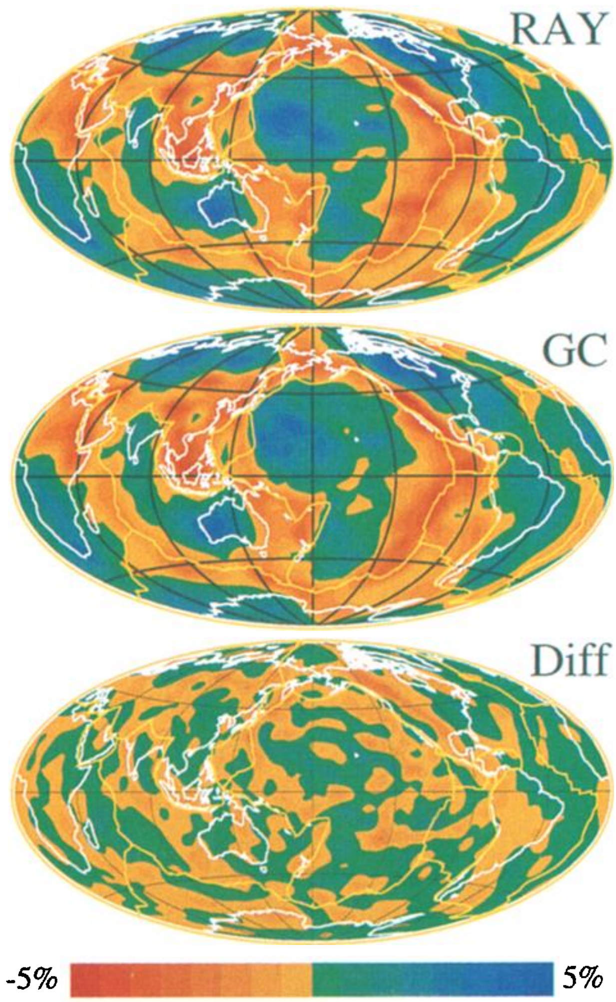

Figure 3. Fractional phase-velocity perturbation map obtained using nonlinear inversion after 4 iterations (*Top*), fractional phase-velocity map obtained based upon the great-0circle approximation (*Middle*), and their difference (*Bottom*).

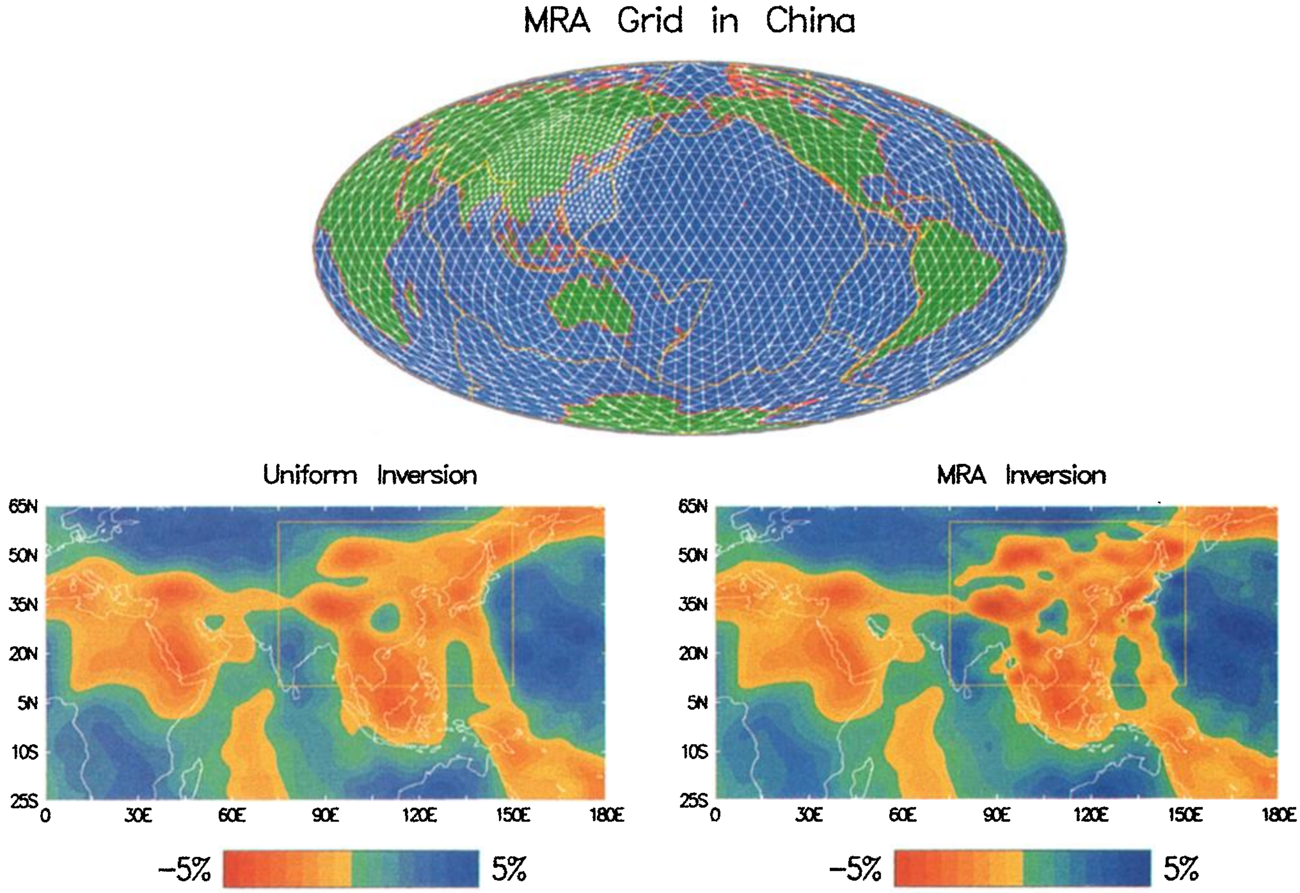

Figure 4. *Top:* Multiresolution grid configuration used in the MRA study of China. The grid consists of 1442 knots with an inter-knot spacing of approximately 6° outside of China and 408 knots with a separation of approximately 3° within China. *Bottom Left*: Inverted fractional phase-velocity perturbations in China using all 11,299 phase anomalies and the same even, global model representation as that for Figure 2 (*Top*). *Bottom Right*: Inverted fractional phase-velocity perturbations in China using all 11,299 phase anomalies and the MRA model representation shown at the top.

Numerical Results

We apply our spherical-spline representation to a data set of phase anomalies collected by *Ekström et al.* [1997] based upon a new single station phase-velocity measurement technique. Their data set consists of Love and Rayleigh wave phase measurements in the period range 35–150 s; here we show results derived from a subset of 11,299 phase measurements for 60 s Rayleigh waves.

The inverted phase-velocity map for 60 s fundamental Rayleigh waves using a spherical-spline parameterization with 1442 parameters is compared with that obtained using a spherical-harmonic representation up to degree 36 with 1369 coefficients in Figure 2. The inversion is based upon the great-circle approximation; the maps show excellent agreement.

Synthetic studies have shown that the great-circle approximation can result in large discrepancies in phase compared to surface-wave JWKB theory even on a relatively smooth model [*Wang and Dahlen*, 1994]. This suggests that exact ray tracing should be used to perform surface-wave inversions. Because the ray path depends upon the phase-velocity perturbation, we conduct a nonlinear inversion by iteration; at each step, we recalculate the Fréchet kernels by tracing rays on the updated phase-velocity map. The inversion converges after 4 iterations and the final results are shown in Figure 3. In general, the phase-velocity patterns show close similarity to those obtained using the great-circle approximation. The variance reduction of the nonlinear inversion is improved by only 0.5% from 91.5% of the great-circle approximation, indicating that, at least at this period, updating Fréchet derivatives by ray tracing is not warranted by the data. The largest differences occur, not surprisingly, near strong velocity gradients, for example in the western US.

Multiresolution Analysis (MRA)

Due to nonuniform ray coverage, the model resolution one can achieve varies according to the ray density from one region to another. Using a global basis may result in large errors in regions with poor ray coverage and such errors can spread to other areas. The spherical-spline model provides us with an opportunity to conduct a multiresolution analysis because of its localized support.

Our multiresolution approach is to use a model parameterization with variable grid spacing determined by the ray coverage. We illustrate the approach by inverting for phase-velocity variations in China and its surrounding area where the station coverage, and hence the ray coverage, is good. The coarse global grid is the same as that used for the model in Figure 2, and a finer grid is added in the area of interest, as shown in Figure 4 (Top). The inverted regional map shown in Figure 4 (Bottom Right) has good correlation with the tectonics of this region. Fast phase velocities are found in the Indian Plate, the Philippine Plate, the western edge of the Pacific Plate, the Yangtze Craton and the Tarim Basin, and slow velocities are seen in the Tibetan Plateau, the Fold Belts in Northern China and Mongolia, Eastern China, and the South China Sea. The map also correlates with Bouguer anomalies in this region [*Wu and Levshin*, 1994] and is consistent with results from regional body-wave travel time inversions [*Liu et al.*, 1989].

Conclusion

A new spherical-spline inversion method is implemented to analyze surface-wave phase anomalies. Inversions show that the spherical-spline method can produce global phase-anomaly maps with the same resolution as spherical-harmonic representations using approximately the same number of model parameters. In addition, the compactly supported spherical spline basis allows us to perform regional inversions. The differentiability of the B-splines enables us to conduct iterative inversions based upon ray tracing. The spline model representation gives us the flexibility to use a multiresolution analysis in order to cope with nonuniform ray coverage. Multiresolution inversion schemes can be used to determine small-scale structure where ray coverage is good while retaining smooth structures in regions with low ray density.

Acknowledgments. We thank Toshiro Tanimoto and an anonymous reviewer for their useful comments and suggestions. This work was supported by the U.S. National Science Foundation under grant EAR 92-19361 and by the David and Lucile Packard Foundation.

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(received August 26, 1997; revised October 20, 1997; accepted November 18, 1997.)