Longest Common Substring

Shusen Wang

Longest Common Substring

- Example inputs:
  - `x = “IntroductionToAlgorithms”.`
  - `y = “AdvancedAlgorithmDesignImplementation”.`

Longest Common Substring

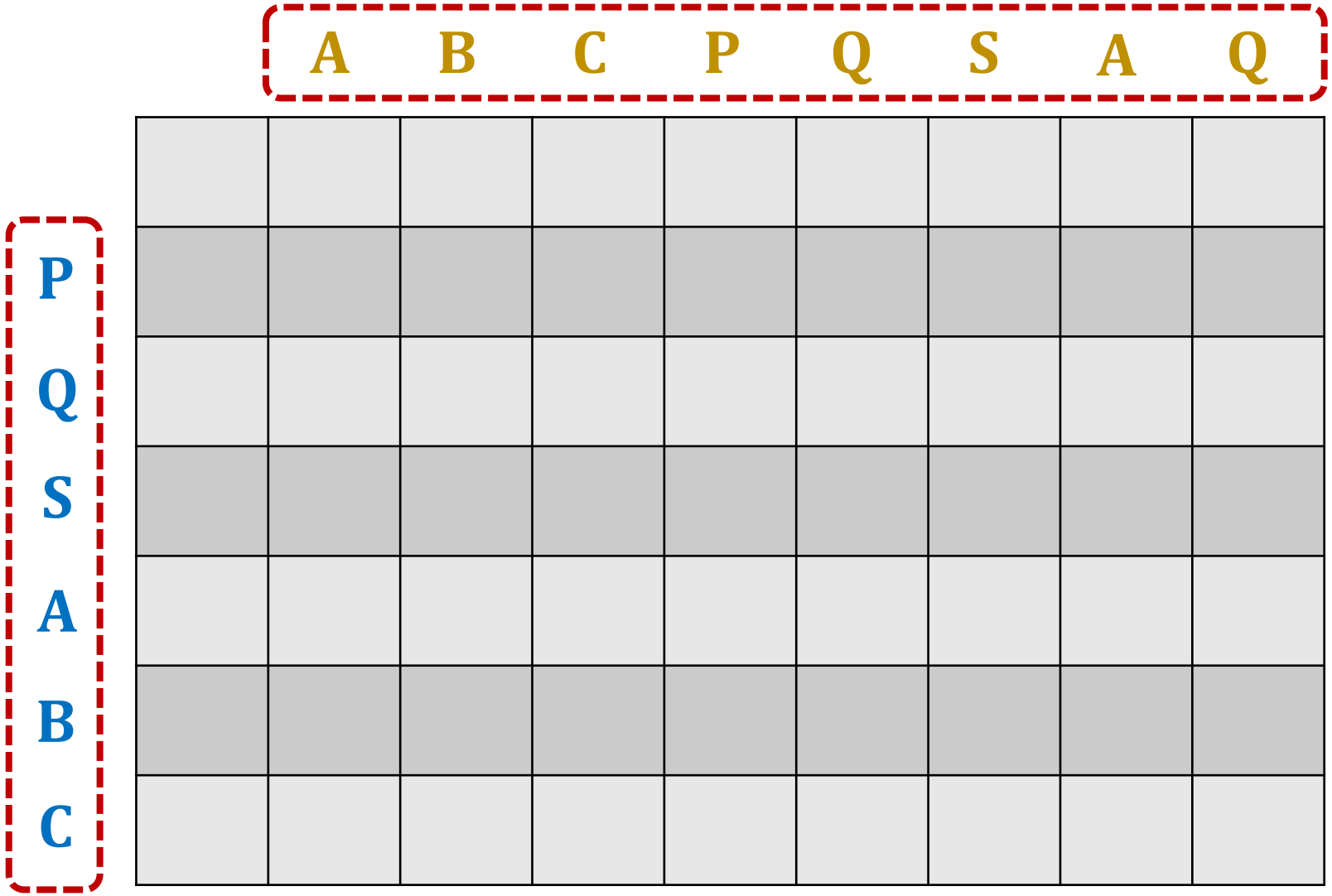
- Example inputs:
  - x = “Introduc**tion**ToAlgorithms”.
  - y = “AdvancedAlgorithmDesignImplementa**tion**”.

Longest Common Substring

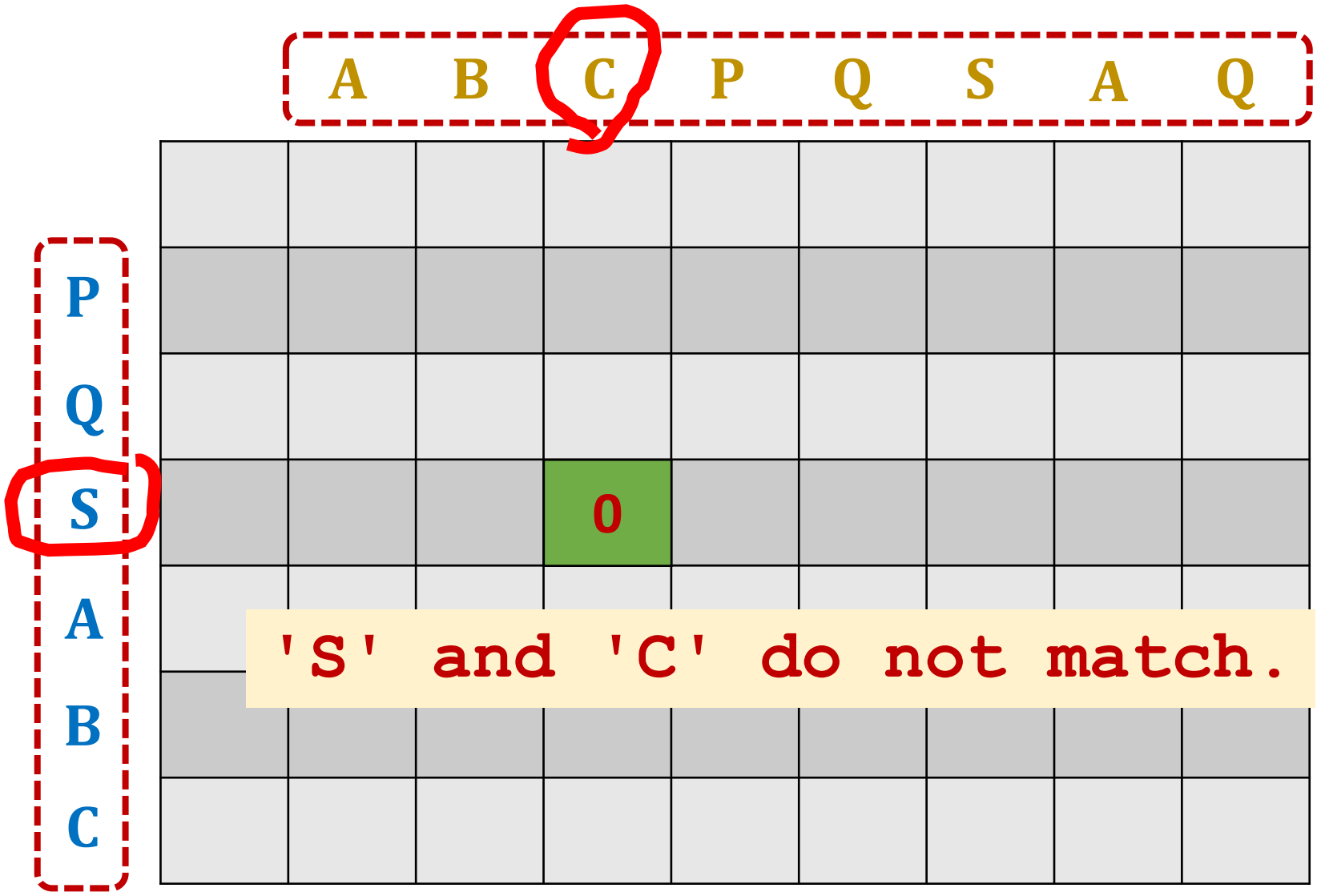
- Example inputs:
  - x = "IntroductionTo**Algorithm**s".
  - y = "Advanced**Algorithm**DesignImplementation".
- Example outputs:
  - Longest common substring: "**Algorithm**".
  - Length of the substring: 9.

Optimal Substructure

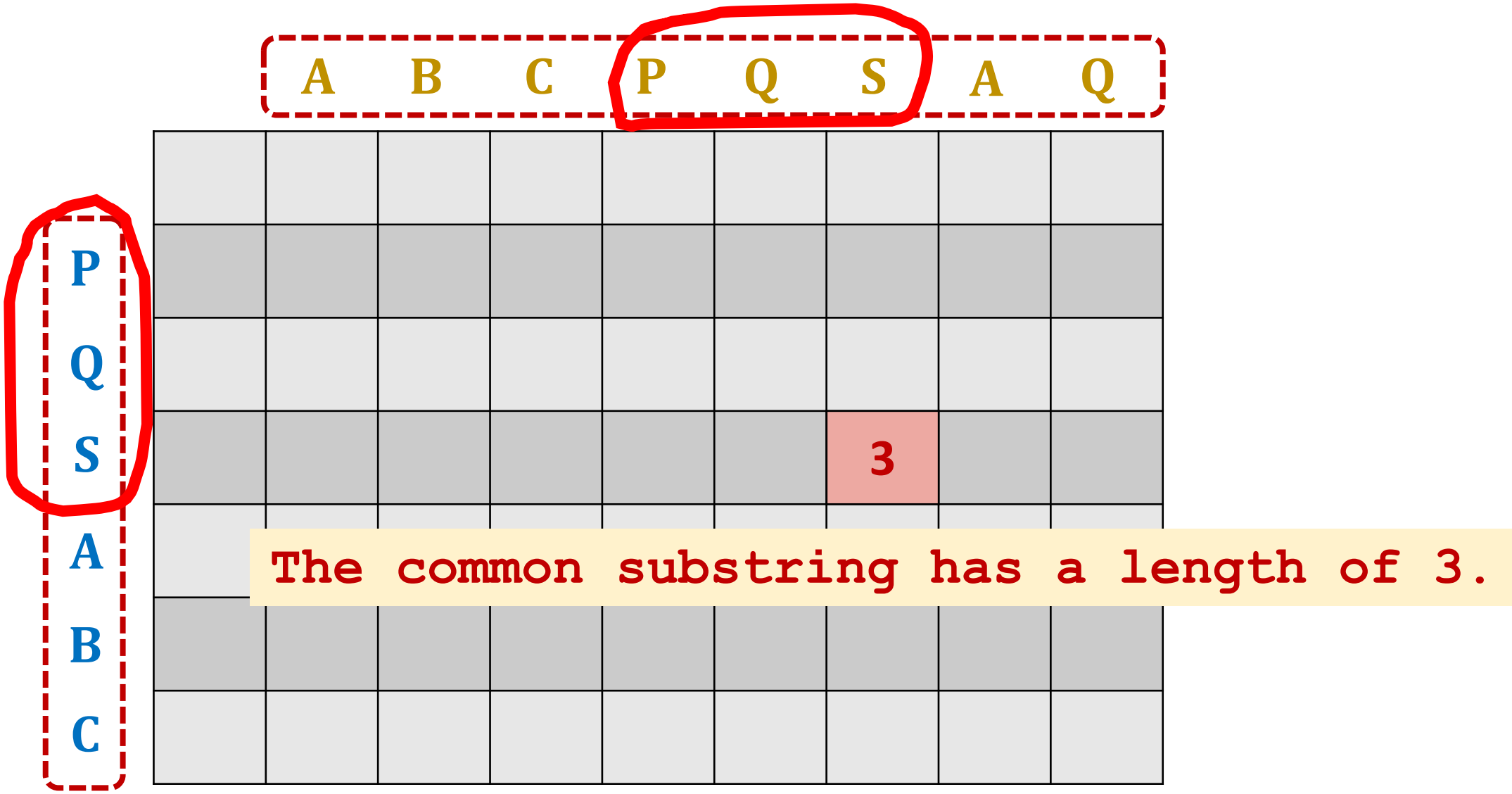
Basic Idea



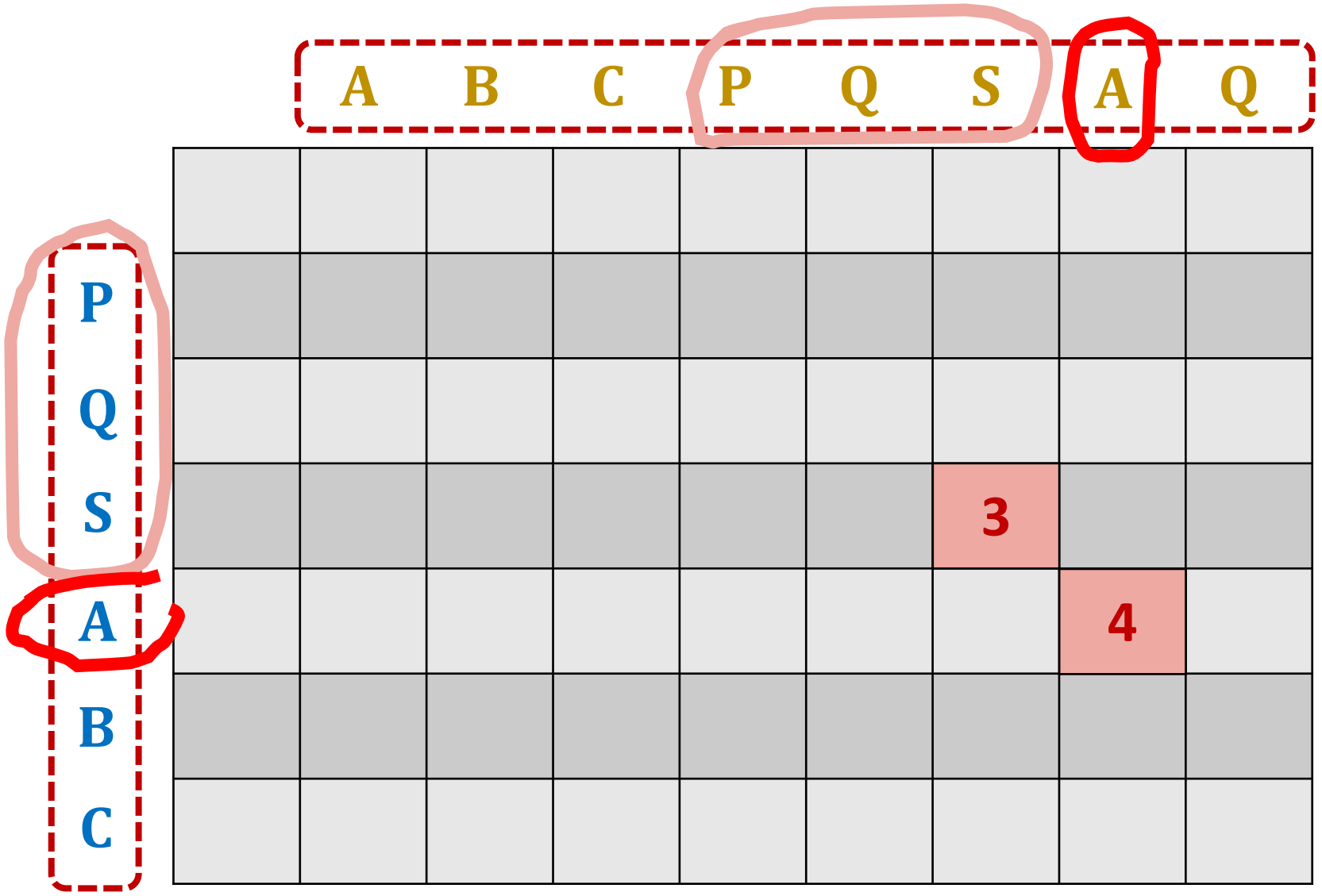
Basic Idea



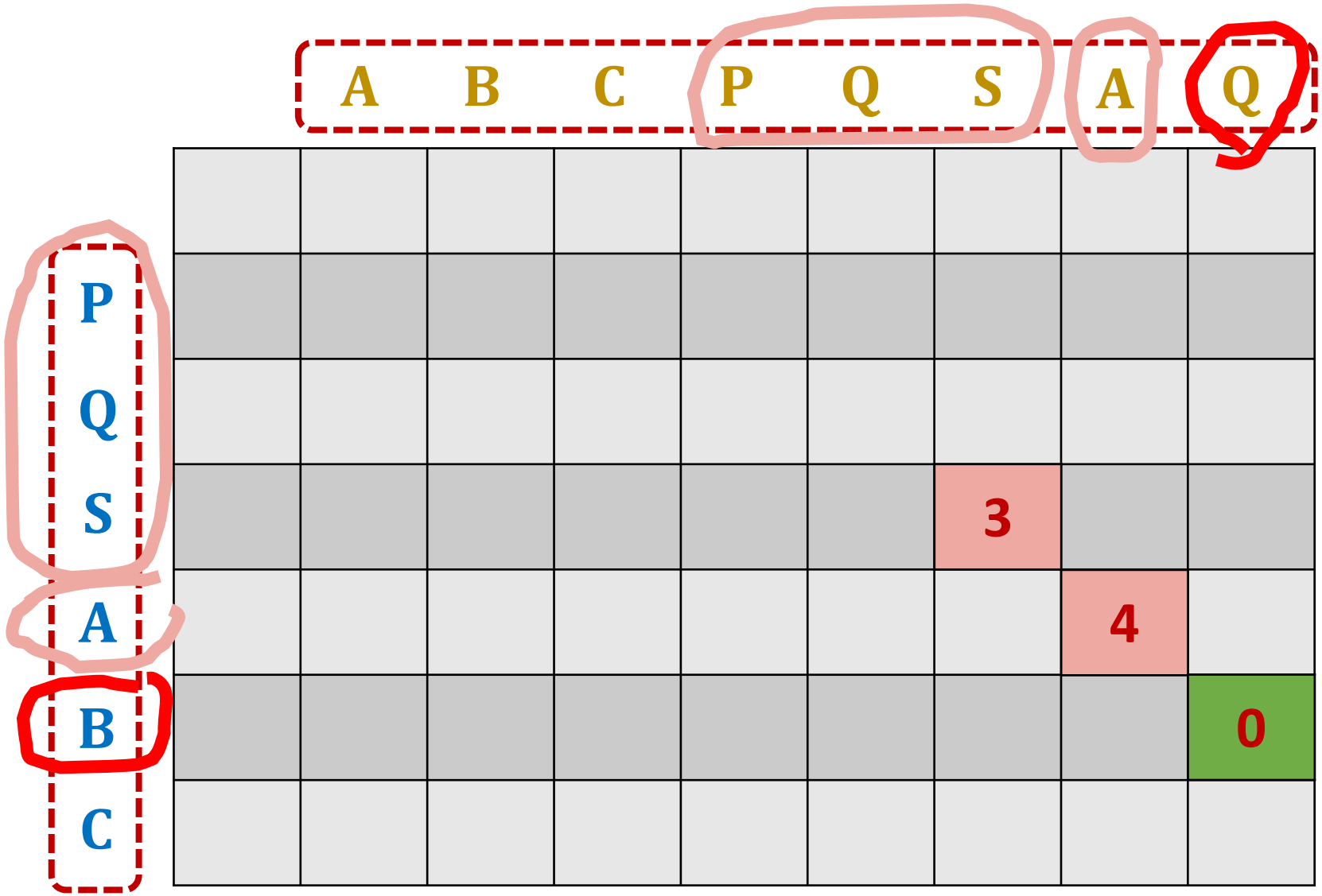
Basic Idea



Basic Idea



Basic Idea



Optimal Substructure

- Inputs: strings `x` and `y`.
- Build a table `L` for storing the lengths of common substrings.
- For indices `i` and `j`: compare `x[i]` and `y[j]`.

Optimal Substructure

- Inputs: strings `x` and `y`.
- Build a table `L` for storing the lengths of common substrings.
- For indices `i` and `j`: compare `x[i]` and `y[j]`.
- If `x[i]` and `y[j]` are the same, then

  `L[i][j] = 1 + L[i-1][j-1]`.

- If `x[i]` and `y[j]` are different, then

  `L[i][j] = 0`.

Optimal Substructure

- Inputs: strings `x` and `y`.
- Build a table `L` for storing the lengths of common substrings.
- For indices `i` and `j`: compare `x[i]` and `y[j]`.
- If `x[i]` and `y[j]` are the same, then

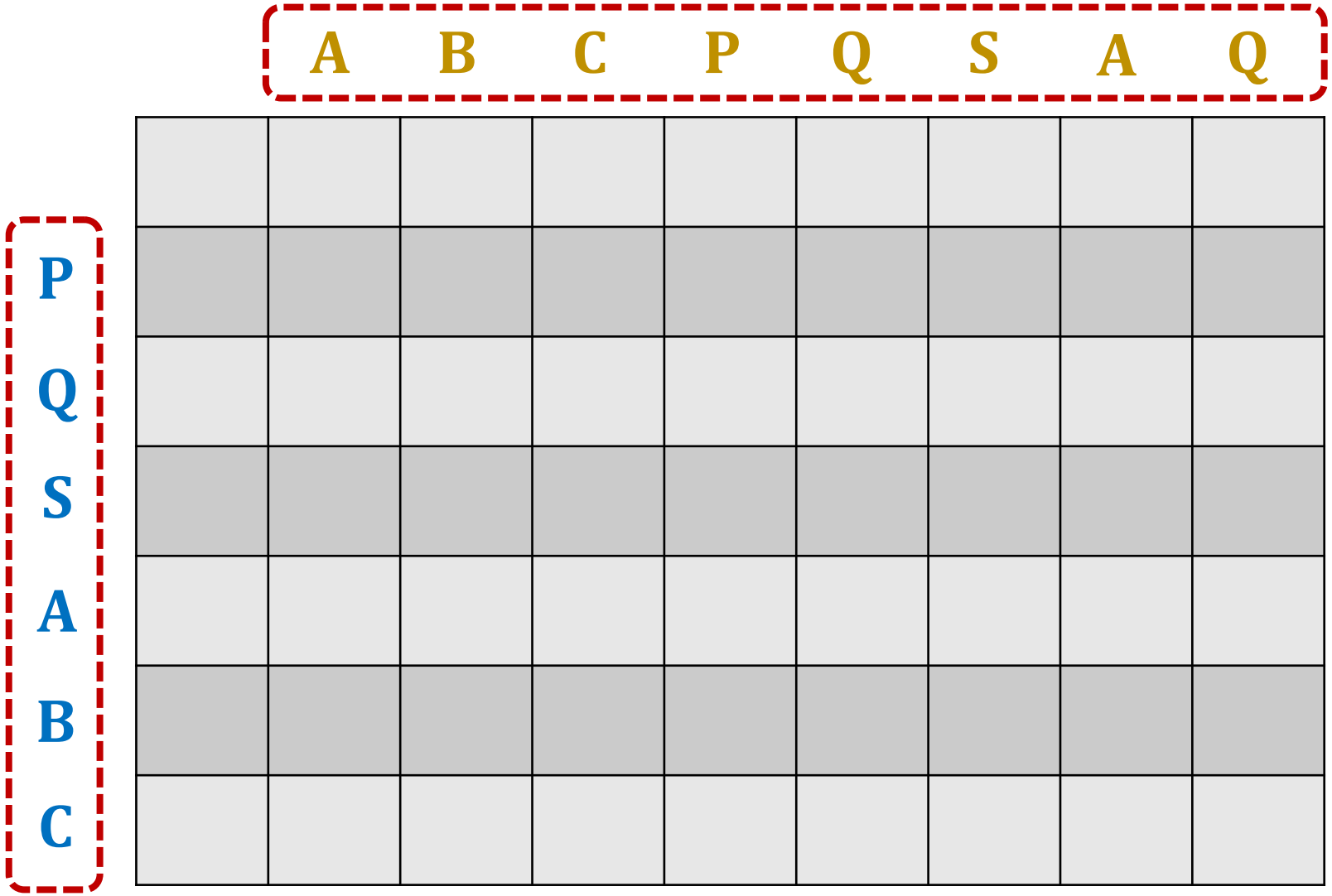
```
L[i][j] = 1 + L[i-1][j-1]
```

- If `x[i]` and `y[j]` are different, then

```
L[i][j] = 0.
```

Dynamic Programming

Inputs: two strings



Base Case: L[0][:] = 0

		A	B	C	P	Q	S	A	Q
NULL	0	0	0	0	0	0	0	0	0
P									
Q									
S									
A									
B									
C									

Base Case: L[:][0] = 0

	NULL	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0								
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[1]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0							
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[2]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0						
Q	0								
S	0								
A	0								
B	0								
C	0								

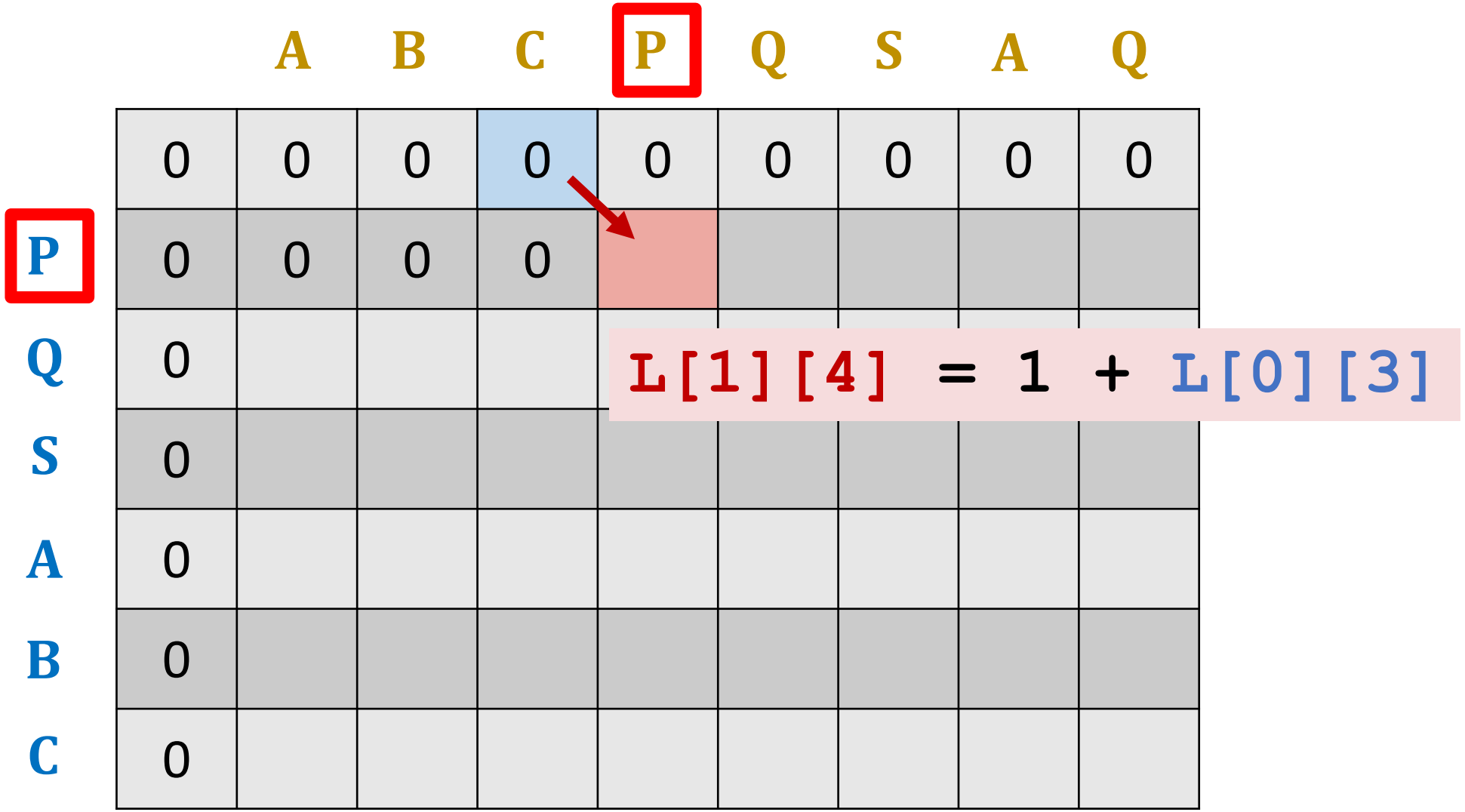
$x[1] \neq y[3]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0					
Q	0								
S	0								
A	0								
B	0								
C	0								

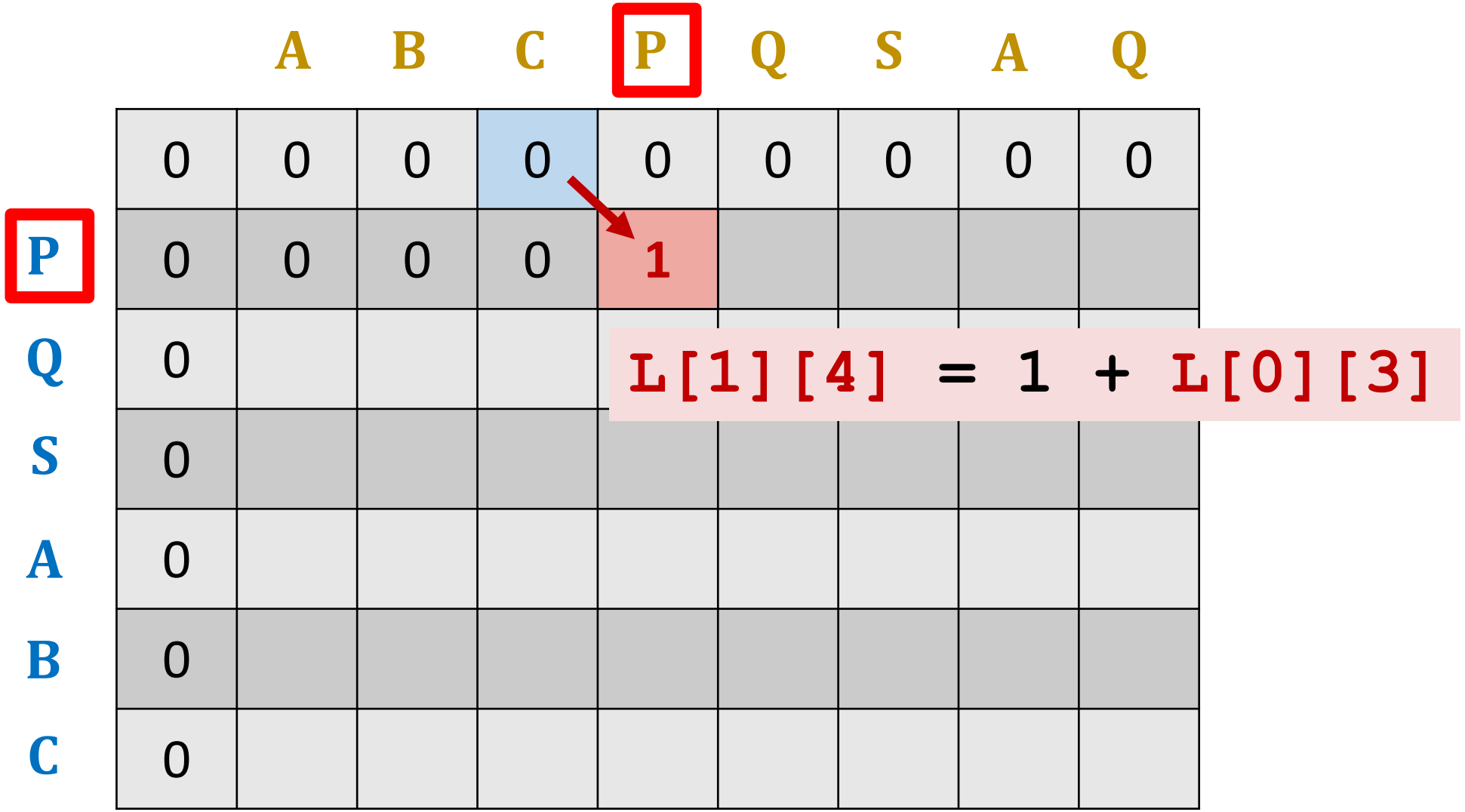
`x[1] = y[4]`

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0					
Q	0								
S	0								
A	0								
B	0								
C	0								

$$x[1] = y[4]$$



x[1] = y[4]



$x[1] \neq y[5]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0			
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0		
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[7]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[8]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[2] \neq y[1]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0							
S	0								
A	0								
B	0								
C	0								

x[2] ≠ y[2]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0						
S	0								
A	0								
B	0								
C	0								

$x[2] \neq y[3]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0					
S	0								
A	0								
B	0								
C	0								

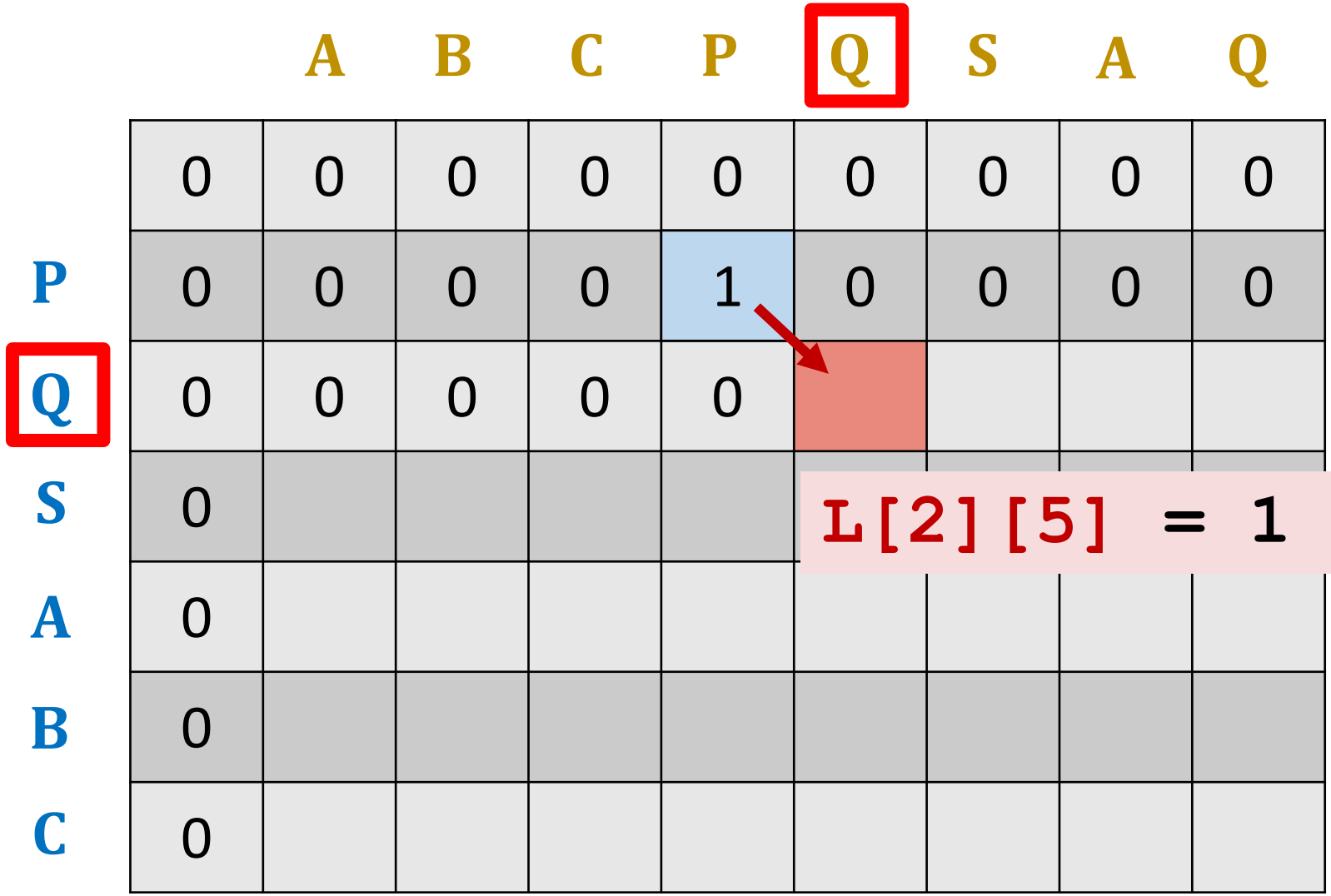
$x[2] \neq y[4]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0				
S	0								
A	0								
B	0								
C	0								

x[2] = y[5]

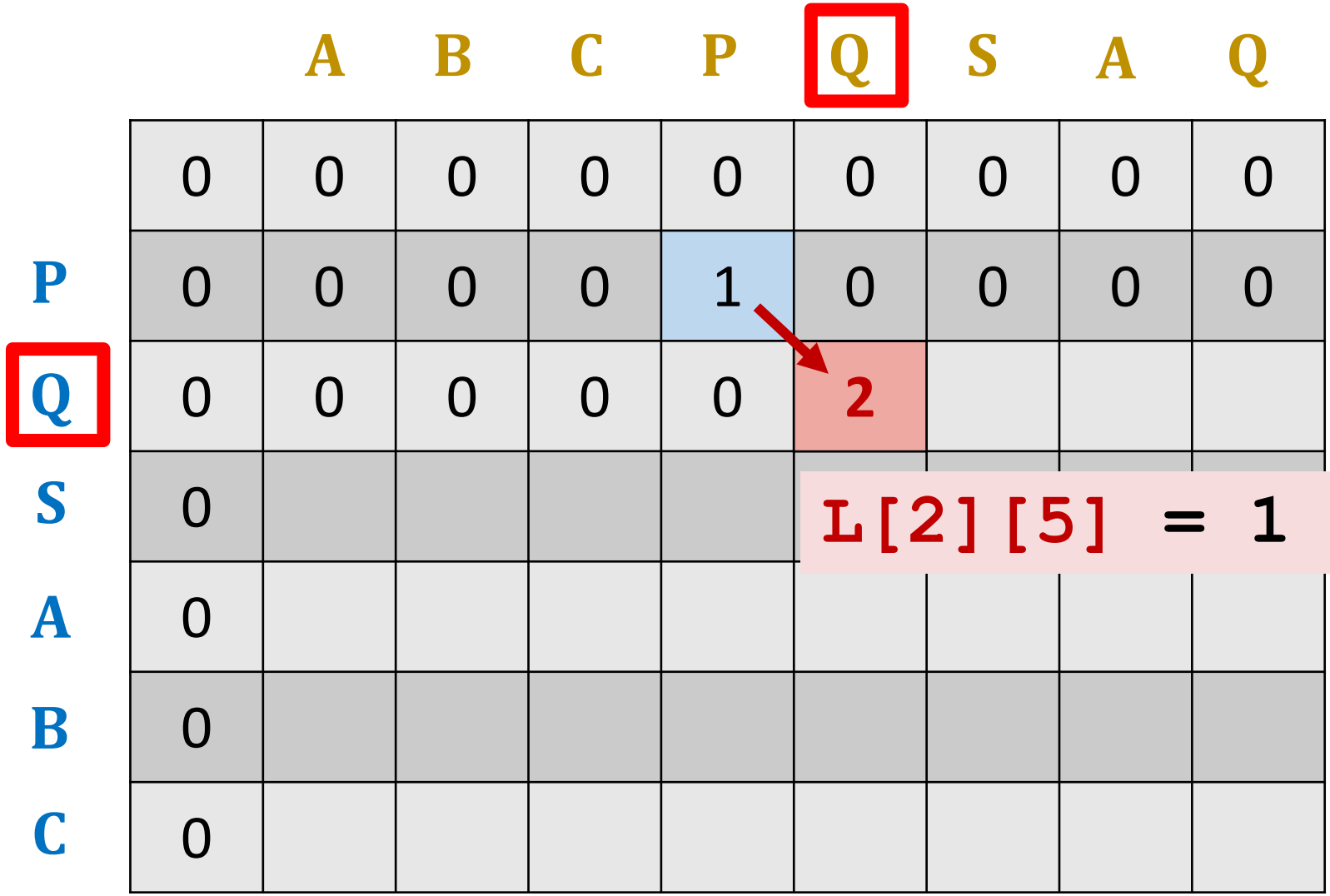
		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0				
S	0								
A	0								
B	0								
C	0								

x[2] = y[5]



L[2][5] = 1 + L[1][4]

x[2] = y[5]



x[2] ≠ y[6]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0		
S	0								
A	0								
B	0								
C	0								

$x[2] \neq y[7]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	
S	0								
A	0								
B	0								
C	0								

x[2] = y[8]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	
S	0								
A	0								
B	0								
C	0								

$$x[2] = y[8]$$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	
S	0								
A	0								
B	0								
C	0								

$$L[2][8] = 1 + L[1][7]$$

x[2] = y[8]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0								
A	0								
B	0								
C	0								

L[2][8] = 1 + L[1][7]

x[3] ≠ y[1]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0							
A	0								
B	0								
C	0								

$x[3] \neq y[2]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0						
A	0								
B	0								
C	0								

$x[3] \neq y[3]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0					
A	0								
B	0								
C	0								

x[3] ≠ y[4]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0				
A	0								
B	0								
C	0								

$x[3] \neq y[5]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0			
A	0								
B	0								
C	0								

x[3] = y[6]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0			
A	0								
B	0								
C	0								

x[3] = y[6]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0			
A	0								
B	0								
C	0								

L[3][6] = 1 + L[2][5]

x[3] = y[6]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3		
A	0								
B	0								
C	0								

L[3][6] = 1 + L[2][5]

x[3] ≠ y[7]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	
A	0								
B	0								
C	0								

$x[3] \neq y[7]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0								
B	0								
C	0								

`x[4] = y[1]`

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0								
B	0								
C	0								

x[4] = y[1]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0								
B	0								
C	0								

L[4][1] = 1 + L[3][0]

x[4] = y[1]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1							
B	0								
C	0								

L[4][1] = 1 + L[3][0]

x[4] ≠ y[2]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0						
B	0								
C	0								

$x[4] \neq y[3]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0					
B	0								
C	0								

$x[4] \neq y[4]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0				
B	0								
C	0								

$x[4] \neq y[5]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0			
B	0								
C	0								

$x[4] \neq y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0		
B	0								
C	0								

x[4] = y[7]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0		
B	0								
C	0								

x[4] = y[7]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0		
B	0								
C	0								

L[4][7] = 1 + L[3][6]

x[4] = y[7]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	
B	0								
C	0								

L[4][7] = 1 + L[3][6]

x[4] ≠ y[8]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0								
C	0								

x[5] ≠ y[1]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0							
C	0								

x[5] = y[2]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0							
C	0								

x[5] = y[2]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0							
C	0	L[5][2] = 1 + L[4][1]							

x[5] = y[2]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2						
C	0	L[5][2] = 1 + L[4][1]							

$x[5] \neq y[3]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0					
C	0								

x[5] ≠ y[4]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0				
C	0								

$x[5] \neq y[5]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0			
C	0								

x[5] ≠ y[6]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0		
C	0								

x[5] ≠ y[7]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	
C	0								

$x[5] \neq y[8]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0								

$x[6] \neq y[1]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0							

x[6] ≠ y[2]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0						

x[6] = y[3]

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	0	3	0
A	0	1	0	0	0	0	0	4
B	0	0	2	0	0	0	0	0
C	0	0	0					

$$x[6] = y[3]$$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0						

$$L[6][3] = 1 + L[5][2]$$

x[6] = y[3]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3					

L[6][3] = 1 + L[5][2]

x[6] ≠ y[4]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0				

$x[6] \neq y[5]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0			

x[6] ≠ y[6]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0		

x[6] ≠ y[7]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0	0	

x[6] ≠ y[8]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0	0	0

Argmax L[i][j]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	**4**	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0	0	0

Argmax L[i][j]

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	**4**	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0	0	0

Output: "PQSA"

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	**4**	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0	0	0

Time Complexity

- Let the lengths of the input strings be m and n.
- The size of the table is $(m+1)\times(n+1)$.
- It takes $O(1)$ time to fill each entry.
- Thus, the overall time complexity is $O(mn)$.

Thank You!