

Prim's Algorithm

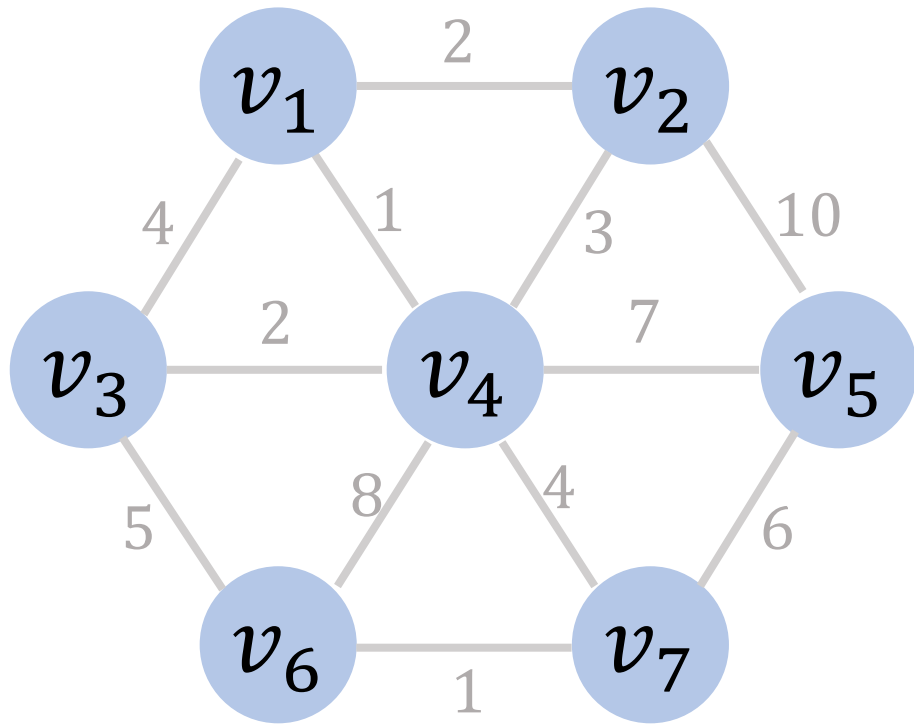
Shusen Wang

Prim's Algorithm

Basic idea: Grow the tree in successive stages.

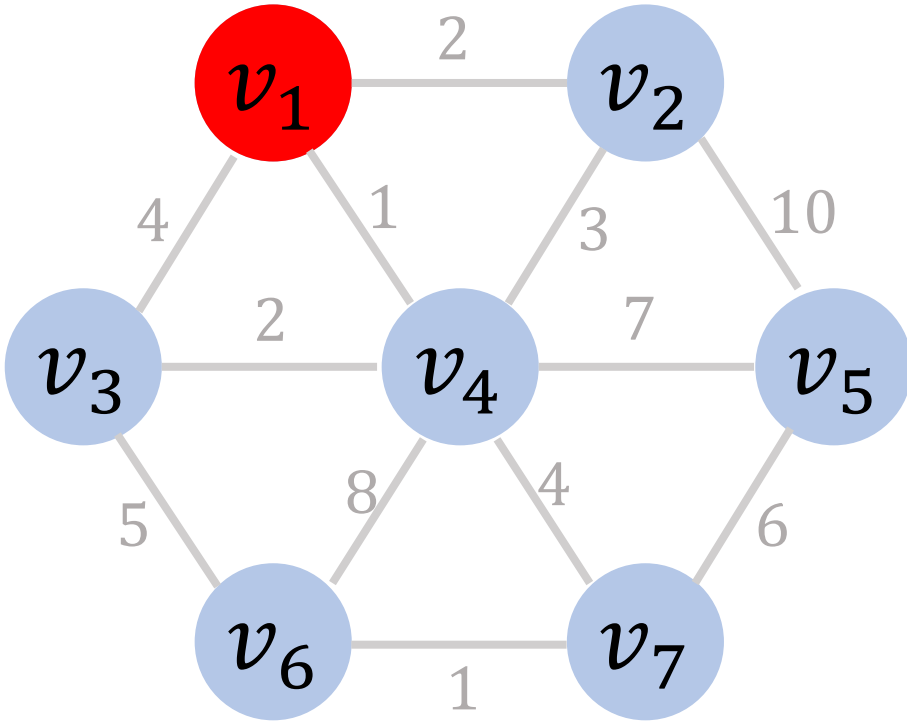
- Initially, the tree has one vertex and no edge.
- In each iteration, add one vertex and one edge to the tree.
- Throughout, maintain the properties of trees:
 - Connectivity.
 - No cycle.
- The algorithm runs in $|V|$ iterations.

Initial State



\mathcal{U} : vertices of spanning tree

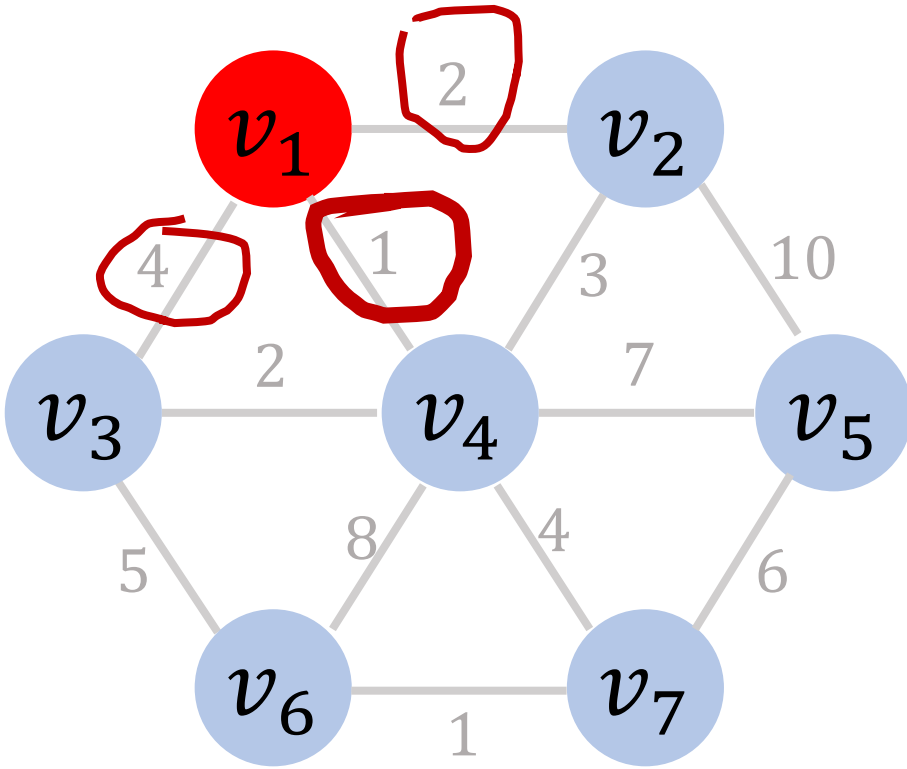
Iteration 1



- Pick any vertex in the graph.
- Maybe pick v_1 .
- Add v_1 to \mathcal{U} .

$$\mathcal{U} = \{v_1\}$$

Iteration 2

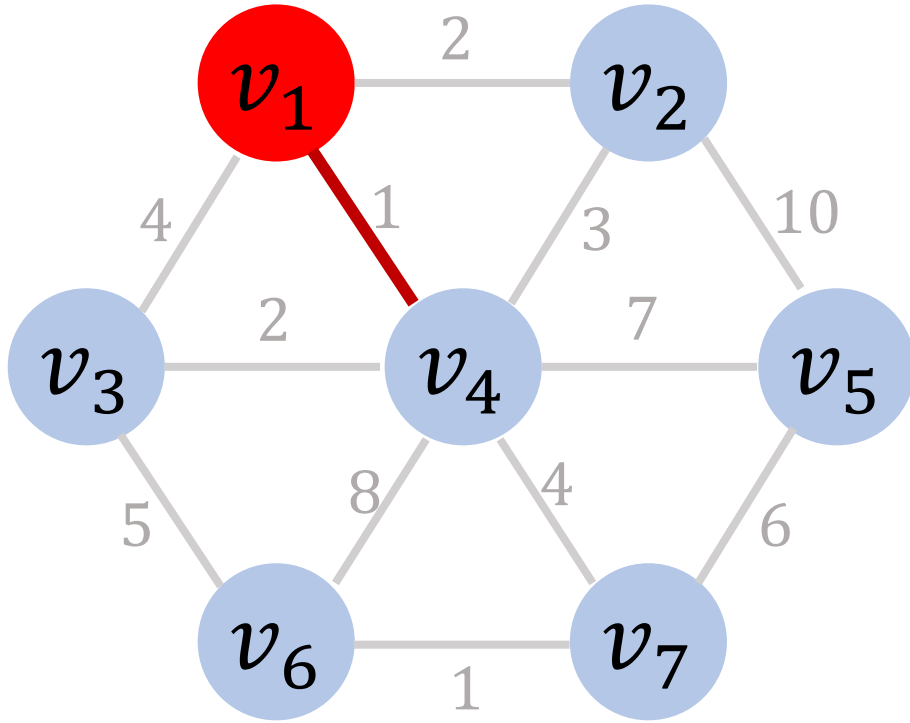


- The edges connecting u to $\mathcal{V} \setminus u$:

$e_{1,2}, e_{1,3}, e_{1,4}$.

$$u = \{v_1\}$$

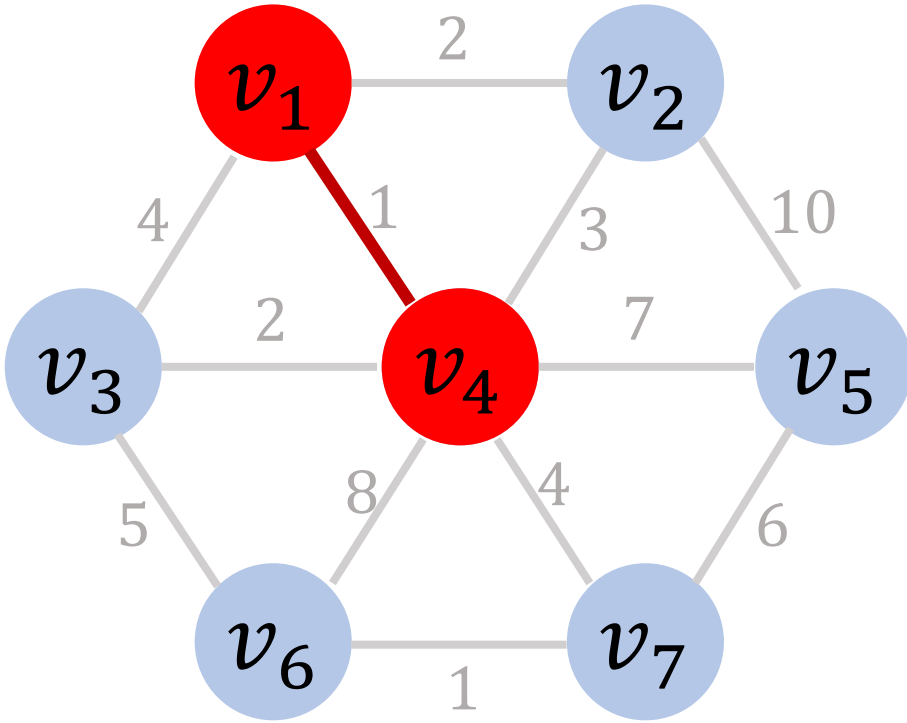
Iteration 2



- The edges connecting u to $\mathcal{V} \setminus u$:
 $e_{1,2}, e_{1,3}, e_{1,4}$.
- Among them, $e_{1,4}$ has the smallest weight.

$$u = \{v_1\}$$

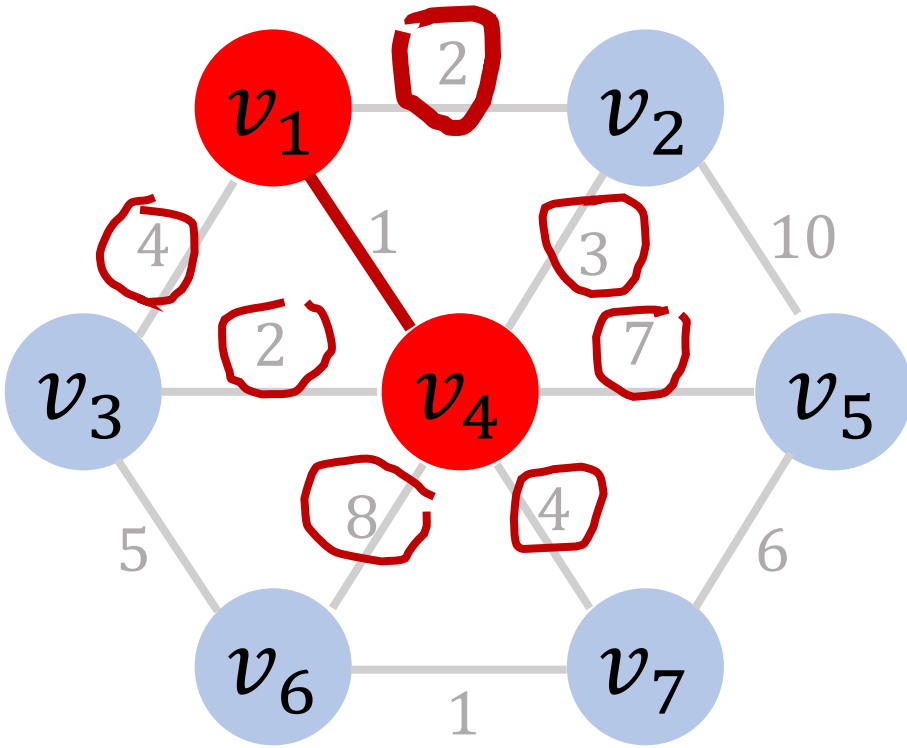
Iteration 2



$$\mathcal{U} = \{v_1, v_4\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{1,2}, e_{1,3}, e_{1,4}$.
- Among them, $e_{1,4}$ has the smallest weight.
- Add v_4 to \mathcal{U} .

Iteration 3



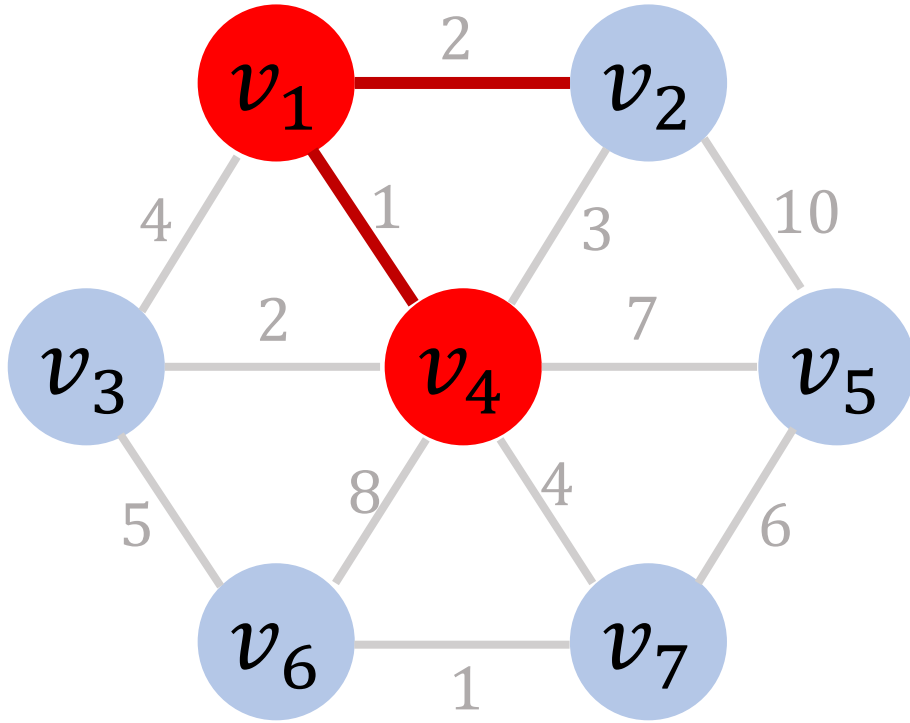
- The edges connecting u to $\mathcal{V} \setminus u$:

$e_{1,2}, e_{1,3},$

$e_{4,2}, e_{4,3}, e_{4,5}, e_{4,6}, e_{4,7}.$

$$u = \{v_1, v_4\}$$

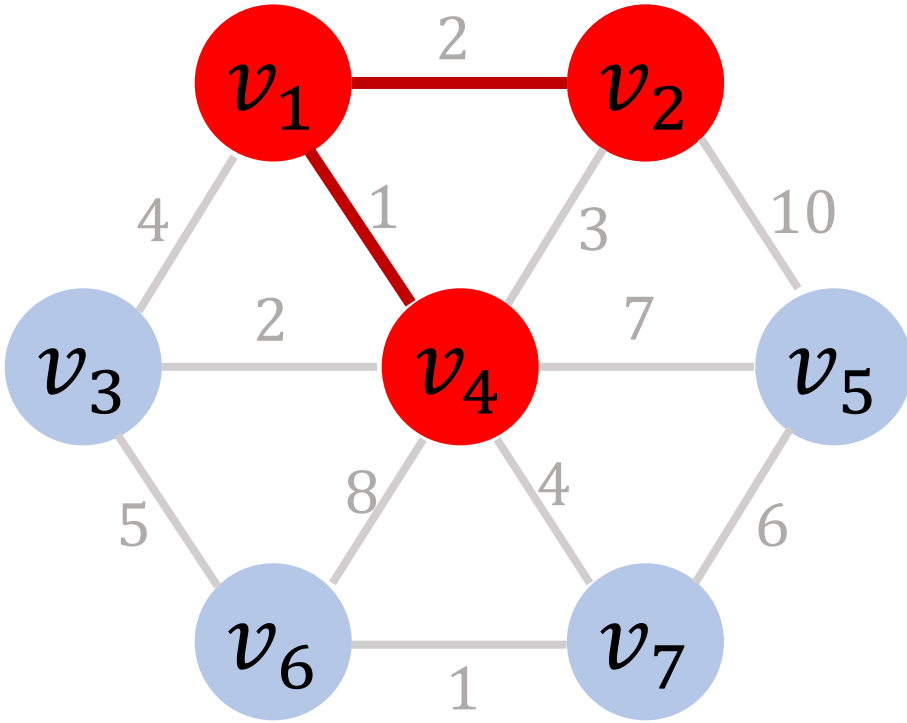
Iteration 3



$$u = \{v_1, v_4\}$$

- The edges connecting u to $\mathcal{V} \setminus u$:
 $e_{1,2}, e_{1,3},$
 $e_{4,2}, e_{4,3}, e_{4,5}, e_{4,6}, e_{4,7}.$
- Among them, $e_{1,2}$ has the smallest weight.

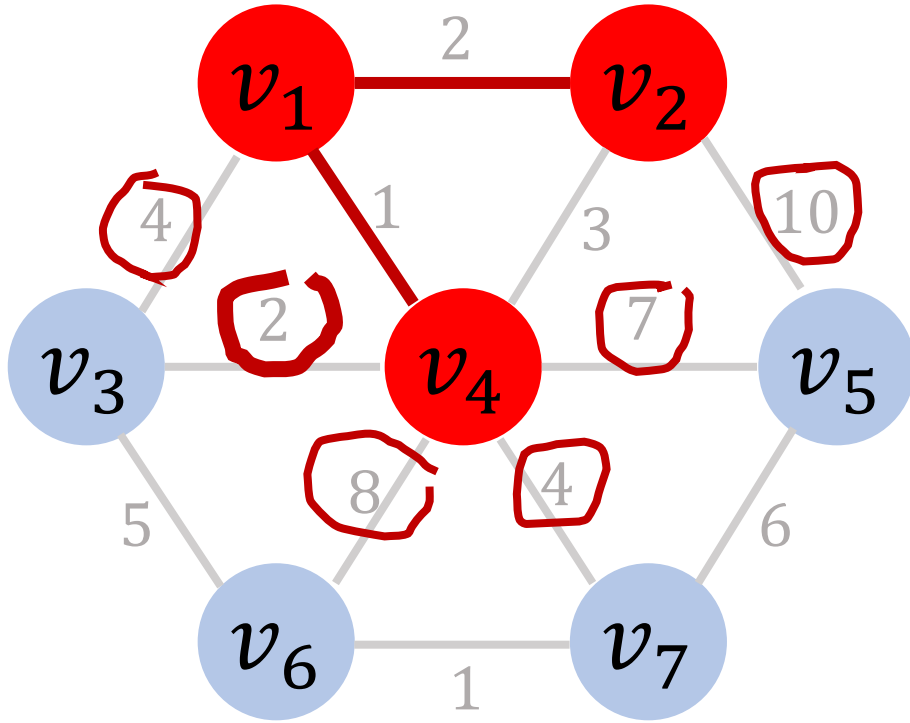
Iteration 3



$$\mathcal{U} = \{v_1, v_4, v_2\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{1,2}, e_{1,3},$
 $e_{4,2}, e_{4,3}, e_{4,5}, e_{4,6}, e_{4,7}.$
- Among them, $e_{1,2}$ has the smallest weight.
- Add v_2 to \mathcal{U} .

Iteration 4



- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:

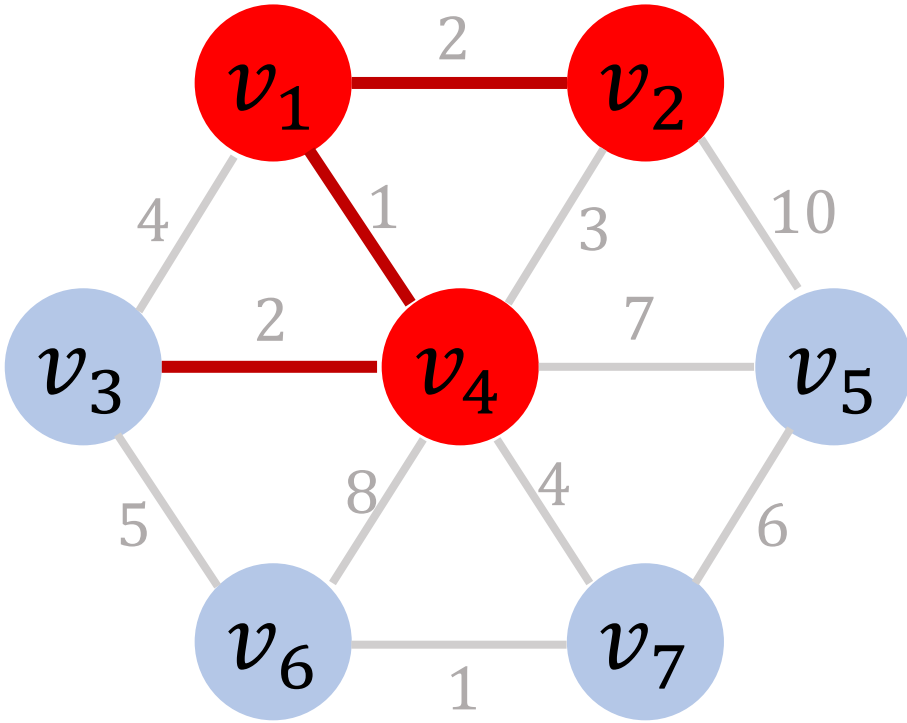
$e_{1,3}$,

$e_{4,3}, e_{4,5}, e_{4,6}, e_{4,7}$,

$e_{2,5}$.

$$\mathcal{U} = \{v_1, v_4, v_2\}$$

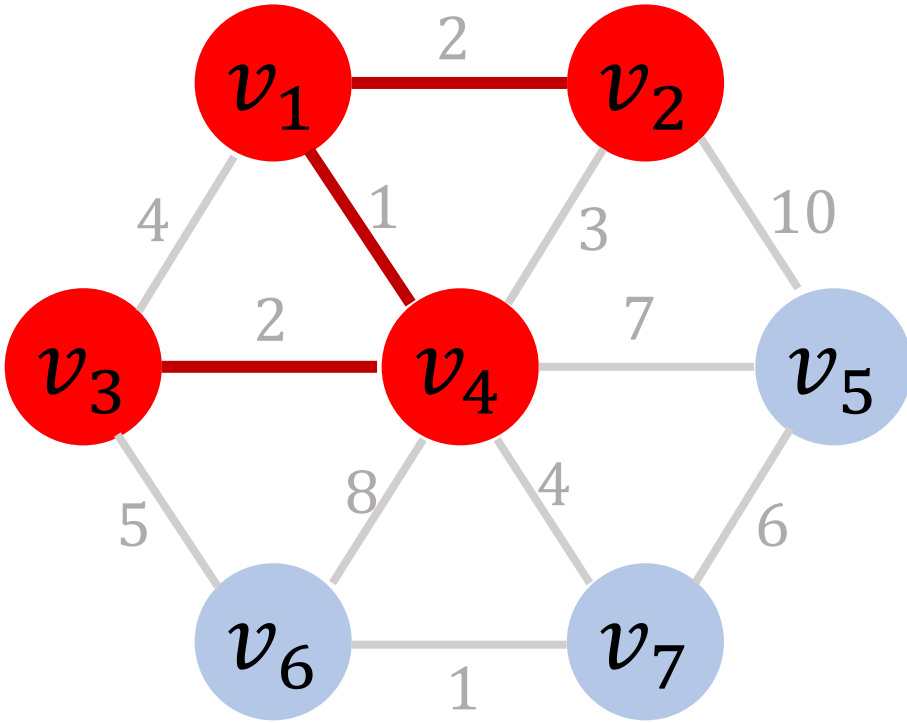
Iteration 4



$$\mathcal{U} = \{v_1, v_4, v_2\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{1,3}$,
 $e_{4,3}, e_{4,5}, e_{4,6}, e_{4,7}$,
 $e_{2,5}$.
- Among them, $e_{4,3}$ has the smallest weight.

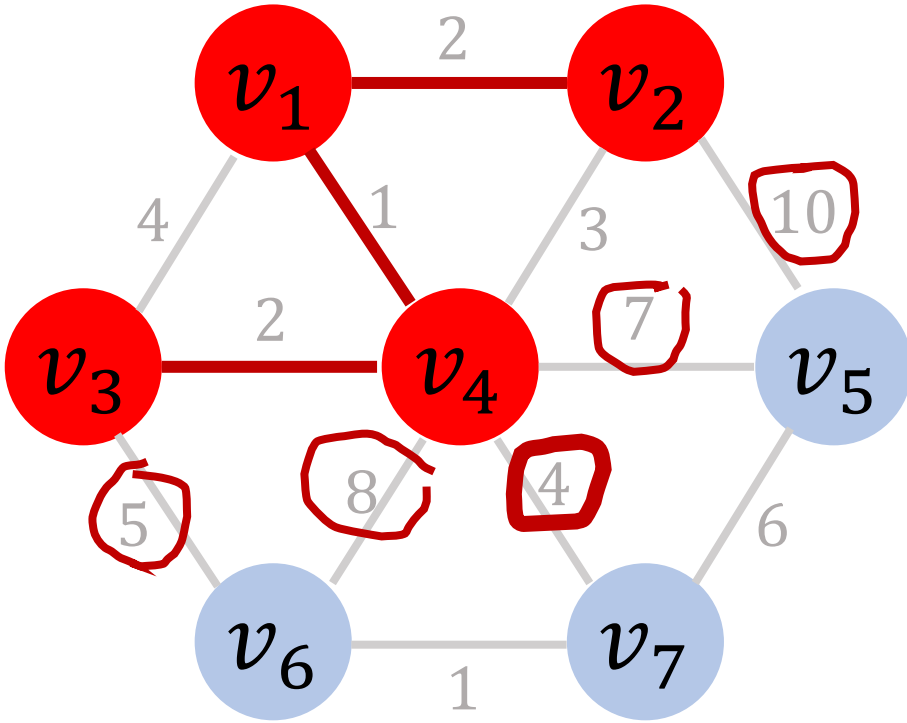
Iteration 4



$$\mathcal{U} = \{v_1, v_4, v_2, v_3\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{1,3}$,
 $e_{4,3}, e_{4,5}, e_{4,6}, e_{4,7}$,
 $e_{2,5}$.
- Among them, $e_{4,3}$ has the smallest weight.
- Add v_3 to \mathcal{U} .

Iteration 5



- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:

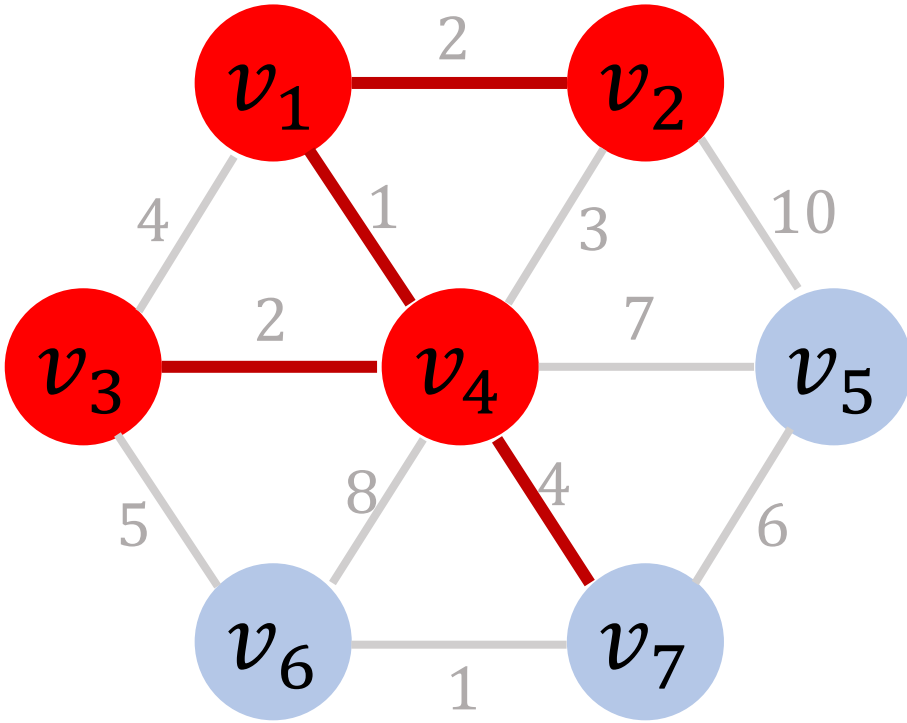
$e_{4,5}, e_{4,6}, e_{4,7},$

$e_{2,5},$

$e_{3,6}.$

$$\mathcal{U} = \{v_1, v_4, v_2, v_3\}$$

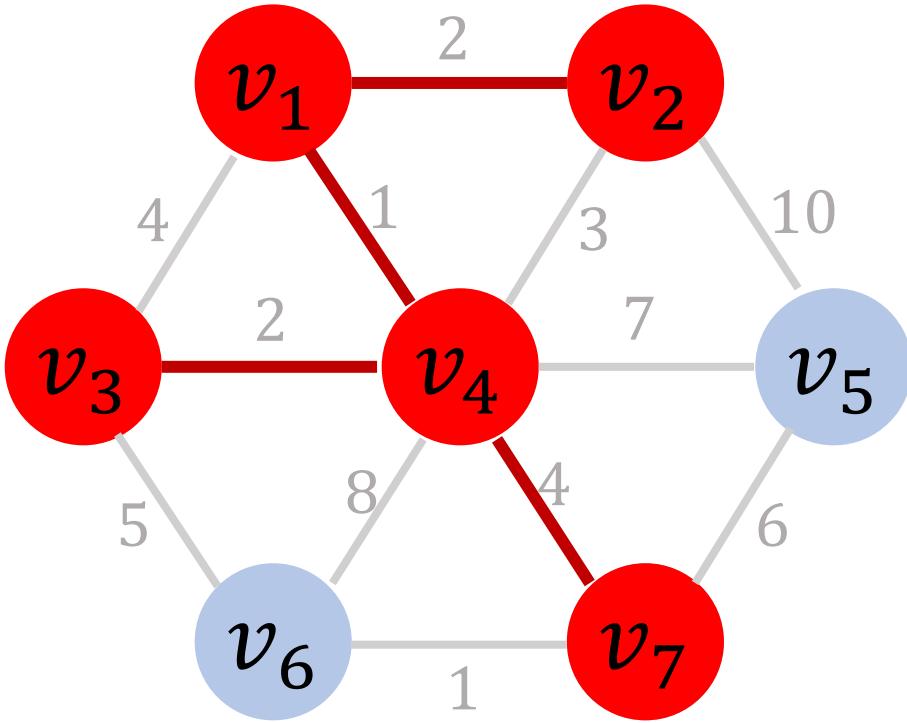
Iteration 5



$$\mathcal{U} = \{v_1, v_4, v_2, v_3\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{4,5}, e_{4,6}, e_{4,7},$
 $e_{2,5},$
 $e_{3,6}.$
- Among them, $e_{4,7}$ has the smallest weight.

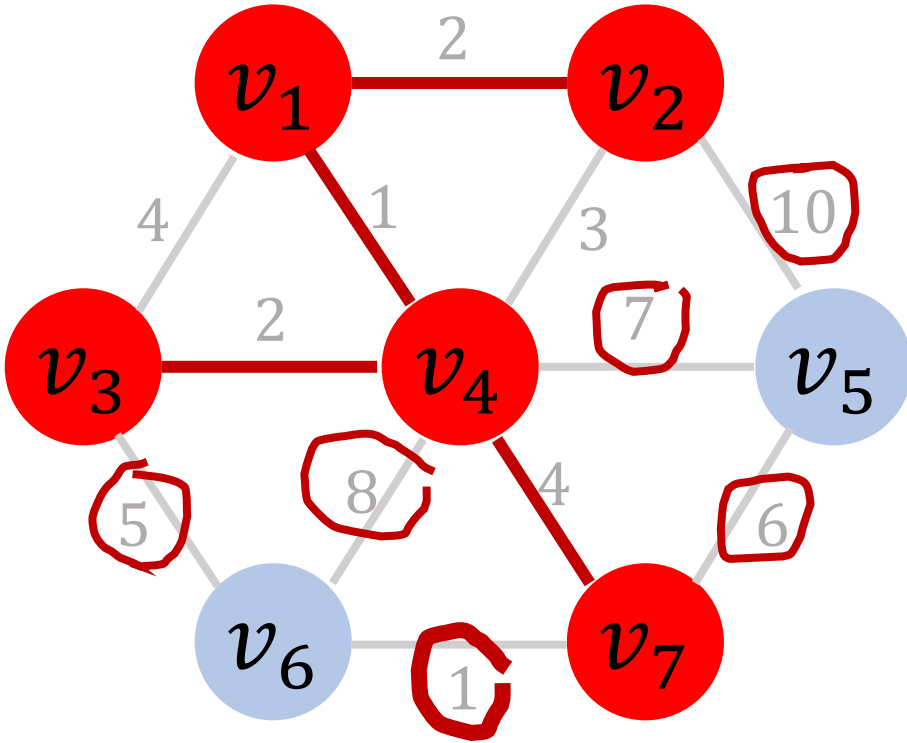
Iteration 5



$$\mathcal{U} = \{v_1, v_4, v_2, v_3, v_7\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{4,5}, e_{4,6}, e_{4,7},$
 $e_{2,5},$
 $e_{3,6}.$
- Among them, $e_{4,7}$ has the smallest weight.
- Add v_7 to \mathcal{U} .

Iteration 6



- The edges connecting u to $\mathcal{V} \setminus u$:

$e_{4,5}, e_{4,6},$

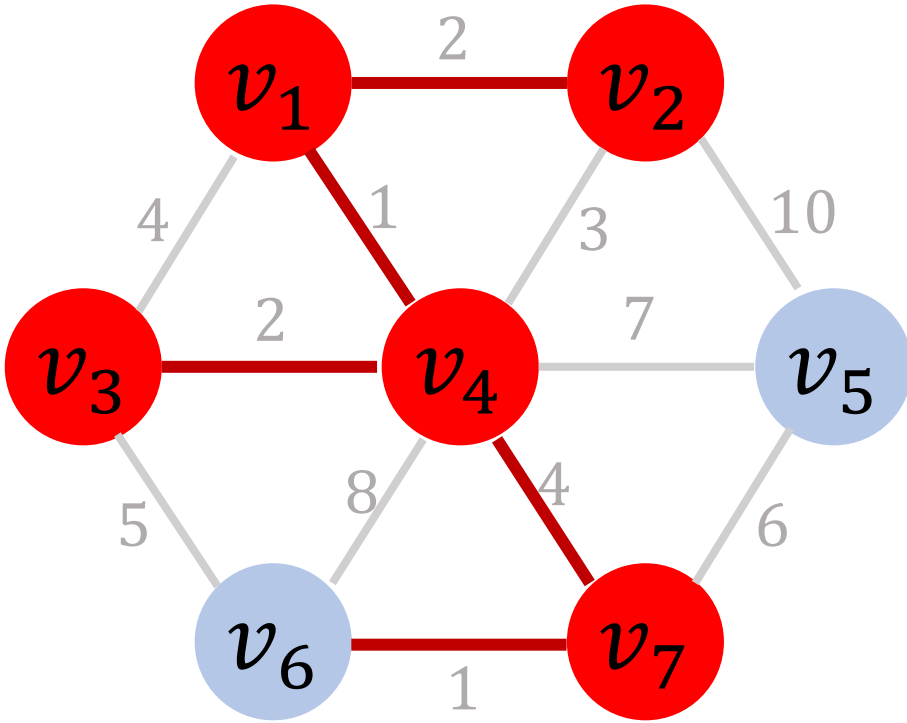
$e_{2,5},$

$e_{3,6},$

$e_{7,5}, e_{7,6}.$

$$u = \{v_1, v_4, v_2, v_3, v_7\}$$

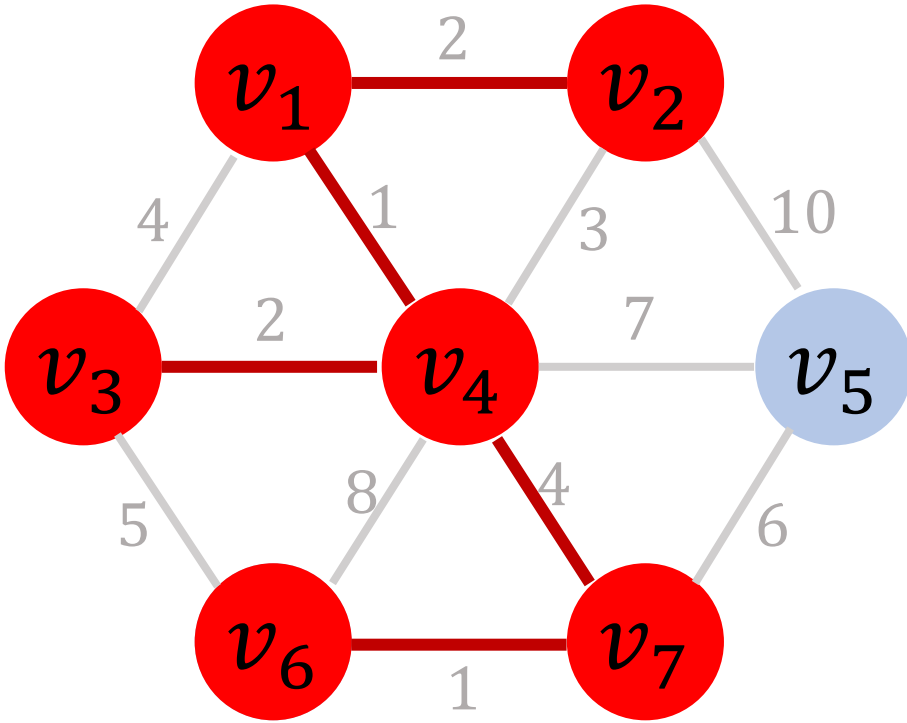
Iteration 6



$$\mathcal{U} = \{v_1, v_4, v_2, v_3, v_7\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{4,5}, e_{4,6},$
 $e_{2,5},$
 $e_{3,6},$
 $e_{7,5}, e_{7,6}.$
- Among them, $e_{7,6}$ has the smallest weight.

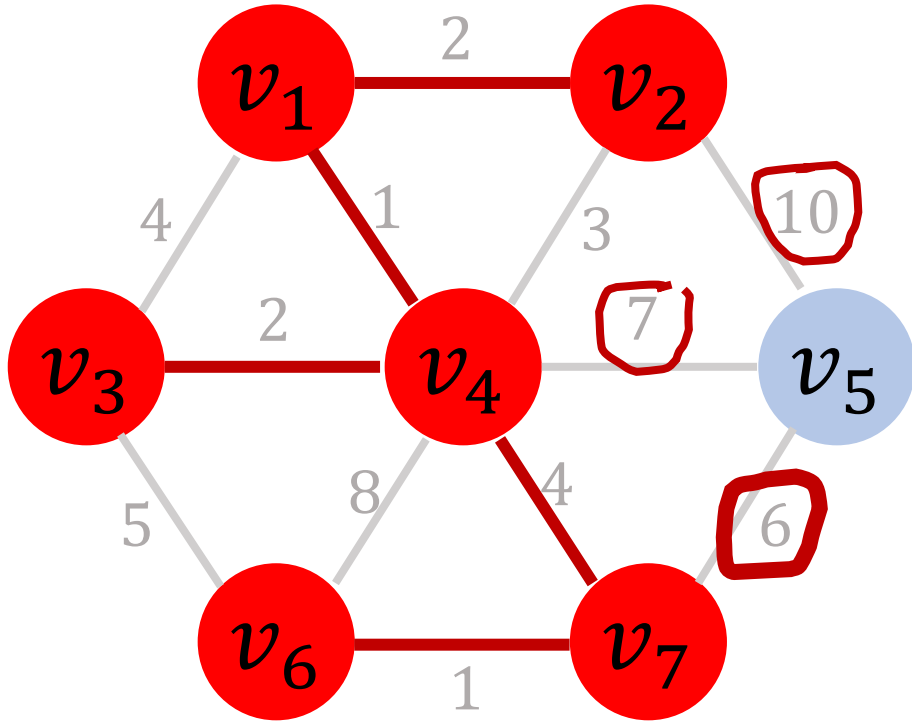
Iteration 6



$$\mathcal{U} = \{v_1, v_4, v_2, v_3, v_7, v_6\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{4,5}, e_{4,6},$
 $e_{2,5},$
 $e_{3,6},$
 $e_{7,5}, e_{7,6}.$
- Among them, $e_{7,6}$ has the smallest weight.
- Add v_6 to \mathcal{U} .

Iteration 7



- The edges connecting u to $\mathcal{V} \setminus u$:

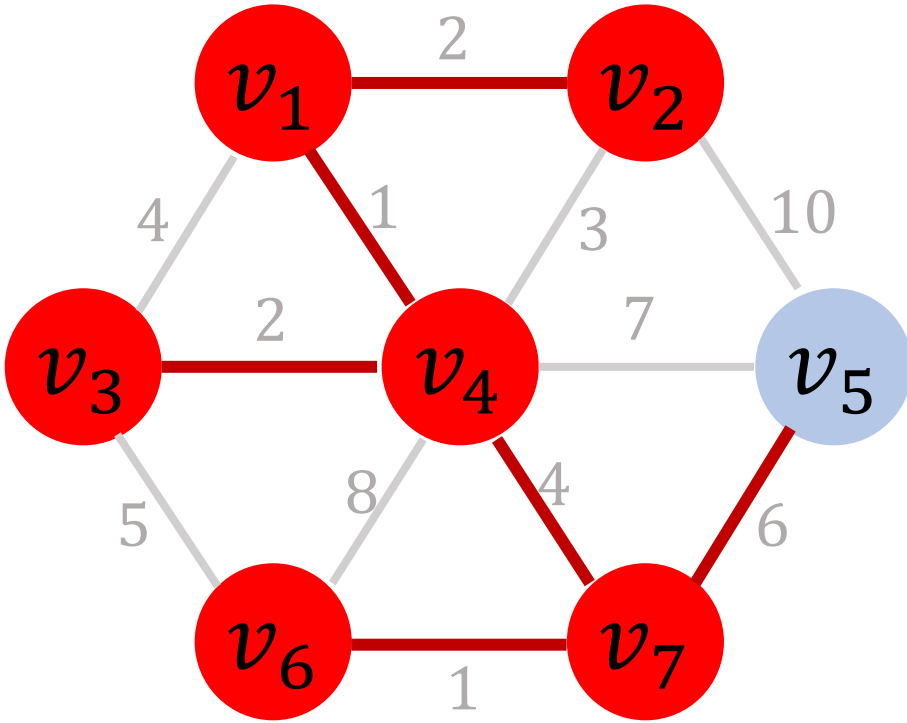
$e_{4,5},$

$e_{2,5},$

$e_{7,5}.$

$$u = \{v_1, v_4, v_2, v_3, v_7, v_6\}$$

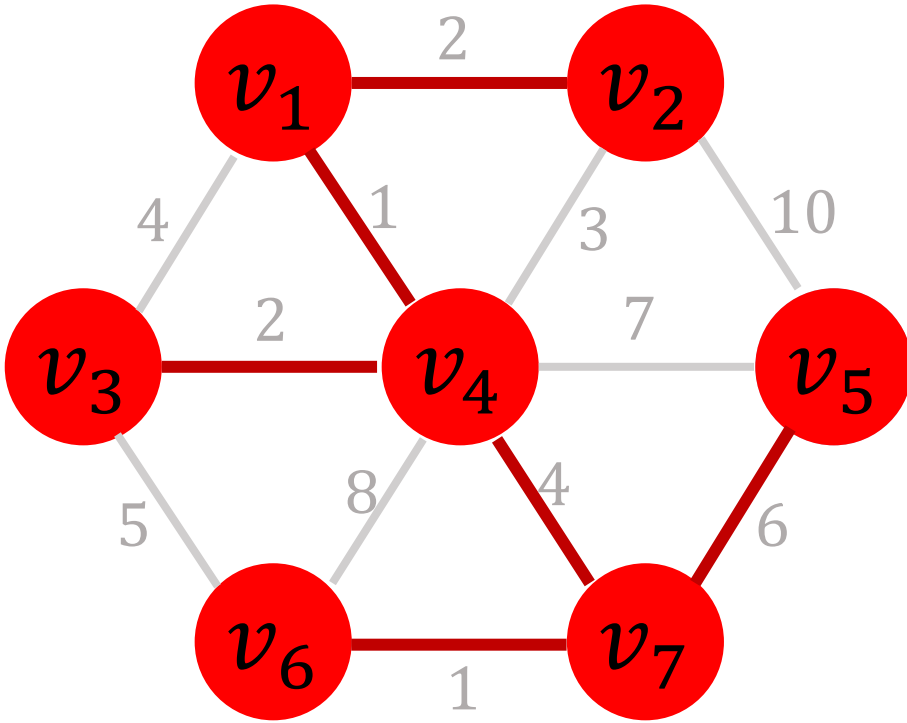
Iteration 7



$$\mathcal{U} = \{v_1, v_4, v_2, v_3, v_7, v_6\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{4,5}$,
 $e_{2,5}$,
 $e_{7,5}$.
- Among them, $e_{7,5}$ has the smallest weight.

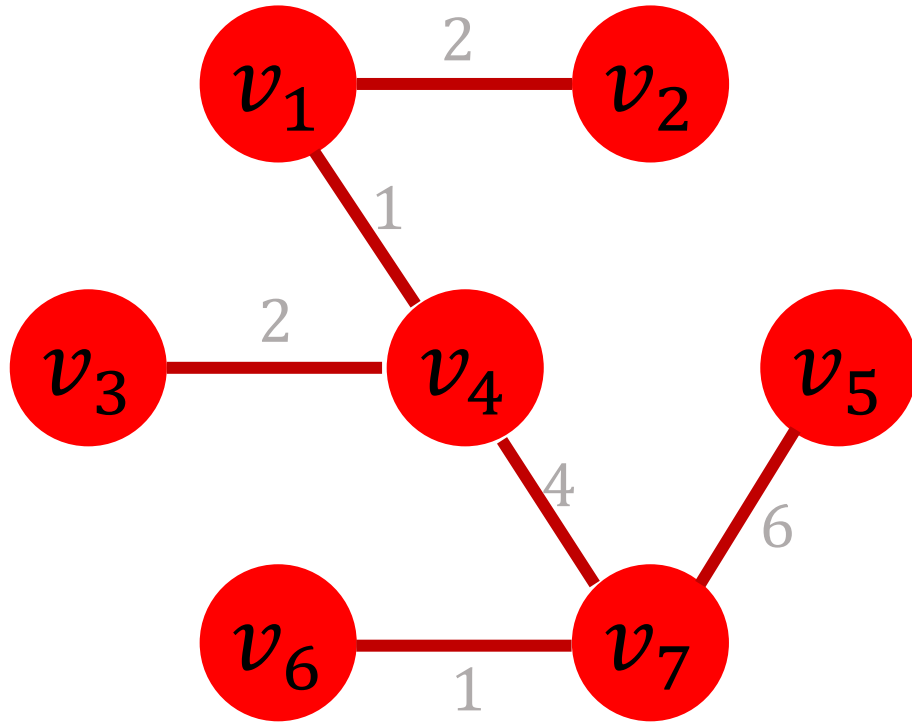
Iteration 7



$$\mathcal{U} = \{v_1, v_4, v_2, v_3, v_7, v_6, v_5\}$$

- The edges connecting \mathcal{U} to $\mathcal{V} \setminus \mathcal{U}$:
 $e_{4,5}$,
 $e_{2,5}$,
 $e_{7,5}$.
- Among them, $e_{7,5}$ has the smallest weight.
- Add v_5 to \mathcal{U} .

End of Procedure



- Now $\mathcal{U} = \mathcal{V}$. (All the vertices have been added to \mathcal{U} .)
- Return the tree.

$$\mathcal{U} = \{v_1, v_4, v_2, v_3, v_7, v_6, v_5\}$$

Prim's Algorithm

1. Initially, the tree has one vertex and no edge.
2. Let set \mathcal{U} contain the vertices of the tree.
3. Let set \mathcal{T} be the empty set.

Prim's Algorithm

1. Initially, the tree has one vertex and no edge.
2. Let set \mathcal{U} contain the vertices of the tree.
3. Let set \mathcal{T} be the empty set.
4. In each iteration, select one vertex and one edge:
 - Define the set $\mathcal{S} = \{e_{xy} \in \mathcal{E} \mid x \in \mathcal{U} \text{ and } y \notin \mathcal{U}\}$.
 - Select from \mathcal{S} the edge (denote e_{uv}) that has the smallest weight.
 - Add edge e_{uv} to \mathcal{T} .
 - Add vertex v to \mathcal{U} .
5. Return the selected edges, \mathcal{T} .

Thank You!