Longest Prefix Suffix Matching

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Prefix

- If string X can be written as $X = PS$ for a nonempty string S, then P is a prefix of X.

Example string: `X = “algorithm”`

- `prefixes = {“algorith”, “algorit”, “algori”, “algor”, “algo”, “alg”, “al”, “a”}.`

Suffix

- If string X can be written as $X = PS$ for a nonempty string P, then S is a suffix of X.

Example string: `X = "algorithm"`

- `suffixes = {"lgorithm", "gorithm", "orithm", "rithm", "ithm", "thm", "hm", "m"}.`

Matching Prefixes and Suffixes

Example string: `X = “aba”`

- `prefixes = { “ab”, “a” }.`
- `suffixes = { “ba”, “a” }.`

Matching Prefixes and Suffixes

Example string: `X = "aba"`

- `prefixes = { "ab", "a" }.`
- `suffixes = { "ba", "a" }.`
- Their intersection: `{ "a" }.`
- The longest matching: `"a"`.

Matching Prefixes and Suffixes

Example string: `X = “ababa”`

- `prefixes = { “abab”, “aba”, “ab”, “a” }.`
- `suffixes = { “baba”, “aba”, “ba”, “a” }.`

Matching Prefixes and Suffixes

Example string: `X = "ababa"`

- `prefixes = { "abab", "aba", "ab", "a" }.`
- `suffixes = { "baba", "aba", "ba", "a" }.`
- Their intersection: `{ "aba", "a" }.`

Matching Prefixes and Suffixes

Example string: **`X = "ababa"`**

- **`prefixes = { "abab", "aba", "ab", "a" }.`**
- **`suffixes = { "baba", "aba", "ba", "a" }.`**
- Their intersection: **`{ "aba", "a" }.`**
- The longest matching: **`"aba"`**.

Longest Prefix Suffix Array

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	?	?	?	?	?	?	?	?

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	?							

- **prefixes = { }.**
- **suffixes = { }.**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	?							

- **prefixes = { }.**
- **suffixes = { }.**
- Their intersection: { }.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	?							

- **prefixes = { }.**
- **suffixes = { }.**
- Their intersection: **{ }.**
- The longest matching: **empty string**. **(Length = 0)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0							

- **prefixes = { }.**
- **suffixes = { }.**
- Their intersection: **{ }.**
- The longest matching: **empty string**. **(Length = 0)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	?						

- **prefixes = {“a”}.**
- **suffixes = {“b”}.**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	?						

- **prefixes = {"a"}.**
- **suffixes = {"b"}.**
- Their intersection: **{ }.**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	?						

- **prefixes = {"a"}.**
- **suffixes = {"b"}.**
- Their intersection: **{ }.**
- The longest matching: **empty string**. **(Length = 0)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0						

- **prefixes = {“a”}.**
- **suffixes = {“b”}.**
- Their intersection: **{ }.**
- The longest matching: **empty string**. **(Length = 0)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	?					

- prefixes = {“ab”, “a”}.
- suffixes = {“ba”, “a”}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	?					

- prefixes = {"ab", "a"}.
- suffixes = {"ba", "a"}.
- Their intersection: {"a"}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	?					

- **prefixes = {“ab”, “a”}.**
- **suffixes = {“ba”, “a”}.**
- Their intersection: {**“a”**}.
- The longest matching: **“a”**. **(Length = 1)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1					

- `prefixes = {"ab", "a"}.`
- `suffixes = {"ba", "a"}.`
- Their intersection: `{"a"}.`
- The longest matching: `"a"`. `(Length = 1)`

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	?				

- prefixes = {“aba”, “ab”, “a”}.
- suffixes = {“bab”, “ab”, “a”}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	?				

- **prefixes = {"aba", "ab", "a"}.**
- **suffixes = {"bab", "ab", "a"}.**
- Their intersection: **{"ab", "a"}.**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	?				

- **prefixes = {“aba”, “ab”, “a”}.**
- **suffixes = {“bab”, “ab”, “a”}.**
- Their intersection: **{“ab”, “a”}.**
- The longest matching: **“ab”.** **(Length = 2)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2				

- **prefixes = {"aba", "ab", "a"}.**
- **suffixes = {"bab", "ab", "a"}.**
- Their intersection: **{"ab", "a"}.**
- The longest matching: **"ab"**. **(Length = 2)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	?			

- prefixes = {“abab”, “aba”, “ab”, “a”}.
- suffixes = {“baba”, “aba”, “ba”, “a”}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	?			

- prefixes = {“abab”, “aba”, “ab”, “a”}.
- suffixes = {“baba”, “aba”, “ba”, “a”}.
- Their intersection: {“aba”, “a”}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	?			

- **prefixes = {"abab", "aba", "ab", "a"}.**
- **suffixes = {"baba", "aba", "ba", "a"}.**
- Their intersection: **{"aba", "a"}.**
- The longest matching: **"aba"**. **(Length = 3)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3			

- **prefixes = {"abab", "aba", "ab", "a"}.**
- **suffixes = {"baba", "aba", "ba", "a"}.**
- Their intersection: **{"aba", "a"}.**
- The longest matching: **"aba"**. **(Length = 3)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	?		

- prefixes = {“ababa”, “abab”, “aba”, “ab”, “a”}.
- suffixes = {“babab”, “abab”, “bab”, “ab”, “b”}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	?		

- `prefixes = {"ababa", "abab", "aba", "ab", "a"}.`
- `suffixes = {"babab", "abab", "bab", "ab", "b"}.`
- Their intersection: `{"abab", "ab"}.`

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	?		

- **prefixes = {"ababa", "abab", "aba", "ab", "a"}.**
- **suffixes = {"babab", "abab", "bab", "ab", "b"}.**
- Their intersection: **{"abab", "ab"}.**
- The longest matching: **"abab"**. **(Length = 4)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	**4**		

- **prefixes = {"ababa", "abab", "aba", "ab", "a"}.**
- **suffixes = {"babab", "abab", "bab", "ab", "b"}.**
- Their intersection: **{"abab", "ab"}.**
- The longest matching: **"abab"**. **(Length = 4)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	?	

- **prefixes = {“ababab”, “ababa”, “abab”, “aba”, “ab”, “a”}.**
- **suffixes = {“bababc”, “ababc”, “babc”, “abc”, “bc”, “c”}.**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	?	

- `prefixes = {"ababab", "ababa", "abab", "aba", "ab", "a"}.`
- `suffixes = {"bababc", "ababc", "babc", "abc", "bc", "c"}.`
- Their intersection: `{ }.`

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	?	

- **prefixes = {"ababab", "ababa", "abab", "aba", "ab", "a"}.**
- **suffixes = {"bababc", "ababc", "babc", "abc", "bc", "c"}.**
- Their intersection: **{ }.**
- The longest matching: **empty string**. **(Length = 0)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	

- **prefixes = {“ababab”, “ababa”, “abab”, “aba”, “ab”, “a”}.**
- **suffixes = {“bababc”, “ababc”, “babc”, “abc”, “bc”, “c”}.**
- Their intersection: **{ }.**
- The longest matching: **empty string**. **(Length = 0)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	?

- prefixes = {"abababc", "ababab", "ababa", "abab", "aba", "ab", "a"}.
- suffixes = {"bababca", "ababca", "babca", "abca", "bca", "ca", "a"}.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	?

- **prefixes = {"abababc", "ababab", "ababa", "abab", "aba", "ab", "a"}.**
- **suffixes = {"bababca", "ababca", "babca", "abca", "bca", "ca", "a"}.**
- Their intersection: { **"a"** }.

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	?

- **prefixes = {"abababc", "ababab", "ababa", "abab", "aba", "ab", "a"}.**
- **suffixes = {"bababca", "ababca", "babca", "abca", "bca", "ca", "a"}.**
- Their intersection: { **"a"** }.
- The longest matching: **"a"**. **(Length = 1)**

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	1

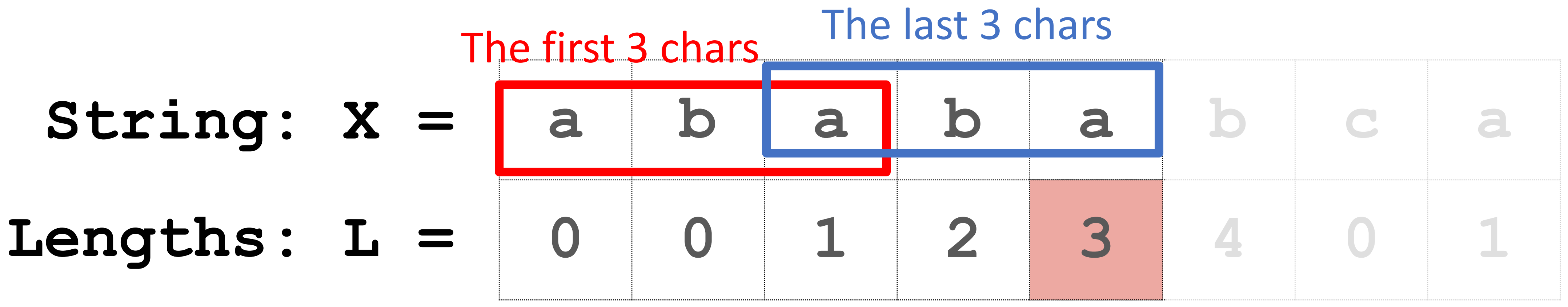
- `prefixes = {"abababc", "ababab", "ababa", "abab", "aba", "ab", "a"}.`
- `suffixes = {"bababca", "ababca", "babca", "abca", "bca", "ca", "a"}.`
- Their intersection: { "a" }.
- The longest matching: "a". (Length = 1)

Longest Prefix Suffix Array

String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	1

What does the number mean?

Longest Prefix Suffix Array



String: X =	a	b	a	b	a	b	c	a
Lengths: L =	0	0	1	2	3	4	0	1

What does the number mean?

Application

Why is the longest prefix suffix array interesting?

- The array is used by the Knuth–Morris–Pratt (KMP) algorithm [1].
- KMP algorithm solves the string matching problem.

Reference:

1. Knuth, Morris, & Pratt. Fast pattern matching in strings. *SIAM Journal on Computing*, 6 (2): 323–350, 1977.

Linear-Time Algorithm

Longest Prefix Suffix Array

String: X =	a	b	c	⋯	a	b
Lengths: L =	0	0	0	⋯	1	2

Longest Prefix Suffix Array

String: X =	a	b	c	…	a	b
Lengths: L =	0	0	0	…	1	2

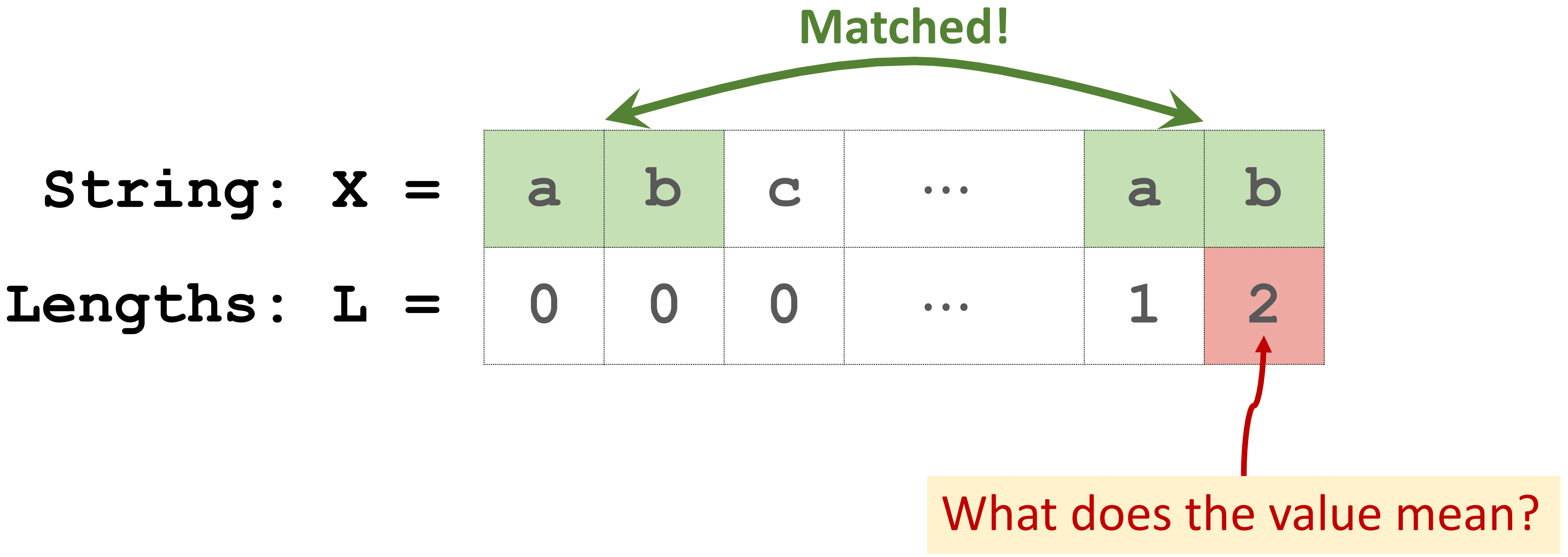
What does the value mean?

Longest Prefix Suffix Array

	Prefix					Suffix
String: X =	a	b	c	…	a	b
Lengths: L =	0	0	0	…	1	2

What does the value mean?

Longest Prefix Suffix Array



`String: X =`	a	b	c	…	a	b
`Lengths: L =`	0	0	0	…	1	2

What does the value mean?

What is the next element in the array?

String: X =	a	b	c	⋯	a	b
Lengths: L =	0	0	0	⋯	1	2

What is the next element in the array?

Append a new character to **X**

String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	

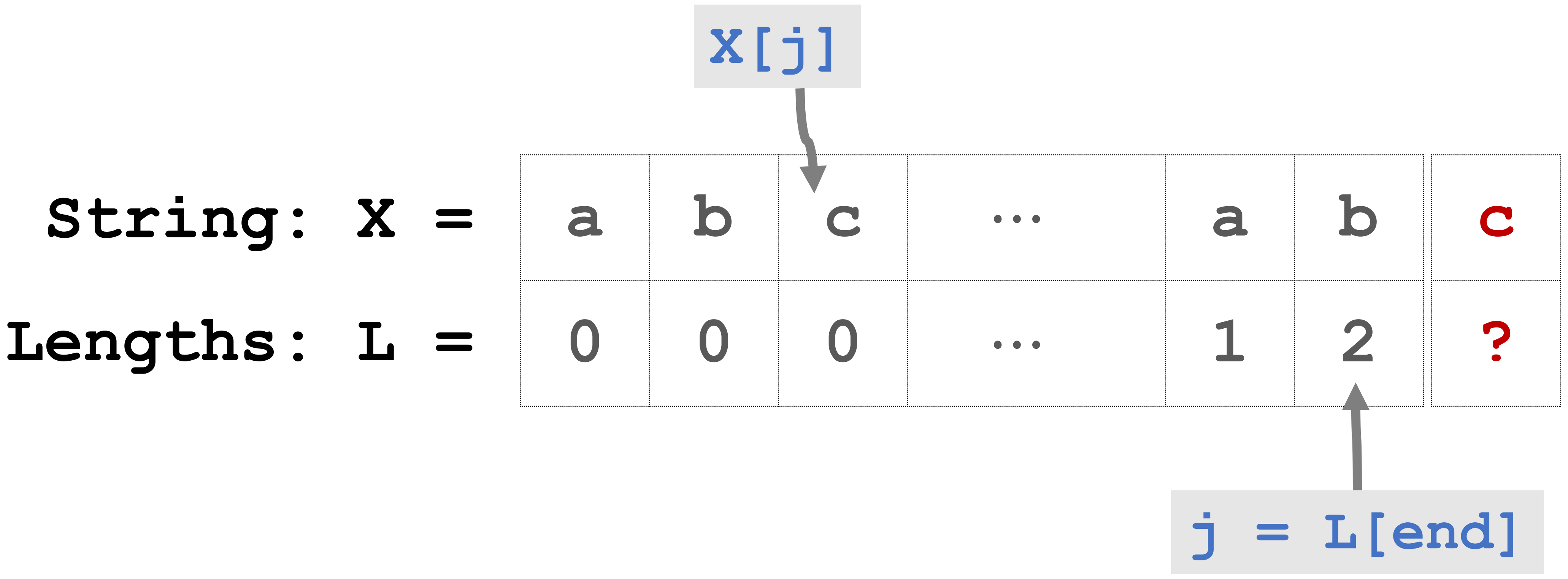
What is the next element in the array?

Append a new character to **`X`**

`String: X =`	`a`	`b`	`c`	…	`a`	`b`	`c`
`Lengths: L =`	0	0	0	…	1	2	`?`

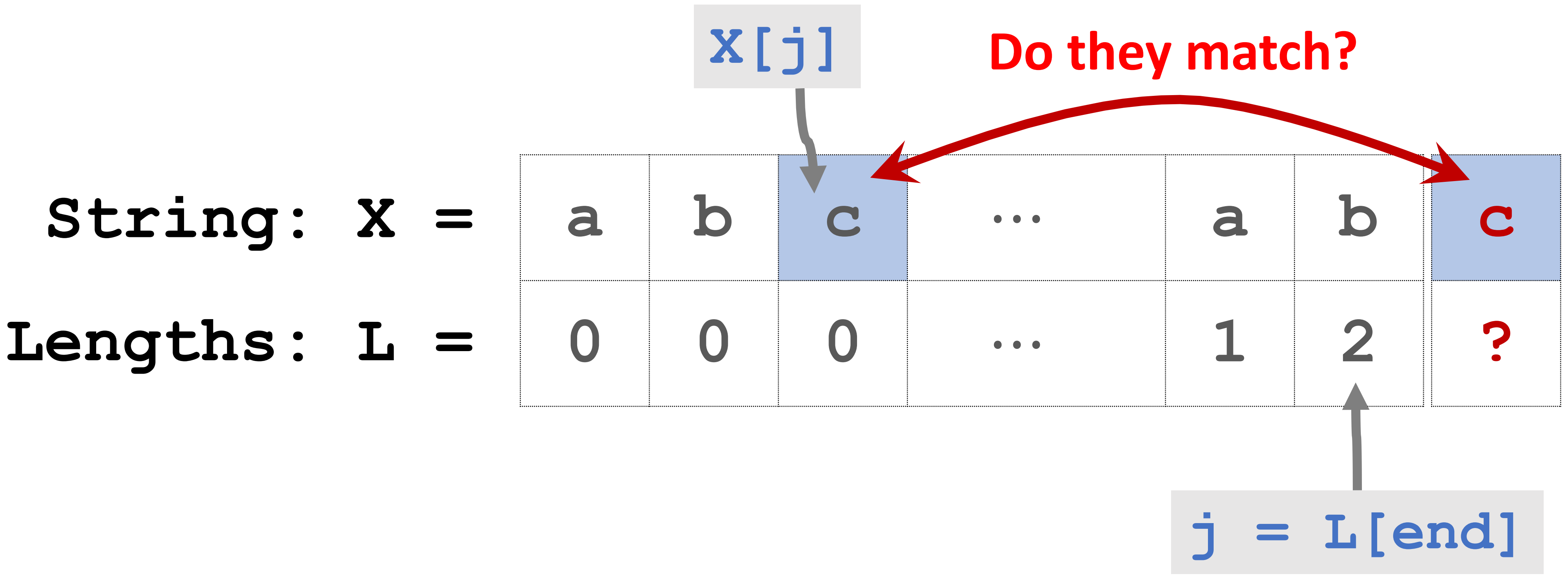
What is the new value?

Three Cases

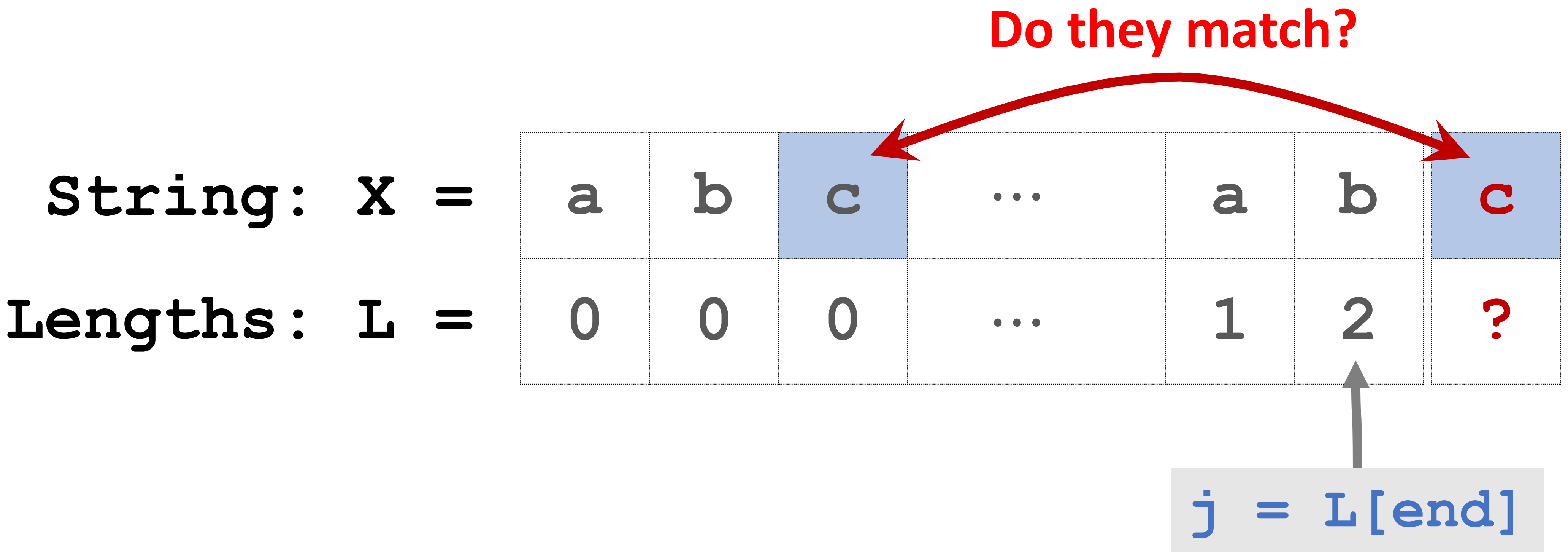


String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	?

Three Cases

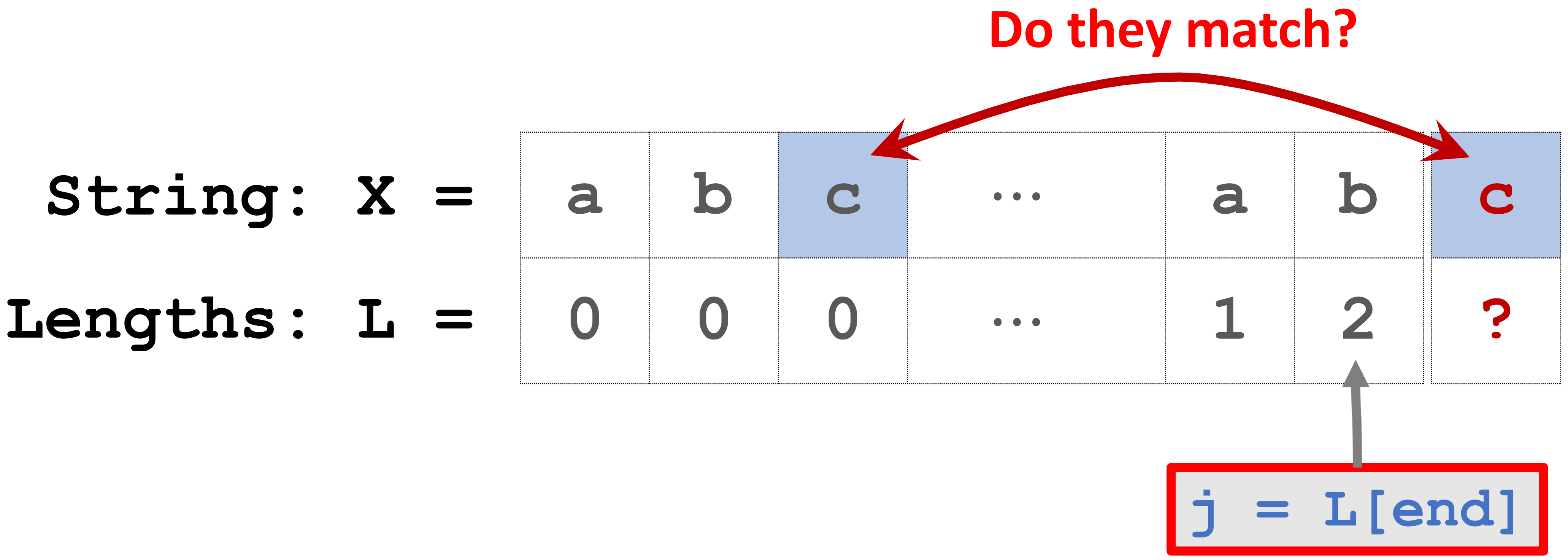


Three Cases



- **Match ==> Case 1;**

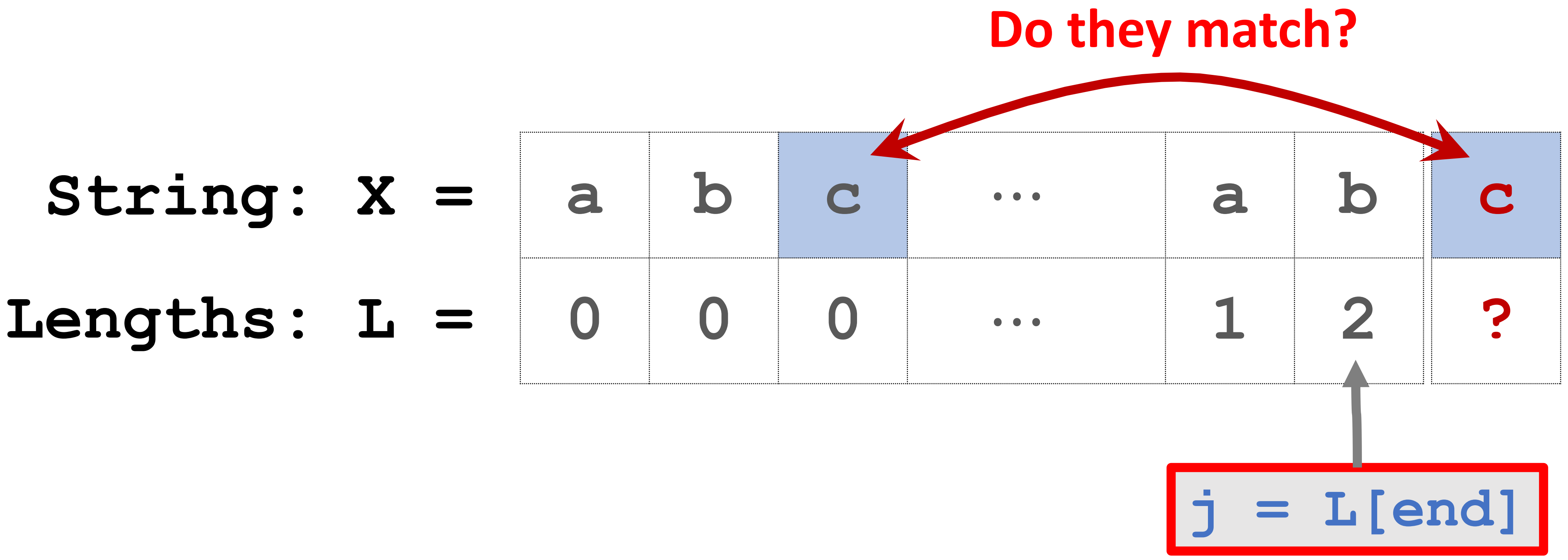
Three Cases



String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	?

- Match ==> Case 1;
- Mismatch ==>

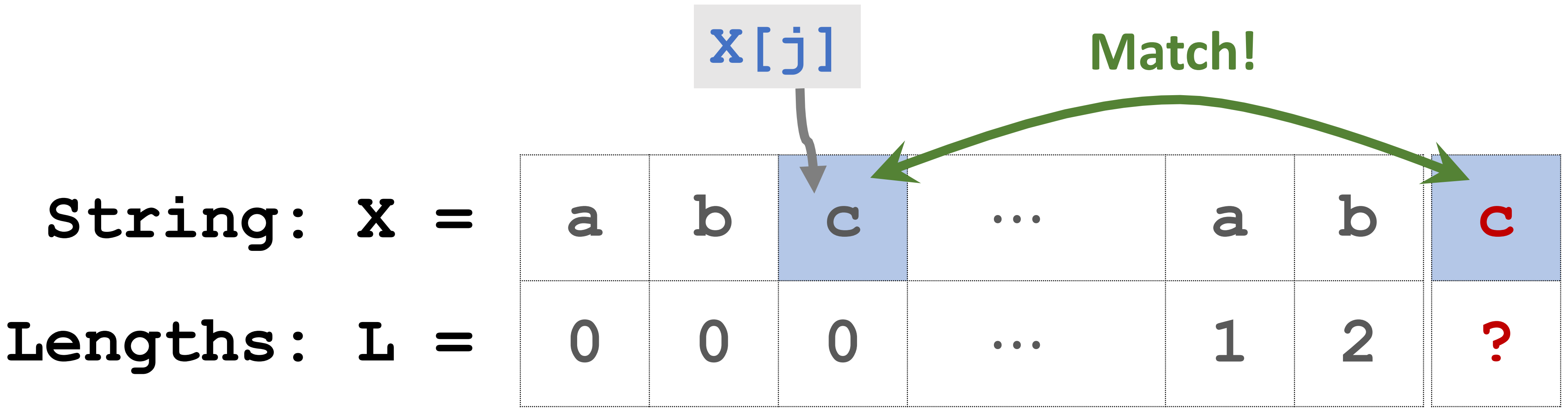
Three Cases



String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	?

- Match ==> Case 1;
- Mismatch ==> $\begin{cases} j = 0 \quad ==> \quad \textbf{Case 2}; \\ j \neq 0 \quad ==> \quad \textbf{Case 3}. \end{cases}$

Case 1: A New Match



- Case 1: the new char is equal to x[j].

Case 1: A New Match

String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	?

j = L[end]

- Case 1 ==> Let the new value in L be $\boxed{j+1}$

Case 1: A New Match

String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	3

- Case 1 ==> Let the new value in L be j+1.

Case 1: A New Match

String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	3

`j = L[end]`

Question: Why is the new value equal to `j+1`?

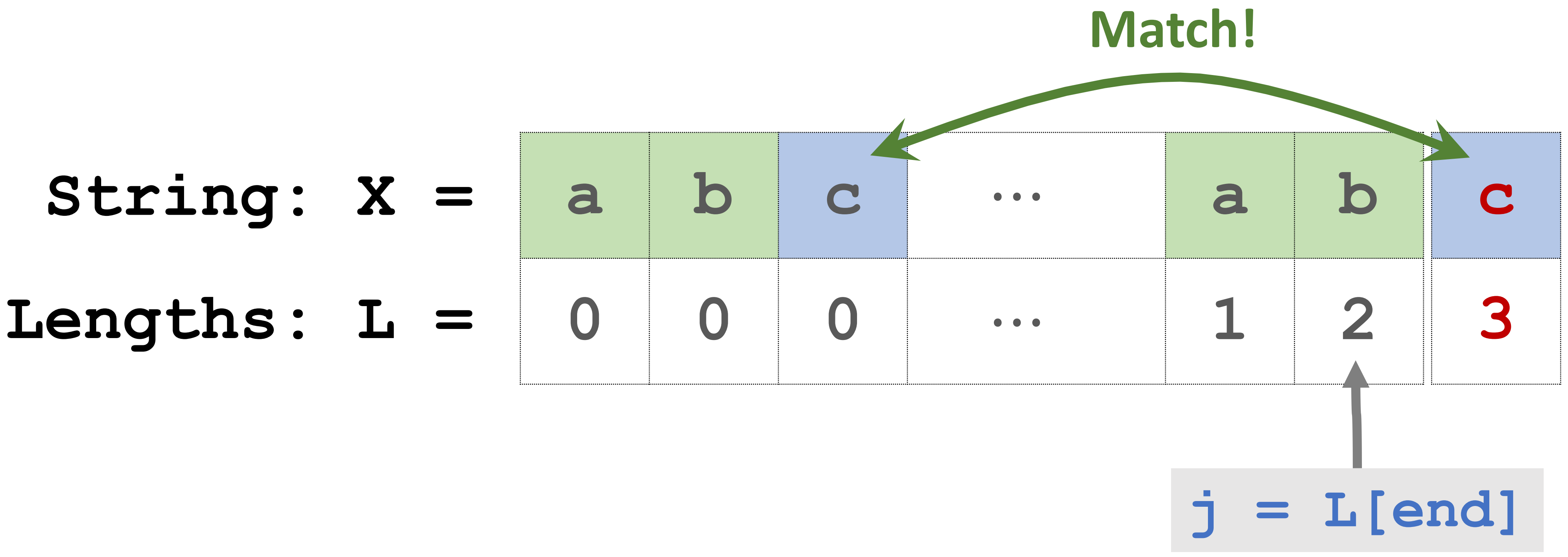
Case 1: A New Match

String: X =	a	b	c	…	a	b	c
Lengths: L =	0	0	0	…	1	2	3

`j = L[end]`

Question: Why is the new value equal to `j+1`?

Case 1: A New Match



Question: Why is the new value equal to `j+1`?

Case 1: A New Match

Another Example:

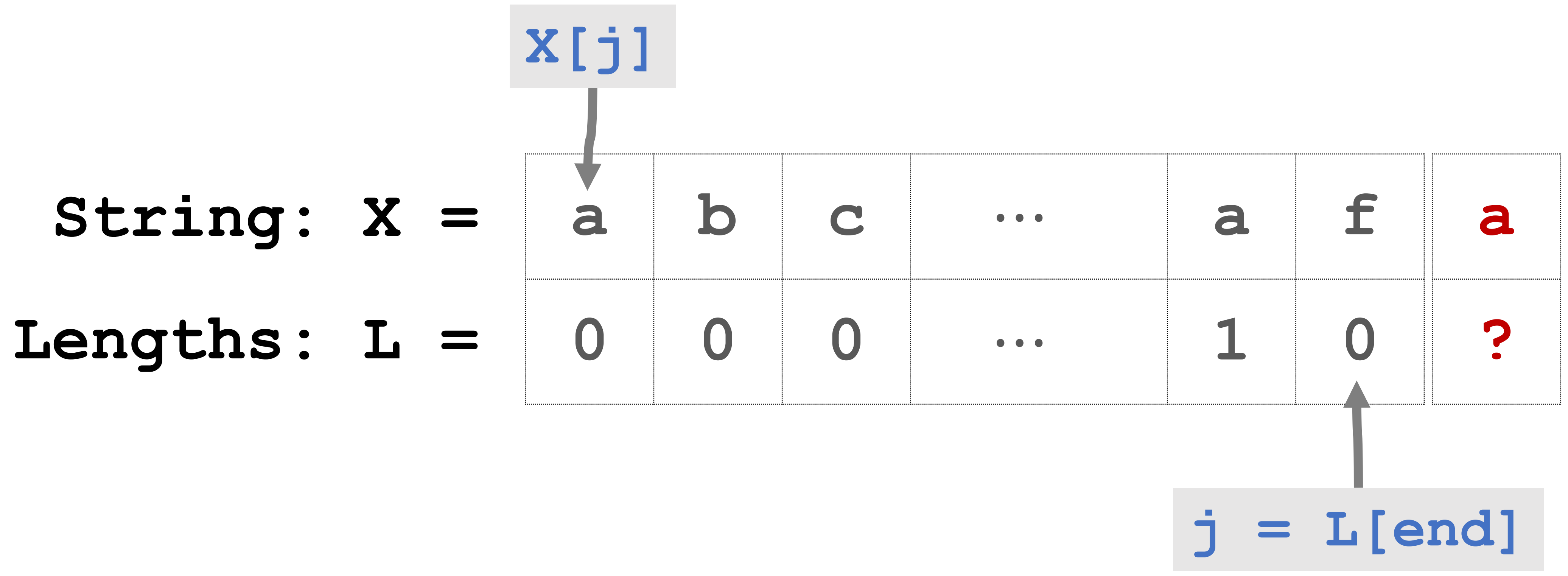
String: X =	a	b	c	…	a	f
Lengths: L =	0	0	0	…	1	0

Case 1: A New Match

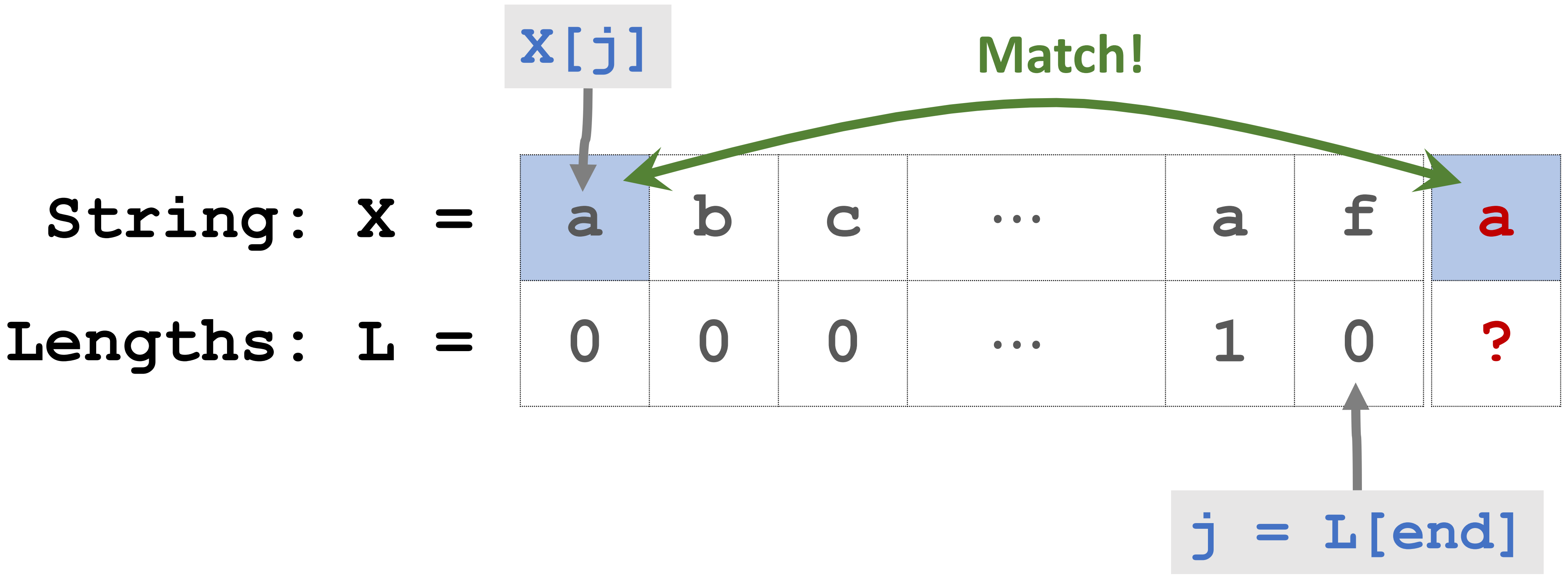
Append a new character to **X**

String: X =	a	b	c	…	a	f	a
Lengths: L =	0	0	0	…	1	0	?

Case 1: A New Match



Case 1: A New Match



- Case 1: the new char is equal to x[j].

Case 1: A New Match

String: X =	a	b	c	…	a	f	a
Lengths: L =	0	0	0	…	1	0	1

j = L[end]

- Case 1 ==> Let the new value of L be j+1.

Case 2: Mismatch and L[end]==0

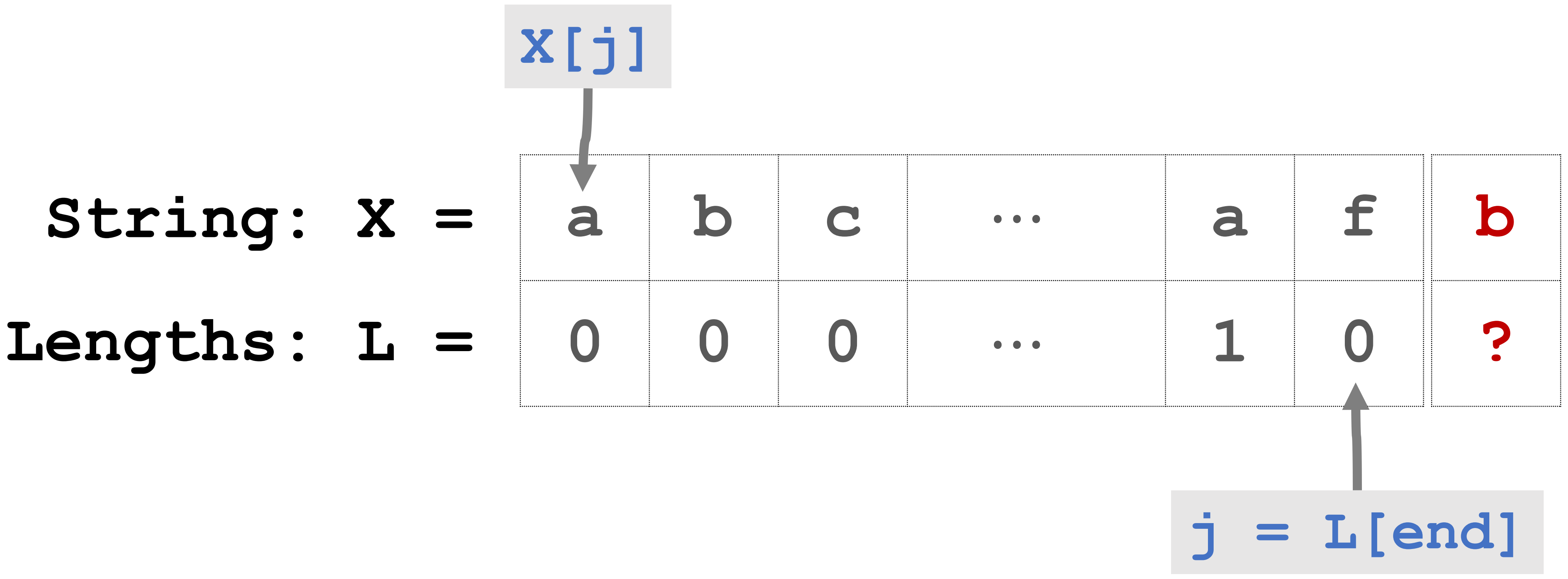
String: X =	a	b	c	…	a	f
Lengths: L =	0	0	0	…	1	0

Case 2: Mismatch and L[end]==0

Append a new character to **X**

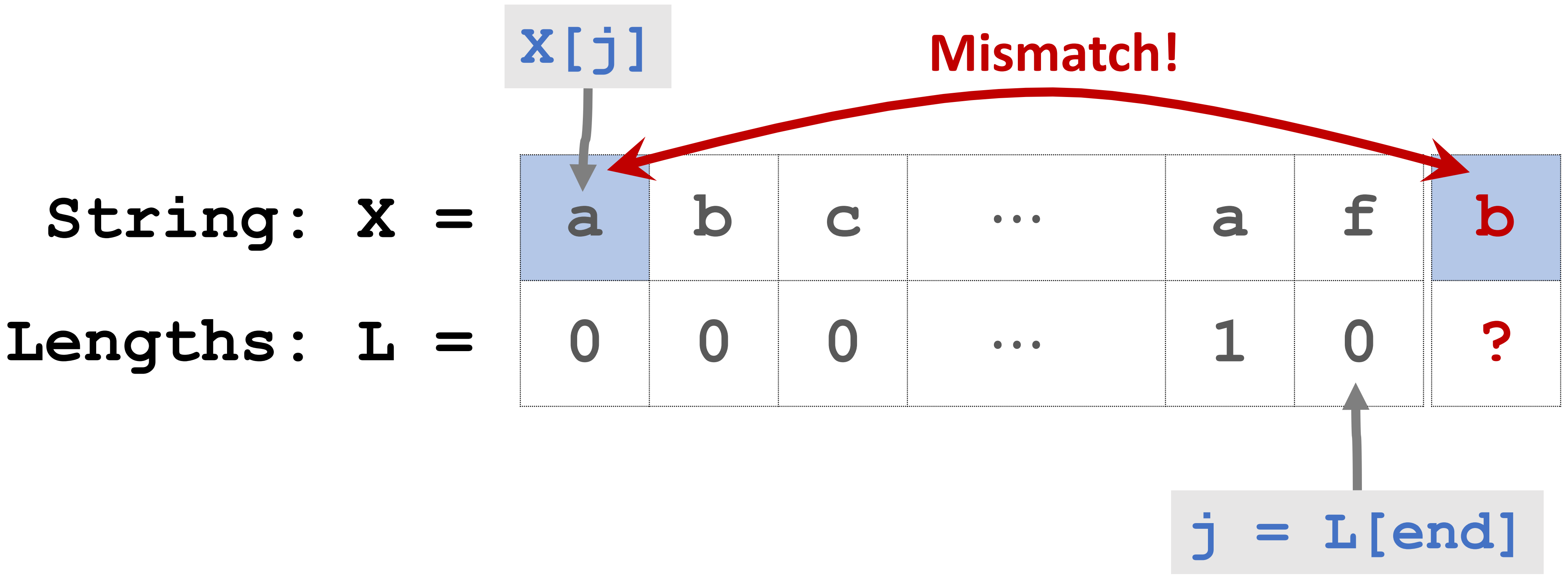
String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	?

Case 2: Mismatch and L[end]==0



String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	?

Case 2: Mismatch and L[end]==0



- Mismatch ==> $\begin{cases} j = 0 & \text{==>} \quad \textbf{Case 2}; \\ j \neq 0 & \text{==>} \quad \textbf{Case 3}. \end{cases}$

Case 2: Mismatch and L[end]==0

String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	?

j = L[end]

- Mismatch ==> $\begin{cases} j = 0 \quad \text{==>} \quad \textbf{Case 2}; \\ j \neq 0 \quad \text{==>} \quad \textbf{Case 3}. \end{cases}$

Case 2: Mismatch and L[end]==0

String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	?

- Case 2 ==> Let the new value be 0.

Case 2: Mismatch and L[end]==0

String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	0

- Case 2 ==> Let the new value be 0.

Case 2: Mismatch and L[end]==0

String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	0

- Case 2 ==> Let the new value be 0.

Case 3: Mismatch and L[end]≠0

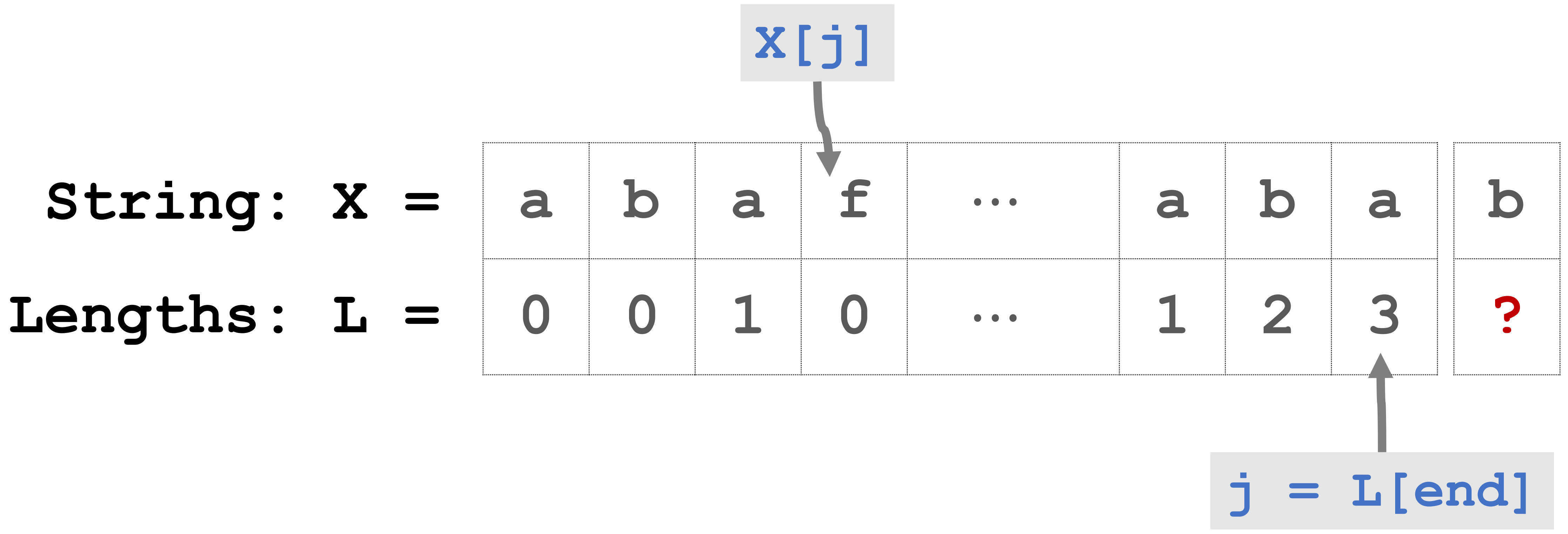
String: X =	a	b	a	f	…	a	b	a
Lengths: L =	0	0	1	0	…	1	2	3

Case 3: Mismatch and L[end]≠0

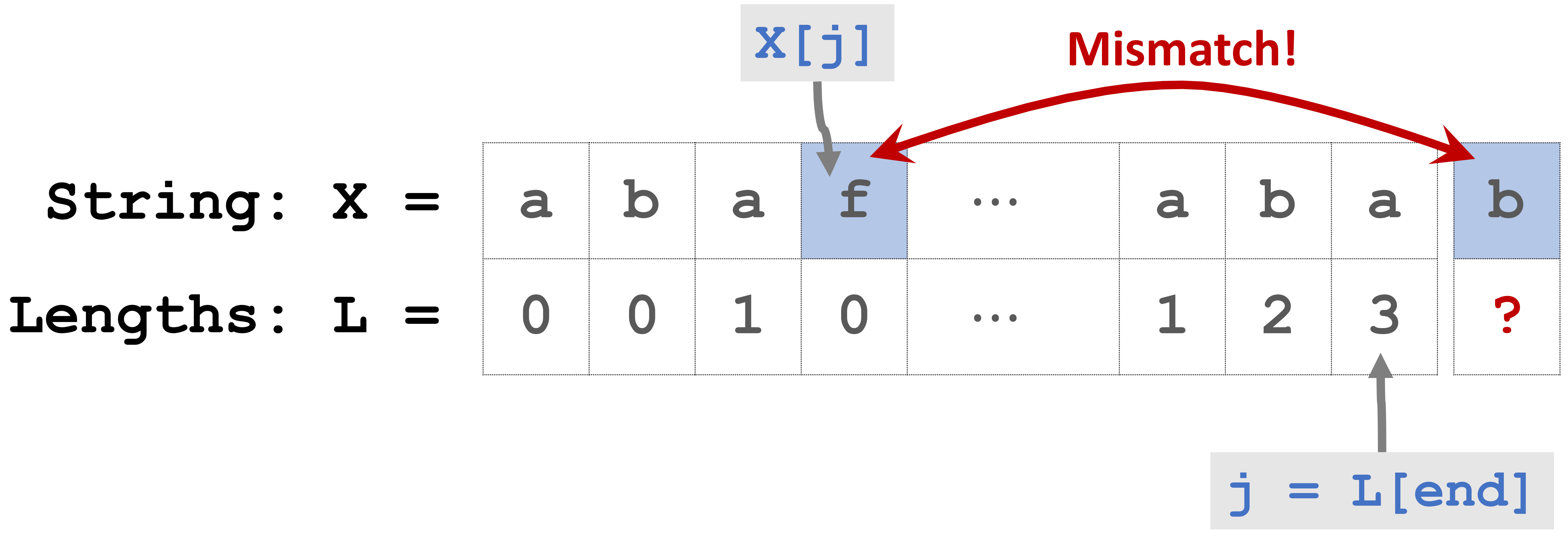
Append a new character to **X**

String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	?

Case 3: Mismatch and L[end]≠0



Case 3: Mismatch and L[end]≠0



- **Mismatch ==>** $\begin{cases} j = 0 & \text{==>} \quad \textbf{Case 2}; \\ j \neq 0 & \text{==>} \quad \textbf{Case 3}. \end{cases}$

Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	?

j = L[end]

- **Mismatch ==>** $\begin{cases} j = 0 \quad ==> \quad \text{Case 2}; \\ \boxed{j \neq 0 \quad ==> \quad \text{Case 3}.} \end{cases}$

Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	?

j = L[end]

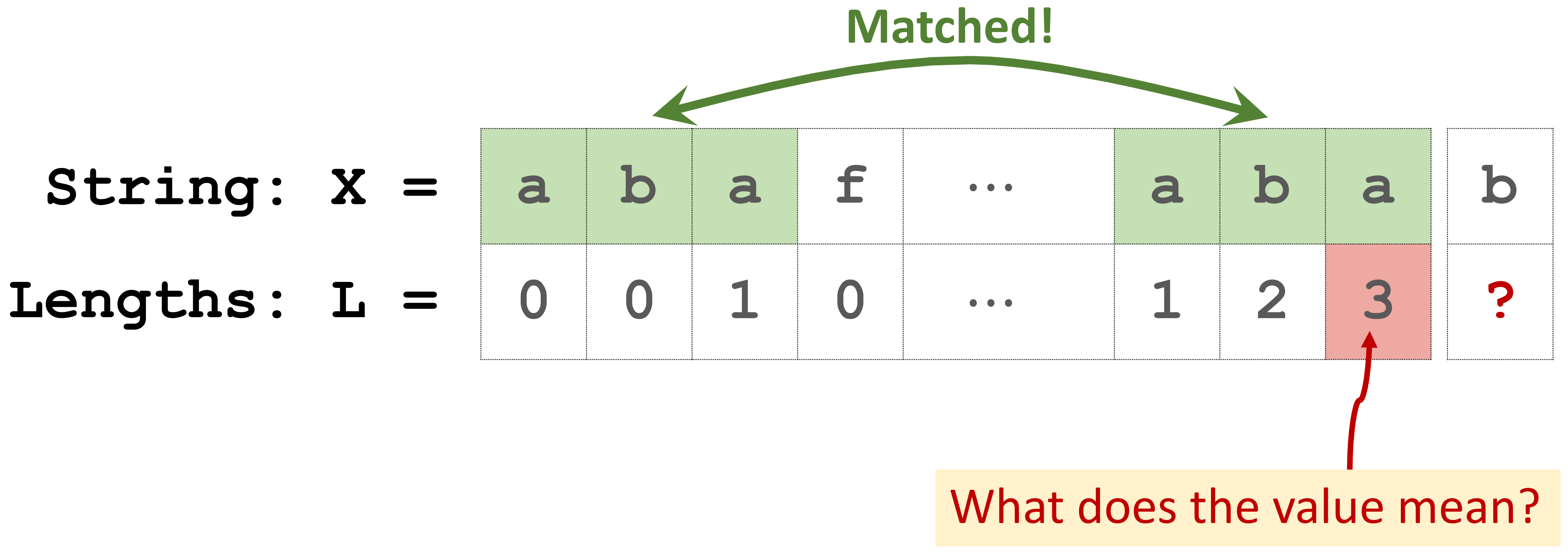
- Case 3 ==> Reduce the big problem to a smaller one.

Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	?

What does the value mean?

Case 3: Mismatch and L[end]≠0



Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	?

String: X′ =

Lengths: L′ =

Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	?

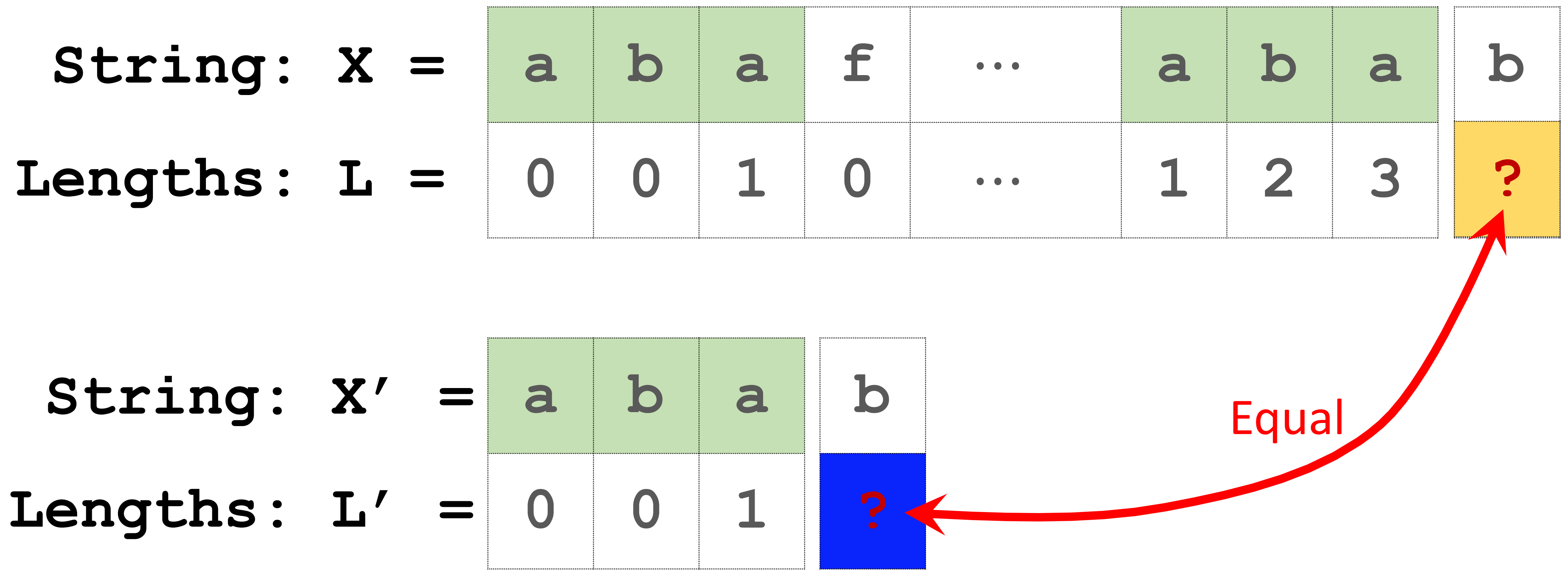
String: X′ =	a	b	a
Lengths: L′ =	0	0	1

Case 3: Mismatch and L[end]≠0

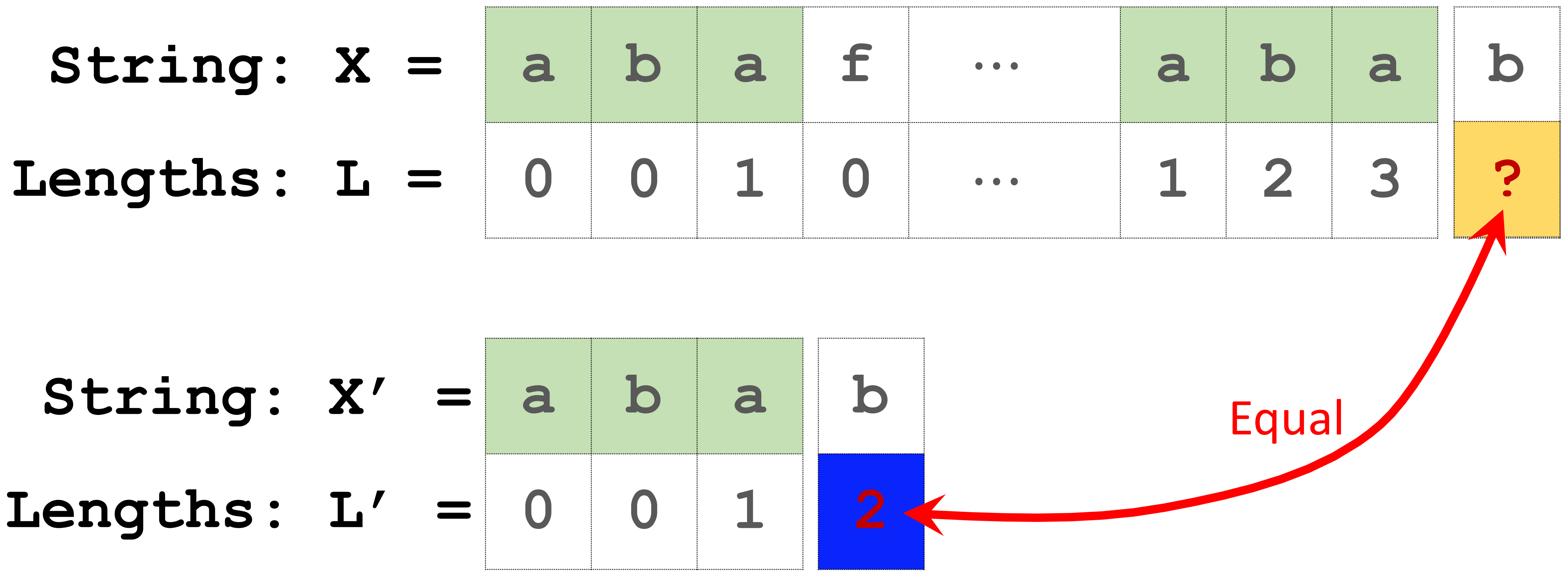
String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	?

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0



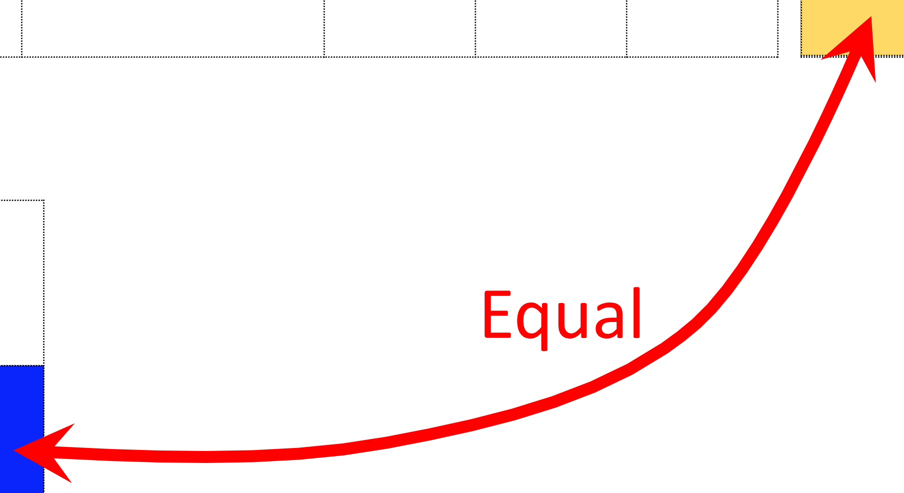
Case 3: Mismatch and L[end]≠0



Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	2

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	2

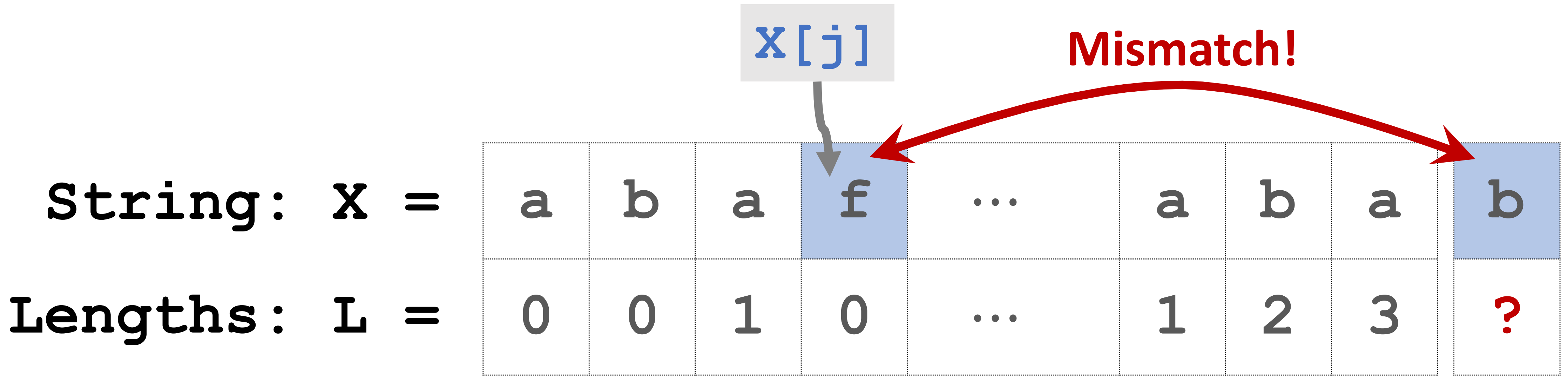


Case 3: Mismatch and L[end]≠0

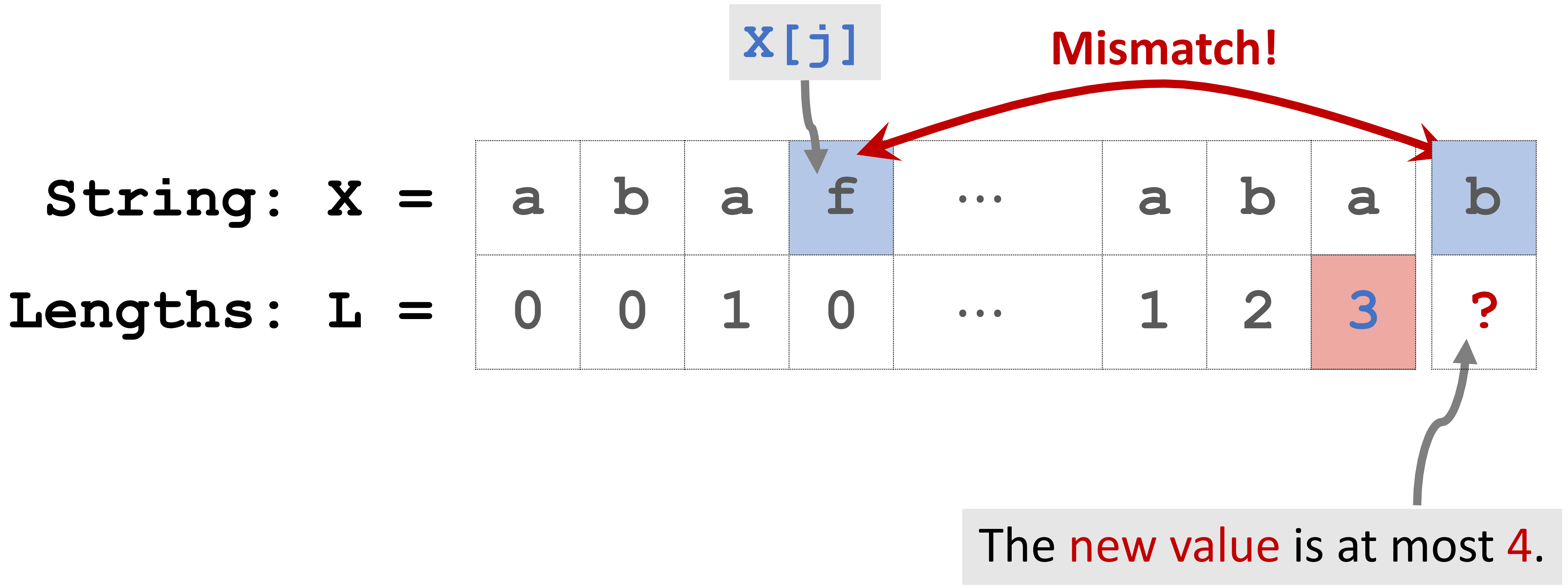
String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	?

Question: Why can Case 3 be solved in this way?

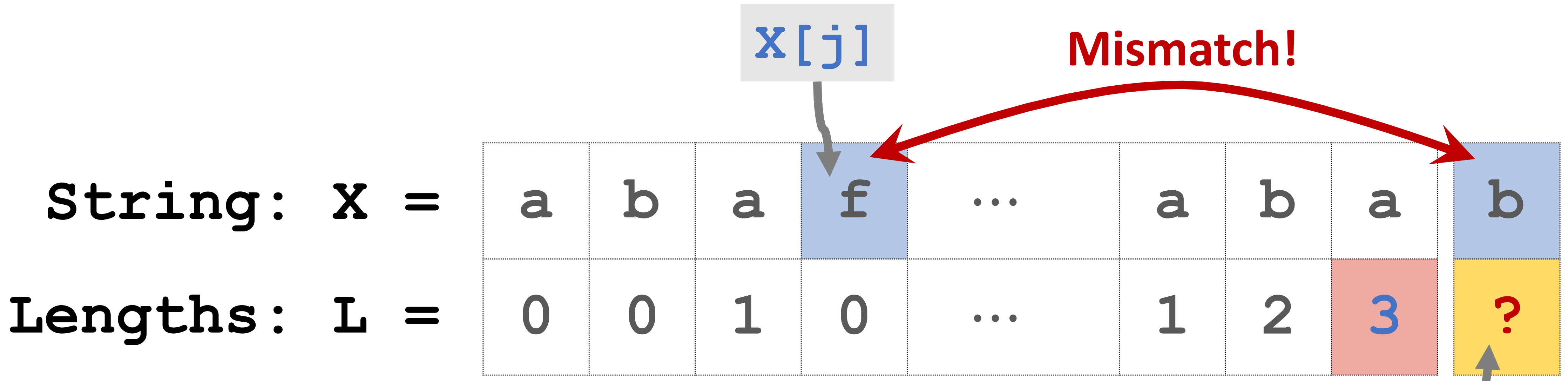
Case 3: Mismatch and L[end]≠0



Case 3: Mismatch and L[end]≠0

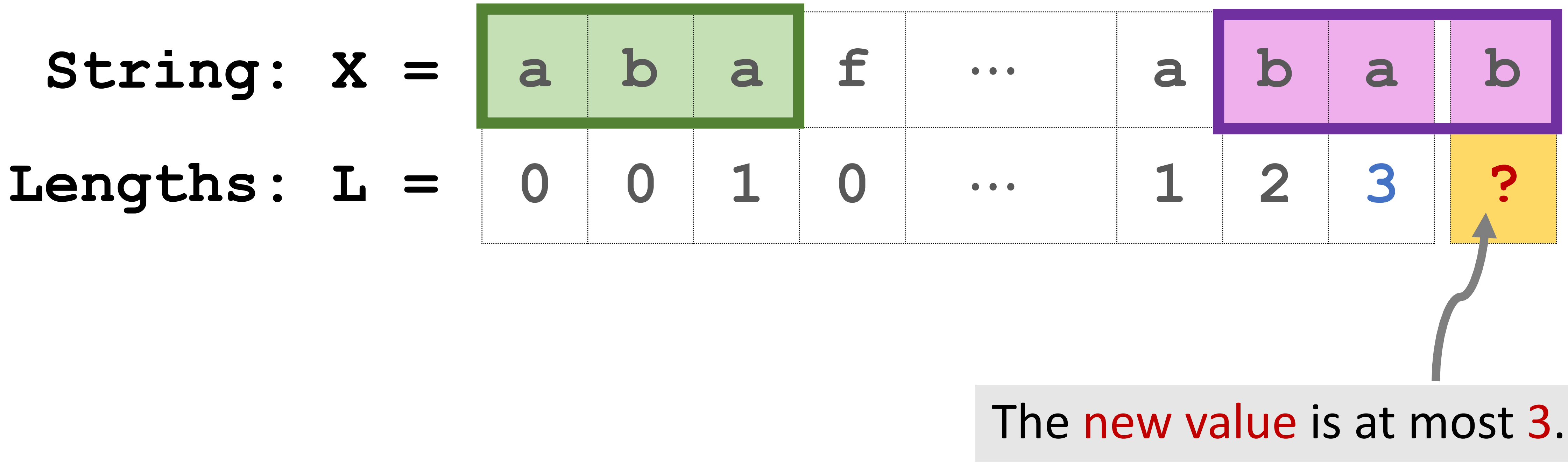


Case 3: Mismatch and L[end]≠0



The new value is at most 3.

Case 3: Mismatch and L[end]≠0

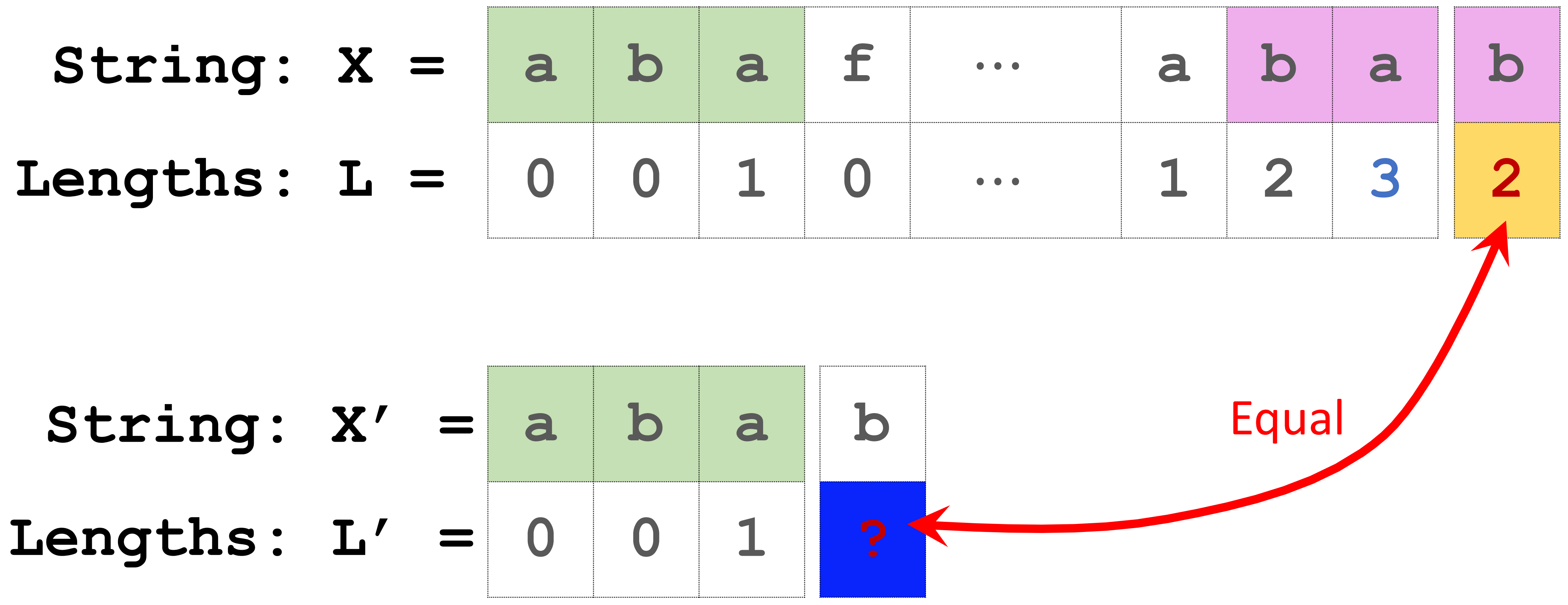


Case 3: Mismatch and L[end]≠0

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?



String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	2

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	2

X and X′ have the same prefixes.

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	2

X and X′ have the same prefixes.

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?

String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	2

X and X′ have the same prefixes.

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?

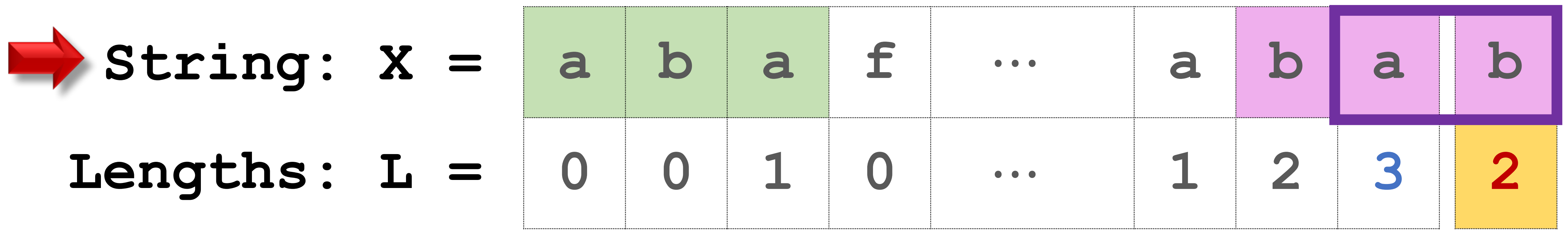
String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	2

X and **X′** have the same suffixes.

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

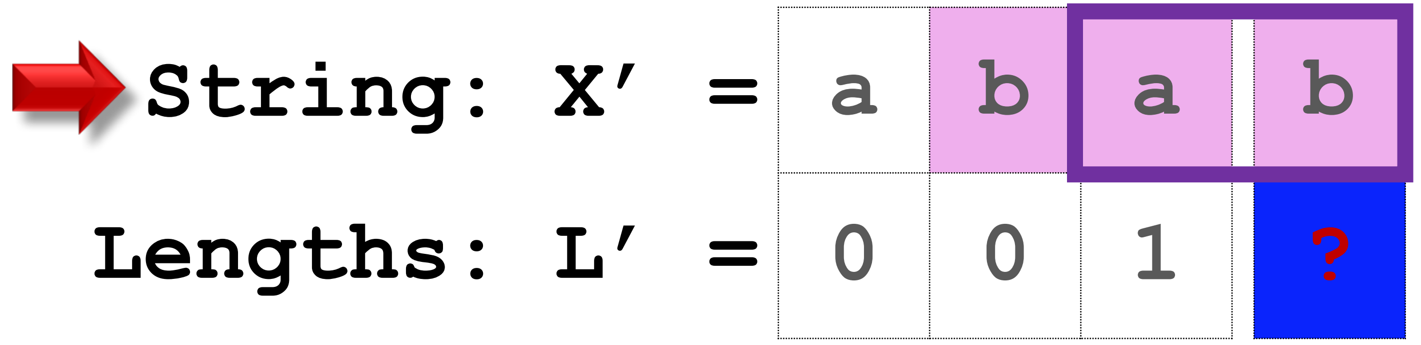
Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?



String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	2

X and X′ have the same suffixes.



String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?

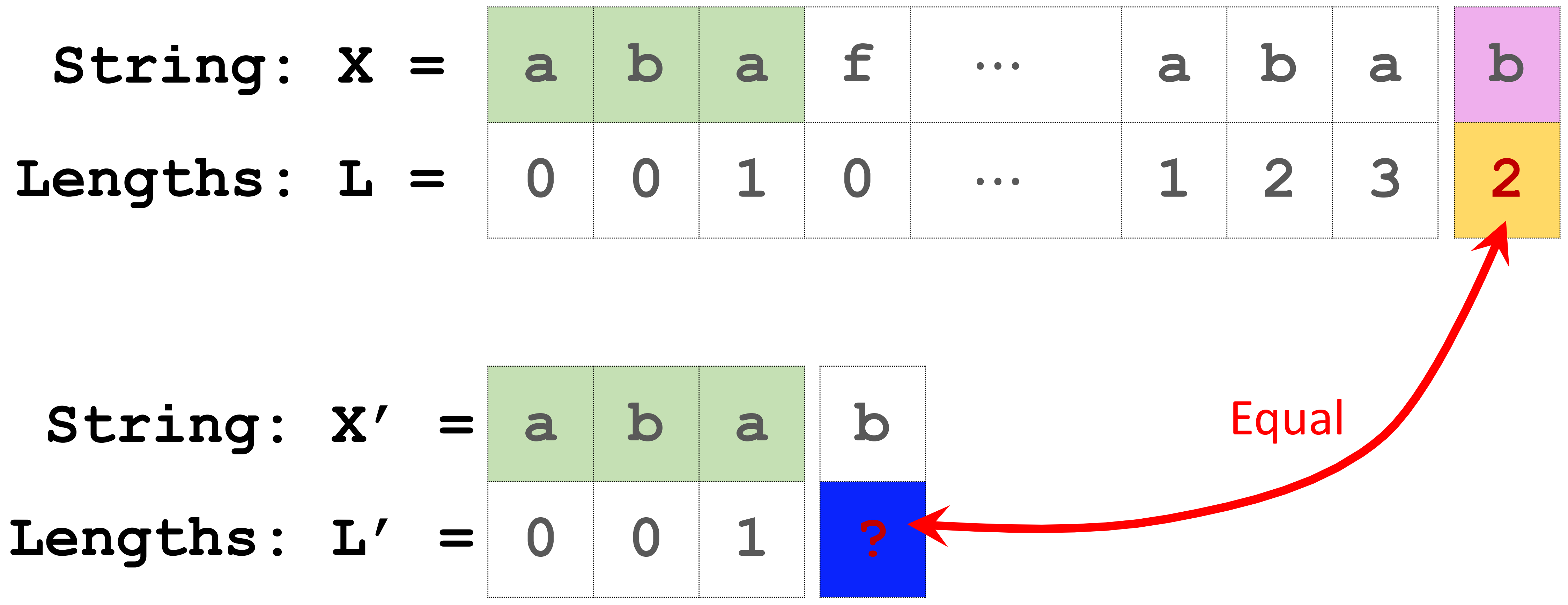
String: X =	a	b	a	f	⋯	a	b	a	b
Lengths: L =	0	0	1	0	⋯	1	2	3	2

X and X′ have the same suffixes.

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Case 3: Mismatch and L[end]≠0

Question: Why are the yellow entry and blue entry equal?



String: X =	a	b	a	f	…	a	b	a	b
Lengths: L =	0	0	1	0	…	1	2	3	2

String: X′ =	a	b	a	b
Lengths: L′ =	0	0	1	?

Summary

Longest Prefixes Suffixes Matching

Example string: `X = "ababa"`

- `prefixes = { "abab", "aba", "ab", "a" }.`
- `suffixes = { "baba", "aba", "ba", "a" }.`

Longest Prefixes Suffixes Matching

Example string: `X = “ababa”`

- `prefixes = { “abab”, “aba”, “ab”, “a” }.`
- `suffixes = { “baba”, “aba”, “ba”, “a” }.`
- Their intersection: `{ “aba”, “a” }.`

Longest Prefixes Suffixes Matching

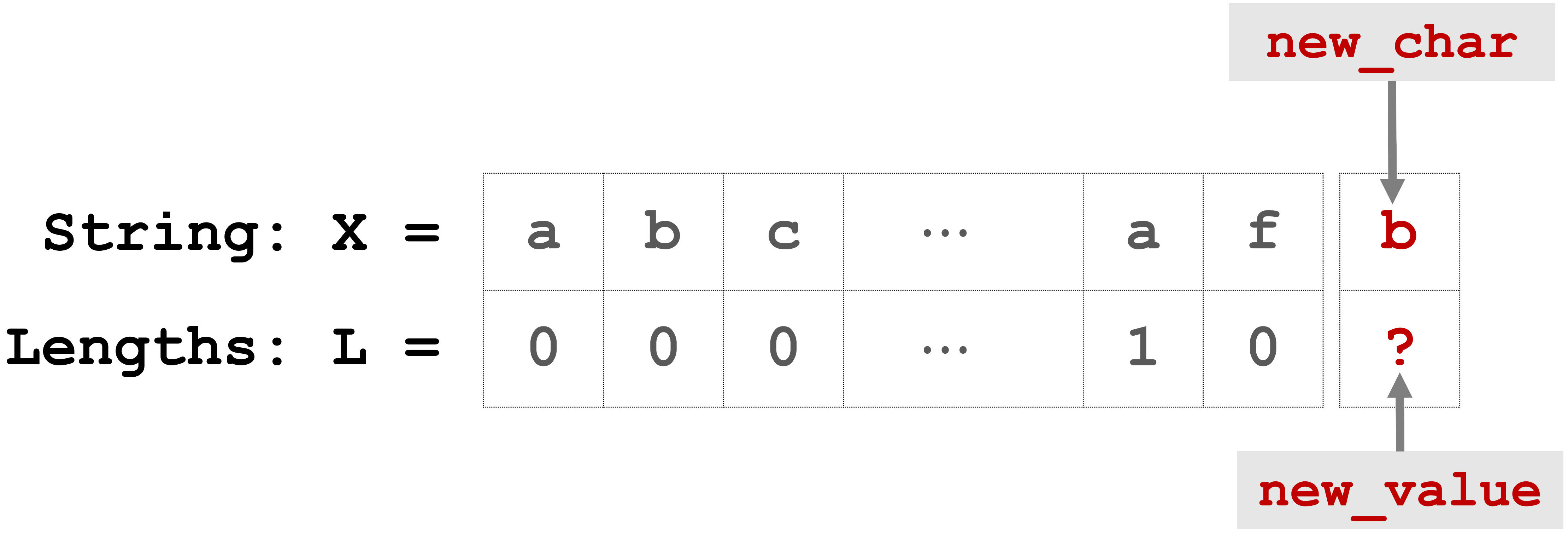
Example string: `X = "ababa"`

- `prefixes = { "abab", "aba", "ab", "a" }.`
- `suffixes = { "baba", "aba", "ba", "a" }.`
- Their intersection: `{ "aba", "a" }.`
- The longest matching: `"aba"`.

Longest Prefix Suffix Array

String: X =	a	b	c	⋯	a	f
Lengths: L =	0	0	0	⋯	1	0

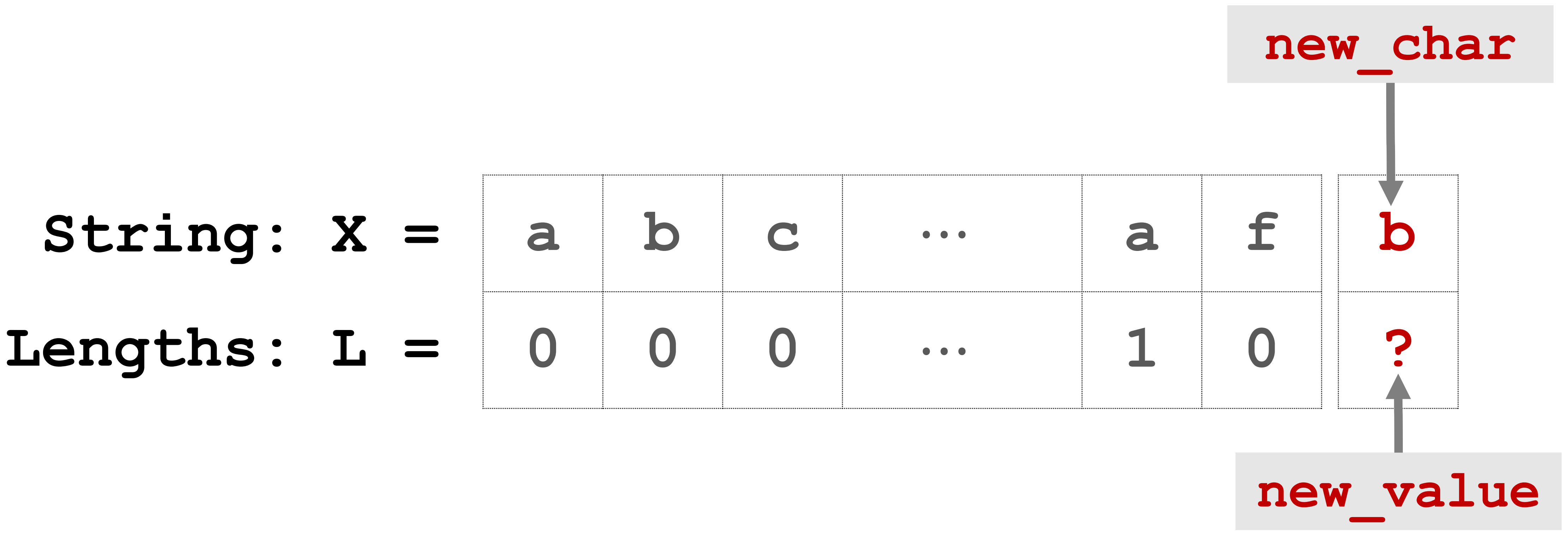
Longest Prefix Suffix Array



String: X =	a	b	c	…	a	f		b
Lengths: L =	0	0	0	…	1	0		?

Efficient Algorithm

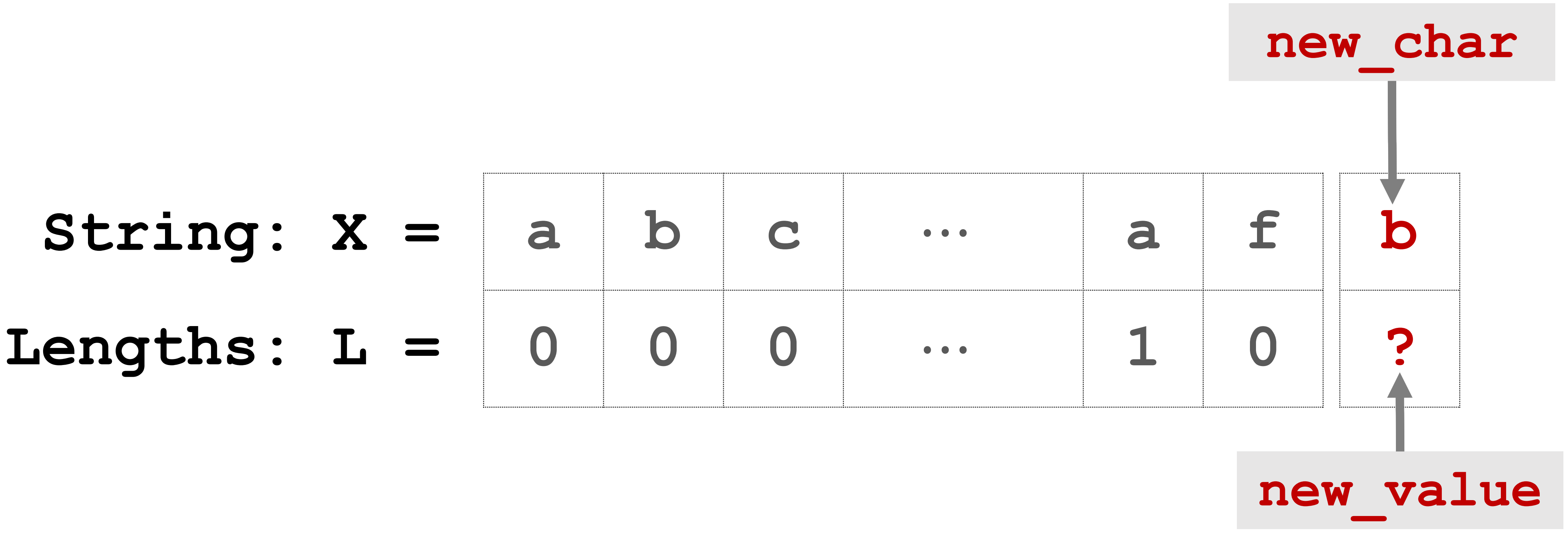
Function: `new_value = f(X, L, new_char)`



String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	?

Efficient Algorithm

Function: `new_value = f(X, L, new_char)`



String: X =	a	b	c	…	a	f	b
Lengths: L =	0	0	0	…	1	0	?

Time complexity: $O(1)$ time (amortized) for running the function once.

Efficient Algorithm

`Function:  new_value = f(X, L, new_char)`

`Step 1:  Decide the 3 cases`

- `Let j = L[end].`

Efficient Algorithm

```
Function:  new_value = f(X, L, new_char)
```

```
Step 1:  Decide the 3 cases
```

- `Let j = L[end].`
- `new_char == X[j]?`

Efficient Algorithm

```
Function:  new_value = f(X, L, new_char)
```

Step 1: Decide the 3 cases

- `Let j = L[end].`
- `new_char == X[j]?`
- `If equal ==> Case 1;`

Efficient Algorithm

`Function: new_value = f(X, L, new_char)`

Step 1: Decide the 3 cases

- `Let j = L[end].`
- `new_char == X[j]?`
- `If equal ==> Case 1;`
- `If unequal ==>` $\begin{cases} j = 0 \implies \text{Case 2;} \\ j \neq 0 \implies \text{Case 3.} \end{cases}$

Efficient Algorithm

```
Function:  value = f(X, L, new_char)
```

```
Step 2:  Different solutions to the 3 cases
```

- **Case 1** ==> **Return** `j + 1`.
- **Case 2** ==> **Return** `0`.
- **Case 3** ==> **Return** `f(X′, L′, new_char)`.

`X′ = X[0:j] and  L′ = L[0:j]`

Questions

Question 1:

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	?	?	?	1	2	3	4	?

Question 2:

X =	a	a	a	a	b	...	a	a	a	a	a
L =	0	1	2	3	?	...	1	2	3	4	?

Question 3:

X =	a	b	a	b	c	a	b	a	b	a	b
L =	0	0	1	2	?	?	2	3	4	?	?

Question 4:

X =	b	a	b	a	a	…	a	b	a	b	a	c	a
L =	0	0	1	2	?	…	2	3	?	?	?	0	?

Question 5: Fill in the red entries.

String: X =	a	?	?	?	f	…	?	?	?	b
Lengths: L =	0	0	1	2	?	…	?	?	?	4

Thank You!

Solution to Question 1

Solution to Question 1

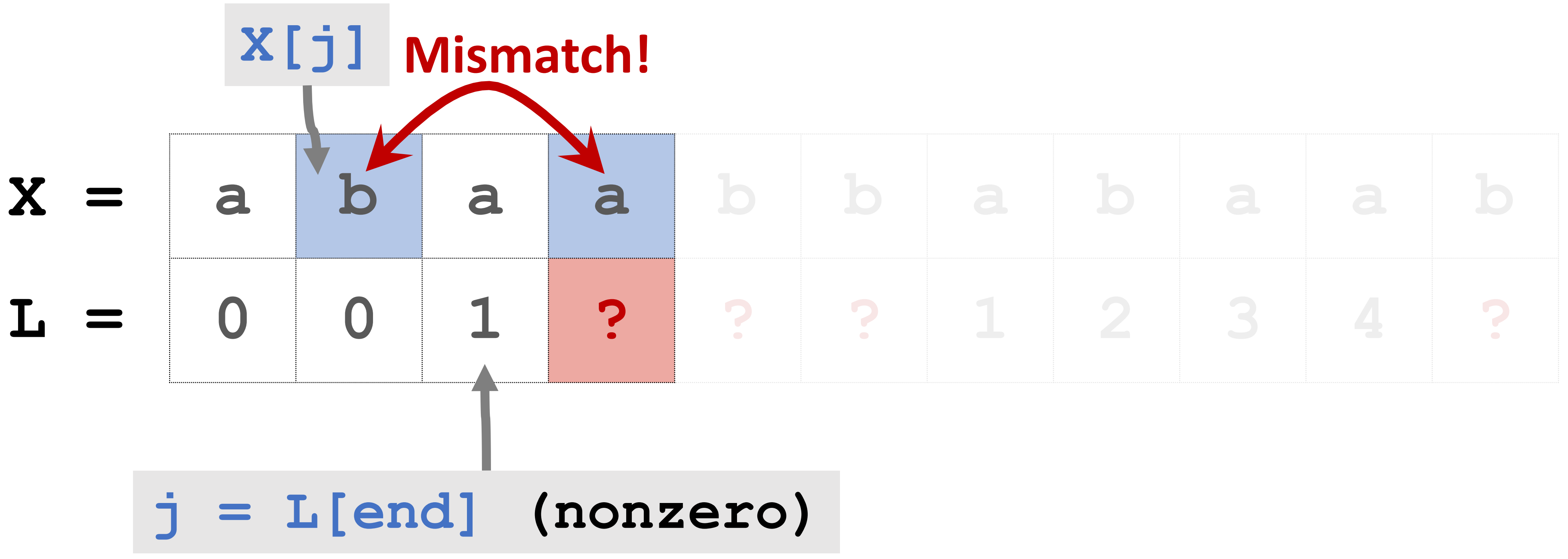
X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	?	?	?	1	2	3	4	?

Solution to Question 1(i)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	?	?	?	1	2	3	4	?

`j = L[end] (nonzero)`

Solution to Question 1(i)



- This is Case 3.
- Reduce the problem to a smaller problem.

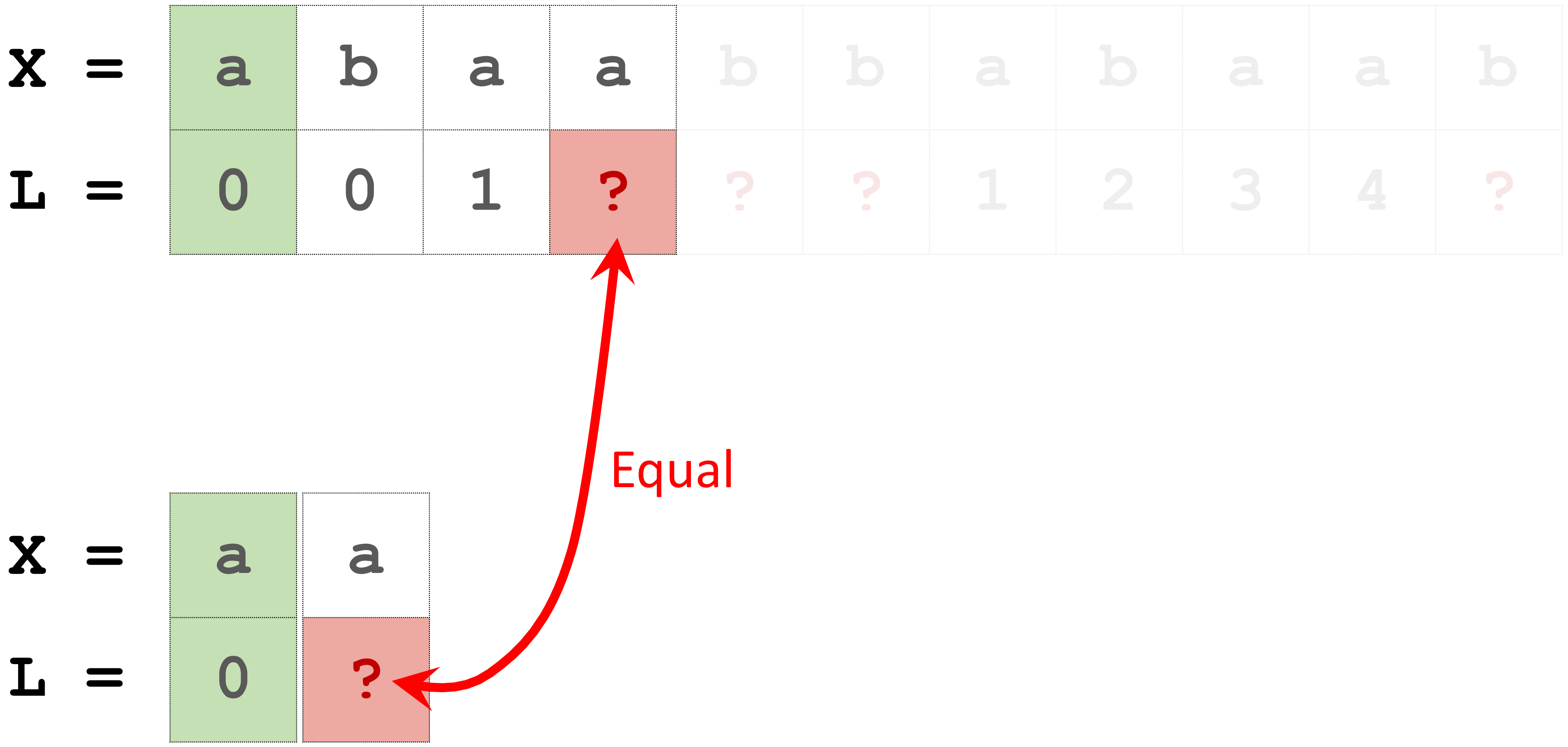
Solution to Question 1(i)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	?	?	?	1	2	3	4	?

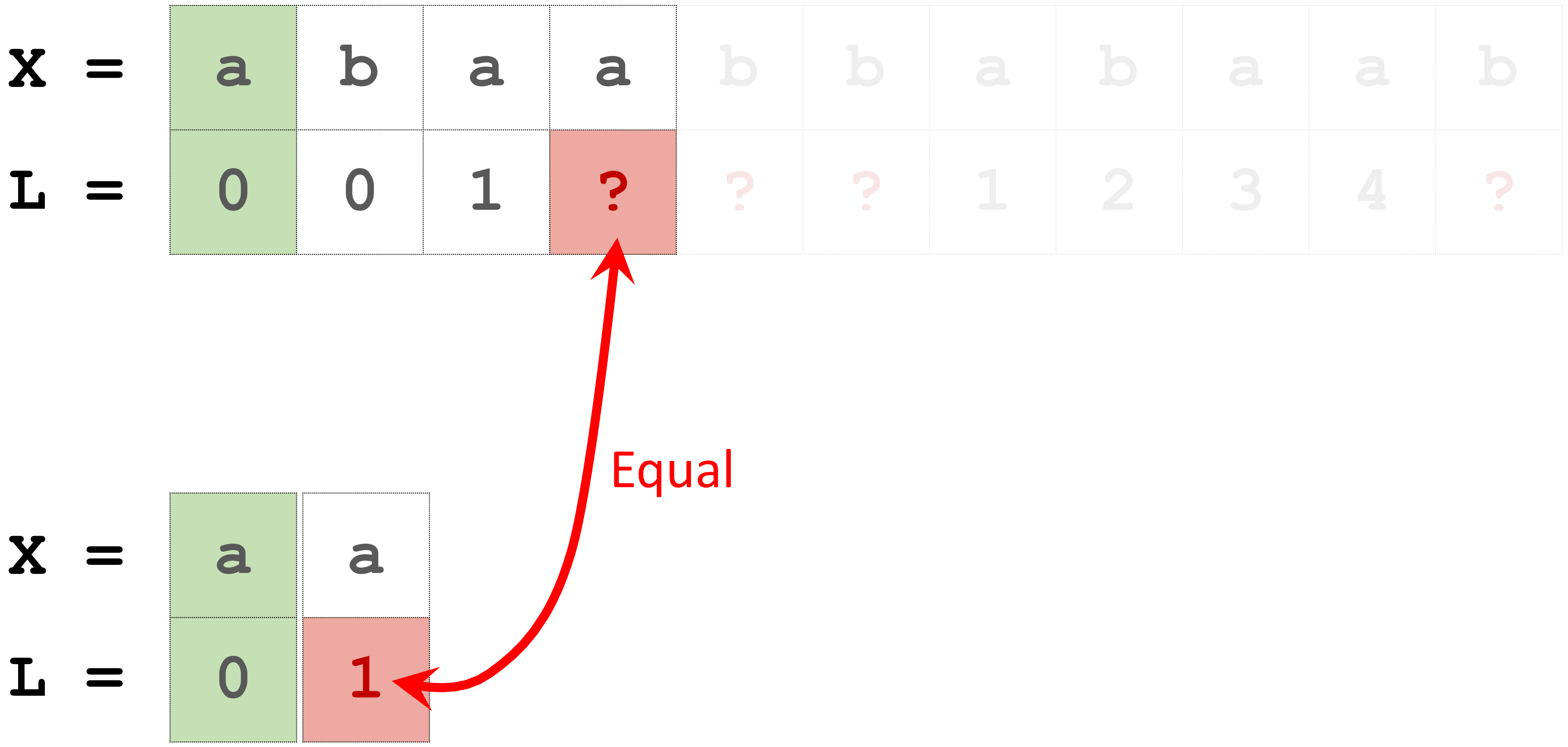
```
j = L[end]  (nonzero)
```

X =	a	a
L =	0	?

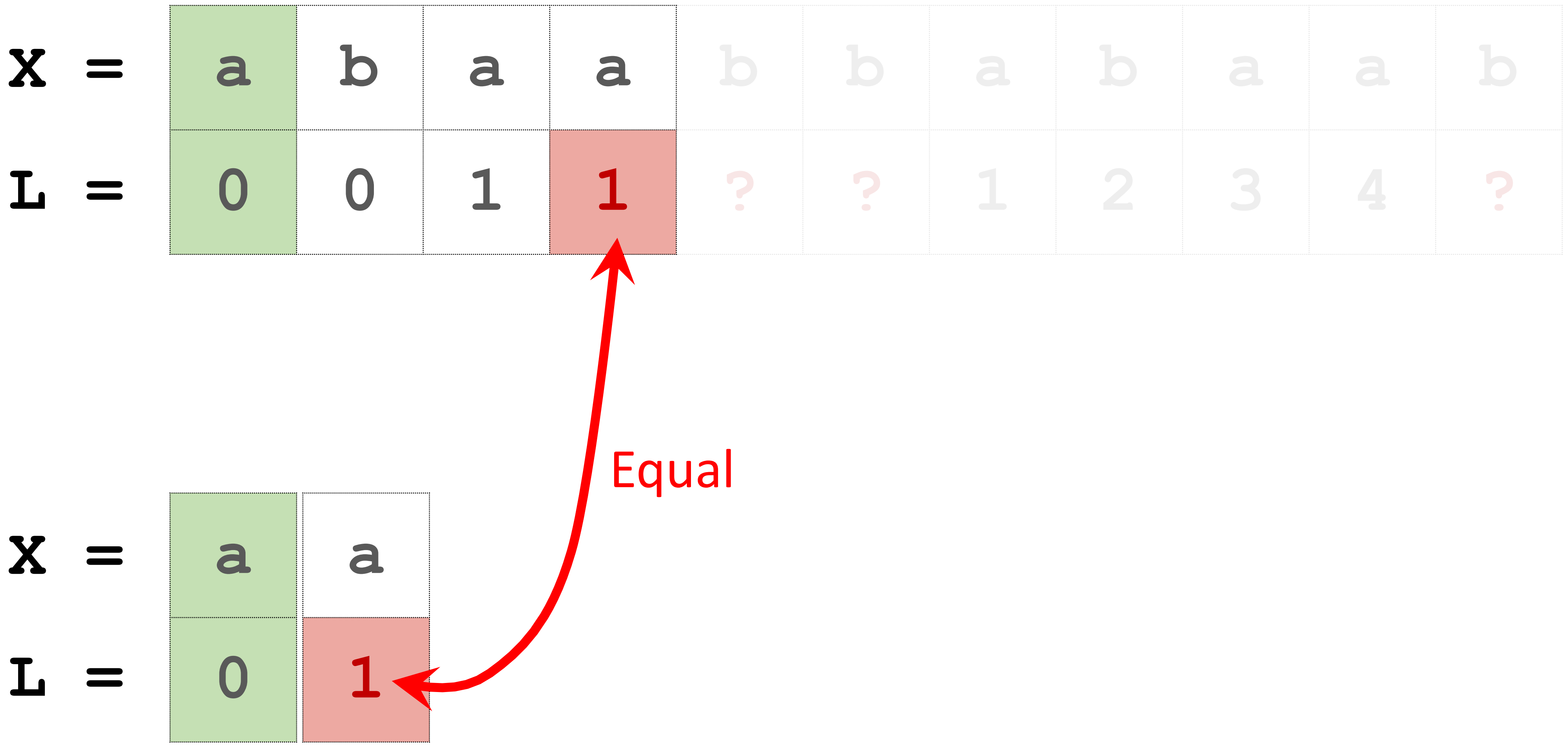
Solution to Question 1(i)



Solution to Question 1(i)



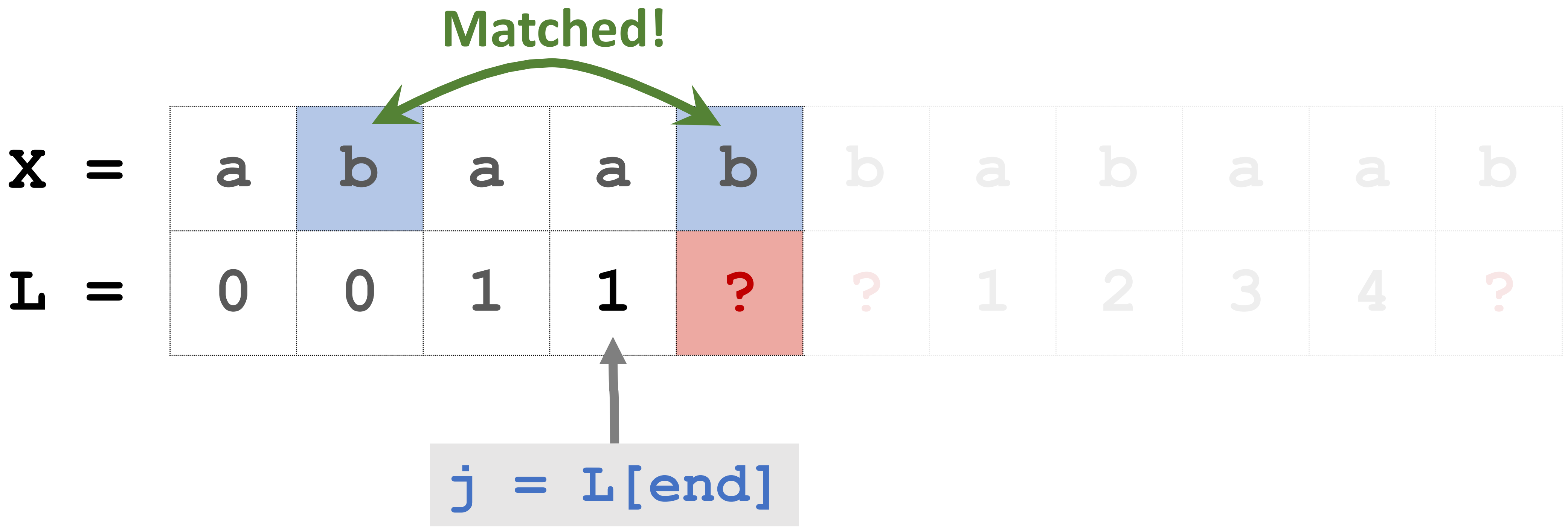
Solution to Question 1(i)



Solution to Question 1(ii)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	?	?	1	2	3	4	?

Solution to Question 1(ii)



- Case 1: the new char is equal to X[j].
- Then the new value is j+1.

Solution to Question 1(ii)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	?	1	2	3	4	?

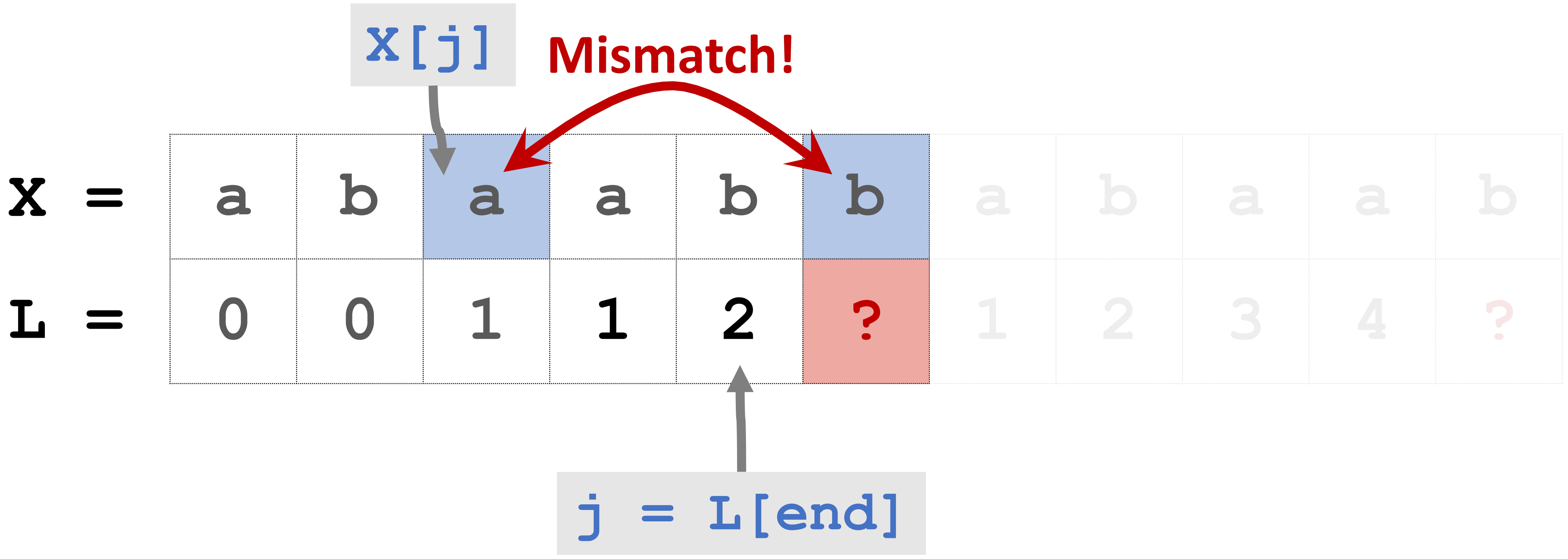
`j = L[end]`

- Case 1: the new char is equal to X[j].
- Then the new value is j+1.

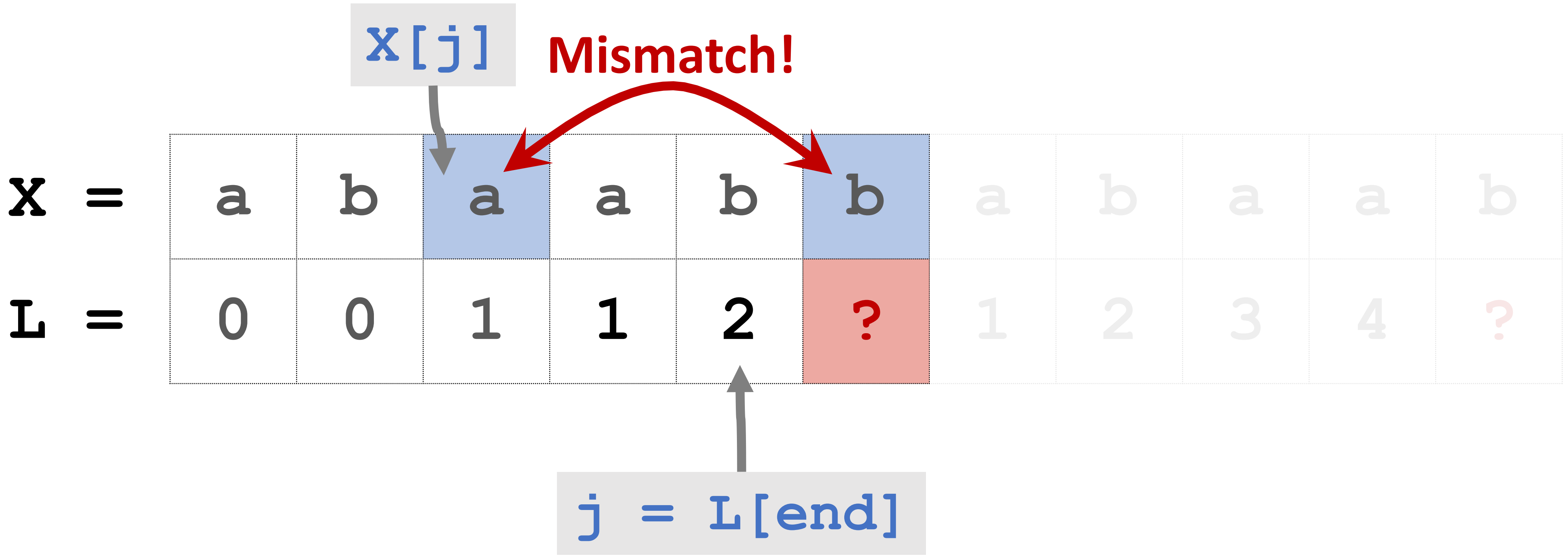
Solution to Question 1(iii)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	🟥 ?	1	2	3	4	?

Solution to Question 1(iii)

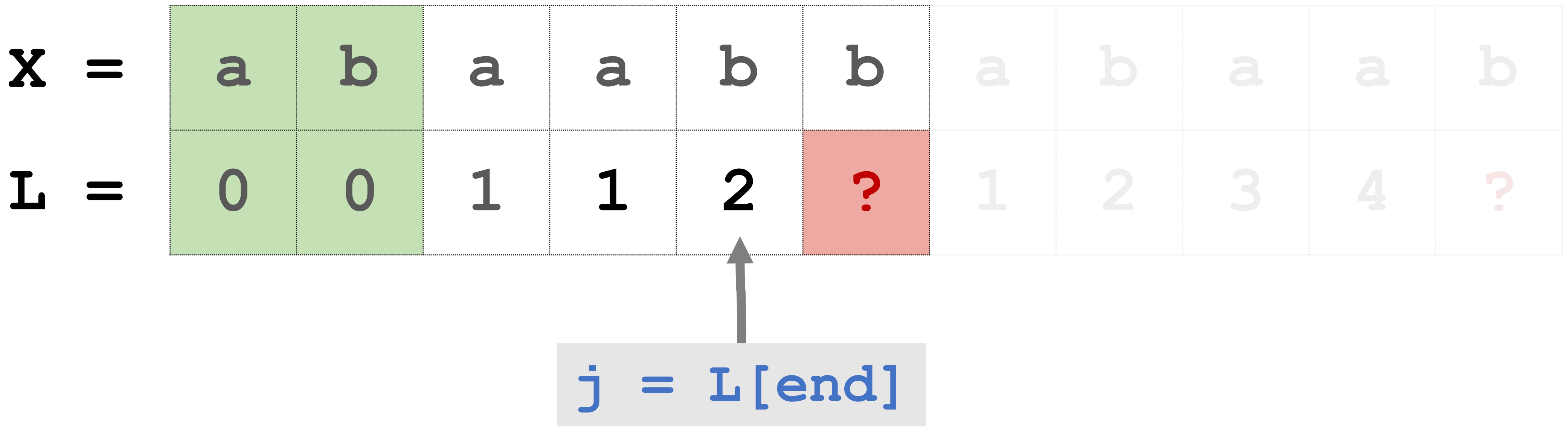


Solution to Question 1(iii)



- This is Case 3.
- Reduce the problem to a smaller problem.

Solution to Question 1(iii)



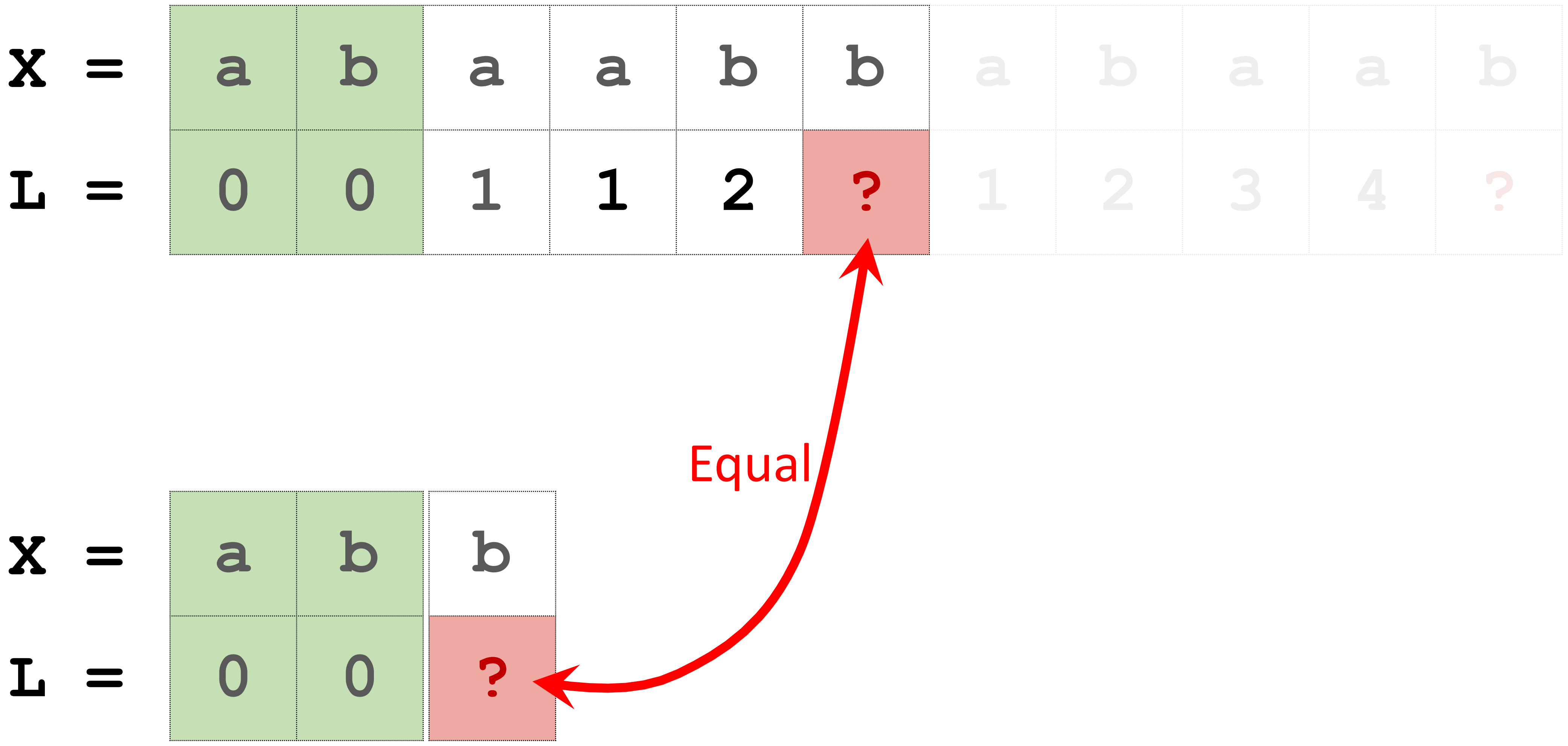
- This is Case 3.
- Reduce the problem to a smaller problem.

Solution to Question 1(iii)

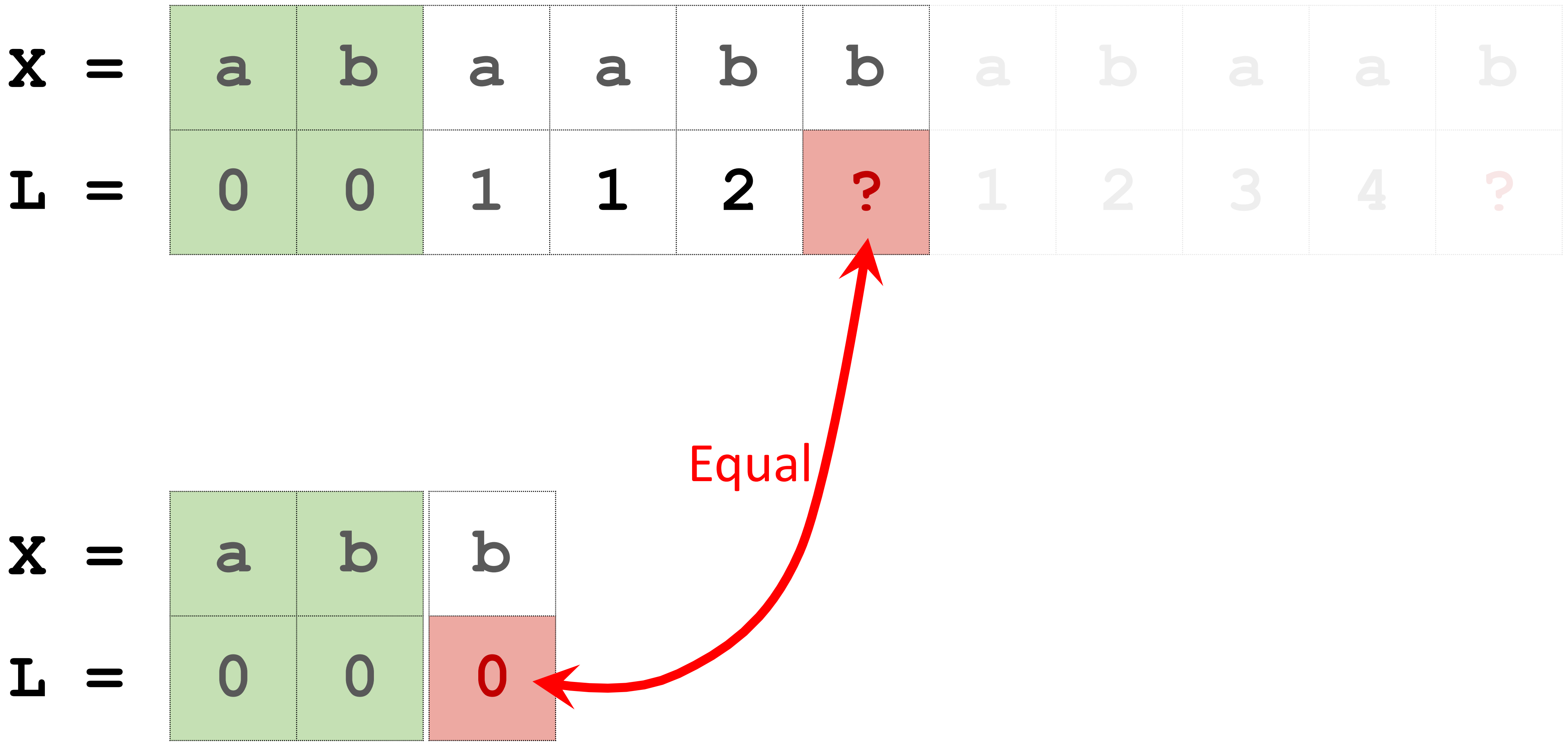
X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	?	1	2	3	4	?

X =	a	b
L =	0	0

Solution to Question 1(iii)



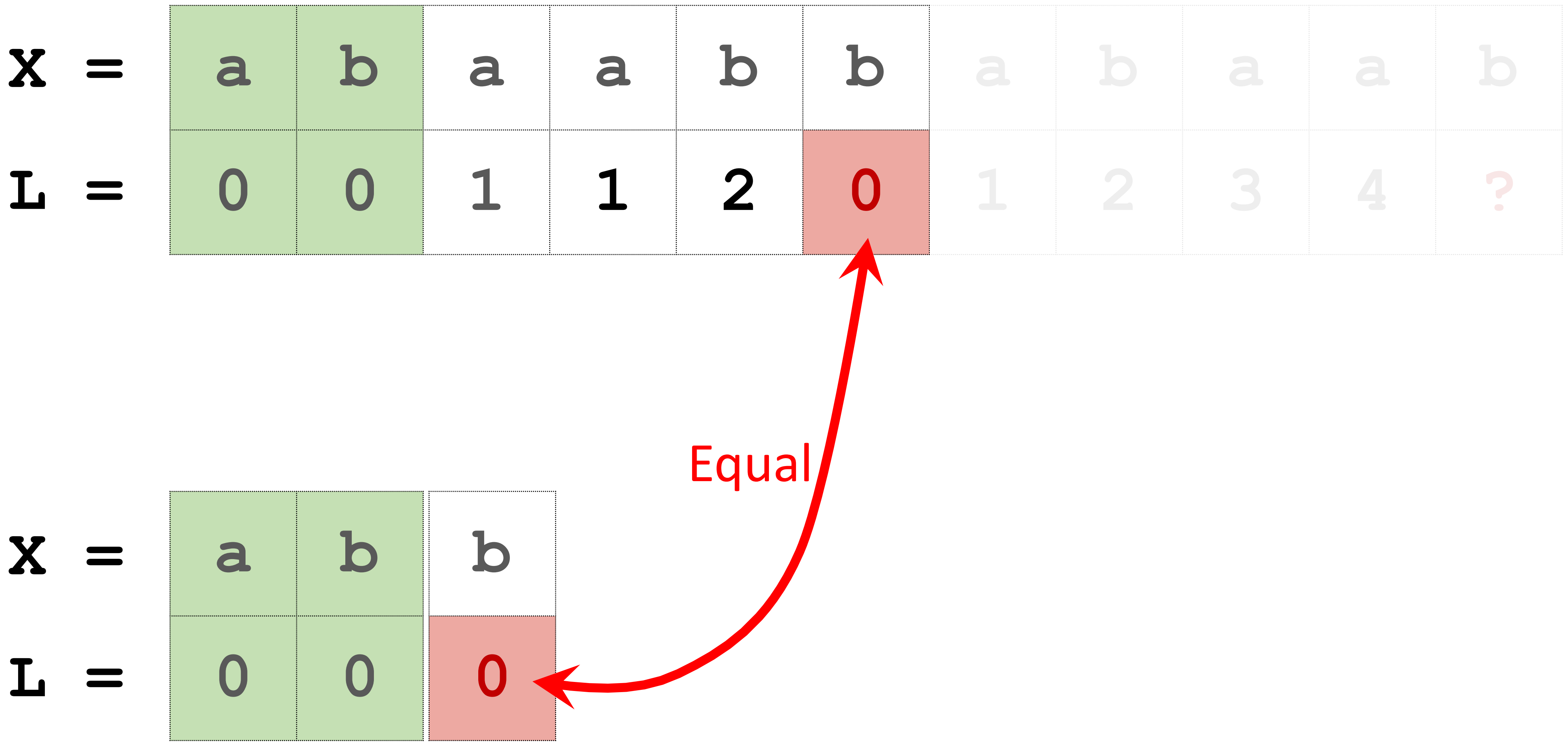
Solution to Question 1(iii)



X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	?	1	2	3	4	?

X =	a	b	b
L =	0	0	0

Solution to Question 1(iii)



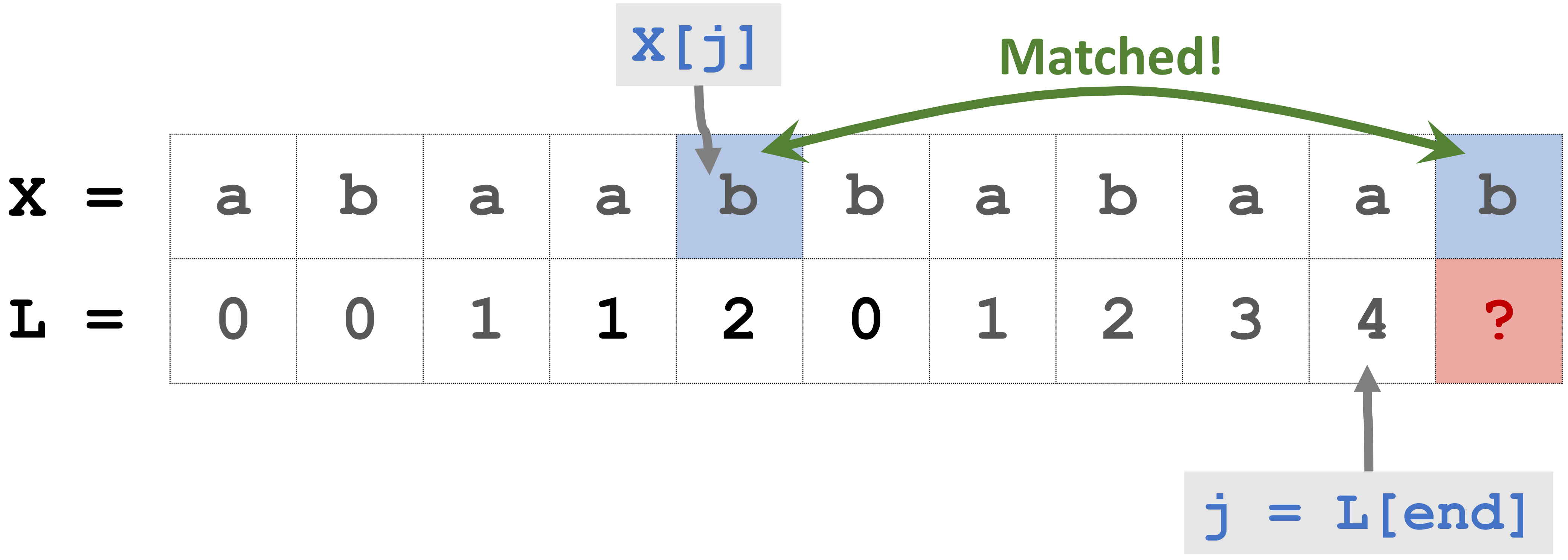
X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	0	1	2	3	4	?

X =	a	b	b
L =	0	0	0

Solution to Question 1(iv)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	0	1	2	3	4	?

Solution to Question 1(iv)



- **Case 1:** the **new char** is equal to X[j].
- Then the **new value** is j+1.

Solution to Question 1(iv)

X =	a	b	a	a	b	b	a	b	a	a	b
L =	0	0	1	1	2	0	1	2	3	4	5

`j = L[end]`

- **Case 1:** the **new char** is equal to `X[j]`.
- Then the **new value** is `j+1`.