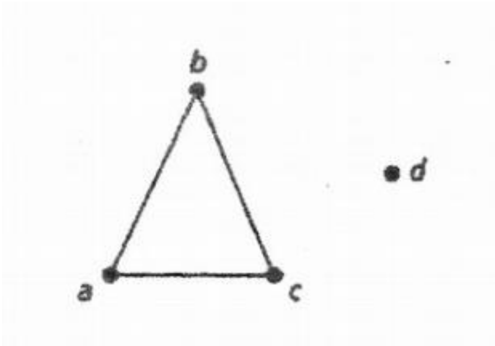
12.1

4. $e = v - \kappa$

6. **(a)** Since a tree contains no cycles it cannot have a subgraph homeomorphic to either K_5 or $K_{3,3}$.

(b) If $T = (V, E)$ is a tree then T is connected and, by part (a), T is planar. By Theorem 11.6, $|V| - |E| + 1 = 2$ or $|V| = |E| + 1$.

7.



8. **(a)** Let x be the number of pendant vertices. Then $2|E| = \sum_{v \in V} deg(v) = x + 4(2) + 1(3) + 2(4) + 1(5)$ and $|E| = |V| - 1 = x + 4 + 1 + 2 + 1 - 1 = x + 7$.

So, $2(x + 7) = x + 24$ and $x = 10$.

(b) $2|E| = \sum_{v \in V} deg(v) = v_1 + v_2(2) + v_3(3) + \cdots + v_m(m)$

$|E| = |V| - 1 = (v_1 + v_2 + v_3 + \cdots + v_m) - 1$

$2(v_1 + v_2 + v_3 + \cdots + v_m - 1) = v_1 + 2v_2 + 3v_3 + \cdots + mv_m$, so $v_1 = v_3 + 2v_4 + 3v_5 + \cdots + (m-2)v_m + 2$, and $|V| = v_1 + v_2 + v_3 + \cdots + v_m = [v_3 + 2v_4 + \cdots + (m-2)v_m + 2] + v_2 + v_3 + \cdots + v_m = v_2 + 2v_3 + 3v_4 + \cdots + (m-1)v_m + 2$ and $|E| = |V| - 1 = v_2 + 2v_3 + 3v_4 + \cdots + (m-1)v_m + 1$

10. 31

11. Since T is a tree, there is a unique path connecting any two distinct vertices of T. Hence there are $\binom{n}{2}$ distinct paths in T.

12. If G contains no cycle then G is a tree. But then G must have at least two pendant vertices. This graph has only one pendant vertex.

17. **(a)** $n \geq m + 1$

(b) Let k be the number of pendant vertices in T. From Theorem 11.2 and Theorem 12.3 we have

$$2(n-1) = 2|E| = \sum_{v \in V} deg(v) \geq k + m(n-k)$$

Consequently, $[2(n-1) \geq k + m(n-k)] \Rightarrow [2n - 2 \geq k + mn - mk] \Rightarrow$ $[k(m-1) \geq 2 - 2n + mn = 2 + (m-2)n \geq 2 + (m-2)(m+1) = 2 + m^2 - m - 2 = m^2 - m = m(m-1)]$, so $k \geq m$.

12.4

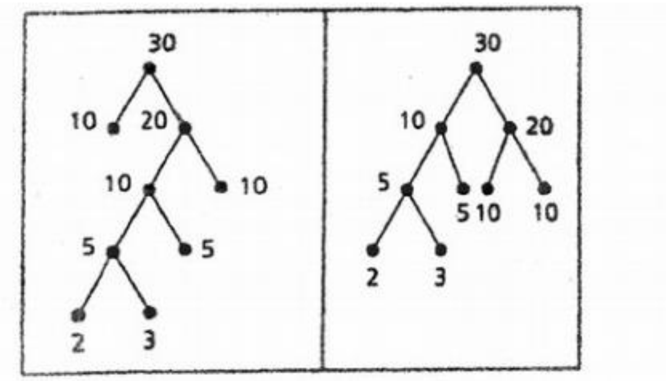
2. $x = y = z = 1$

3.

a: 111	e: 10	h: 010
b: 110101	f: 0111	i: 00
c: 0110	g: 11011	j: 110100
d: 0001		

4. **(a)** 2^3 **(b)** 2^7 **(c)** 2^{12} **(d)** 2^h

7.



Amend part (a) of Step 2 for the Huffman tree algorithm as follows. If there are $n(> 2)$ such trees with smallest root weights w and w', then

(i) if $w < w'$ and $n - 1$ of these trees have root weight w', select a tree (of root weight w') with smallest height; and

(ii) if $w = w'$ (and all n trees have the same smallest root weight), select two trees (of root weight w) of smallest height.