Combinations of Coins

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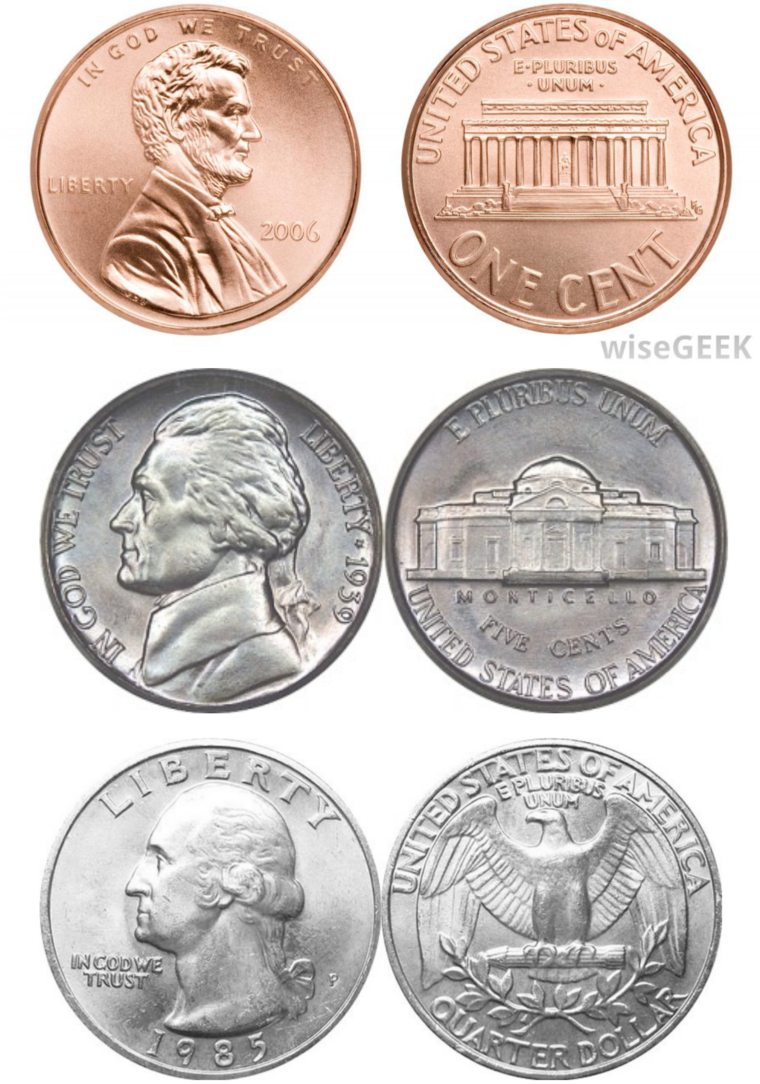
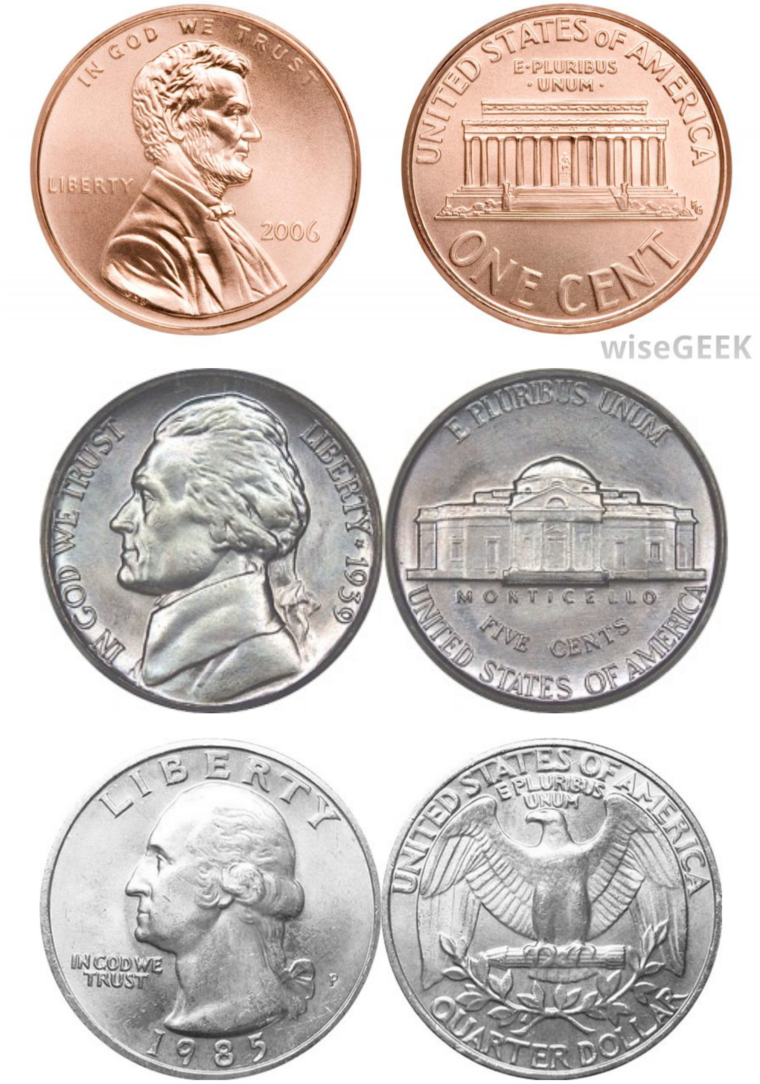
Combinations of Coins



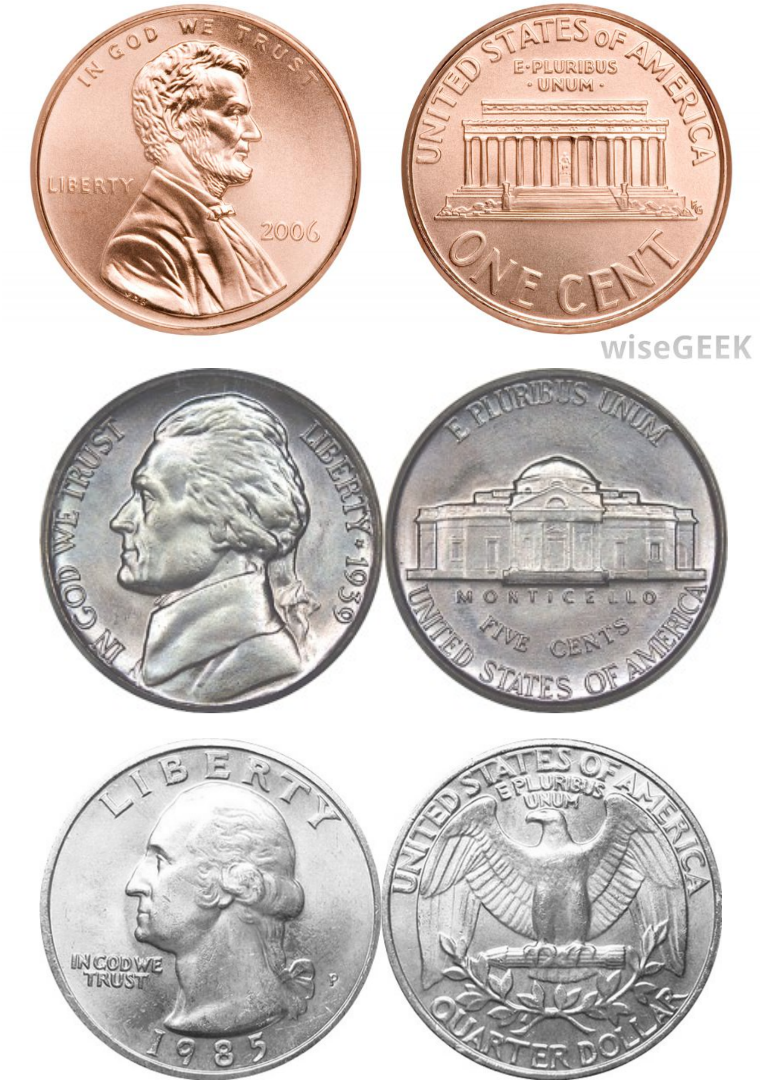
- **Inputs:**
  - Coins of different values, e.g., 1 cent, 2 cents, 5 cents, 25 cents, and 100 cents.
  - A total amount of money, e.g., 200 cents.
- **Output:** The number of combinations that make up that amount.
- **Assumption:** There are infinite number of each kind of coin.

Combinations of Coins



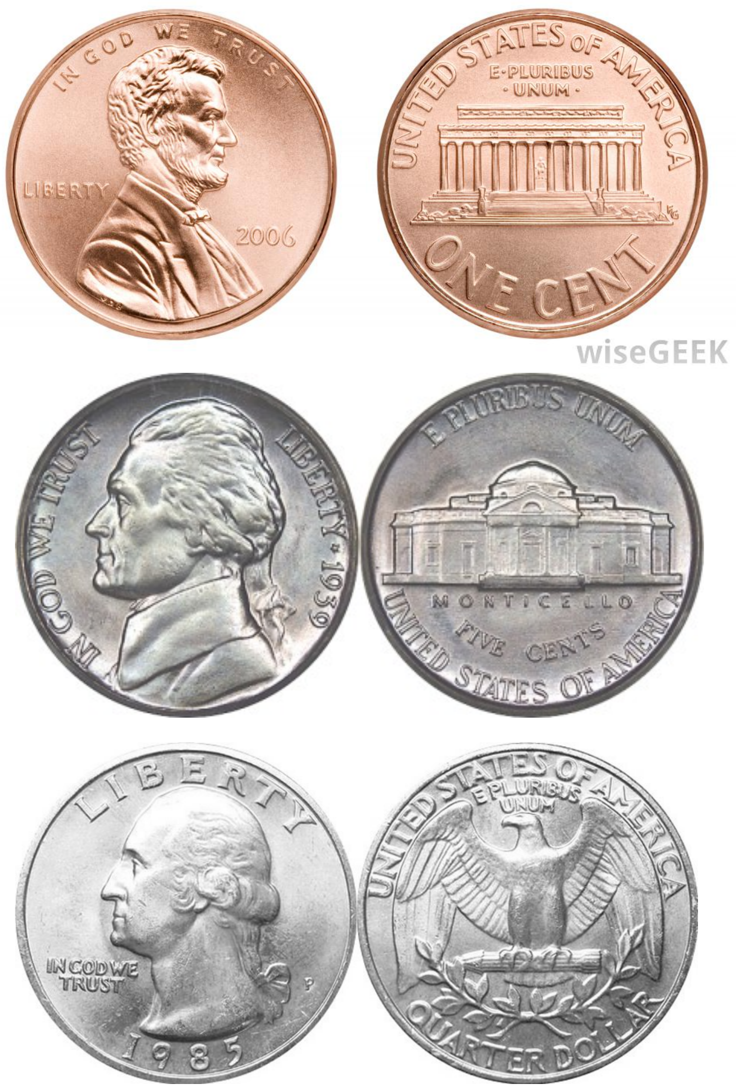
- **Example inputs:**
  - Amount: $n = 5$.
  - Coins' values: $\mathbf{v} = [1, 2, 5]$.

Combinations of Coins



- **Example inputs:**
  - Amount: $n = 5$.
  - Coins' values: $\mathbf{v} = [1, 2, 5]$.
- **Example output:** 4.
- Why? There are 4 ways to make up the amount:
  - $5 = 1 + 1 + 1 + 1 + 1$,
  - $5 = 1 + 1 + 1 + 2$,
  - $5 = 1 + 2 + 2$,
  - $5 = 5$.

Coin Change Problem



- **Example inputs:**
  - Amount: $n = 3$.
  - Coins' values: $\mathbf{v} = [2, 5, 10]$.
- **Example output:** 0.

Problem Formulation

- **Inputs:**
  - n: the total amount. (E.g., $n = 10$.)
  - $\mathbf{v} = [v_1, v_2, \cdots, v_m]$: values of coins. (E.g.,$\mathbf{v} = [1, 2, 5]$.)

Problem Formulation

- **Inputs:**
  - n: the total amount. (E.g., $n = 10$.)
  - $\mathbf{v} = [v_1, v_2, \cdots, v_m]$: values of coins. (E.g.,$\mathbf{v} = [1, 2, 5]$.)
- Let c_i be the number of the coin of value v_i.
- **Constraint:** $c_1, c_2, \cdots, c_m$ are such integers that

$$n = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m .$$

Problem Formulation

- **Inputs:**
  - n: the total amount. (E.g., $n = 10$.)
  - $\mathbf{v} = [v_1, v_2, \cdots, v_m]$: values of coins. (E.g.,$\mathbf{v} = [1, 2, 5]$.)
- Let c_i be the number of the coin of value v_i.
- **Constraint:** $c_1, c_2, \cdots, c_m$ are such integers that

$$n = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m.$$

- **Goal:** Find the number of unique vector $\mathbf{c} = [c_1, c_2, \cdots, c_m]$.

Problem Formulation

- **Inputs:**
  - $n = 5$: the total amount.
  - $\mathbf{v} = [v_1, v_2, v_3] = [1, 2, 5]$: values of coins.

Problem Formulation

- **Inputs:**
  - $n = 5$: the total amount.
  - $\mathbf{v} = [v_1, v_2, v_3] = [1, 2, 5]$: values of coins.

- There are four ways of combinations:
  - $n = 5 \times v_1 + 0 \times v_2 + 0 \times v_3$, $(\mathbf{c} = [5, 0, 0].)$
  - $n = 3 \times v_1 + 1 \times v_2 + 0 \times v_3$, $(\mathbf{c} = [3, 1, 0].)$
  - $n = 1 \times v_1 + 2 \times v_2 + 0 \times v_3$, $(\mathbf{c} = [1, 2, 0].)$
  - $n = 0 \times v_1 + 0 \times v_2 + 1 \times v_3$, $(\mathbf{c} = [0, 0, 1].)$

Optimal Substructure

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Constraint: $n = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m$.

Question: What are the possible choices of c_i?

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Constraint: $n = c_1 v_1 + c_2 v_2 + \cdots + c_m v_m$.

Question: What are the possible choices of c_i?

- The integer c_i is at least 0 and at most $\left\lfloor \frac{n}{v_i} \right\rfloor$.
- Thus, c_i must be in the set $\left\{0, 1, \cdots, \left\lfloor \frac{n}{v_i} \right\rfloor\right\}$.

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.
- c_m is chosen from the set $\left\{0, 1, \cdots, \left\lfloor \frac{n}{v_m} \right\rfloor\right\}$.

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.
- c_m is chosen from the set $\left\{0, 1, \cdots, \left\lfloor \frac{n}{v_m} \right\rfloor\right\}$.

What if I use $c_m = 0$ coins of value v_m?

- The problem is reduced to: *"Changing the amount of n into the coins of values $v_1, v_2, \cdots, v_{m-1}$."*
- The number of combinations is $\mathrm{F}[m-1][n]$.

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.
- c_m is chosen from the set $\left\{0, 1, \cdots, \left\lfloor \frac{n}{v_m} \right\rfloor\right\}$.

What if I use $c_m = 1$ coins of value v_m?

- The problem is reduced to: *"Changing the amount of* $n - v_m$ *into the coins of values* $v_1, v_2, \cdots, v_{m-1}$*."*
- The number of combinations is $\mathrm{F}[m-1][n - v_m]$.

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.
- c_m is chosen from the set $\left\{0, 1, \cdots, \left\lfloor \frac{n}{v_m} \right\rfloor\right\}$.

What if I use $c_m = 2$ coins of value v_m?

- The problem is reduced to: *"Changing the amount of $n - 2 \times v_m$ into the coins of values $v_1, v_2, \cdots, v_{m-1}$."*
- The number of combinations is $\mathrm{F}[m-1][n - 2 \times v_m]$.

Derive Optimal Substructure

- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.
- c_m is chosen from the set $\left\{0, 1, \cdots, \left\lfloor \frac{n}{v_m} \right\rfloor\right\}$.

What if I use $\left\lfloor \frac{n}{v_m} \right\rfloor$ coins of value v_m?

- The problem is reduced to: *"Changing the amount of* $n - \left\lfloor \frac{n}{v_m} \right\rfloor \times v_m$ *into the coins of values* $v_1, v_2, \cdots, v_{m-1}$*."*
- The number of combinations is $\mathrm{F}[m-1]\left[n - \left\lfloor \frac{n}{v_m} \right\rfloor \times v_m\right]$.

Optimal Substructure

Formula: $\underline{F[m][n]} = \underline{\sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m]}.$

Optimal Substructure

Formula: $\mathrm{F}[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} \mathrm{F}[m-1][n - c_m \times v_m]$.

There are $m-1$ kinds of coins remaining.

- We use c_m coins of value v_m.
- The remaining amount is $n - c_m \times v_m$.

Dynamic Programming

Example

`Inputs: v=[2,5] and n=10.`

$\mathbf{v} =$

2

5

Example

`Inputs: v=[2,5] and n=10.`

`In the table, the (i, j)-th entry is F[i][j].`

Total amount:

		0	1	2	3	4	5	6	7	8	9	10	v =
Coin types:	1												2
	2												5

Base case: only one type of coin

Total amount:

		0	1	2	3	4	5	6	7	8	9	10
Coin types:	1											
	2											

$\mathbf{v} =$

2

5

Base case: only one type of coin

- How to change change the sum of $n = 0$ into 2-cent coins.
- Only 1 combination, $c_1 = 0$, satisfies $0 = c_1 \times 2$.

Total amount:

	0	1	2	3	4	5	6	7	8	9	10
Coin types: 1	**?**										

$\mathbf{v} =$

2

Base case: only one type of coin

- How to change change the sum of $n = 0$ into 2-cent coins.
- Only 1 combination, $c_1 = 0$, satisfies $0 = c_1 \times 2$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	**?**										

v = 2

```
F[1][0] = 1
```

Base case: only one type of coin

- How to change change the sum of $n = 1$ into 2-cent coins.
- No c_1 can satisfies $1 = c_1 \times v_1$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	?									

$\mathbf{v} =$ 2

Base case: only one type of coin

- How to change change the sum of $n = 1$ into 2-cent coins.
- No c_1 can satisfies $1 = c_1 \times v_1$.

Total amount:

Coin types:		0	1	2	3	4	5	6	7	8	9	10
	1	1	?									

v = 2

`F[1][1] = 0`

Base case: only one type of coin

- How to change change the sum of $n = 2$ into 2-cent coins.
- Only 1 combination, $c_1 = 1$, satisfies $2 = c_1 \times 2$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	0	?								

$\mathbf{v} =$ 2

Base case: only one type of coin

- How to change change the sum of $n = 2$ into 2-cent coins.
- Only 1 combination, $c_1 = 1$, satisfies $2 = c_1 \times 2$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	0	?								

$\mathbf{v} =$ 2

`F[1][2] = 1`

Base case: only one type of coin

- $F[1][n] = \begin{cases} 1, & \text{if } n \bmod v_1 = 0; \\ 0, & \text{otherwise.} \end{cases}$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	0	1								

v = 2

Base case: only one type of coin

- $F[1][n] = \begin{cases} 1, & \text{if } n \bmod v_1 = 0; \\ 0, & \text{otherwise.} \end{cases}$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	0	1	0	1	0	1	0	1	0	1

v = 2

m > 1

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	0	1	0	1	0	1	0	1	0	1	2
2											?	5

m > 1

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- c_2 can be 0, 1, or 2. (Because $\frac{n}{v_2} = \frac{10}{5} = 2$.)

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	$\mathbf{v} =$
1	1	0	1	0	1	0	1	0	1	0	1	2
2											**?**	5

m > 1

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- c_2 can be 0, 1, or 2. (Because $\frac{n}{v_2} = \frac{10}{5} = 2$.)
- $F[2][10] = F[1][0] + F[1][5] + F[1][10]$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	**1**	0	1	0	1	**0**	1	0	1	0	**1**	2
2											**?**	5

m > 1

$$\mathrm{F}[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} \mathrm{F}[m-1][n - c_m \times v_m].$$

- c_2 can be 0, 1, or 2. (Because $\frac{n}{v_2} = \frac{10}{5} = 2$.)
- $\mathrm{F}[2][10] = \mathrm{F}[1][0] + \mathrm{F}[1][5] + \mathrm{F}[1][10]$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	**1**	0	1	0	1	**0**	1	0	1	0	**1**
2											**2**

v =

2
5

Another Example

Another Example

`Inputs: v=[1,3,5] and n=10.`

Another Example

`Inputs: v=[1,3,5] and n=10.`

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1												1
2												3
3												5

Base case: only one type of coin

- $F[1][n] = \begin{cases} 1, & \text{if } n \bmod v_1 = 0; \\ 0, & \text{otherwise.} \end{cases}$ (Note that $v_1 = 1$.)

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1											

$\mathbf{v} =$ 1

Base case: only one type of coin

- $F[1][n] = \begin{cases} 1, & \text{if } n \bmod v_1 = 0; \\ 0, & \text{otherwise.} \end{cases}$ (Note that $v_1 = 1$.)

Total amount:

	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1

Coin types:

$\mathbf{v} =$ 1

$\mathbf{m = 2, n = 5}$

Total amount:

		0	1	2	3	4	5	6	7	8	9	10
Coin types:	1	1	1	1	1	1	1	1	1	1	1	1
	2											

$\mathbf{v} =$

1

3

$\mathbf{m} = 2, \mathbf{n} = 5$

$$F[m][n] \;=\; \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

Total amount:

Coin types:	0	1	2	3	4	$\boxed{5}$	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2						?					

v =

1

3

$\mathbf{m} = 2$, $\mathbf{n} = 5$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $c_2 \in \{0, 1\}$. (Because $\left\lfloor \frac{n}{v_2} \right\rfloor = \left\lfloor \frac{5}{3} \right\rfloor = 1$.)
- The remaining amount can be 5 or 2.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2						?					

$\mathbf{v} =$

1

3

$m = 2$, $n = 5$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[2][5] = F[1][5] + F[1][2].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2						?						3

m = 2, n = 5

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[2][5] = F[1][5] + F[1][2].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2						?						3

$\mathbf{m = 2, n = 5}$

$$F[m][n] \;=\; \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[2][5] = F[1][5] + F[1][2].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	1	**1**	1	1	**1**	1	1	1	1	1
2						**2**					

v =

1

3

$\mathbf{m} = 2$, $\mathbf{n} = 9$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

Total amount:

Coin types:	v =	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1	1
2	3						2				?	

$m = 2, n = 9$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $c_2 \in \{0, 1, 2, 3\}$. (Because $\left\lfloor \frac{n}{v_2} \right\rfloor = \left\lfloor \frac{9}{3} \right\rfloor = 3$.)
- The remaining amount can be 9, 6, 3, or 0.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2						2				?		3

$\mathbf{m = 2, n = 9}$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[2][9] = F[1][9] + F[1][6] + F[1][3] + F[1][0].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2						2				?		3

$m = 2, n = 9$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[2][9] = F[1][9] + F[1][6] + F[1][3] + F[1][0].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	**1**	1	1	**1**	1	1	**1**	1	1	**1**	1	1
2						2				**?**		3

m = 2, n = 9

$$F[m][n] \; = \; \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[2][9] = F[1][9] + F[1][6] + F[1][3] + F[1][0].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	**1**	1	1	**1**	1	1	**1**	1	1	**1**	1
2						2				**4**	

v =

1

3

m = 2

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4	3

$\mathbf{m} = 3$, $\mathbf{n} = 4$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4	3
3					?							5

$\mathbf{m} = 3$, $\mathbf{n} = 4$

$$\mathrm{F}[m][n] \ = \ \sum_{c_m=0}^{\lfloor n/v_m \rfloor} \mathrm{F}[m-1][n - c_m \times v_m].$$

- $c_3 \in \{0\}$. (Because $\left\lfloor \frac{n}{v_3} \right\rfloor = \left\lfloor \frac{4}{5} \right\rfloor = 0$.)
- The remaining amount is 4.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4	3
3					**?**							5

$m = 3$, $n = 4$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[3][4] = F[2][4]$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4	3
3					?							5

$\mathbf{m = 3, n = 4}$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[3][4] = F[2][4].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	**2**	2	3	3	3	4	4
3					**?**						

v =

1

3

5

m = 3, n = 4

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[3][4] = F[2][4]$.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	**2**	2	3	3	3	4	4
3					**2**						

v =
1
3
5

m = 3, n = 10

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4	3
3					2						?	5

$\mathbf{m = 3, n = 10}$

$$\mathrm{F}[m][n] \; = \; \sum_{c_m=0}^{\lfloor n/v_m \rfloor} \mathrm{F}[m-1][n - c_m \times v_m].$$

- $c_3 \in \{0, 1, 2\}$. (Because $\left\lfloor \frac{n}{v_3} \right\rfloor = \left\lfloor \frac{10}{5} \right\rfloor = 2$.)
- The remaining amount can be 10, 5, 0.

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4	3
3					2						?	5

$m = 3$, $n = 10$

$$F[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} F[m-1][n - c_m \times v_m].$$

- $F[3][10] = F[2][10] + F[2][5] + F[2][0].$

Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	3	3	3	4	4
3					2						?

v = 1, 3, 5

$\mathbf{m = 3, n = 10}$

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Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10	v =
1	1	1	1	1	1	1	1	1	1	1	1	1
2	**1**	1	1	2	2	**2**	3	3	3	4	**4**	3
3					2						**?**	5

$\mathbf{m = 3, n = 10}$

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Total amount:

Coin types:	0	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1	1
2	**1**	1	1	2	2	**2**	3	3	3	4	**4**
3					2						**7**

$\mathbf{v} =$ 1, 3, 5

Summary

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- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.

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- Change the amount of n into coins of values $v_1, v_2, \cdots, v_m$.
- Let $\mathrm{F}[m][n]$ be the number of combinations.
- Optimal substructure:

$$\mathrm{F}[m][n] = \sum_{c_m=0}^{\lfloor n/v_m \rfloor} \mathrm{F}[m-1][n - c_m \times v_m].$$

- Build an $m \times (n+1)$ table and fill the table.
- Return the bottom right element, $\mathrm{F}[m][n]$.

Question

Fill the table

Inputs: $\mathbf{v} = [2, 3, 4, 5]$ and $n = 10$.

Total amount

Coin types	0	1	2	3	4	5	6	7	8	9	10	$\mathbf{v} =$
1												2
2												3
3												4
4												5

Thank You!