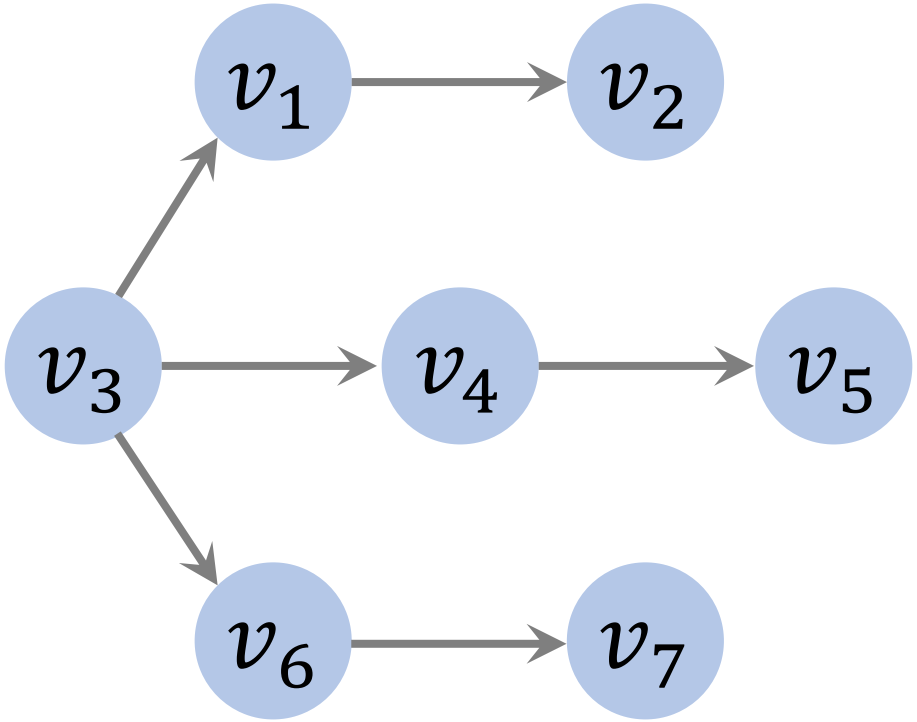
Minimum Spanning Trees

Shusen Wang

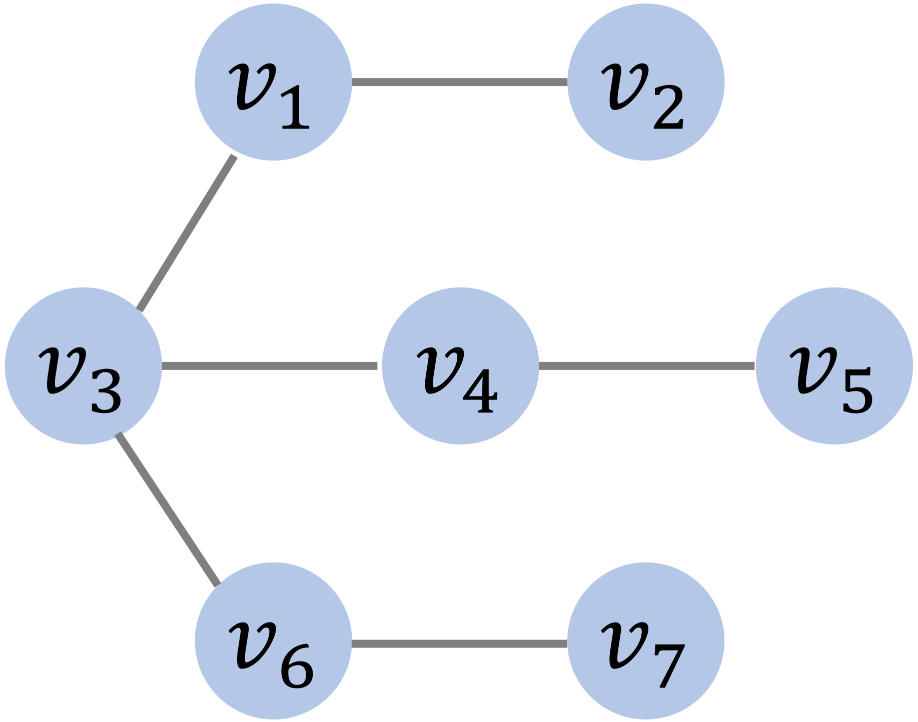
Trees vs Graphs

Trees are graphs

Trees are either directed or undirected graphs.



The tree is directed graph

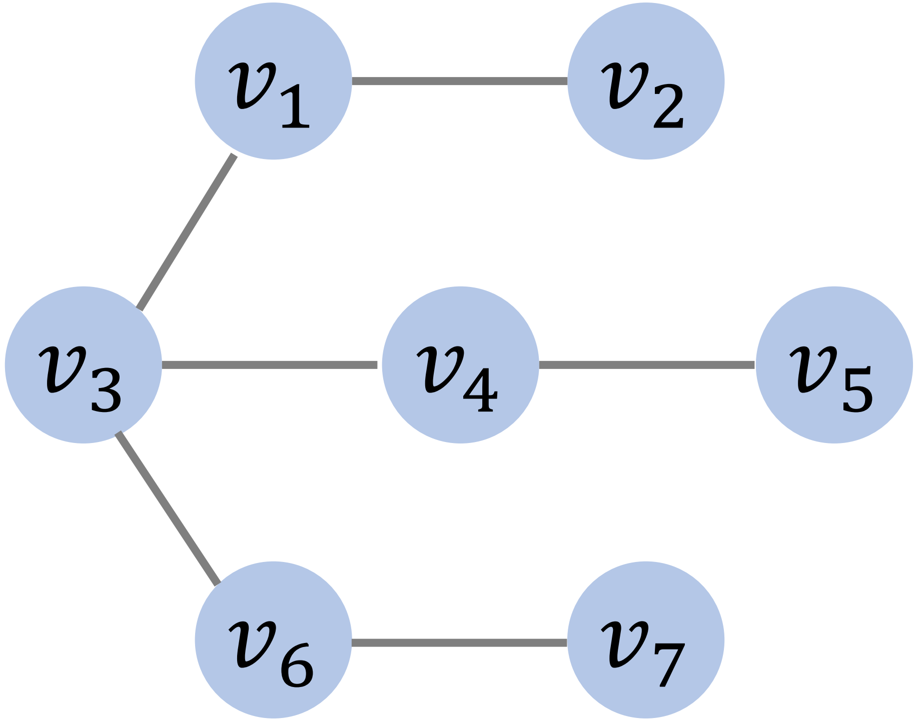


The tree is undirected graph

Trees are graphs

For now on, we study only undirected graphs.

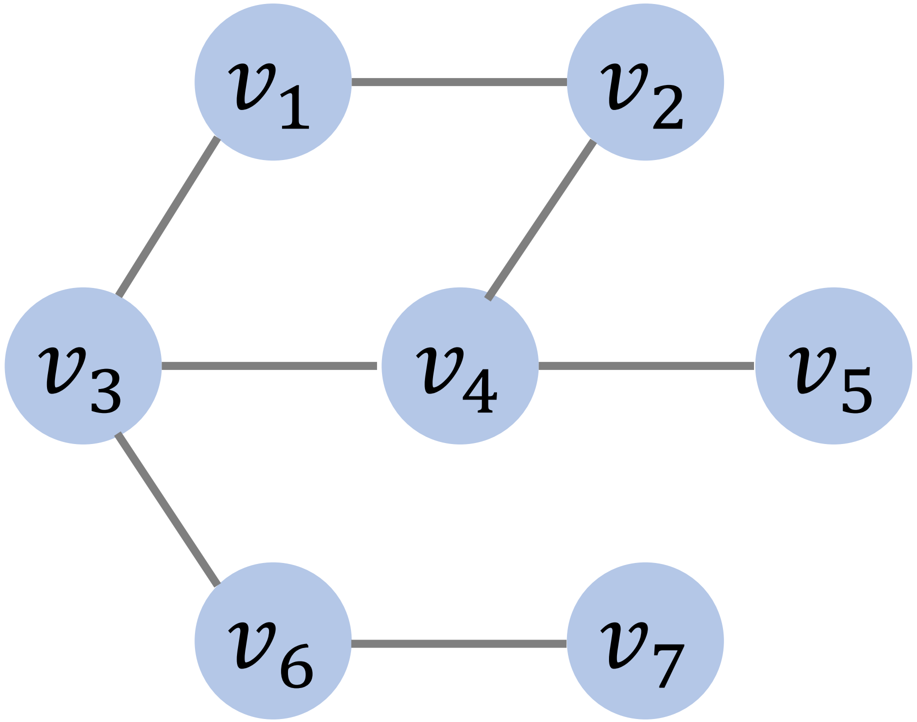
The tree is directed graph



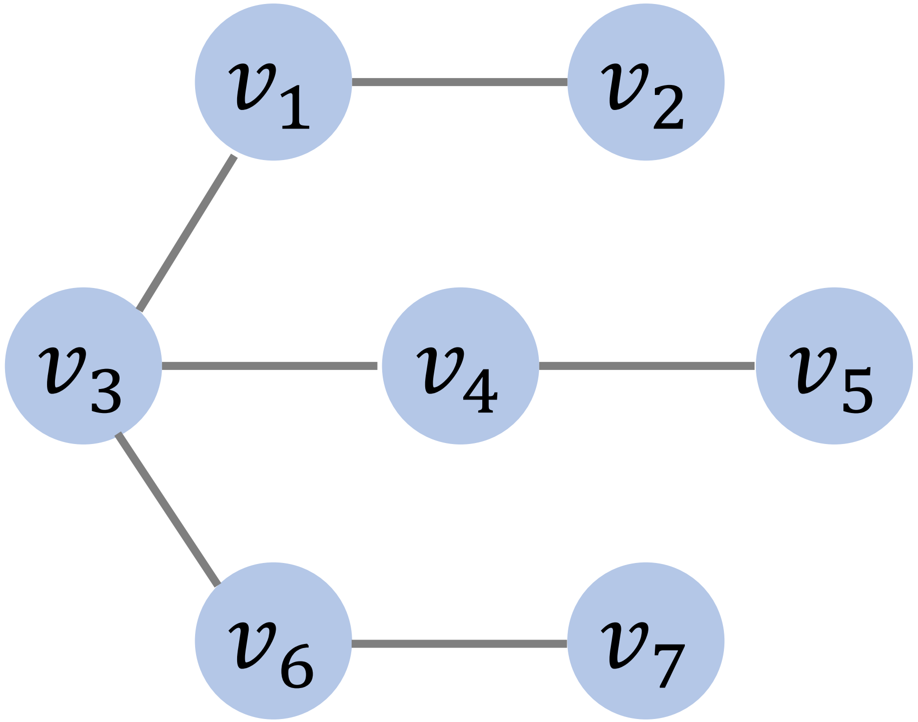
The tree is undirected graph

Trees do not have cycles

Assume all the edges are undirected.



Not a tree



Tree

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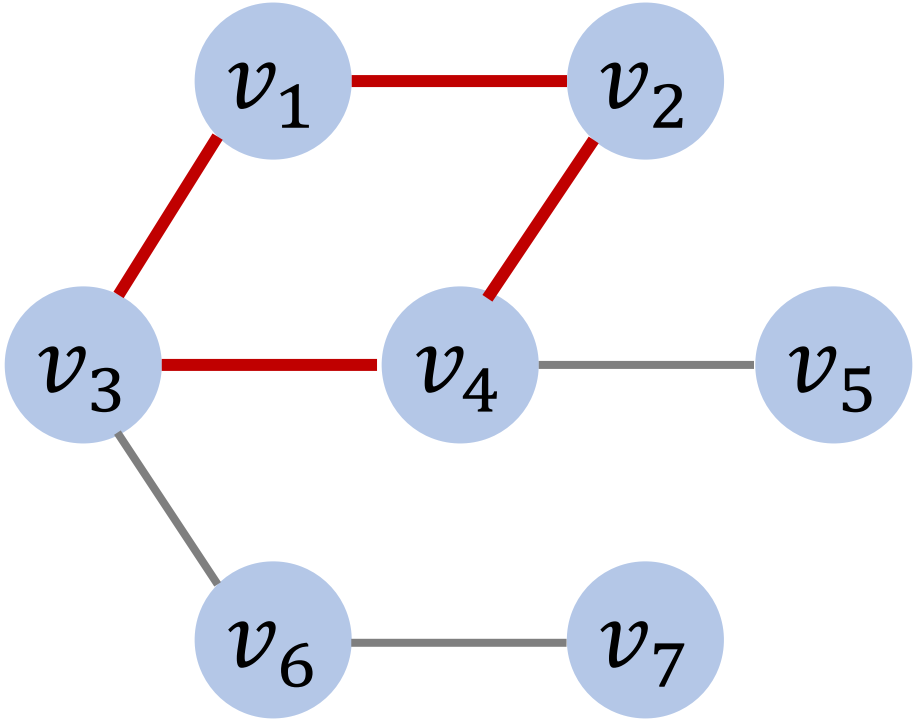
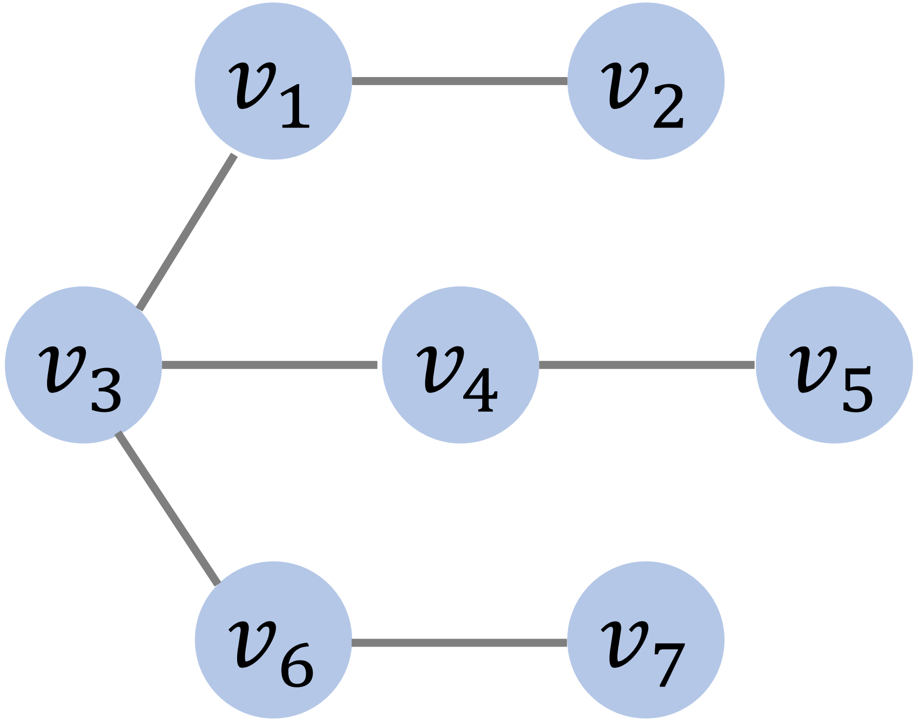


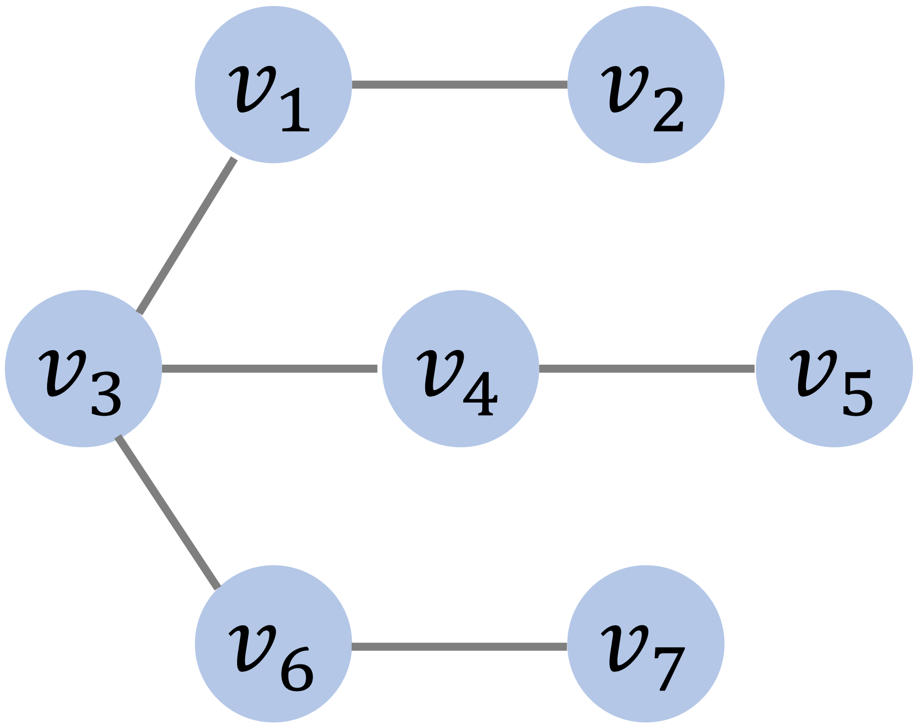
Not a tree



Tree

Properties of trees

- If the tree has n vertices, then it has $n-1$ edges.

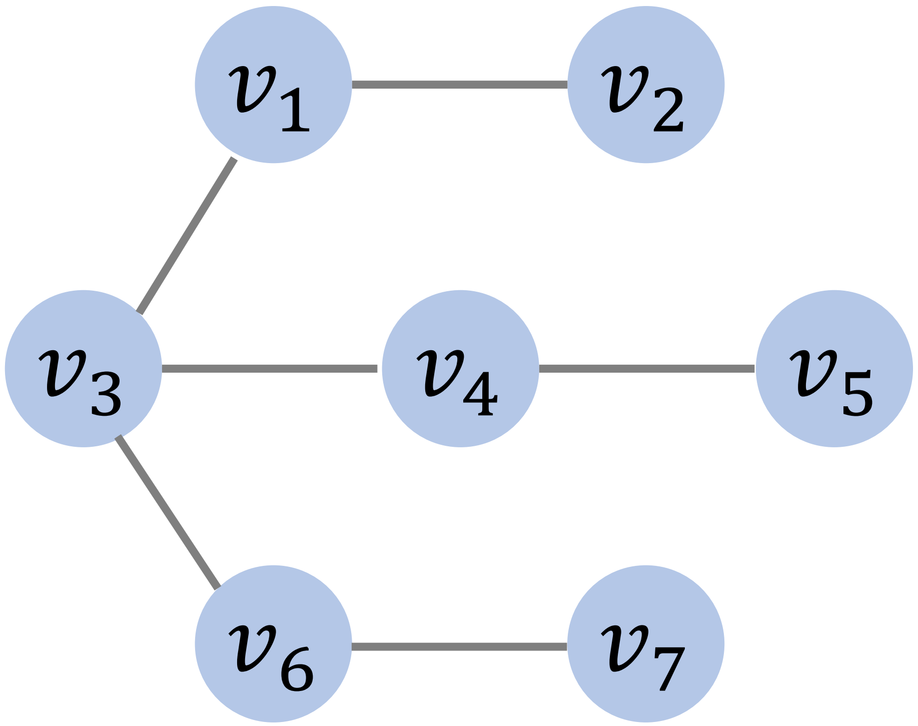


Tree

Properties of trees

- If the tree has n vertices, then it has $n-1$ edges.

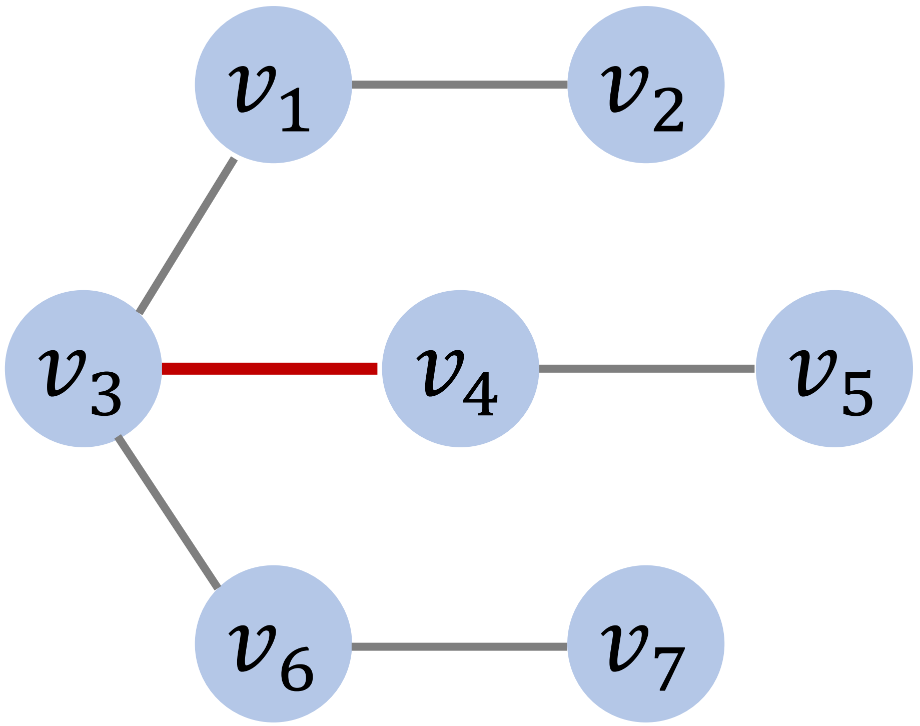
- Assume all the edges are undirected.
- Trees are connected graphs. (There is a path between any two vertices.)



Tree

Properties of trees

- Let n be the number of vertices.
- Less than $n-1$ edges
  ➔ Disconnected.
- More than $n-1$ edges
  ➔ There is a cycle.



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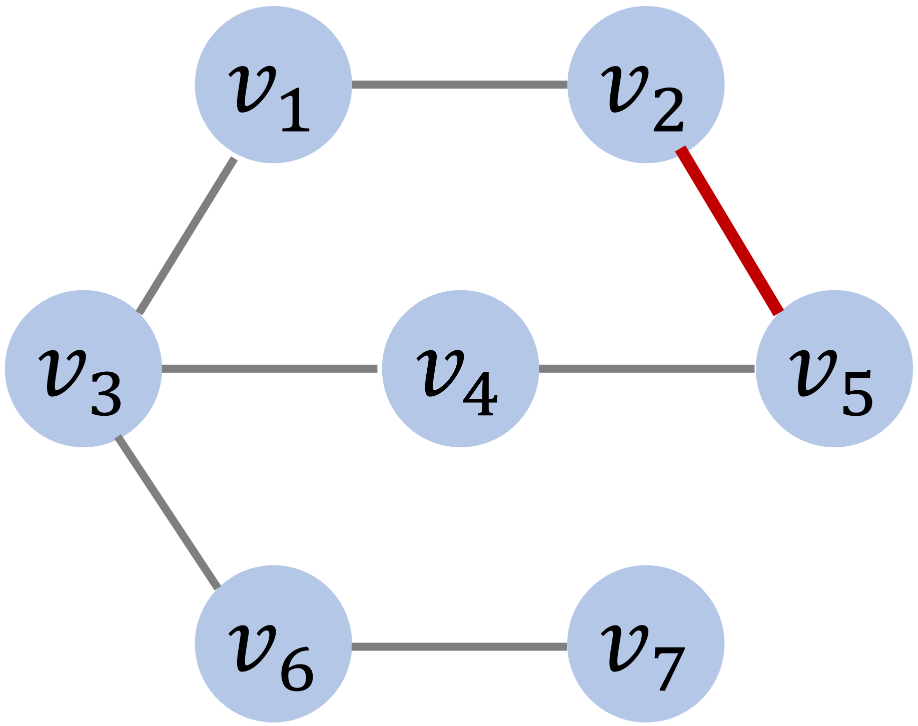
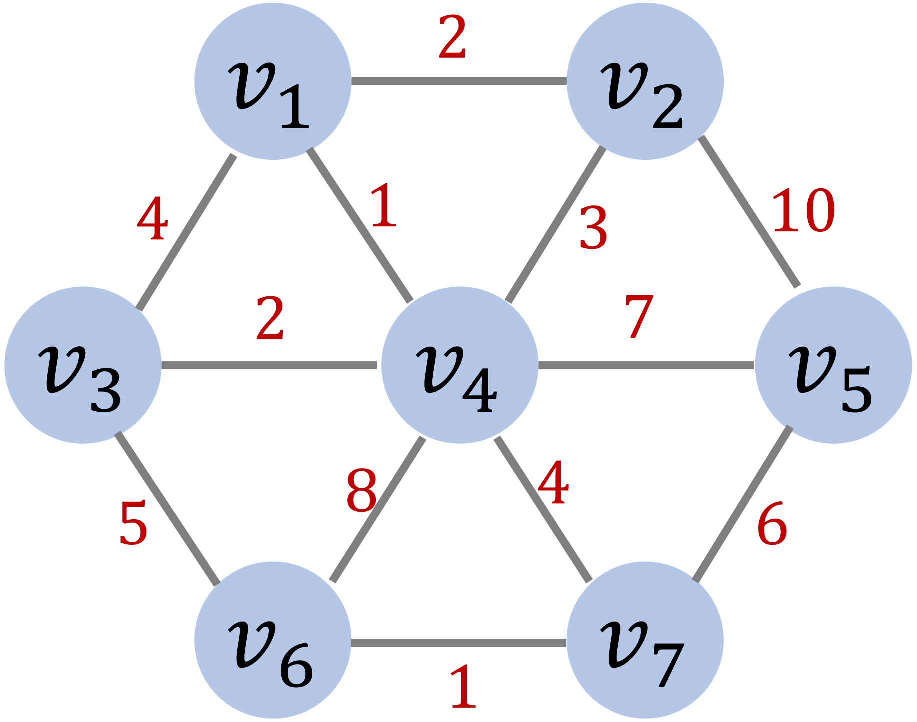


There is a cycle

Spanning Trees in Undirected Graphs

Spanning Trees

A spanning tree of a connected undirected graph $\mathcal{G}$ is a subgraph that is a tree which includes all of the vertices of $\mathcal{G}$.



Original Graph

Spanning Trees

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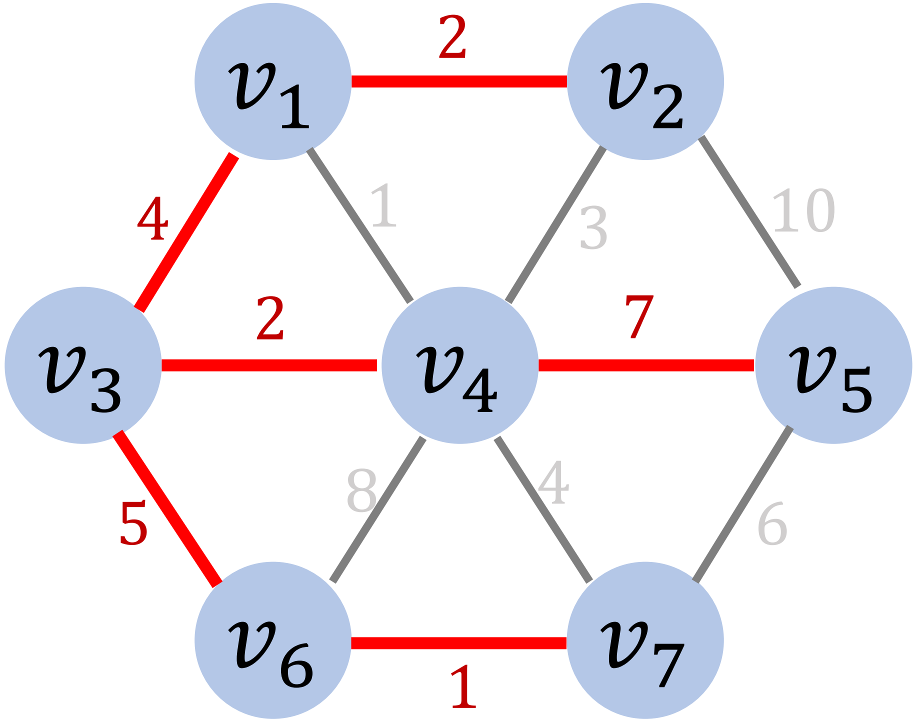

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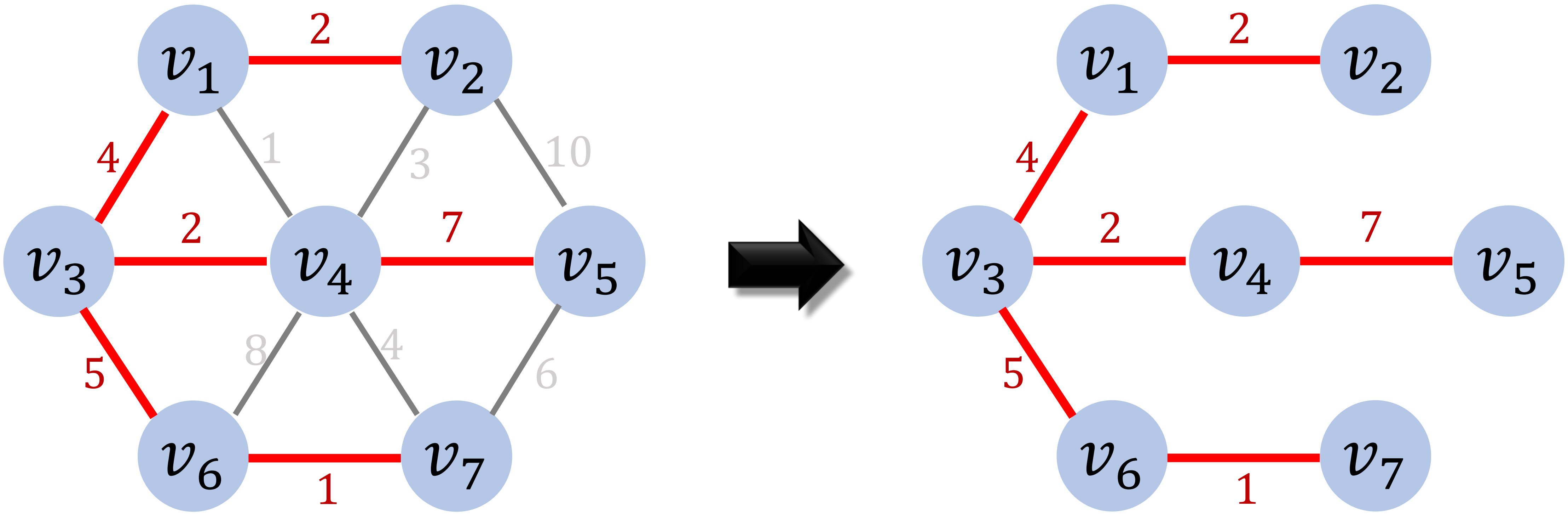
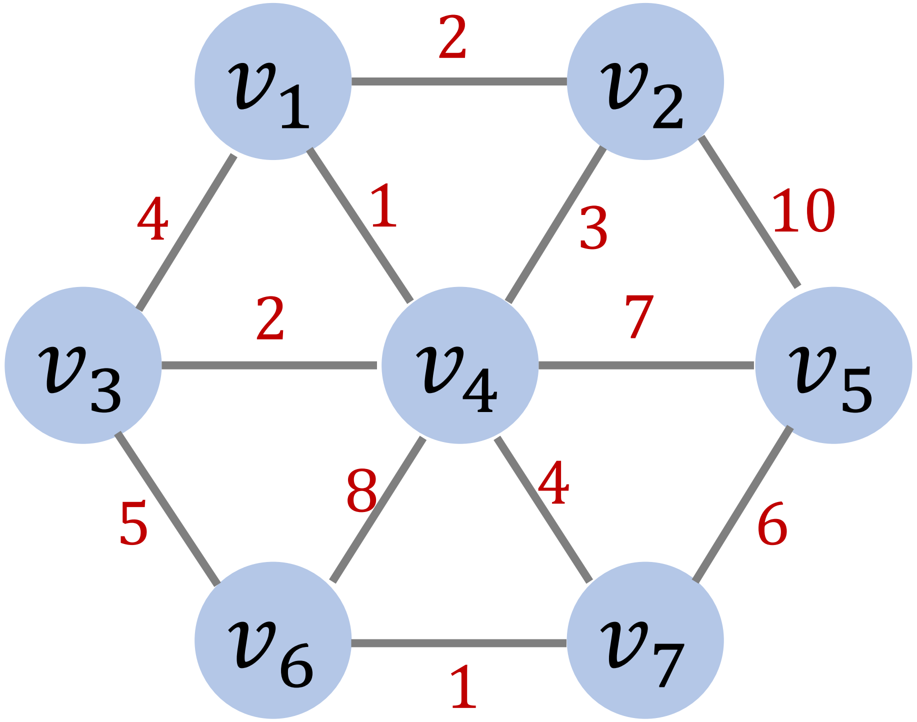


Original Graph

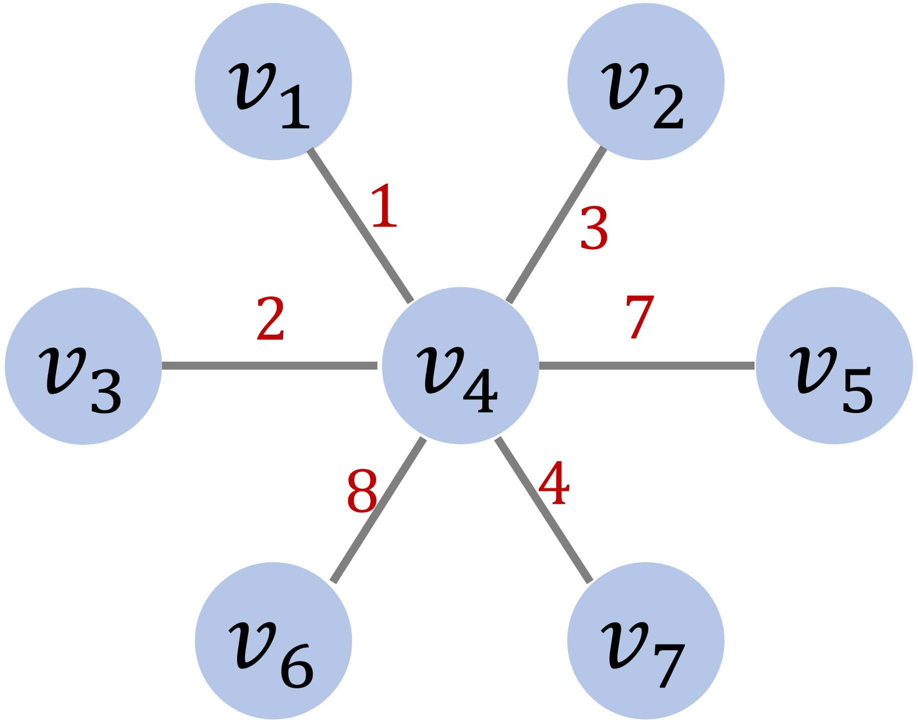
A Spanning Tree

Sum of weights is 21.

Spanning trees are not unique



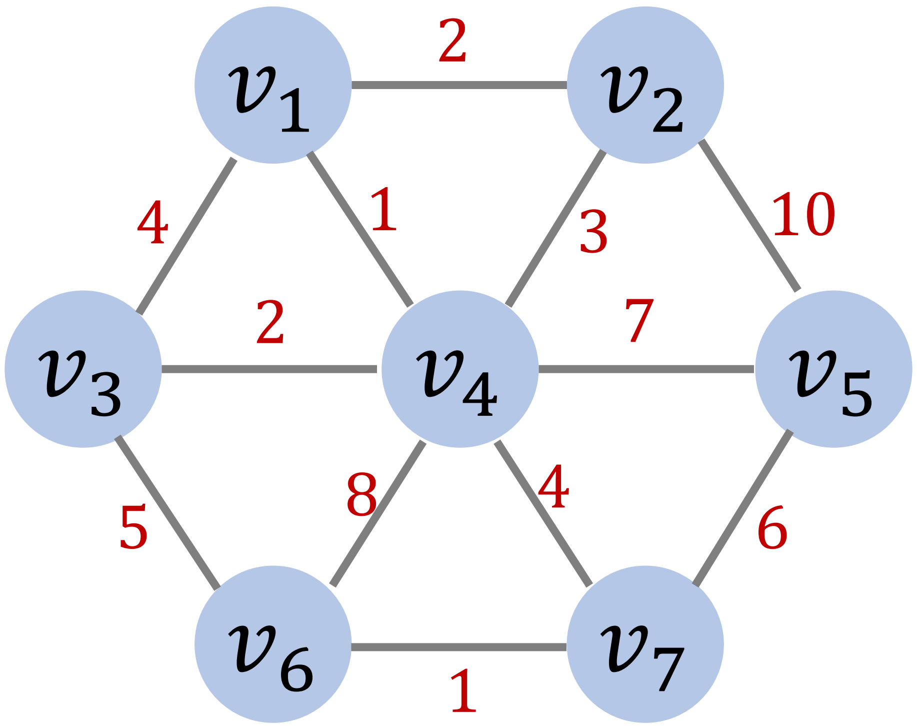
Original Graph



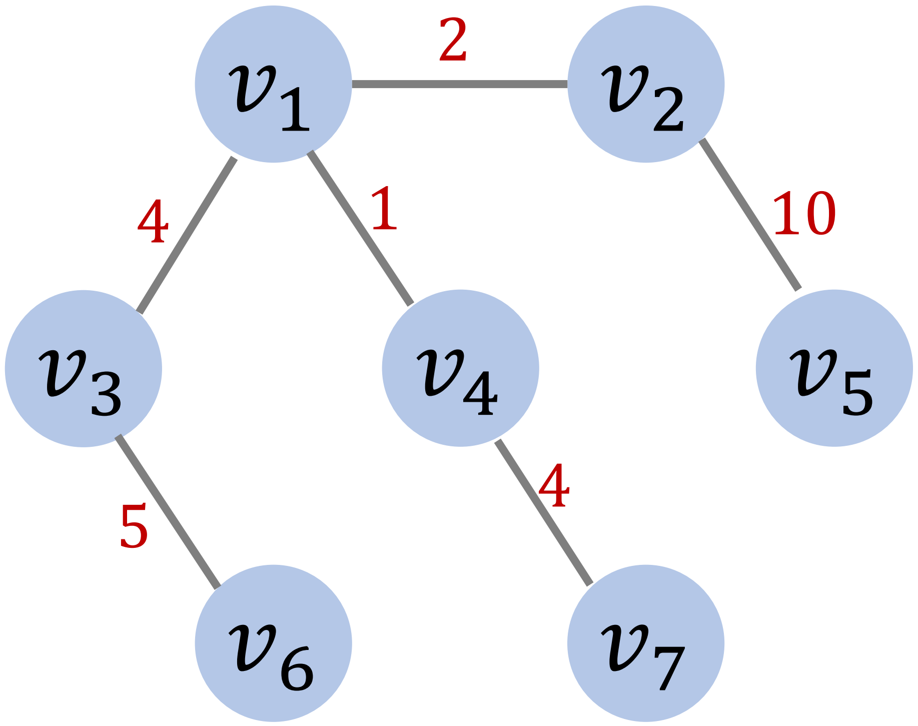
A Spanning Tree

Sum of weights is 25.

Spanning trees are not unique



Original Graph

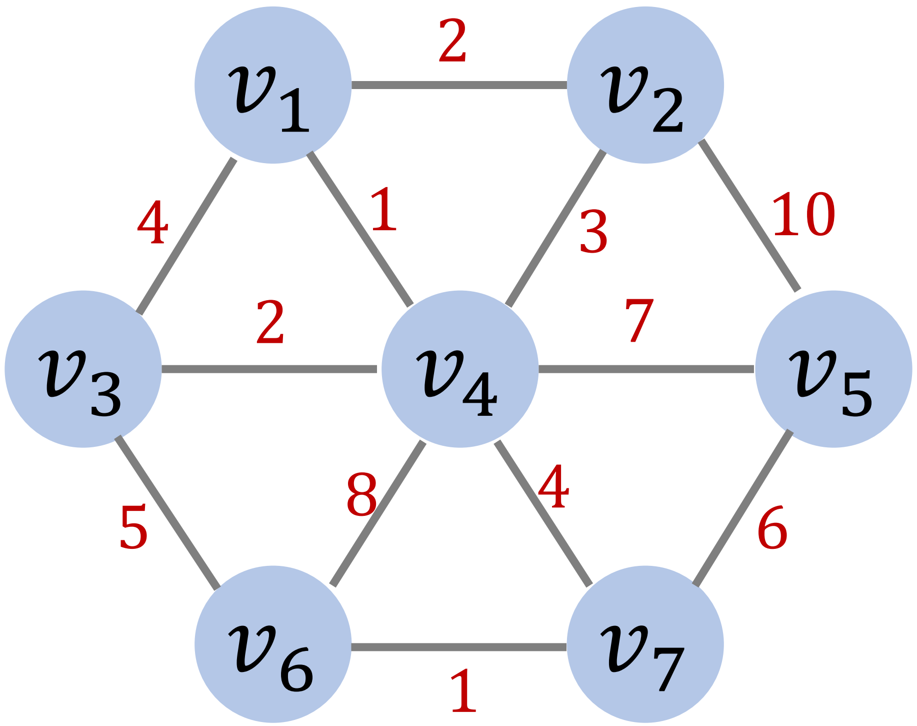


A Spanning Tree

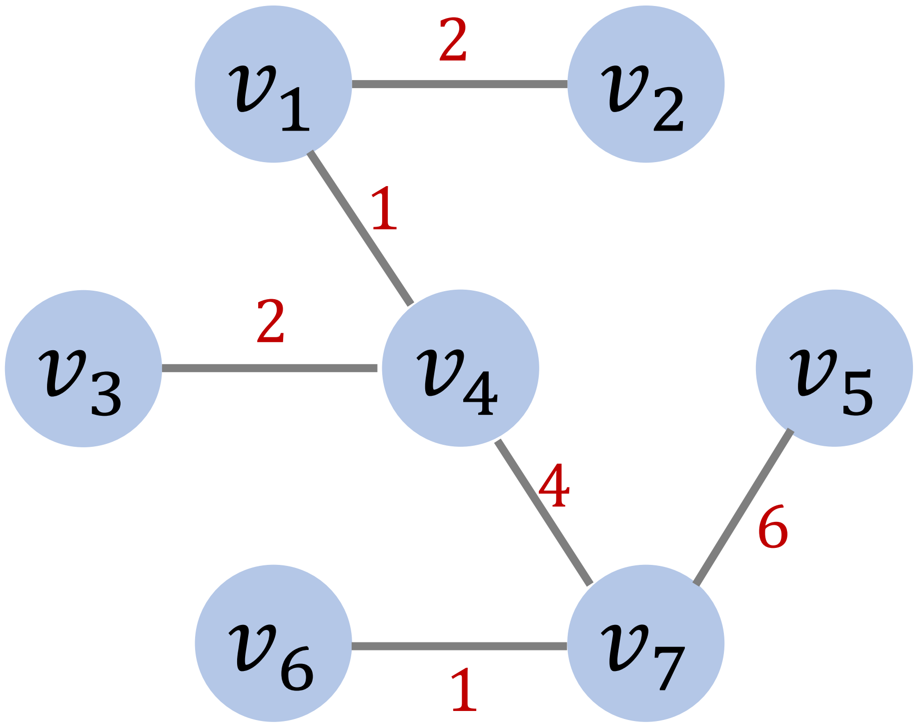
Sum of weights is 26.

Minimum Spanning Trees

Minimum spanning tree is a spanning tree that minimizes the sum of weights.



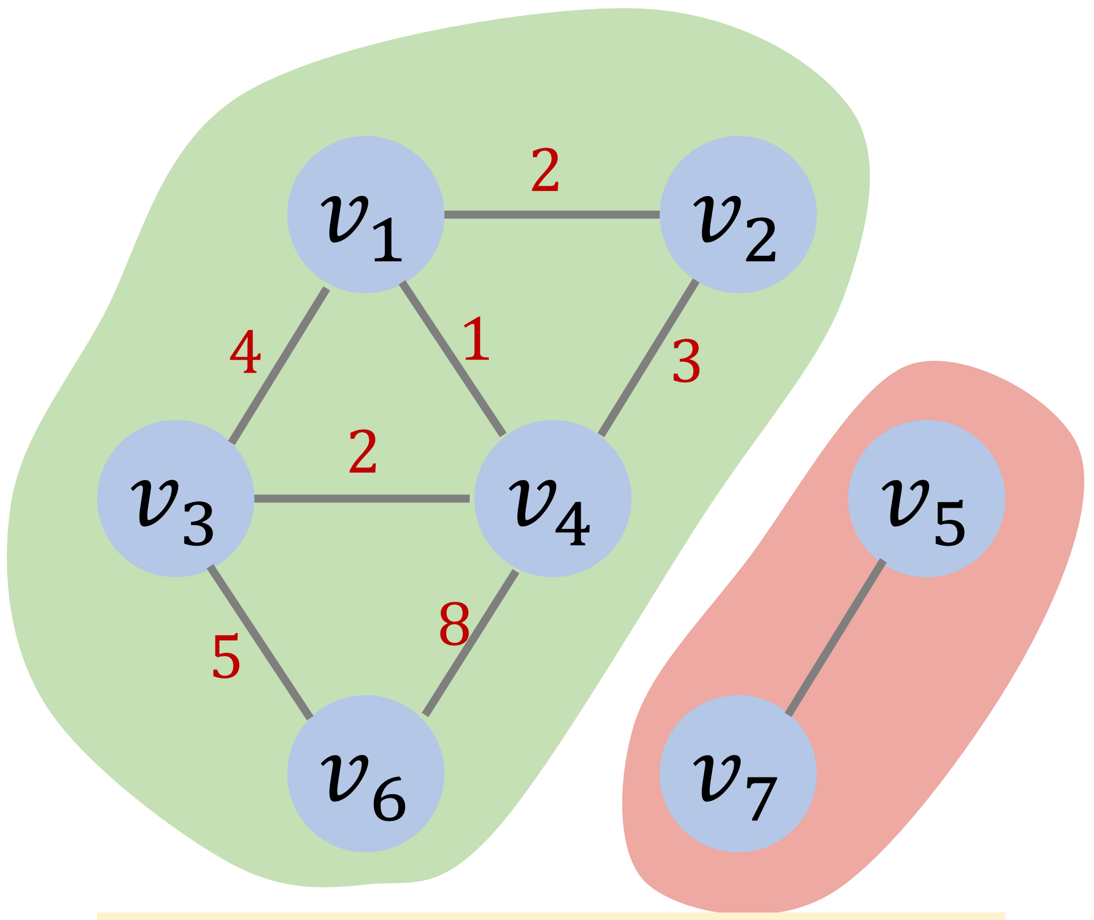
Original Graph



Minimum Spanning Tree

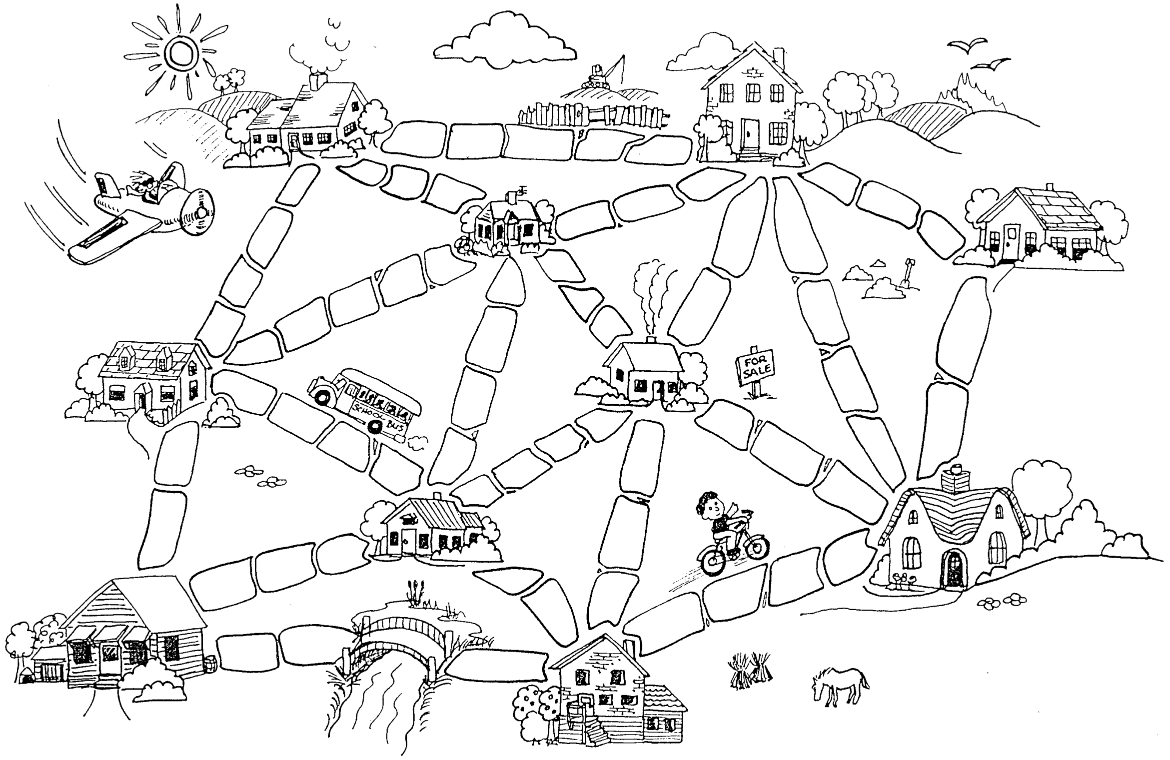
Sum of weights is 16.

A graph may not have spanning tree



Original Graph

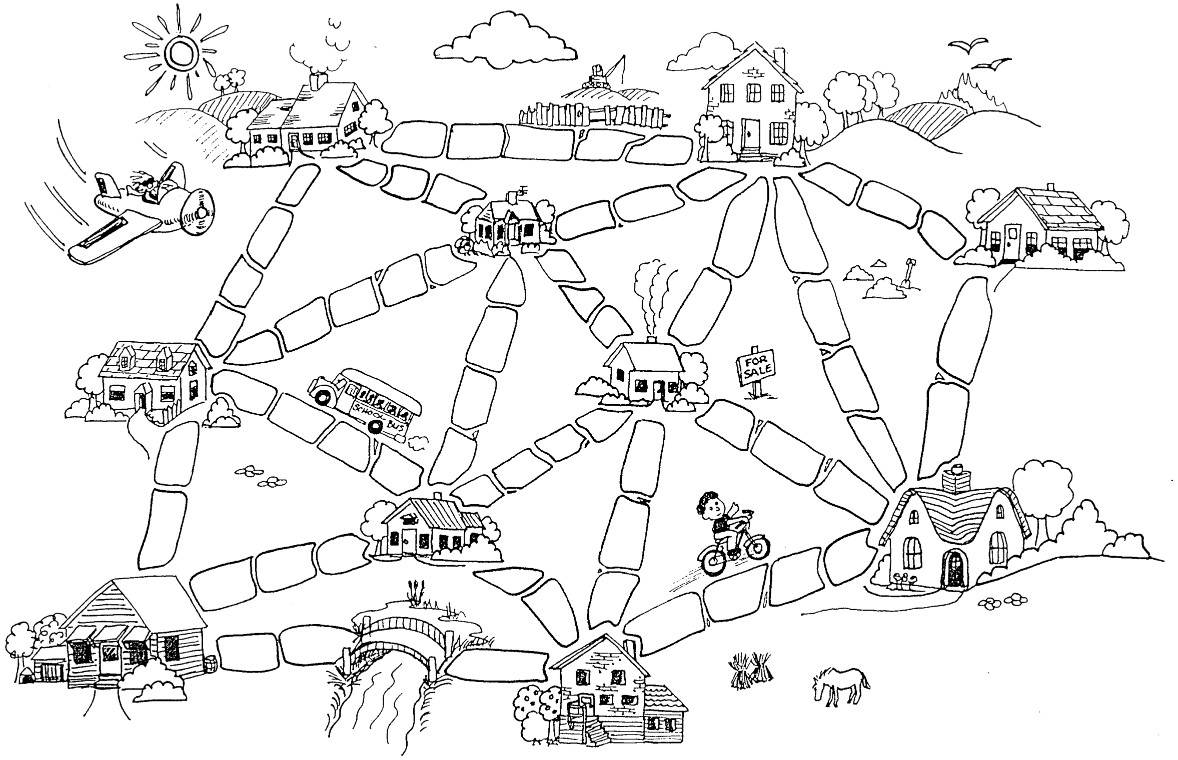
Application: Muddy City Problem



The muddy roads form a graph.

- The city has no road.
- The mayor wants to pave roads.
- Constraints:
    1. Enough roads must be paved so that everyone can travel from his house to anyone else's house.
    2. The paving should cost as little as possible.

Application: Muddy City Problem



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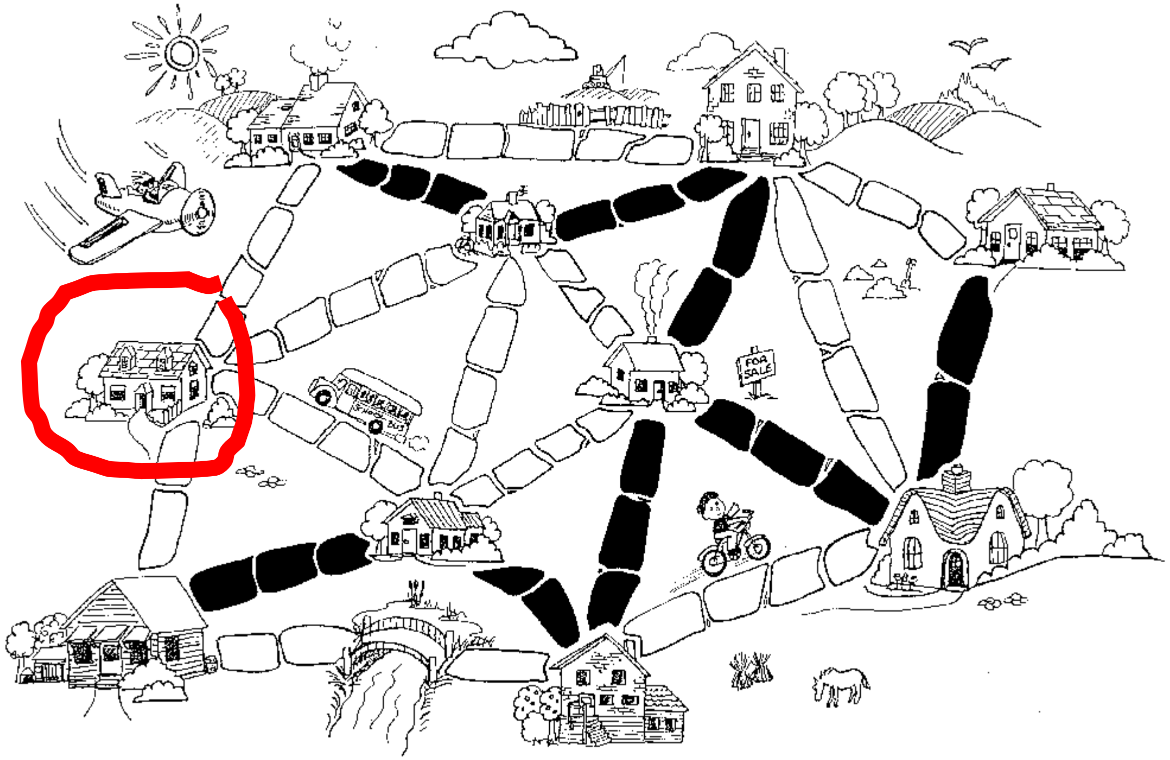
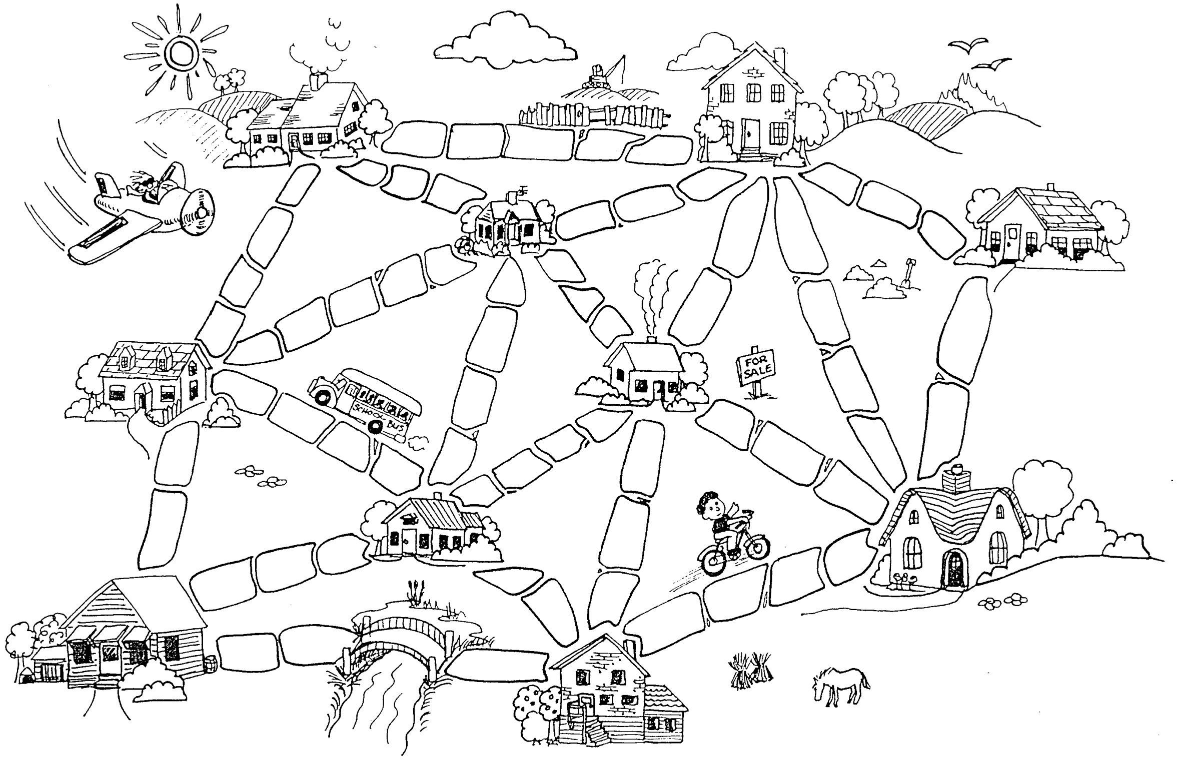


Not a spanning tree.

Application: Muddy City Problem



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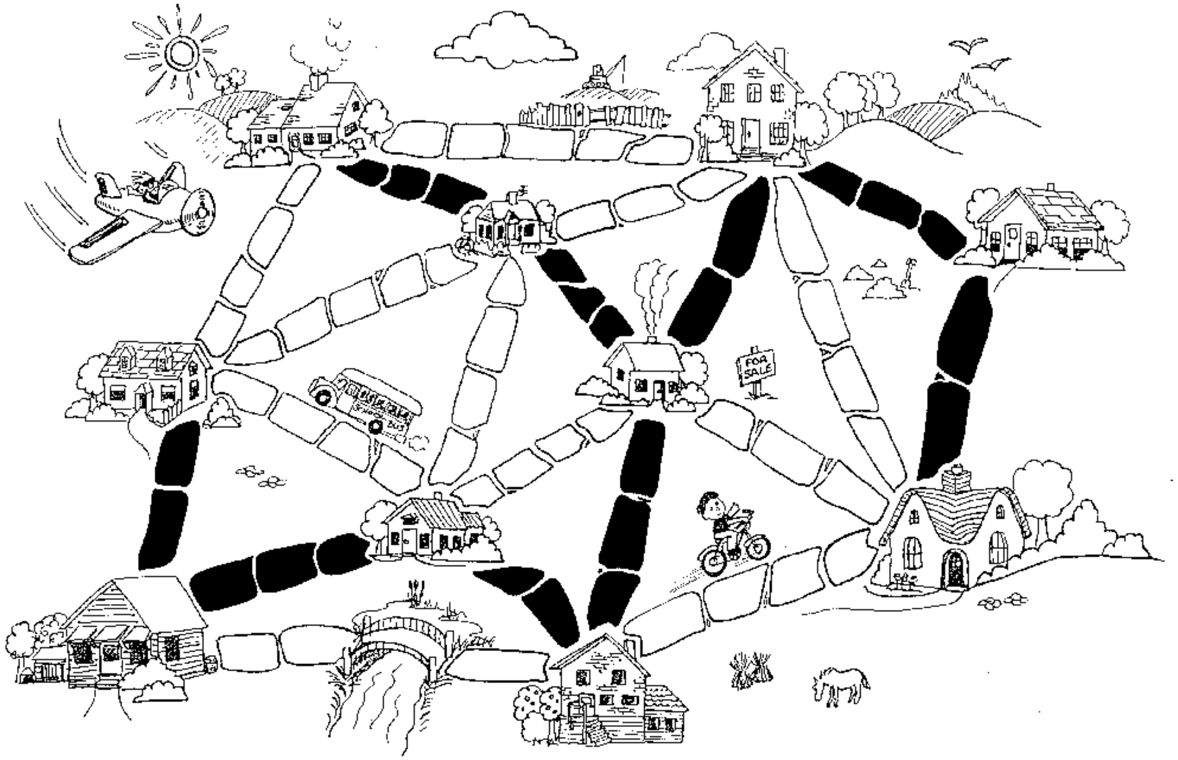

Spanning tree.

Summary

Trees vs Graphs

Assume all the edges are undirected.

- Trees are graphs.
- Trees are connected graphs.
- Trees do not have cycles.
- If there are n vertices, then there must be $n - 1$ edges.

Spanning Trees

Input: A connected undirected graph.

- Keep all the n vertices.
- Keep a subset of $n-1$ edges.
- The subgraph must be connected and have no cycle.

Output: The obtained subgraph is called spanning tree.

Minimum spanning tree: The spanning tree with the minimum sum of weights.

Thank You!