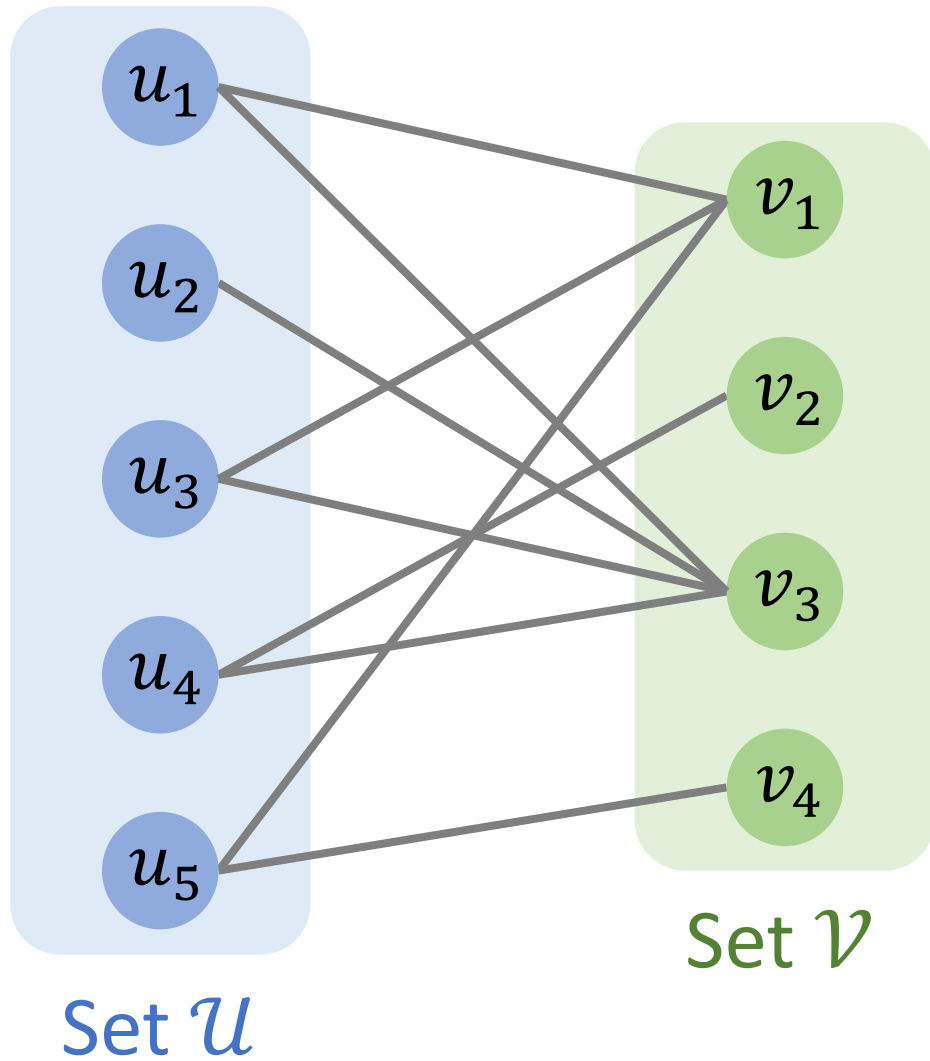


# Maximum-Cardinality Bipartite Matching

Shusen Wang

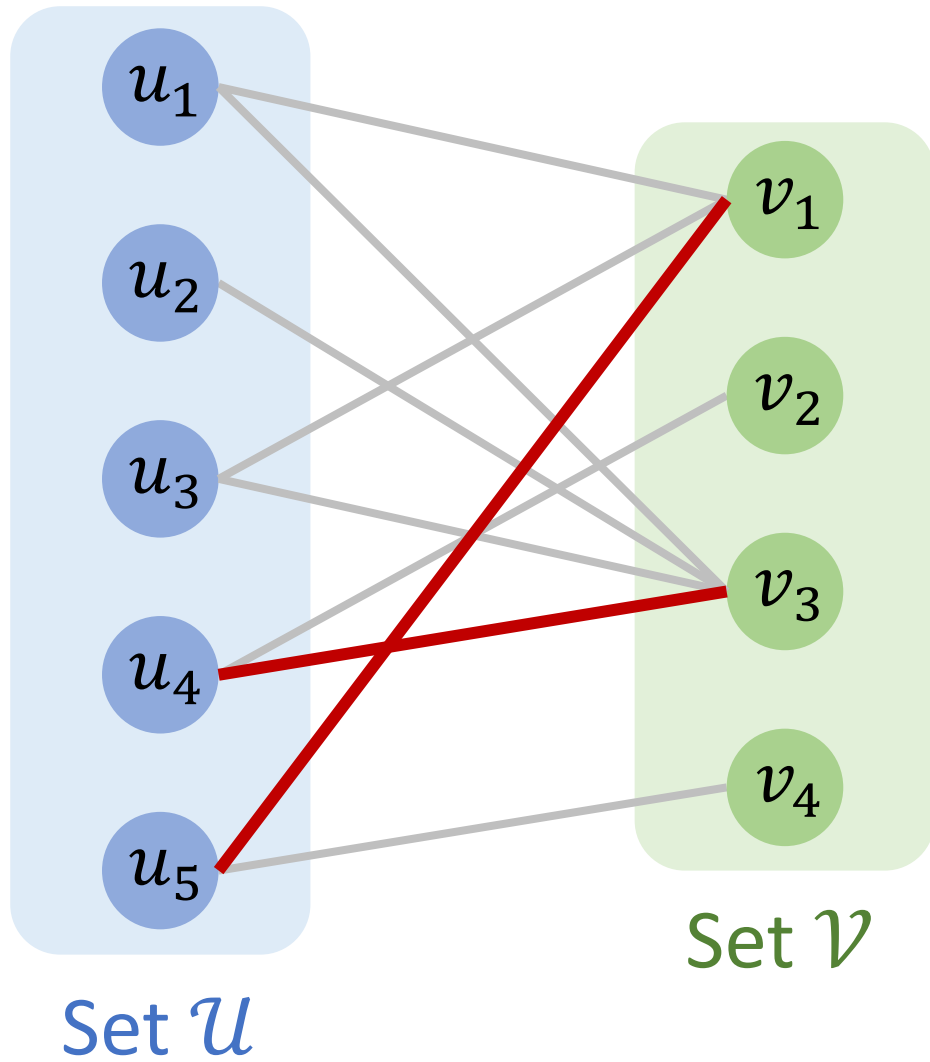
# Definition

# Bipartite Matching



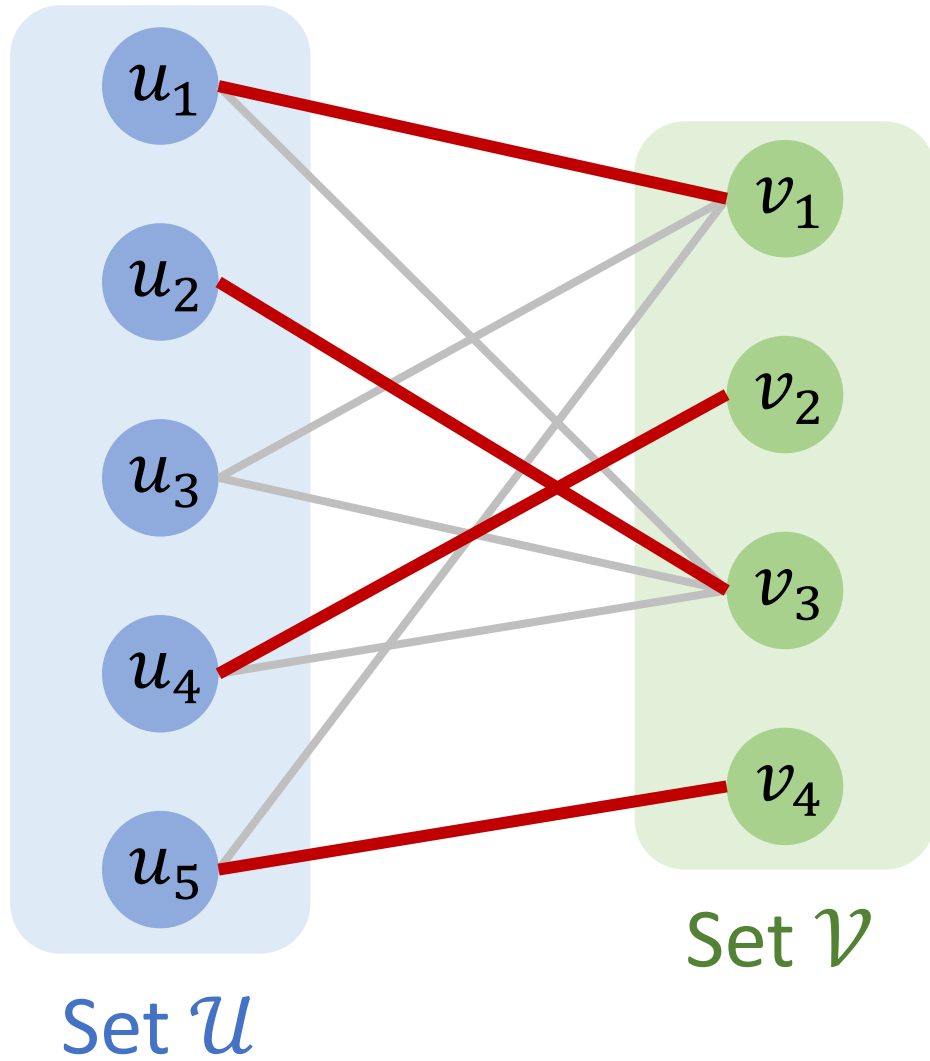
- Bipartite graph:  $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ .
- Matching is a subset of edges without common vertices.
- Denote the matching by set  $\mathcal{S} \subseteq \mathcal{E}$ .

# Bipartite Matching



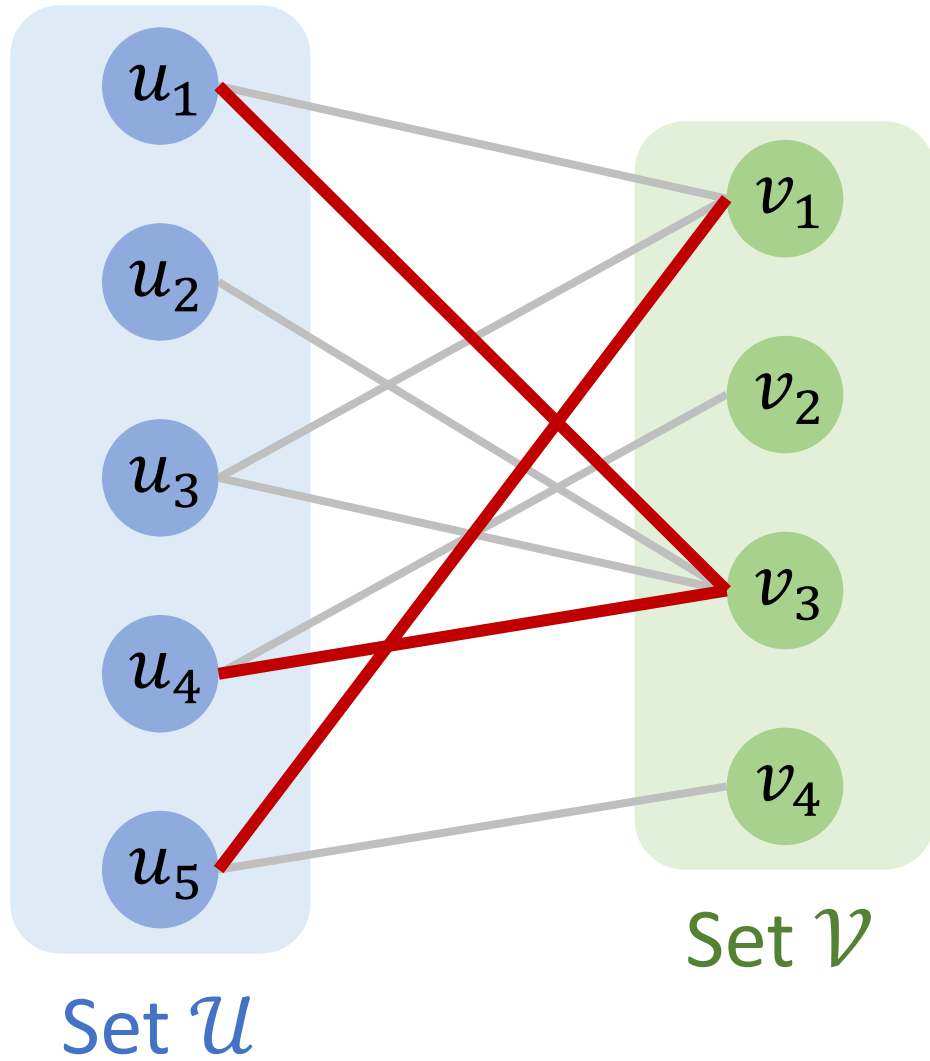
- $\mathcal{S} = \{e_{51}, e_{43}\}$  is a matching. (But not a maximum matching.)

# Bipartite Matching



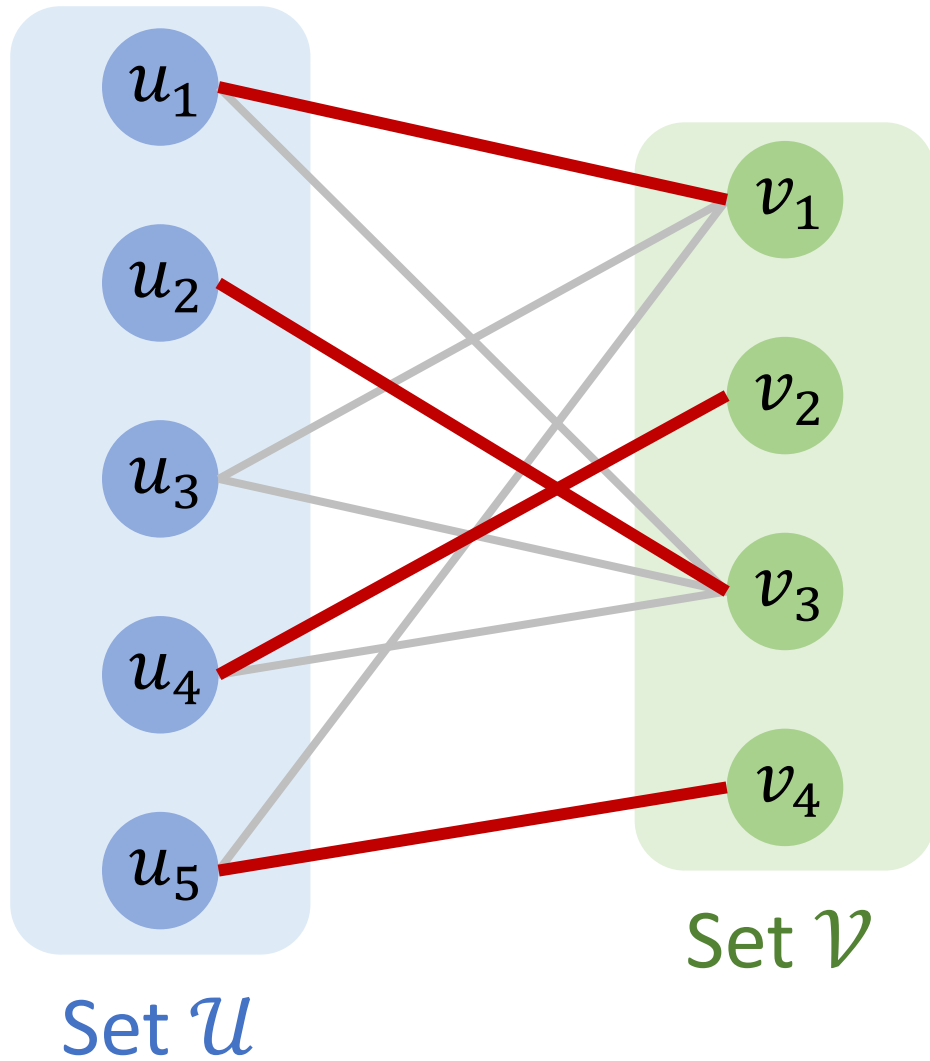
- $\mathcal{S} = \{e_{11}, e_{23}, e_{42}, e_{54}\}$  is a matching.

# Bipartite Matching



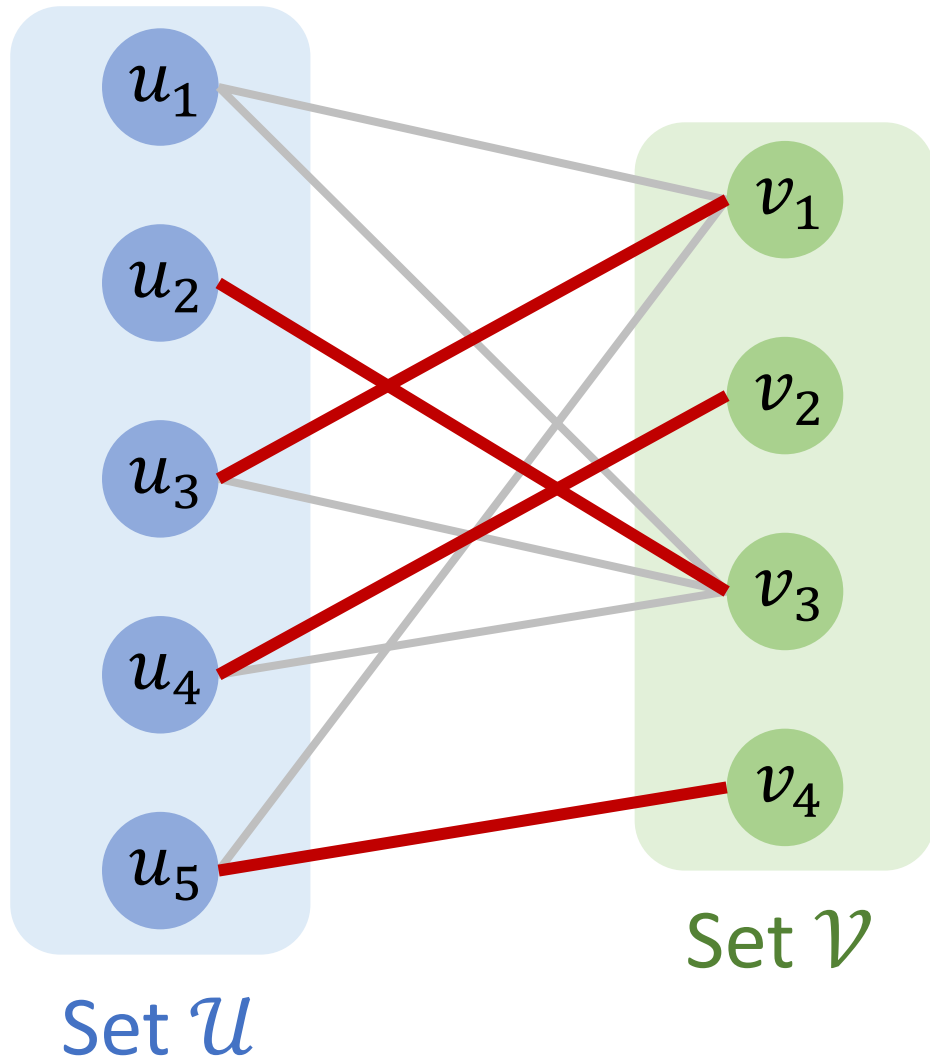
- $\{e_{13}, e_{51}, e_{43}\}$  is not a matching.
- Two edges have common vertex.

# Maximum-Cardinality Bipartite Matching (MCBM)



- **MCBM:** The matching  $\mathcal{S}$  that has the maximum cardinality  $|\mathcal{S}|$ .

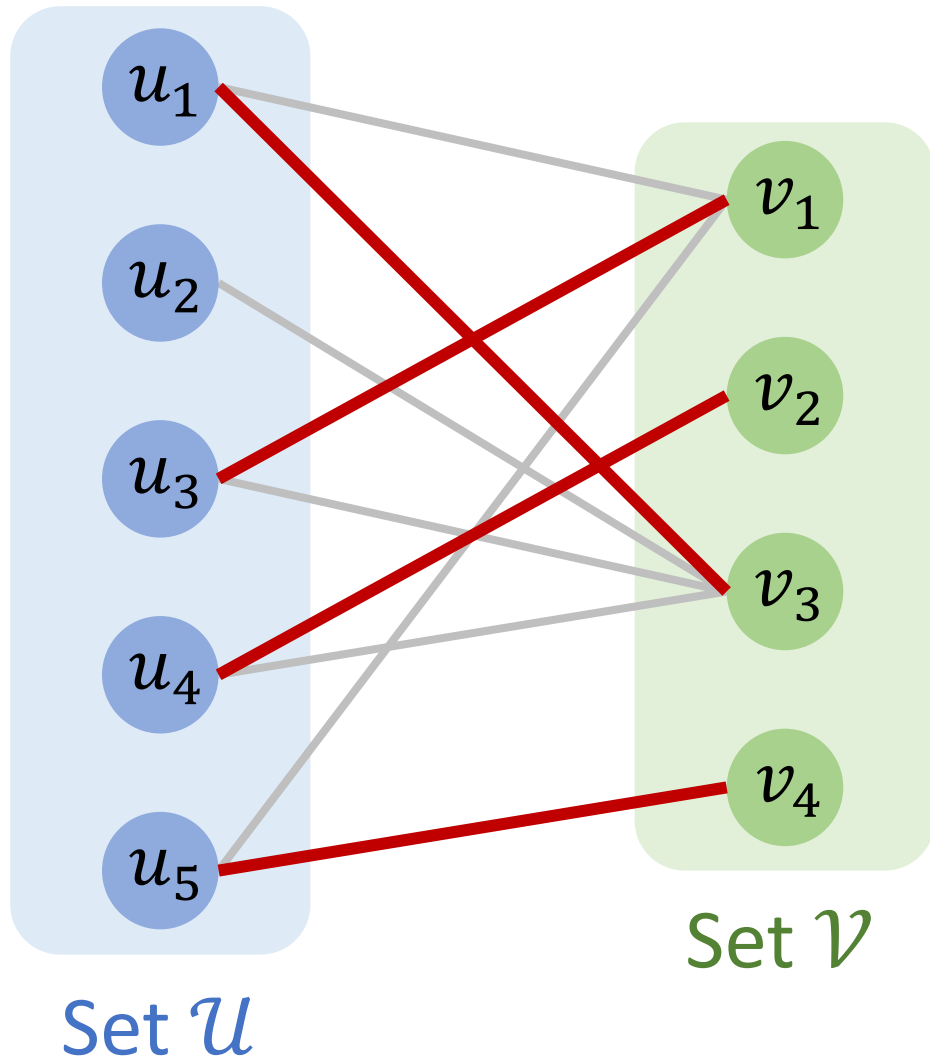
# Maximum-Cardinality Bipartite Matching (MCBM)



- **MCBM:** The matching  $\mathcal{S}$  that has the maximum cardinality  $|\mathcal{S}|$ .
- MCBM may not be unique.



# Maximum-Cardinality Bipartite Matching (MCBM)

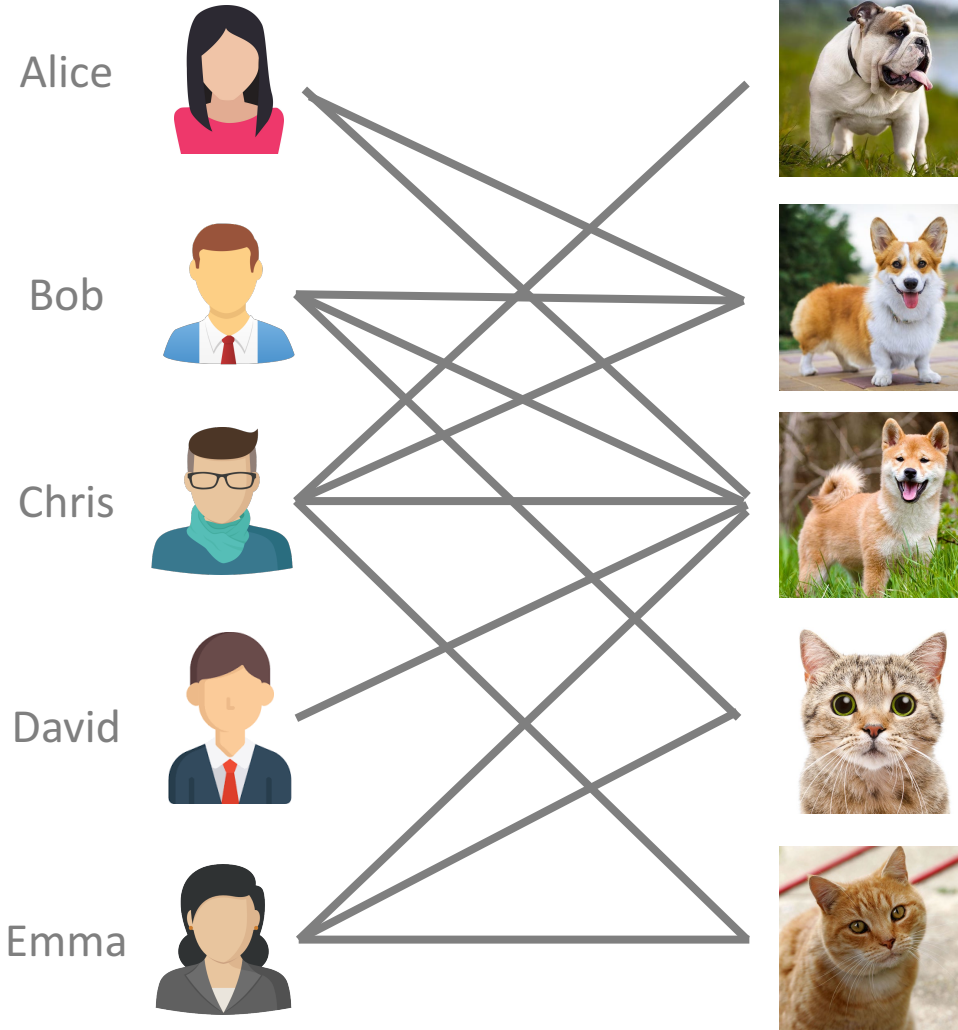


- **MCBM:** The matching  $\mathcal{S}$  that has the maximum cardinality  $|\mathcal{S}|$ .
- MCBM may not be unique.

# Greedy Algorithm

## People

## Pets

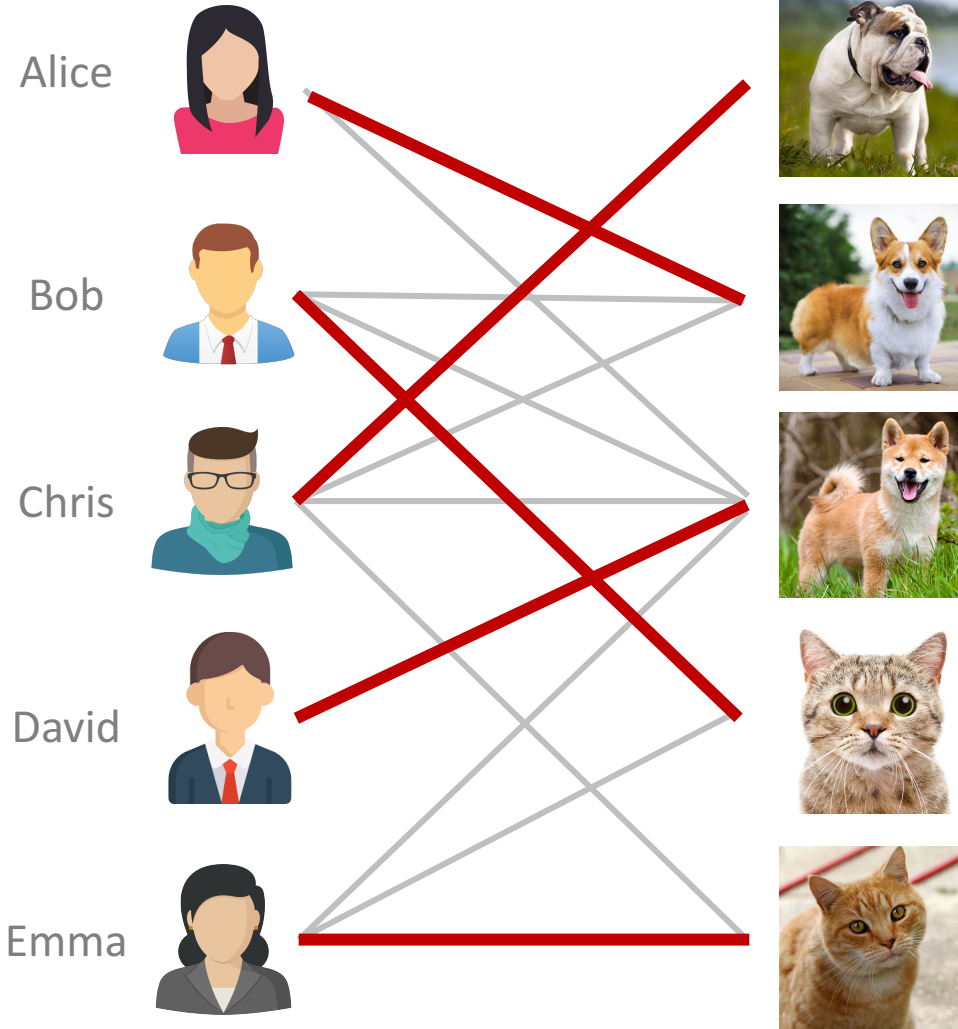


## Pet adoption

- Bipartite graph:  $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ .
- Set  $\mathcal{U}$  contains candidates.
- Set  $\mathcal{V}$  contains jobs.
- Edges in  $\mathcal{E}$  are candidates' skills.
- **Goal:** Maximizing the cardinality of matching.

## People

## Pets

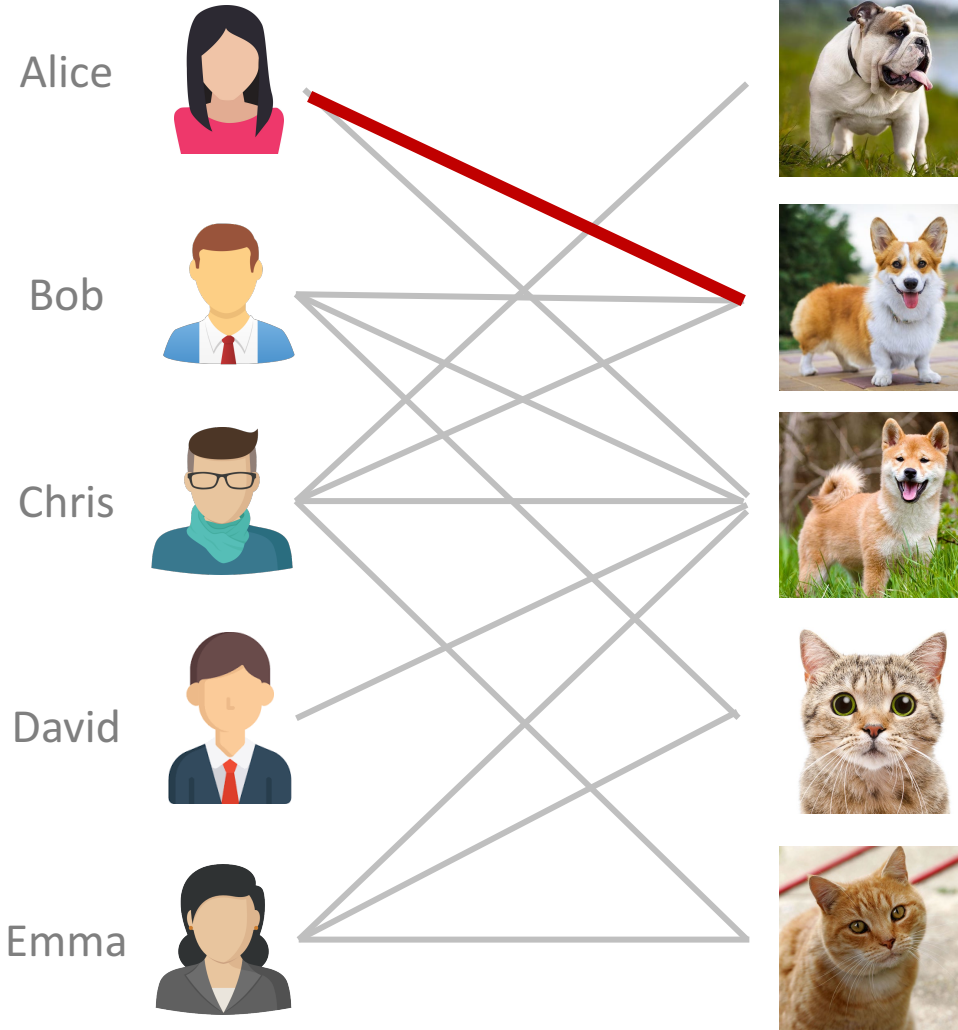


## Pet adoption

- The cardinality of the maximum matching is 5.

## People

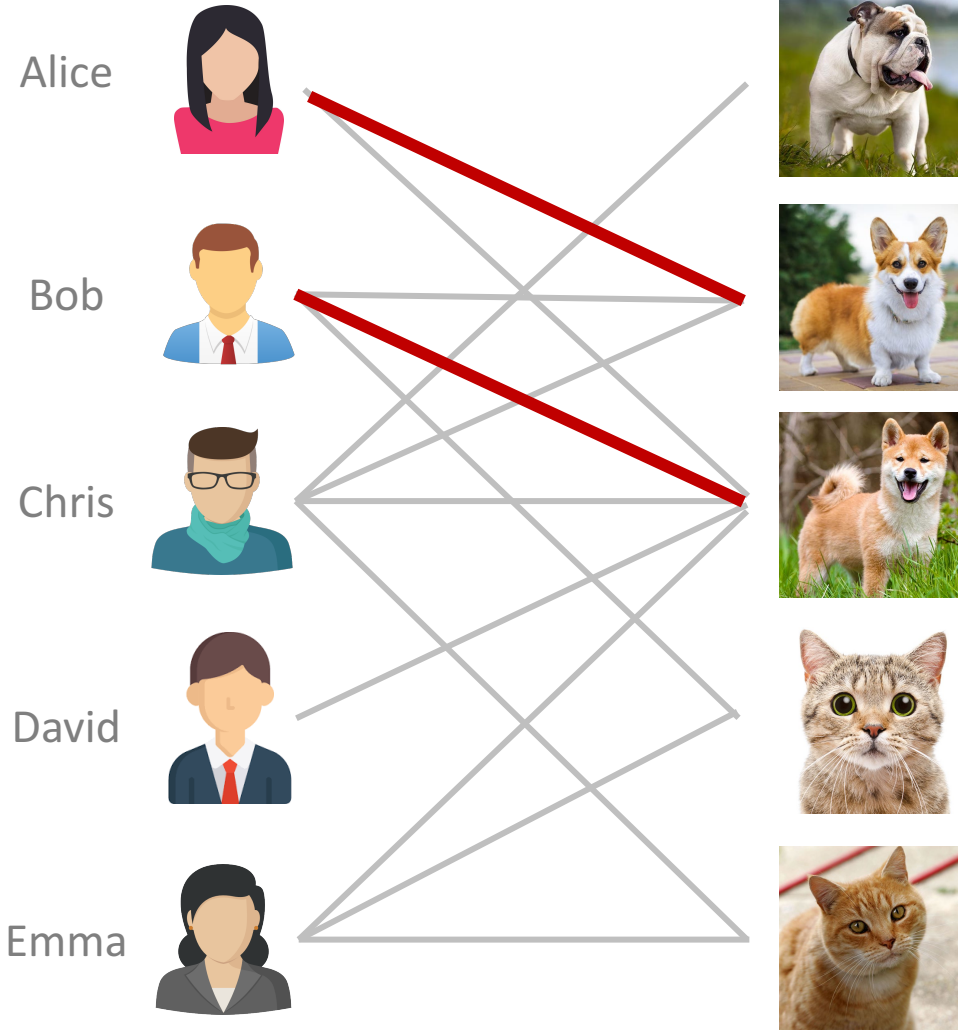
## Pets



## Greedy Algorithm

## People

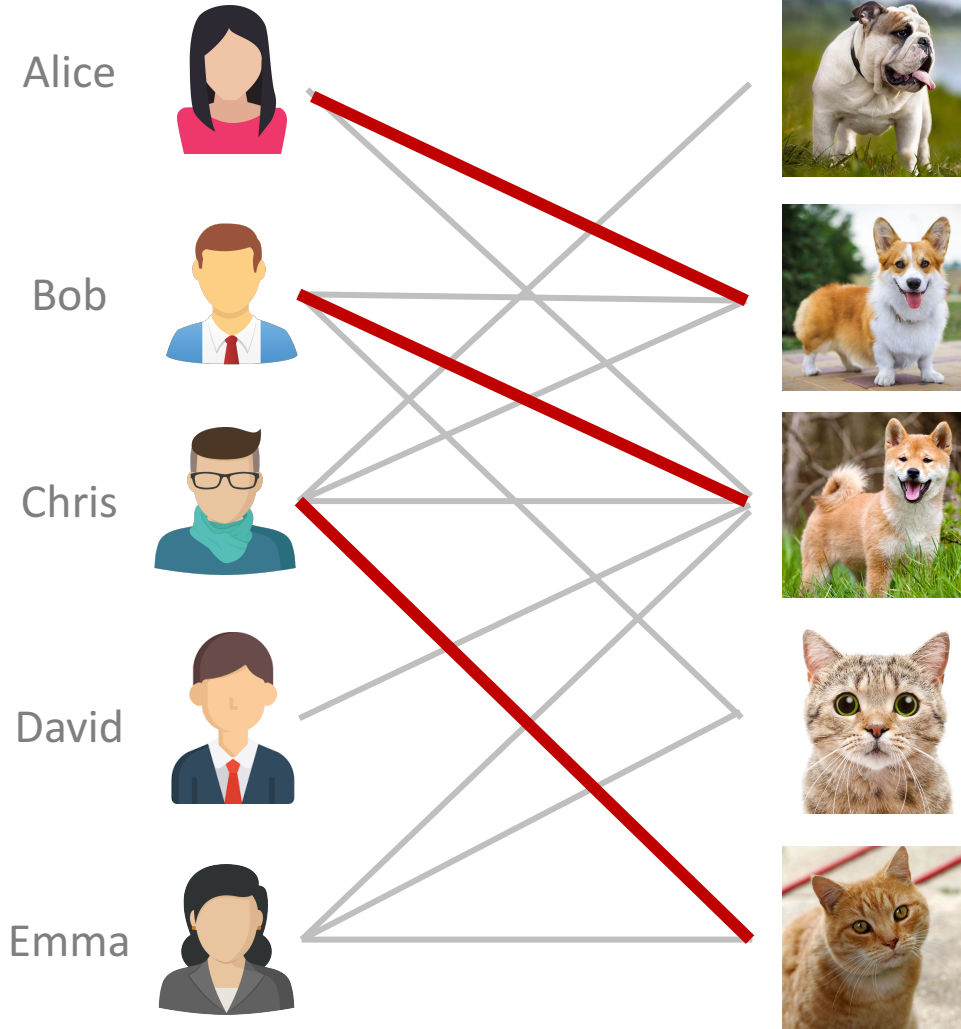
## Pets



## Greedy Algorithm

## People

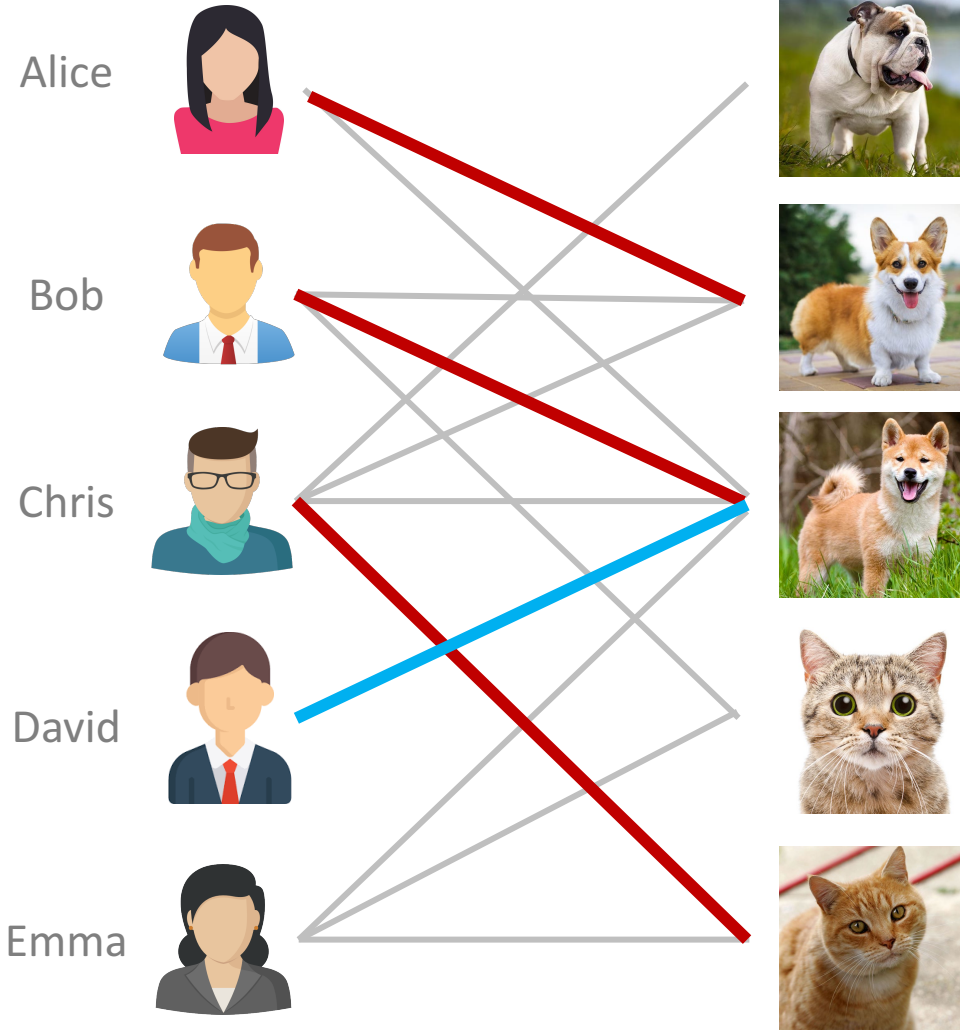
## Pets



## Greedy Algorithm

## People

## Pets

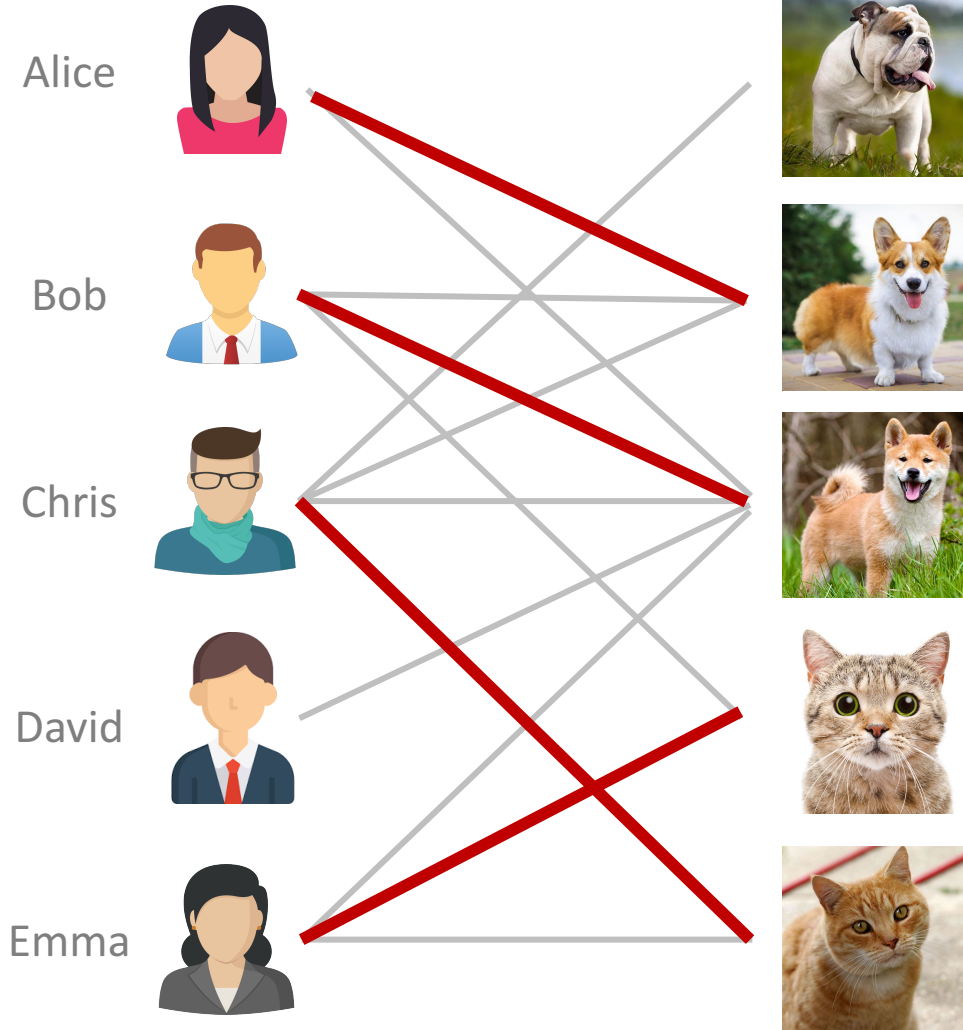


## Greedy Algorithm



## People

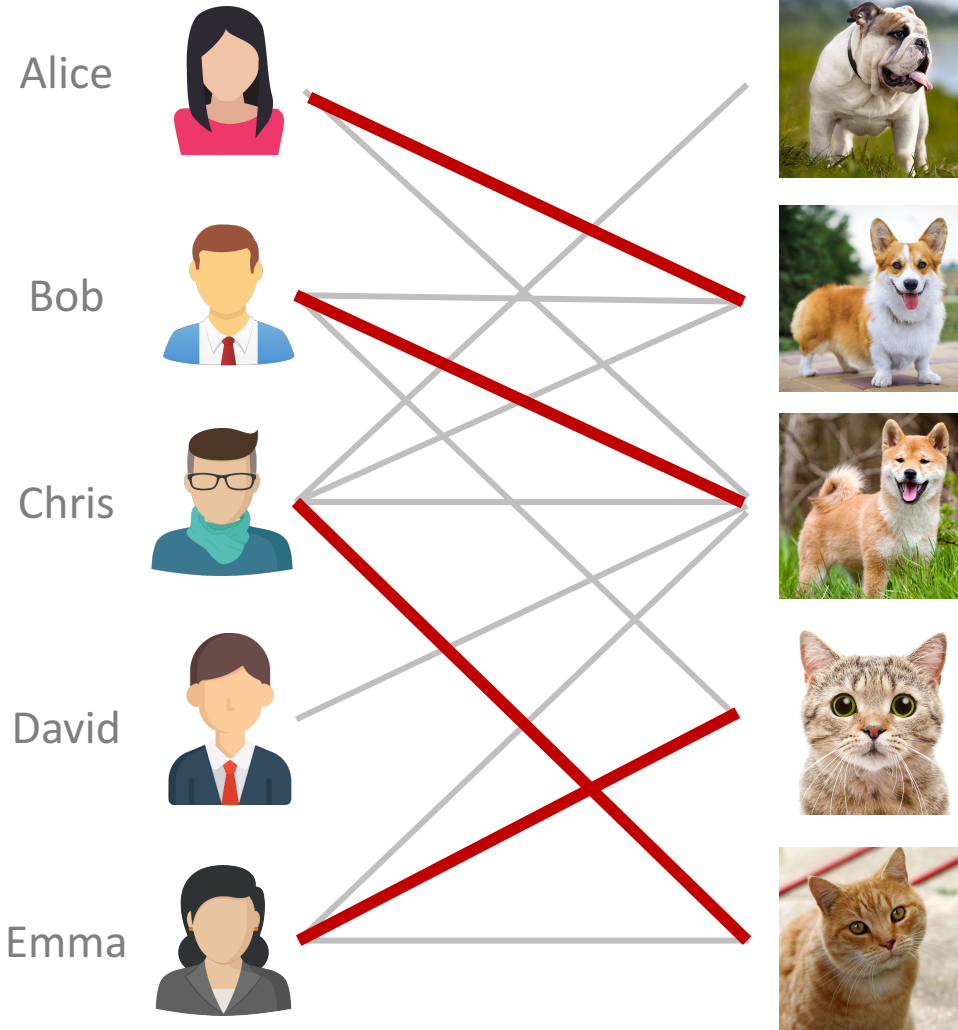
## Pets



## Greedy Algorithm

## People

## Pets



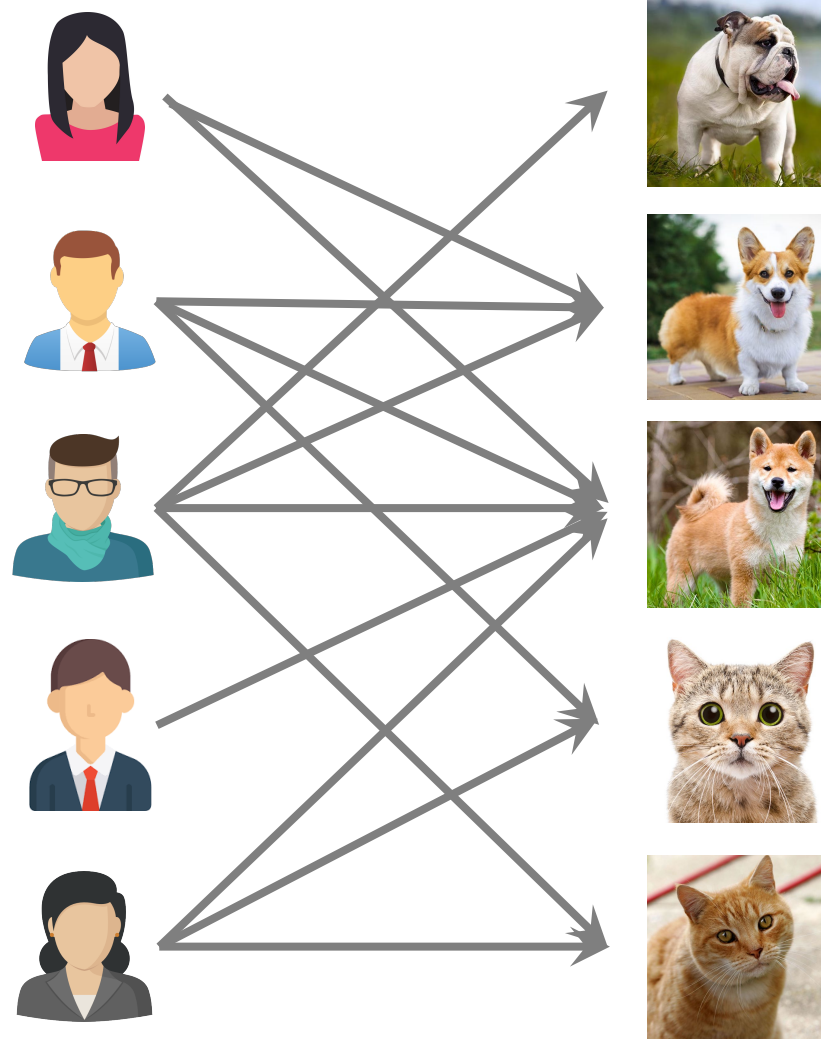
## Greedy Algorithm

- The cardinality of matching is 4.  
(Not the maximum.)

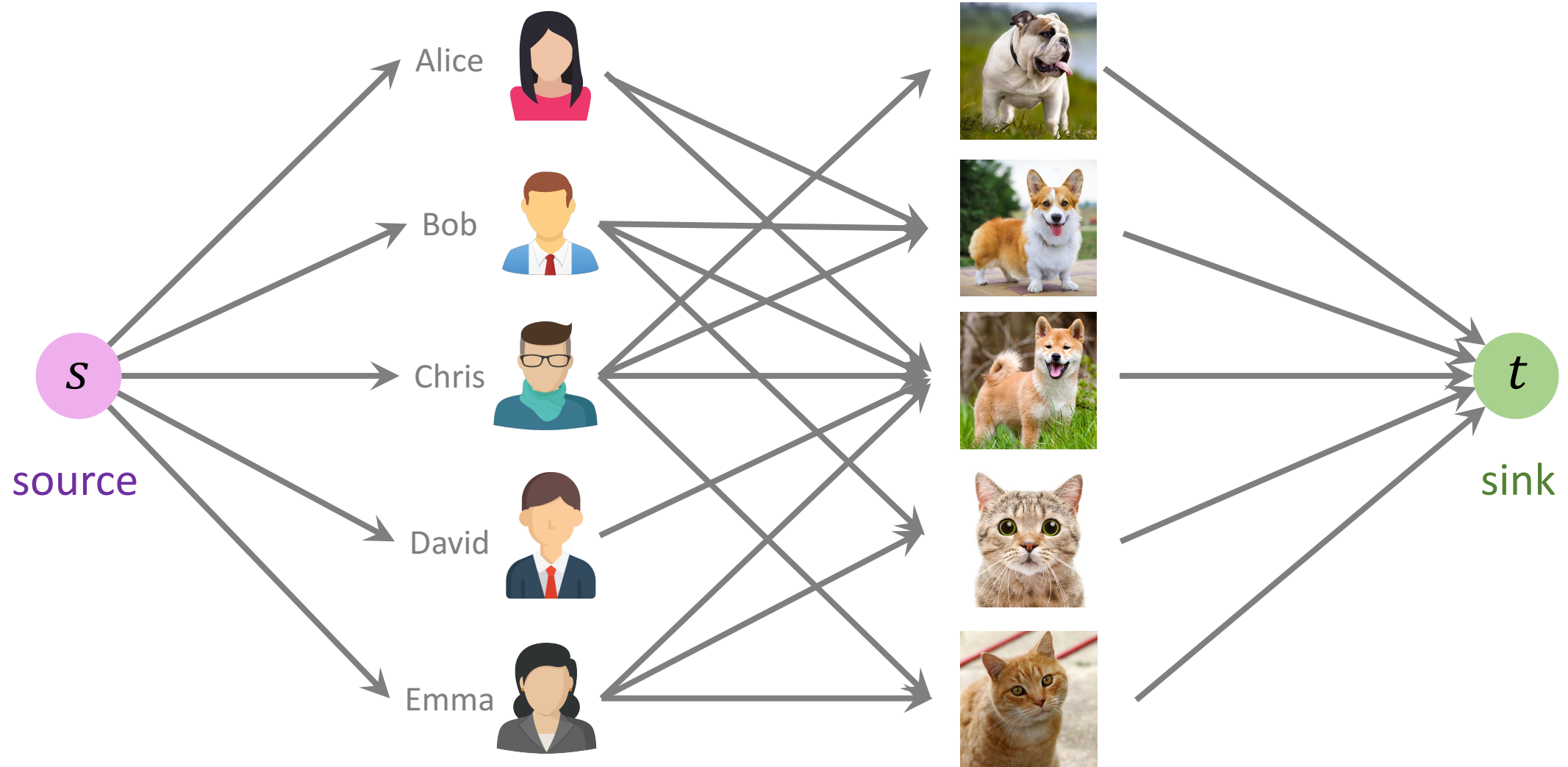
Greedy algorithm can fail!

# **Reducing Bipartite Matching to Network Flow**

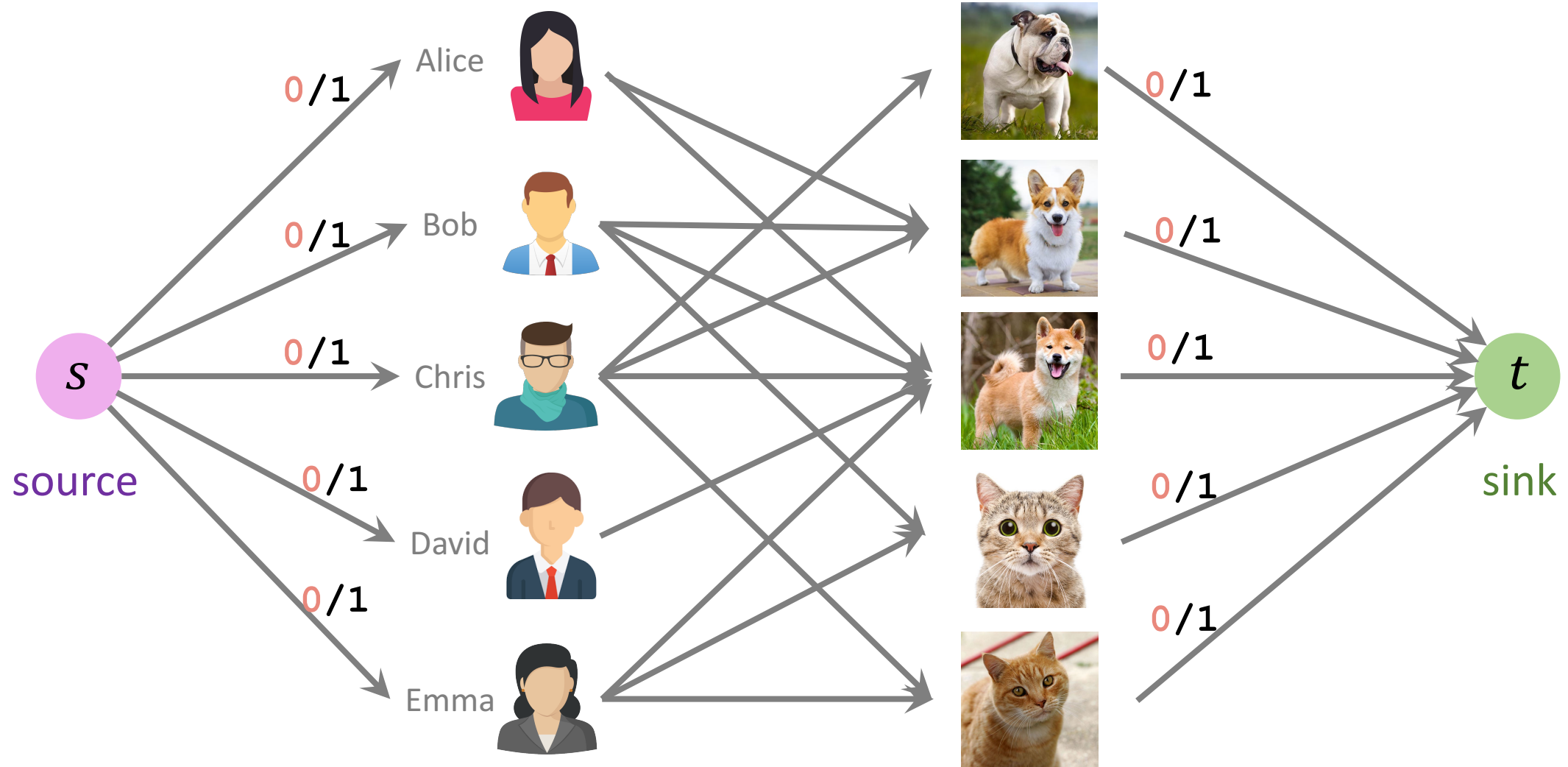
# Make the edges directed



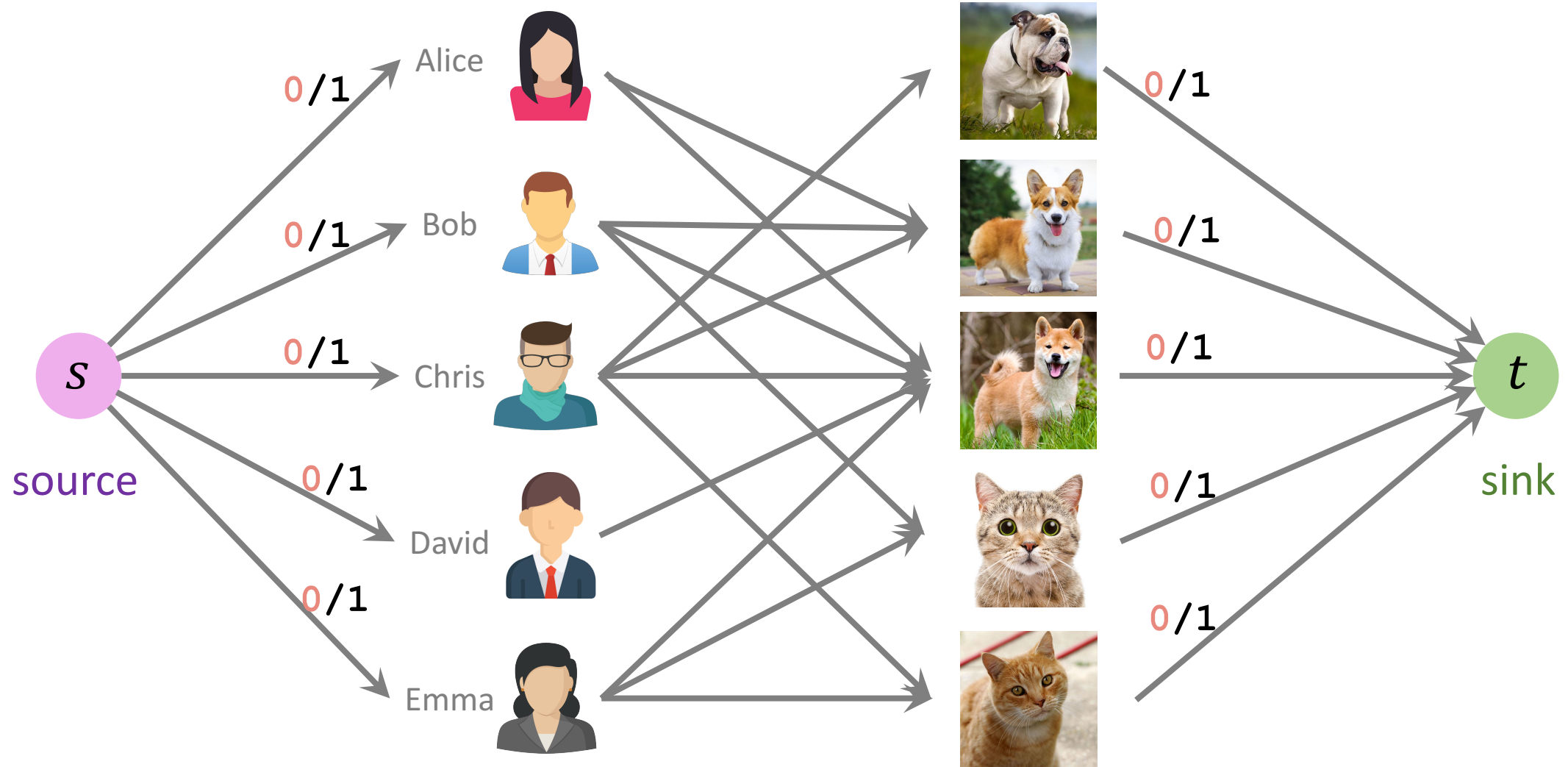
# Add source and sink



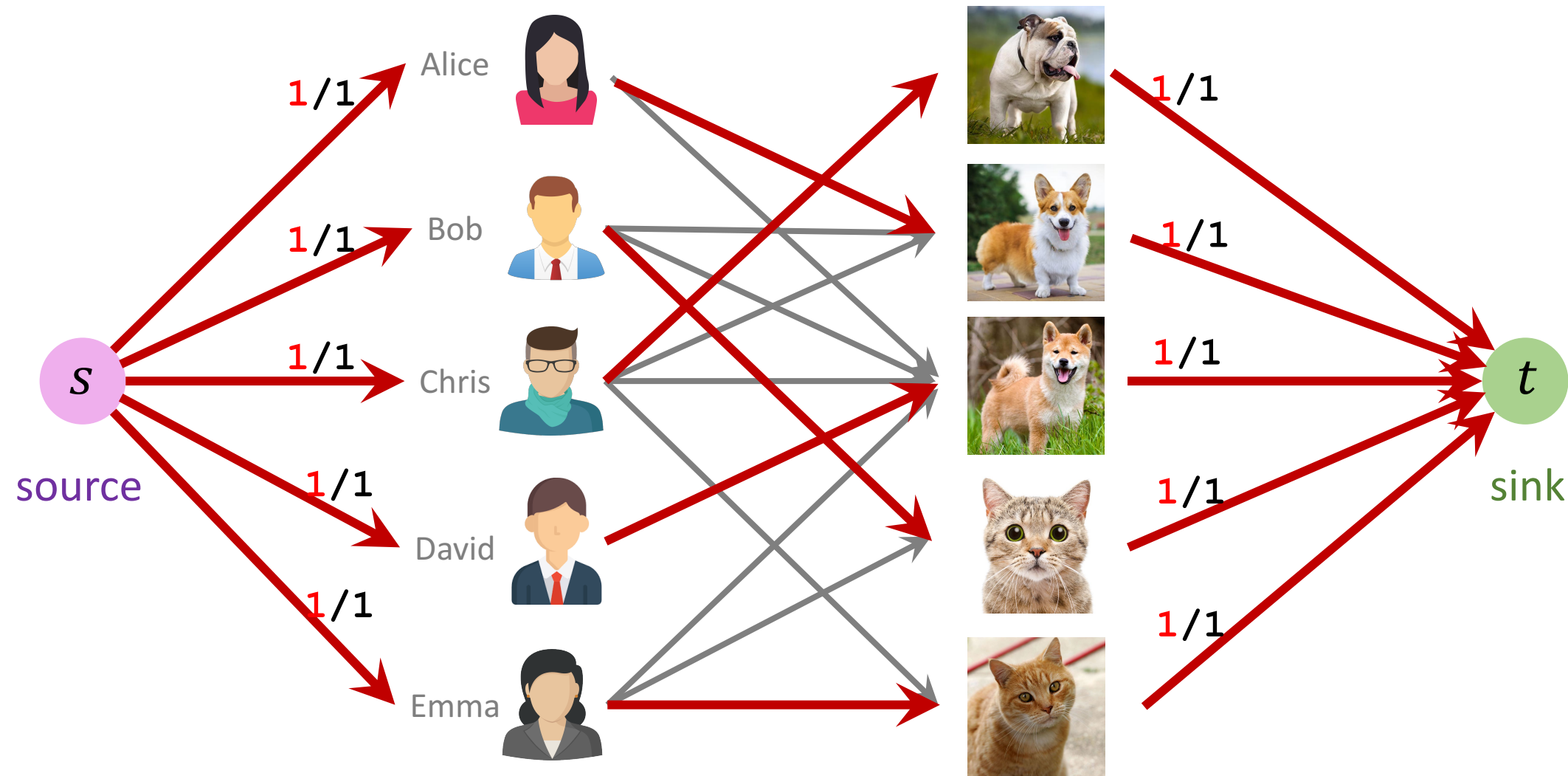
# All the edge weights (capacities) are ones



# Find the max-flow using any algorithm



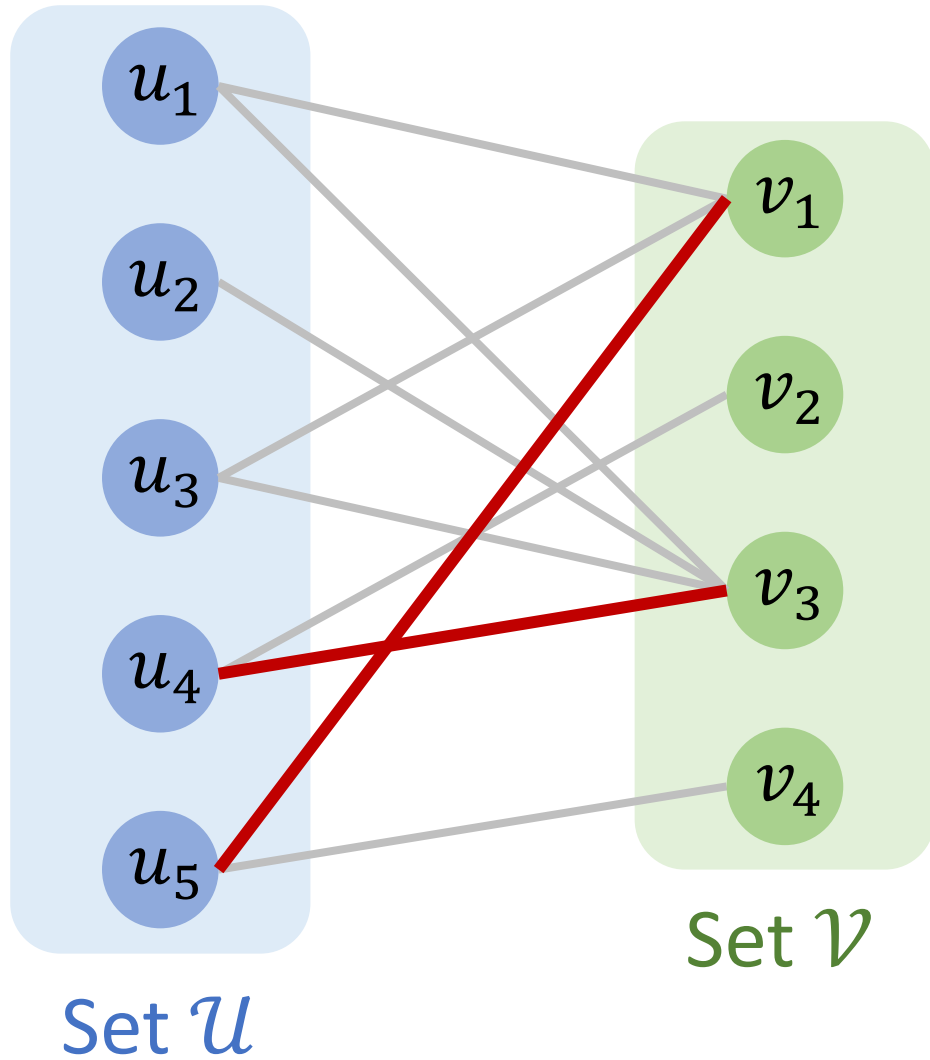
# capacity of max-flow = cardinality of max-matching





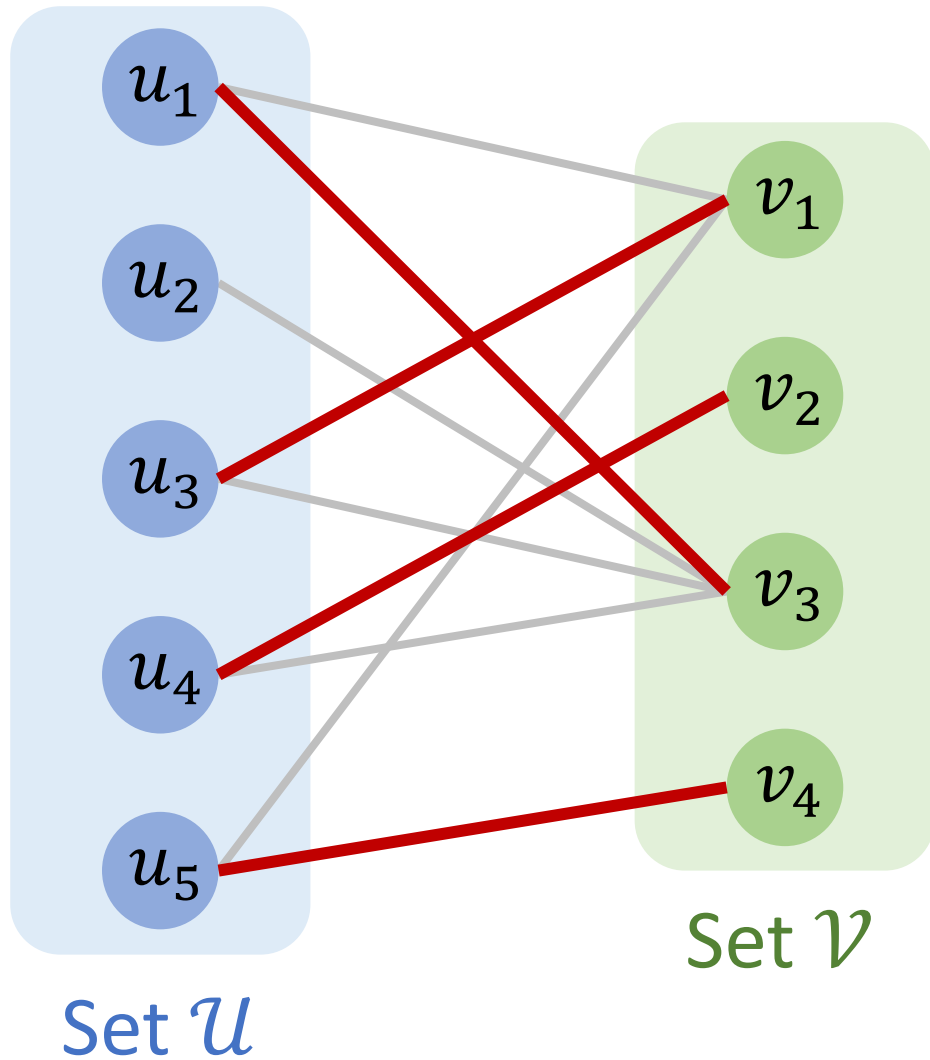
# Summary

# Bipartite Matching



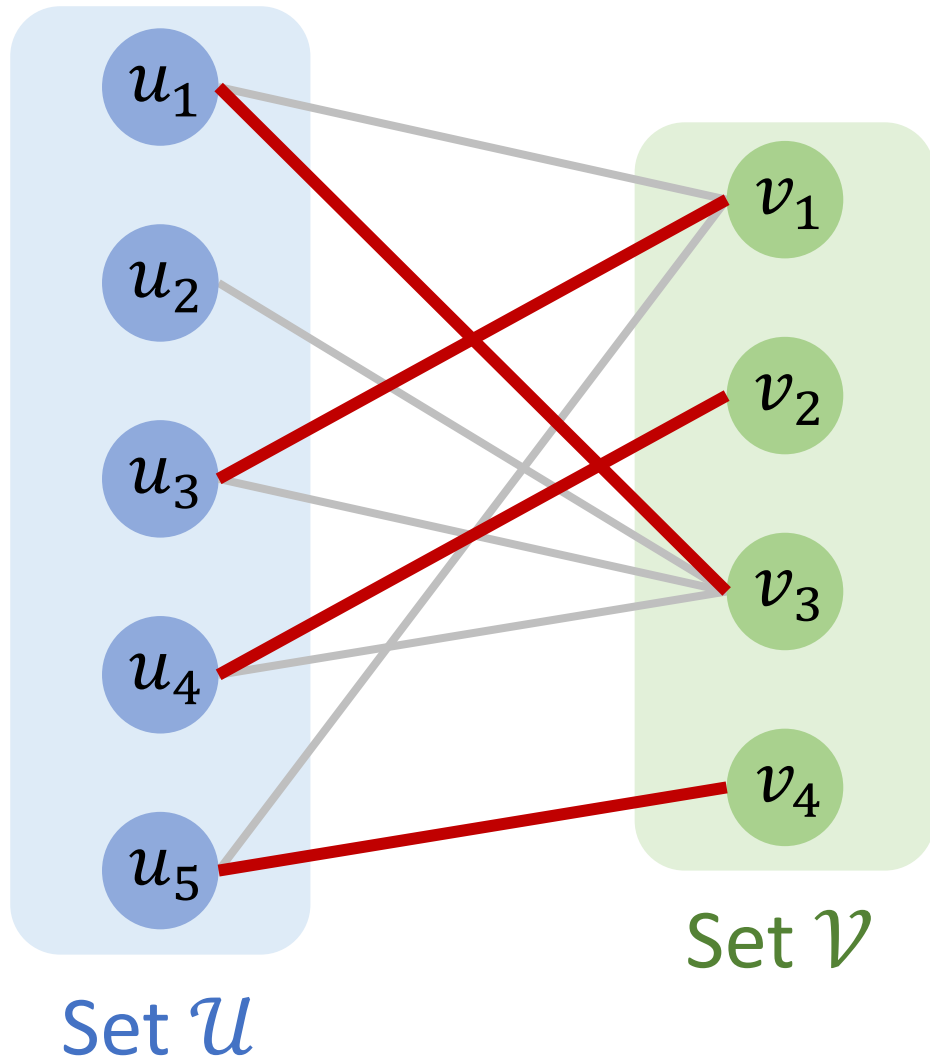
- Bipartite graph:  $\mathcal{G} = (\mathcal{U}, \mathcal{V}, \mathcal{E})$ .
- Matching is a subset of edges without common vertices.
- Denote the matching by set  $\mathcal{S} \subseteq \mathcal{E}$ .

# Maximum Cardinality Bipartite Matching (MCBM)



- Given an **unweighted** bipartite graph.
- MCBM: Find matching  $\mathcal{S}$  that has the maximum cardinality  $|\mathcal{S}|$ .
- Bipartite matching in **unweighted** graphs is **easier** than in **weighted** bipartite graph.

# Algorithms for finding MCBM



- Greedy algorithm can fail.
- Reduce MCBM to the max-flow problem and solve it using any max-flow algorithm.

**Thank You!**