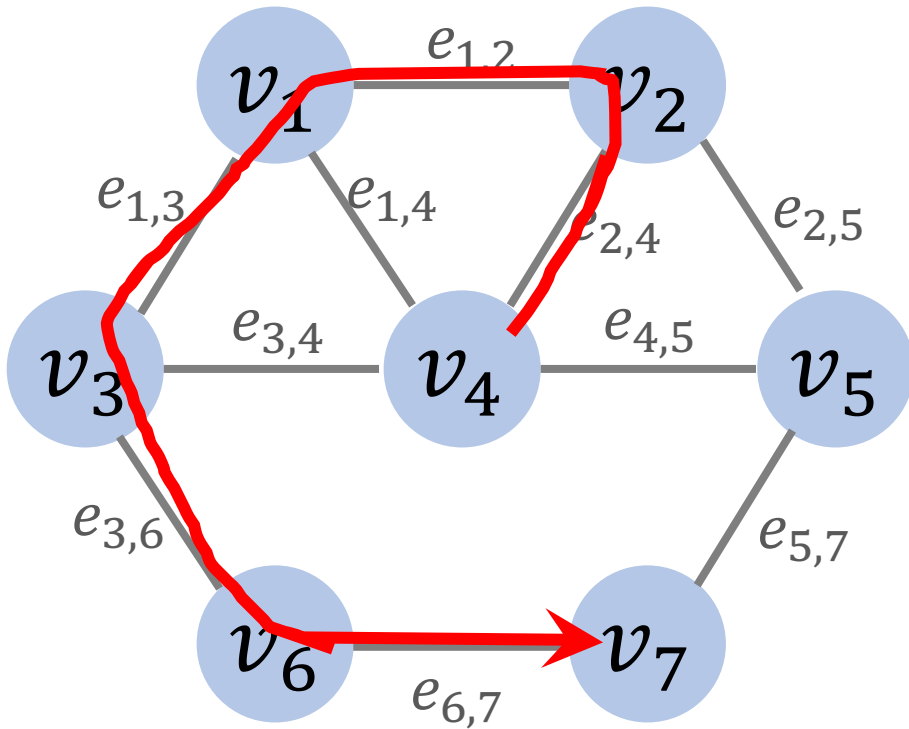


# Shortest Path

Shusen Wang

**Path**

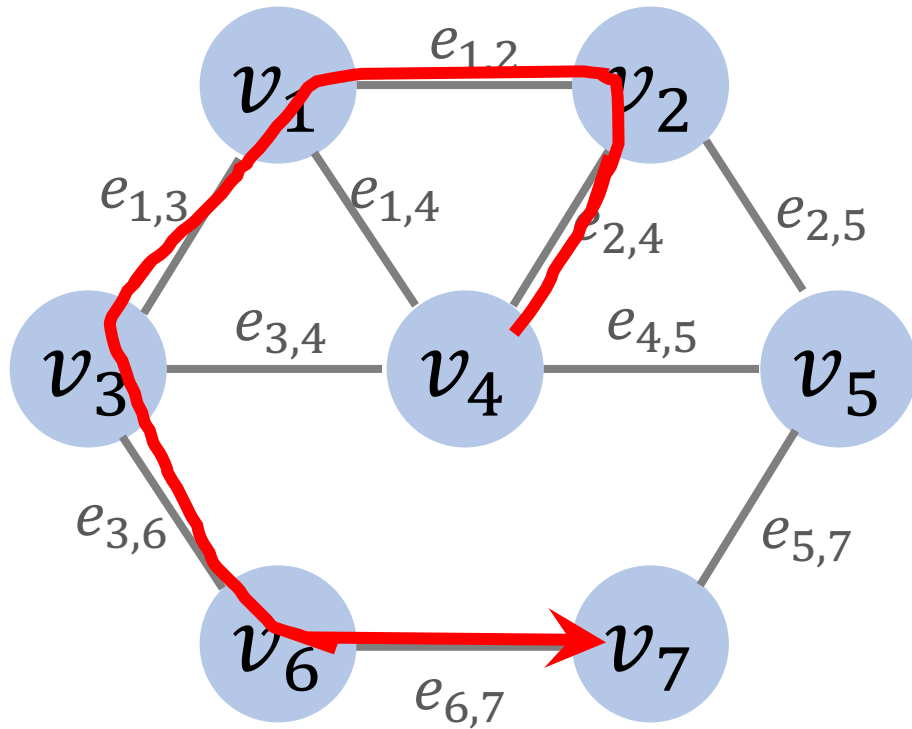
# Path in undirected unweighted graph



## Definition: Path

- A sequence of vertices  
 $(v_4, v_2, v_1, v_3, v_6, v_7)$
- Or a sequence of edges  
 $(e_{4,2}, e_{2,1}, e_{1,3}, e_{3,6}, e_{6,7}).$

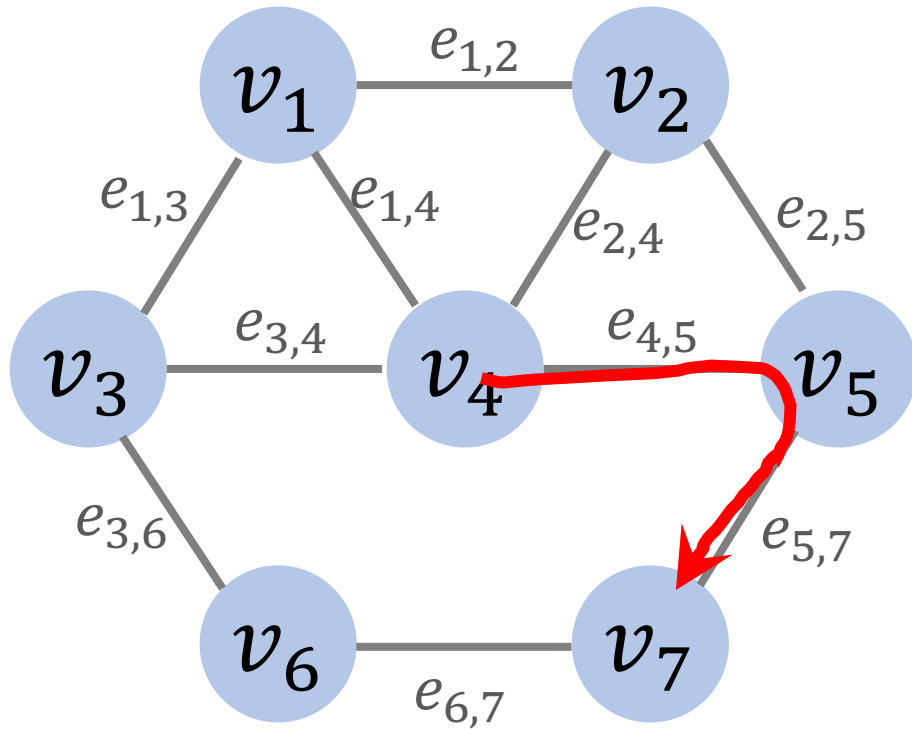
# Path in undirected unweighted graph



Definition: Length of path

- Length of a path is the number of edges on the path.
- In this example, the length is 5.

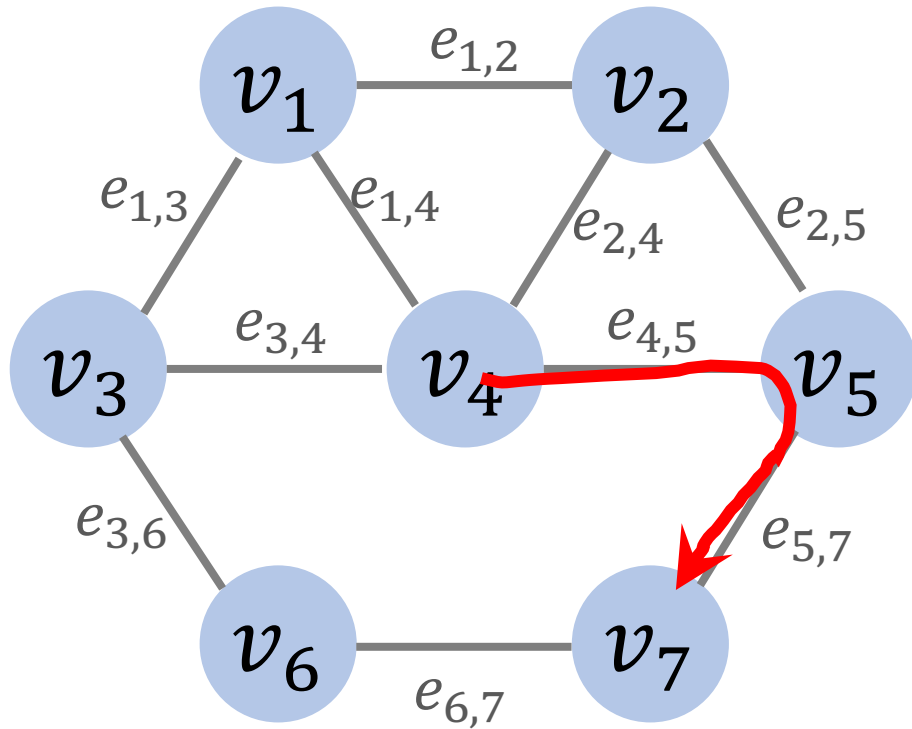
# Path in undirected unweighted graph



## Definition: Path

- A sequence of vertices  
 $(v_4, v_5, v_7)$
- Or a sequence of edges  
 $(e_{4,5}, e_{5,7})$ .

# Path in undirected unweighted graph



Definition: Length of path

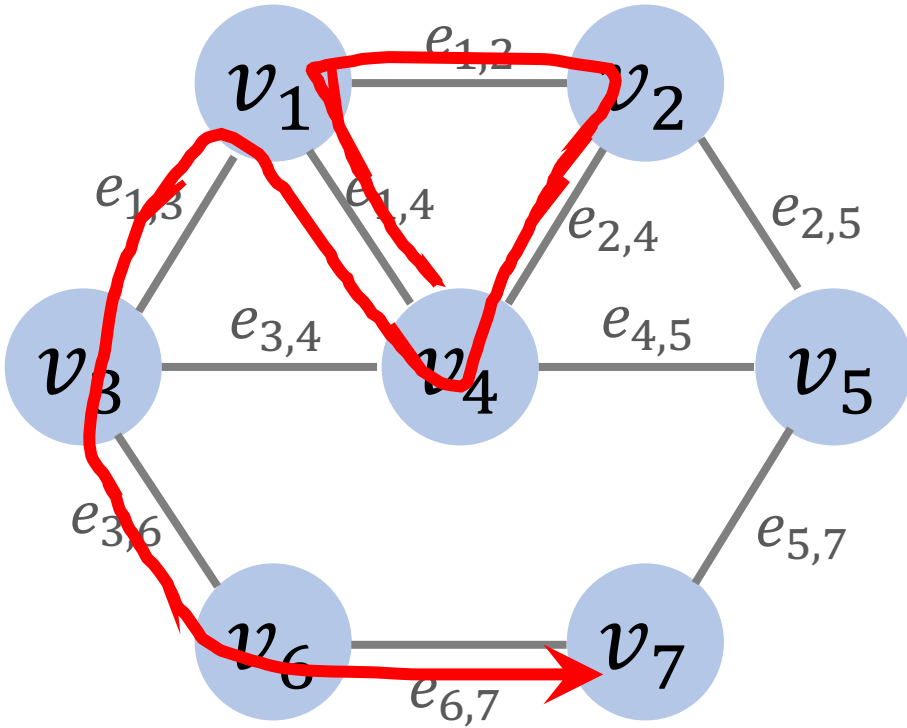
- Length of a path is the number of edges on the path.
- In this example, the length is 2.

# Simple Path

## Definition: Path

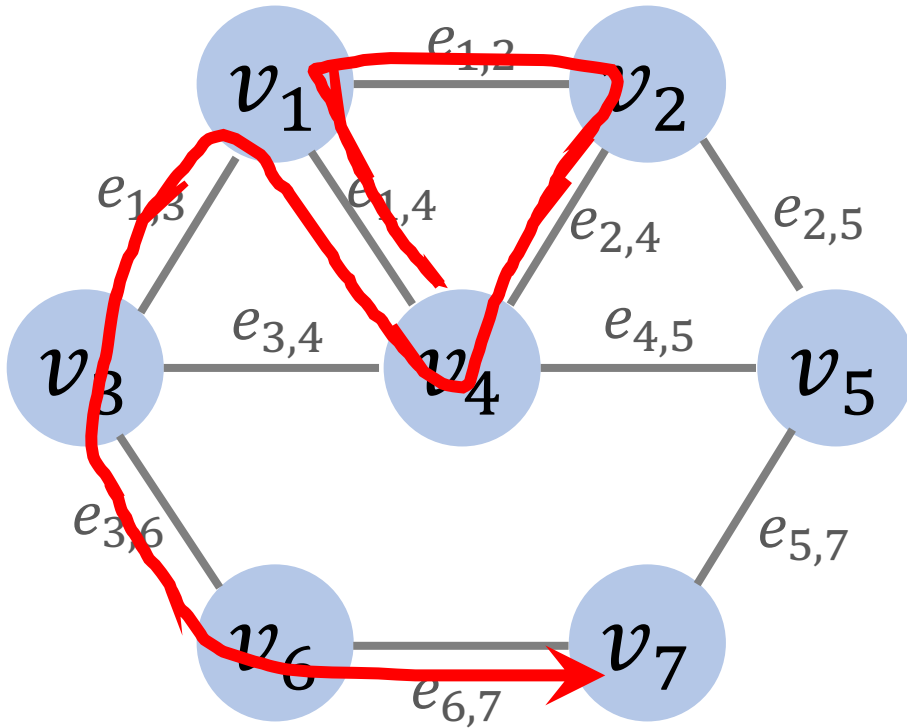
A sequence of vertices

$(v_4, v_1, v_2, v_4, v_1, v_3, v_6, v_7)$  .



# Simple Path

Not a simple path!



Definition: Path

A sequence of vertices

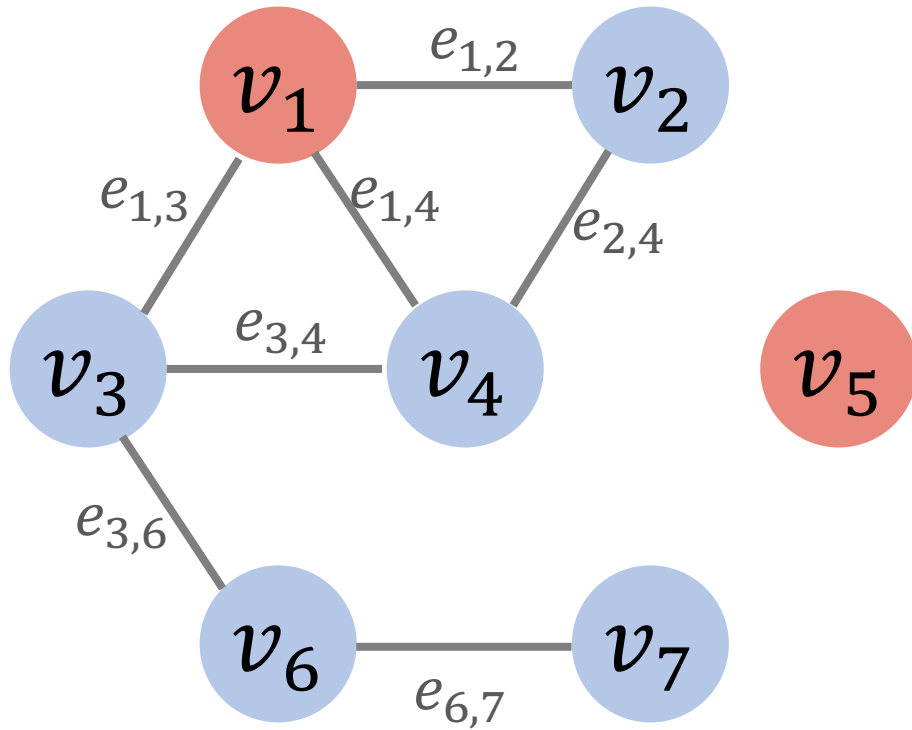
$(v_4, v_1, v_2, v_4, v_1, v_3, v_6, v_7)$ .

Definition: simple path

- A path that does not repeat vertices is called a simple path.

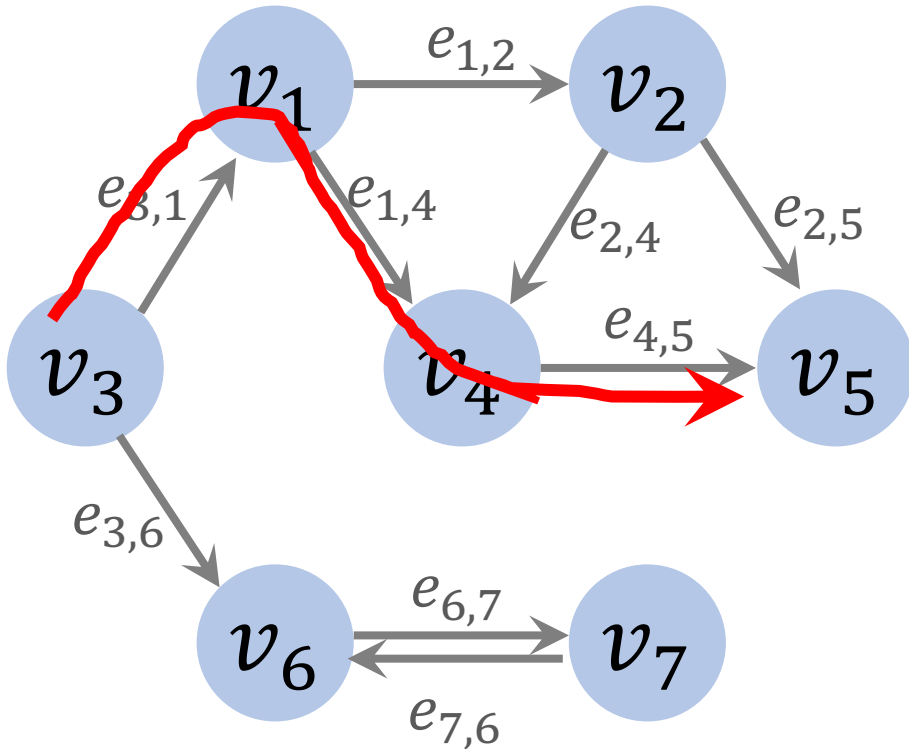


# Path may not always exist



- There is no path from  $v_1$  to  $v_5$ .
- The path from  $v_1$  to  $v_5$  has a length of  $\infty$ .

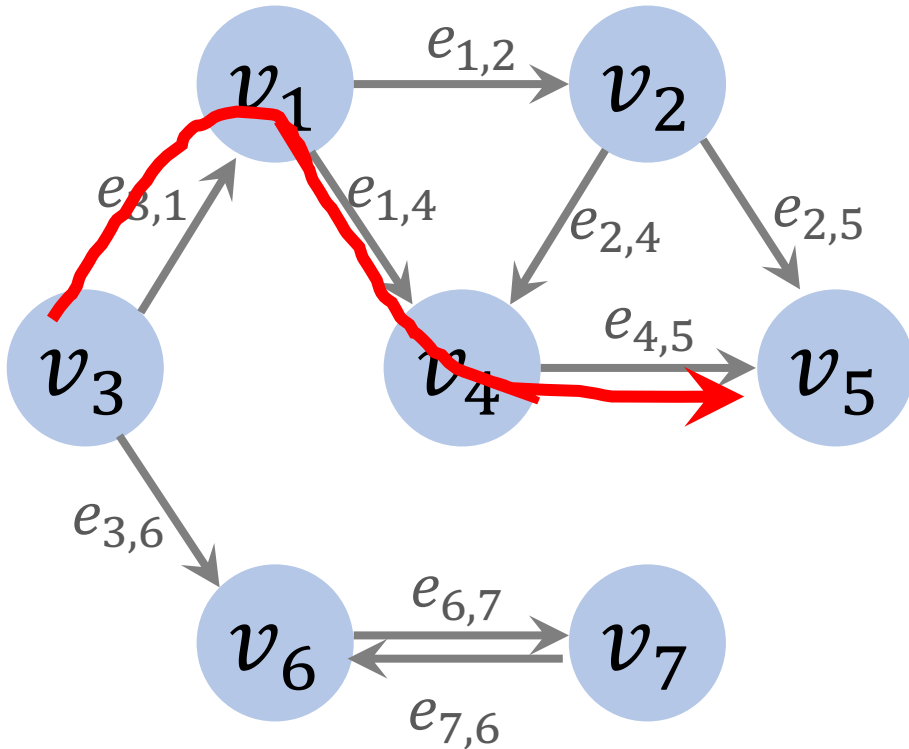
# Path in directed unweighted graph



## Definition: Path

- A sequence of vertices  
 $(v_3, v_1, v_4, v_5)$
- Or a sequence of edges  
 $(e_{3,1}, e_{1,4}, e_{4,5})$ .

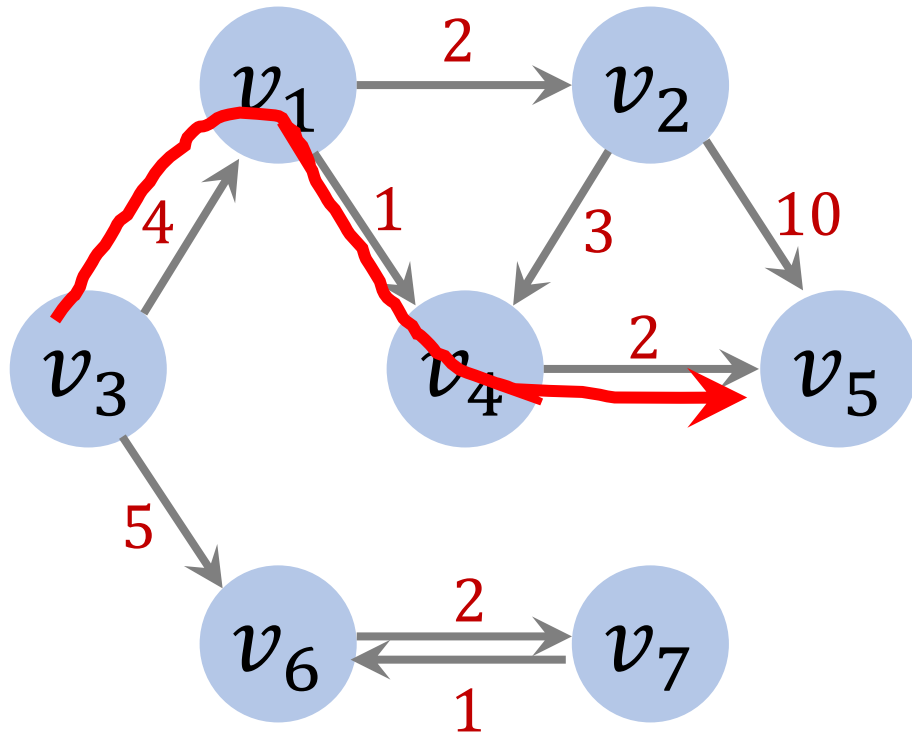
# Path in directed unweighted graph



Definition: Length of path

- Length of a path is the number of edges on the path.
- In this example, the length is 3.

# Path in directed weighted graph

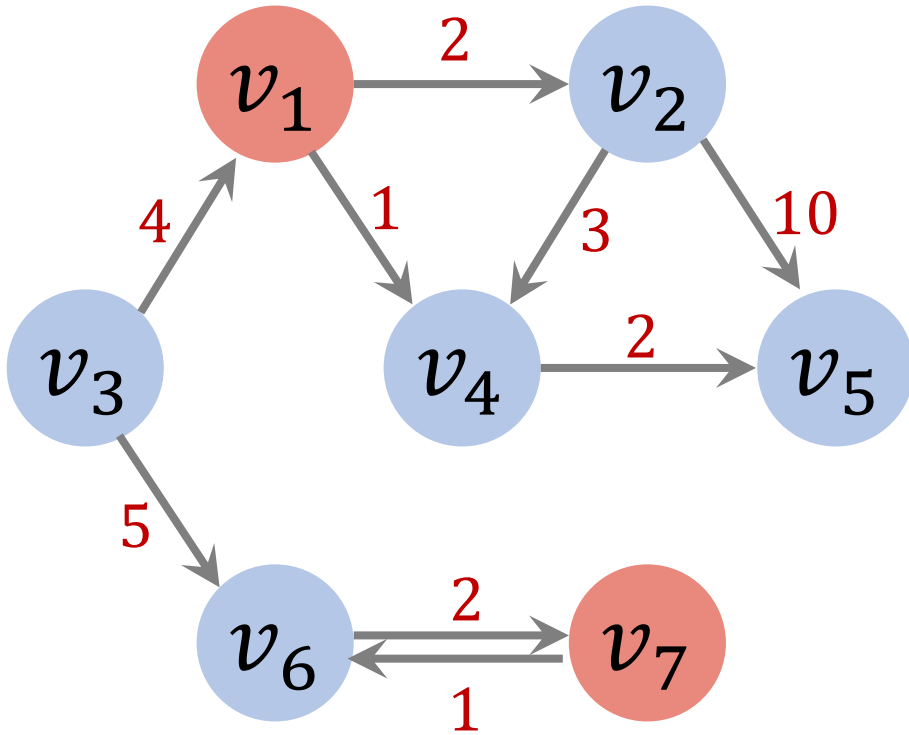


Definition: Length of path

- Length of a path is the number of edges on the path.
- In this example, the length is

$$4 + 1 + 2 = 7.$$

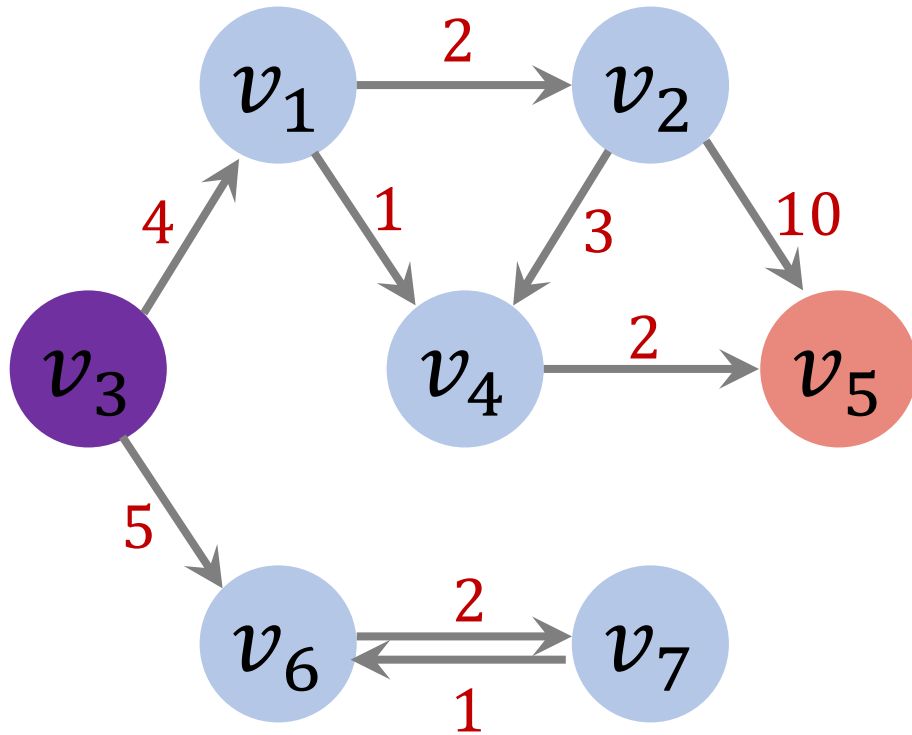
# Path may not always exist



- There is no path from  $v_1$  to  $v_7$ .
- The path from  $v_1$  to  $v_7$  has a length of  $\infty$ .

# Shortest Path

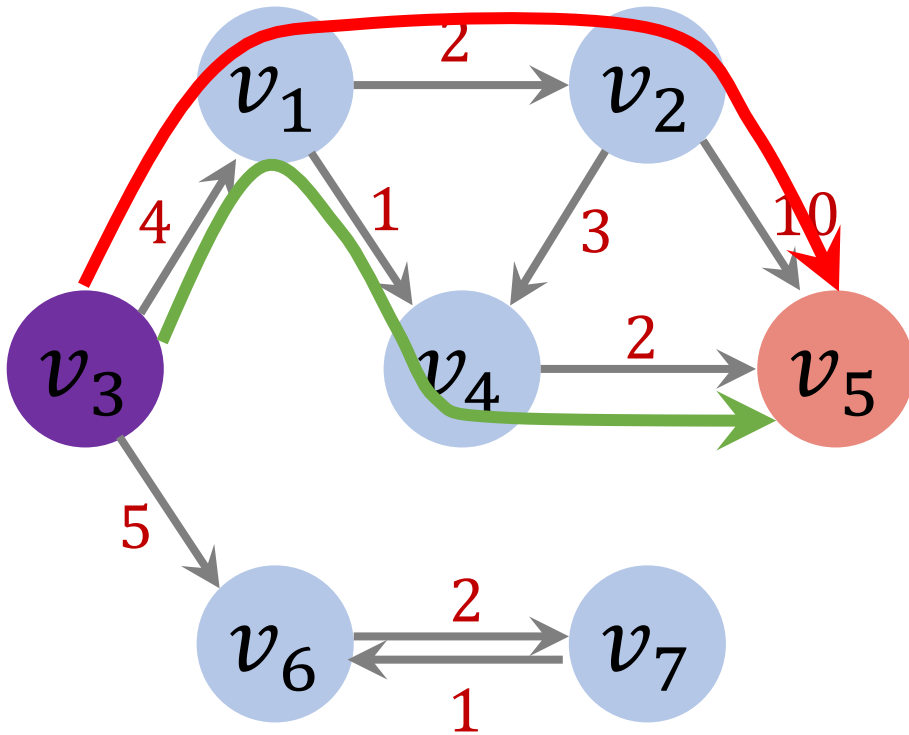
# Shortest Path



## Definition

- **Inputs:** graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , the source vertex,  $s$ , and the destination,  $d$ .

# Shortest Path

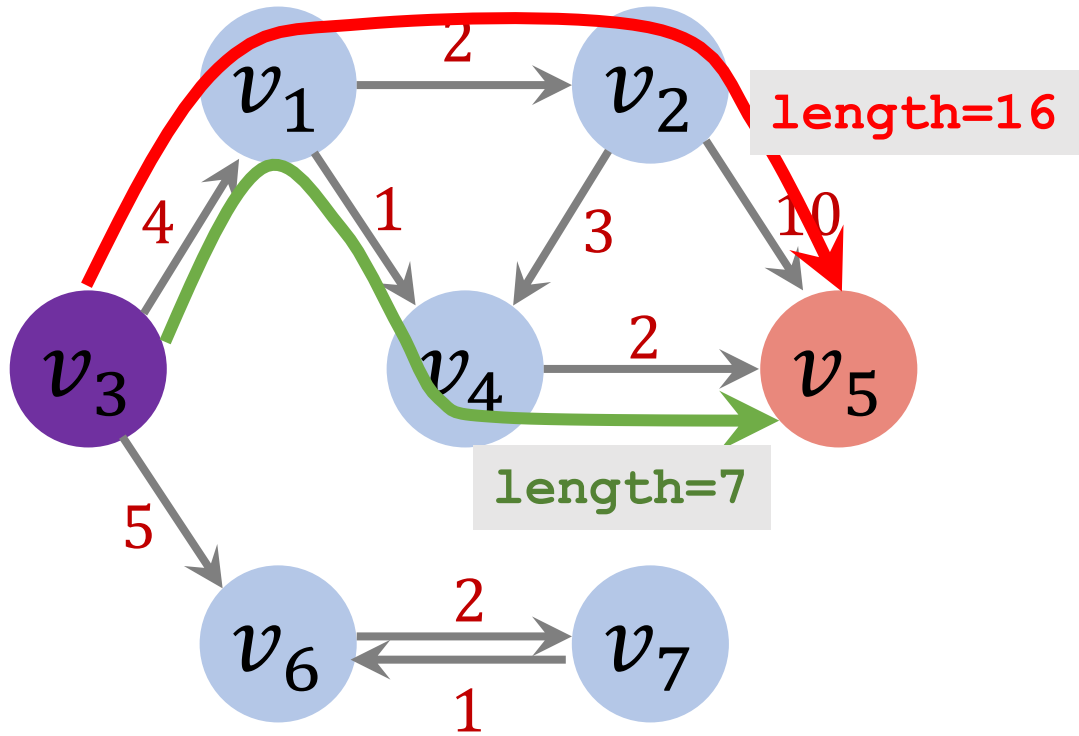


## Definition

- **Inputs:** graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , the source vertex,  $s$ , and the destination,  $d$ .
- There can be multiple paths from  $s$  to  $d$ .



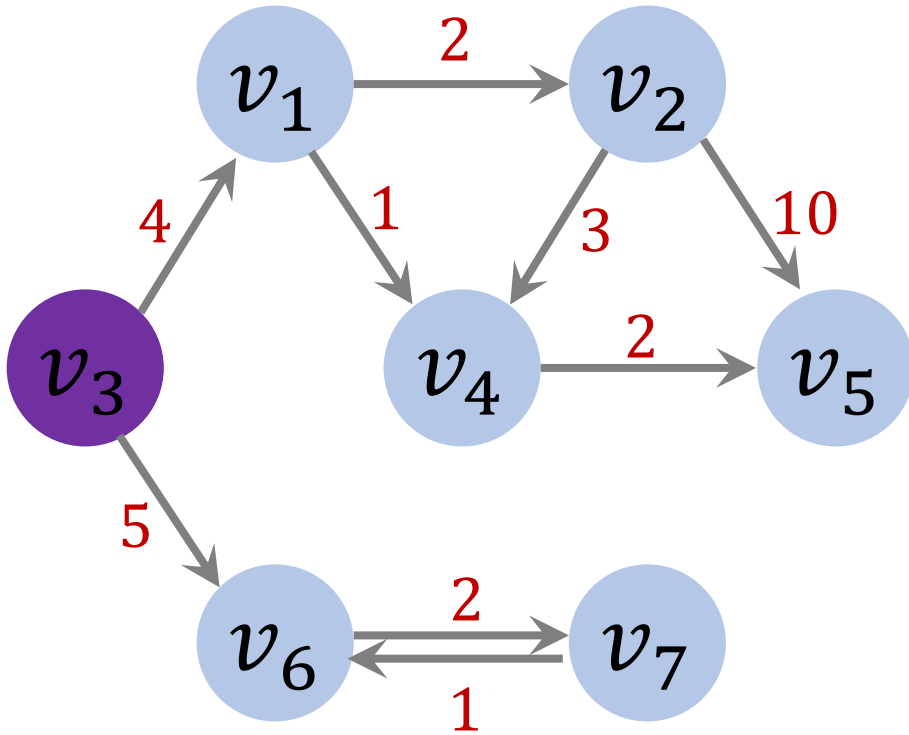
# Shortest Path



## Definition

- **Inputs:** graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ , the source vertex,  $s$ , and the destination,  $d$ .
- There can be multiple paths from  $s$  to  $d$ .
- Among all the paths, the one with the smallest length is called the shortest path.

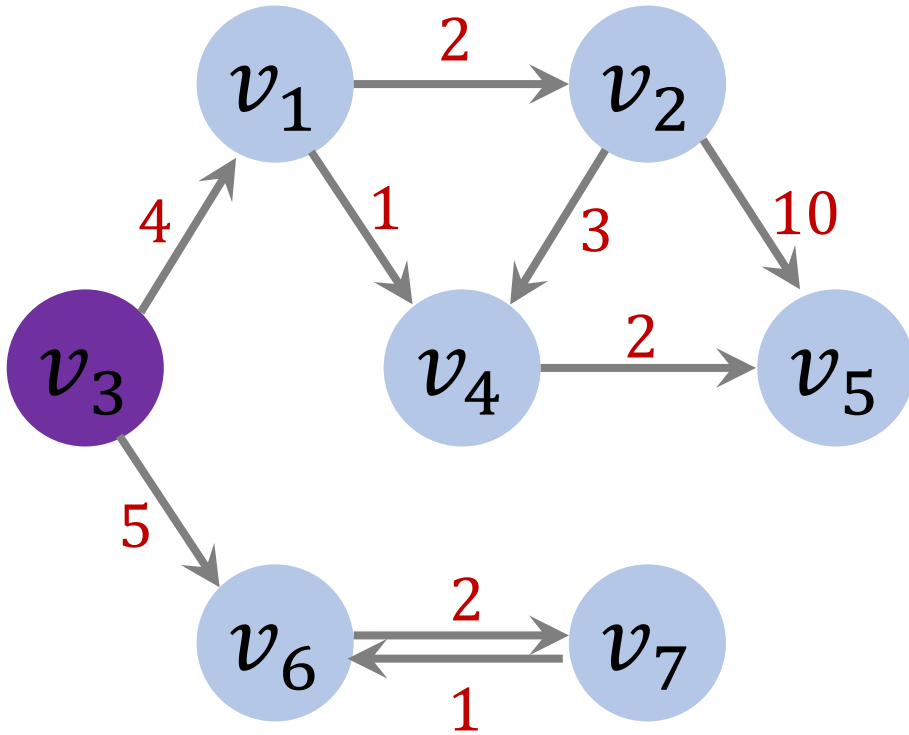
# Single-Source Shortest Path Problem



## Definition

- **Inputs:** graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  and the source vertex,  $s$ .
- **Goal:** find the shortest path from  $s$  to every other vertex in  $\mathcal{G}$ .

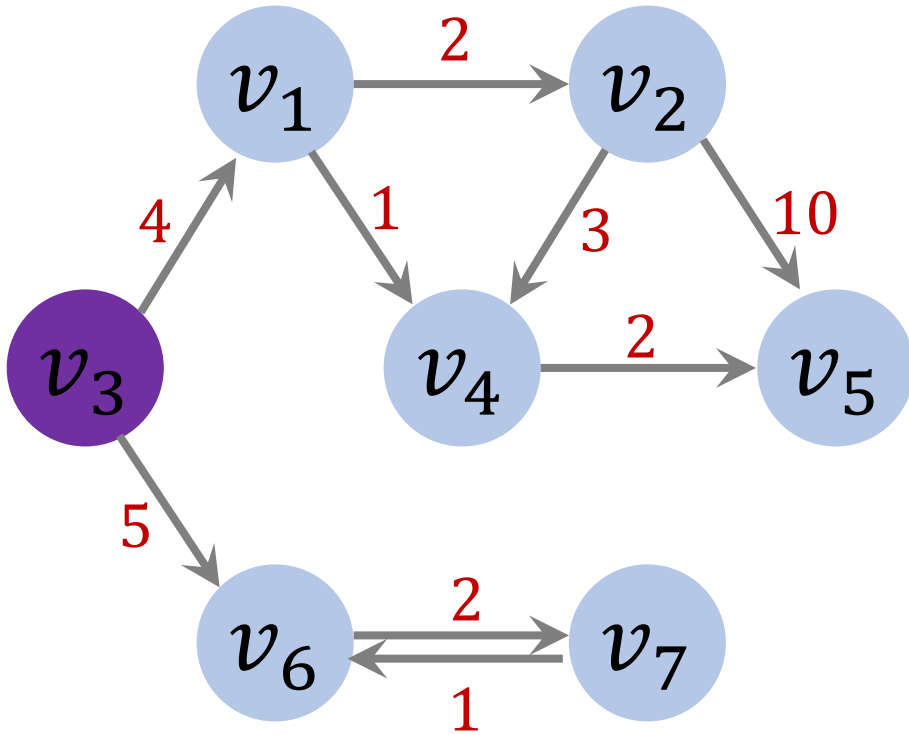
# Single-Source Shortest Path Problem



## Example

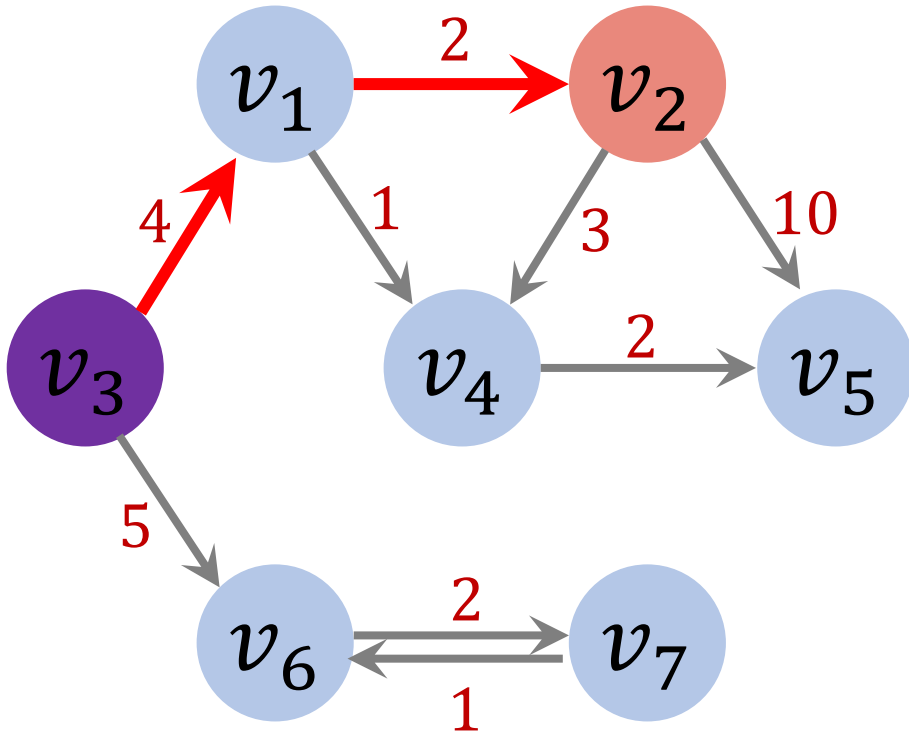
- Source:  $s = v_3$ .
- Find out the shortest path from  $v_3$  to all the other vertices.

# Single-Source Shortest Path Problem



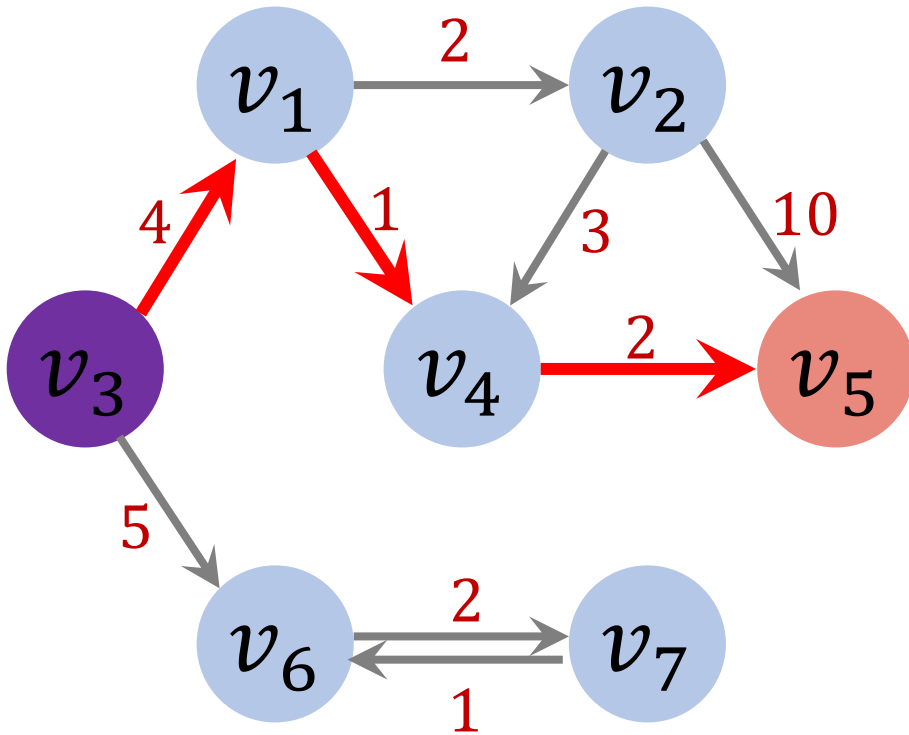
vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

# Lengths of the shortest paths



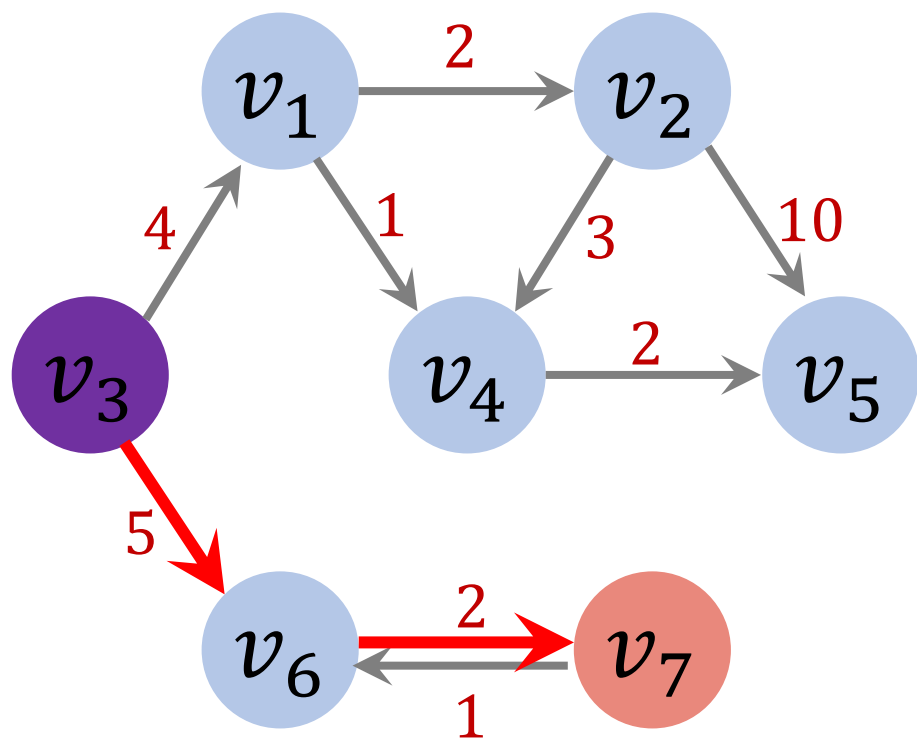
vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

# Lengths of the shortest paths



vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

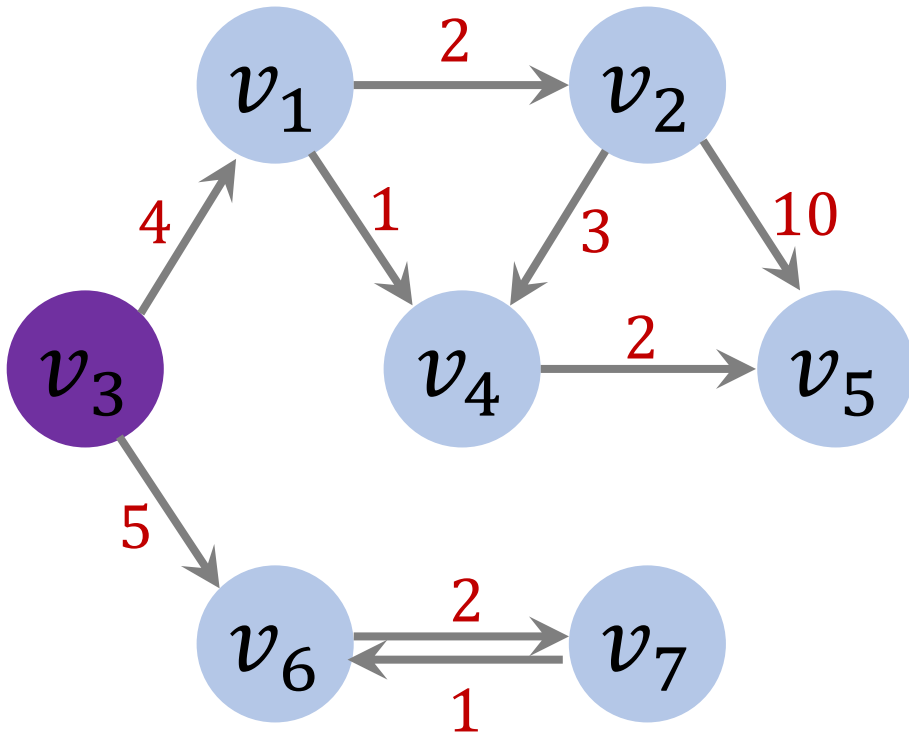
# Lengths of the shortest paths



vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

# Lengths of the shortest paths

Lengths of the shortest paths.

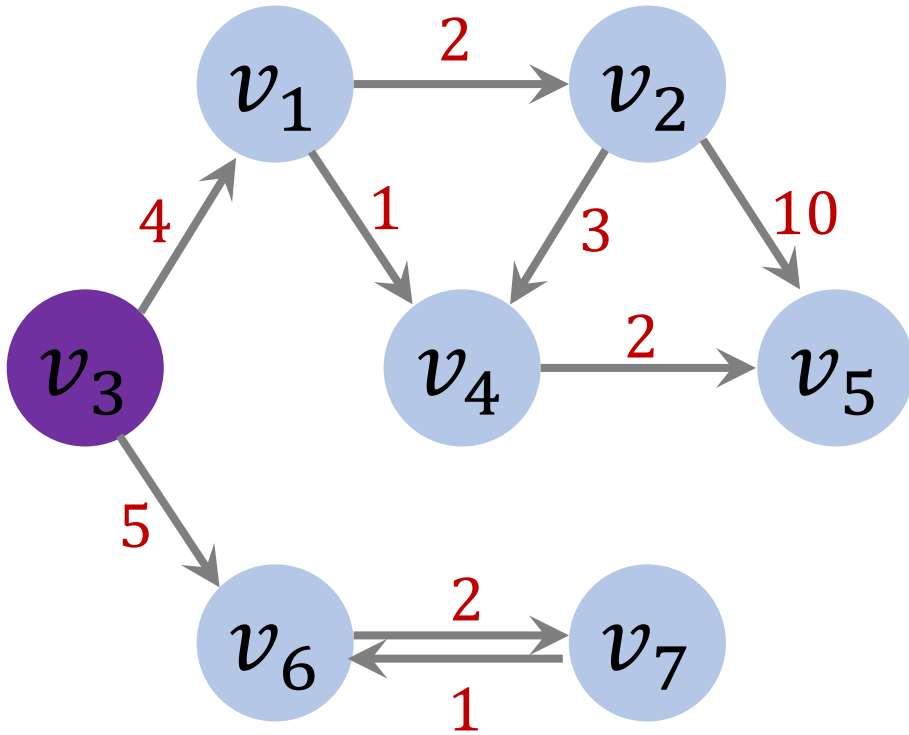


vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$



# Recover the shortest paths

The previous vertex along the path.



vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

# Single-Source Shortest Path Problem

What is the shortest path from  $v_3$  to  $v_2$ ?

- The 2<sup>nd</sup> row in the table:  $v_1 \rightarrow v_2$ .

vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

# Single-Source Shortest Path Problem

What is the shortest path from  $v_3$  to  $v_2$ ?

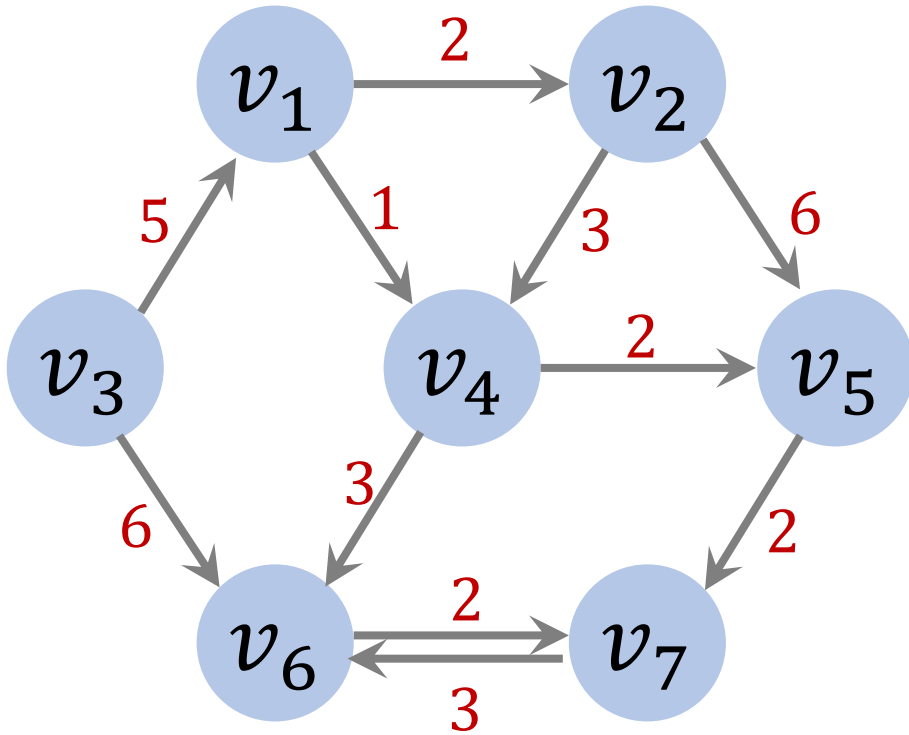
- The 2<sup>nd</sup> row in the table:  $v_1 \rightarrow v_2$ .
- The 1<sup>st</sup> row in the table:  $v_3 \rightarrow v_1$ .
- Thus, the shortest path is

$v_3 \rightarrow v_1 \rightarrow v_2$ .

vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

# Questions

# Question 1



- **Q:** What is the shortest paths?
  - From  $v_1$  to  $v_1$ .
  - From  $v_1$  to  $v_3$ .
  - From  $v_1$  to  $v_7$ .
- **Q:** What are the lengths of these shortest paths?

## Question 2

- The graph is directed and weighted.
- Let  $v_3$  be the source vertex.
- The table reflects the single-source shortest paths.
- **Q:** What is the shortest path from  $v_3$  to  $v_7$ ?
- **Q:** What is the length of this shortest path?

vertex	dist	path
$v_1$	4	$v_3$
$v_2$	6	$v_1$
$v_3$	0	0
$v_4$	5	$v_1$
$v_5$	7	$v_4$
$v_6$	5	$v_3$
$v_7$	7	$v_6$

**Thank You!**