Dinic's Algorithm

Shusen Wang

Comparisons

- m: the number of edges.
- n: the number of vertices.
- Time complexity of Edmonds–Karp algorithm [1] is $O(m^2 \cdot n)$.
- Time complexity of Dinic's algorithm [2] is $O(m \cdot n^2)$. (Faster because m is bigger than n.)

Reference

1. Jack Edmonds and Richard M Karp. Theoretical improvements in algorithmic efficiency for network flow problems. *Journal of the ACM.* 19 (2): 248–264, 1972.
2. Yefim Dinitz. Algorithm for solution of a problem of maximum flow in a network with power estimation. *Proceedings of the USSR Academy of Sciences*, 11: 1277–1280, 1970.

Dinic's Algorithm

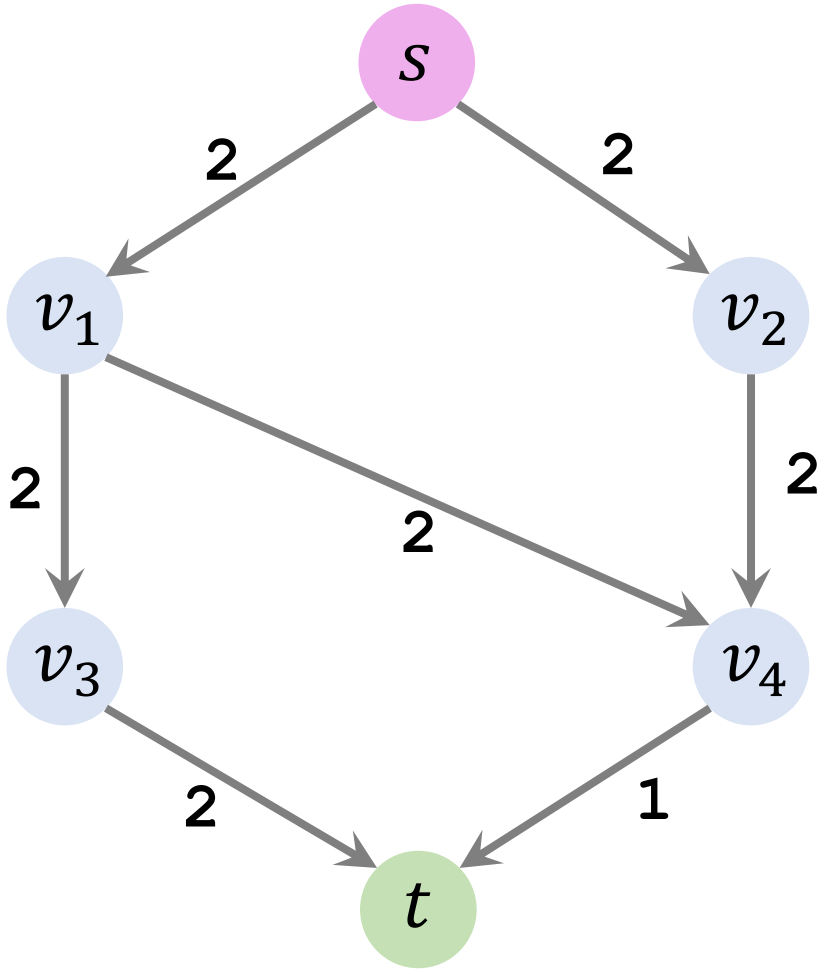
- Yefim Dinitz published "Dinitz's Algorithm" in USSR, 1970 [1].
- "Dinitz's Algorithm" was introduced to the westerners by Even & Tarjan's 1975 paper [2].
- Even & Tarjan misspelled Dinitz's name as "Dinic" [2].

Reference

1. Yefim Dinitz. Algorithm for solution of a problem of maximum flow in a network with power estimation. *Proceedings of the USSR Academy of Sciences*, 11: 1277–1280, 1970.
2. Shimon Even and R. Endre Tarjan. Network Flow and Testing Graph Connectivity. *SIAM Journal on Computing*, 4 (4): 507–518, 1975.

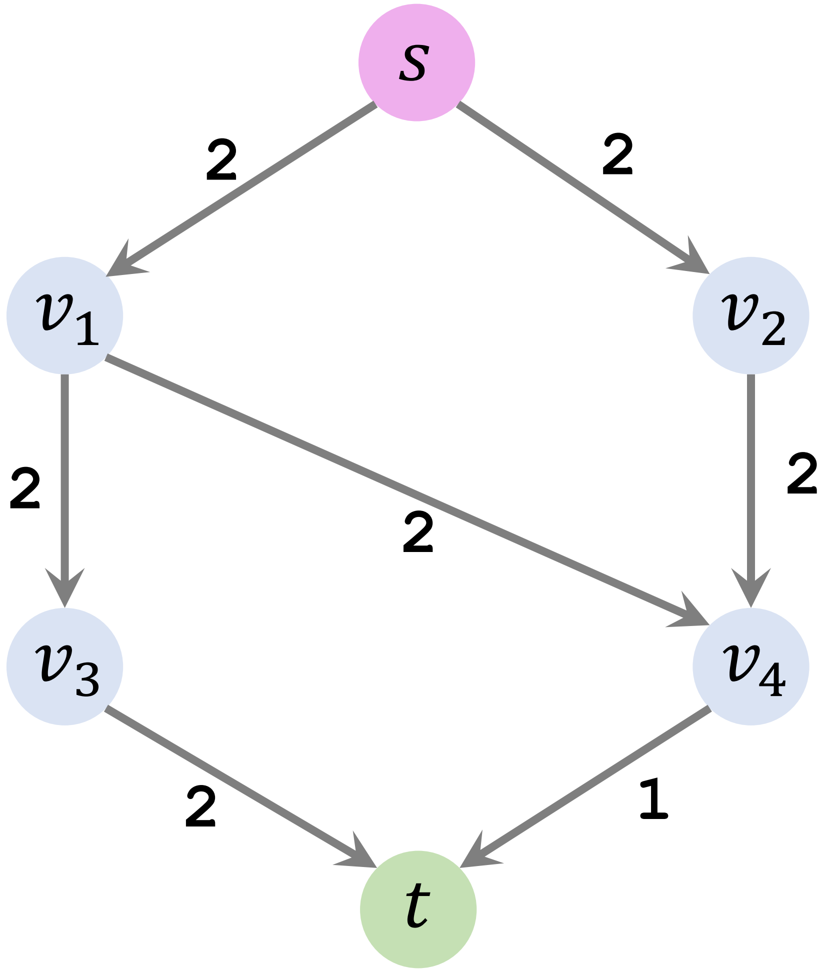
Key Concept: Blocking Flow

Blocking Flow

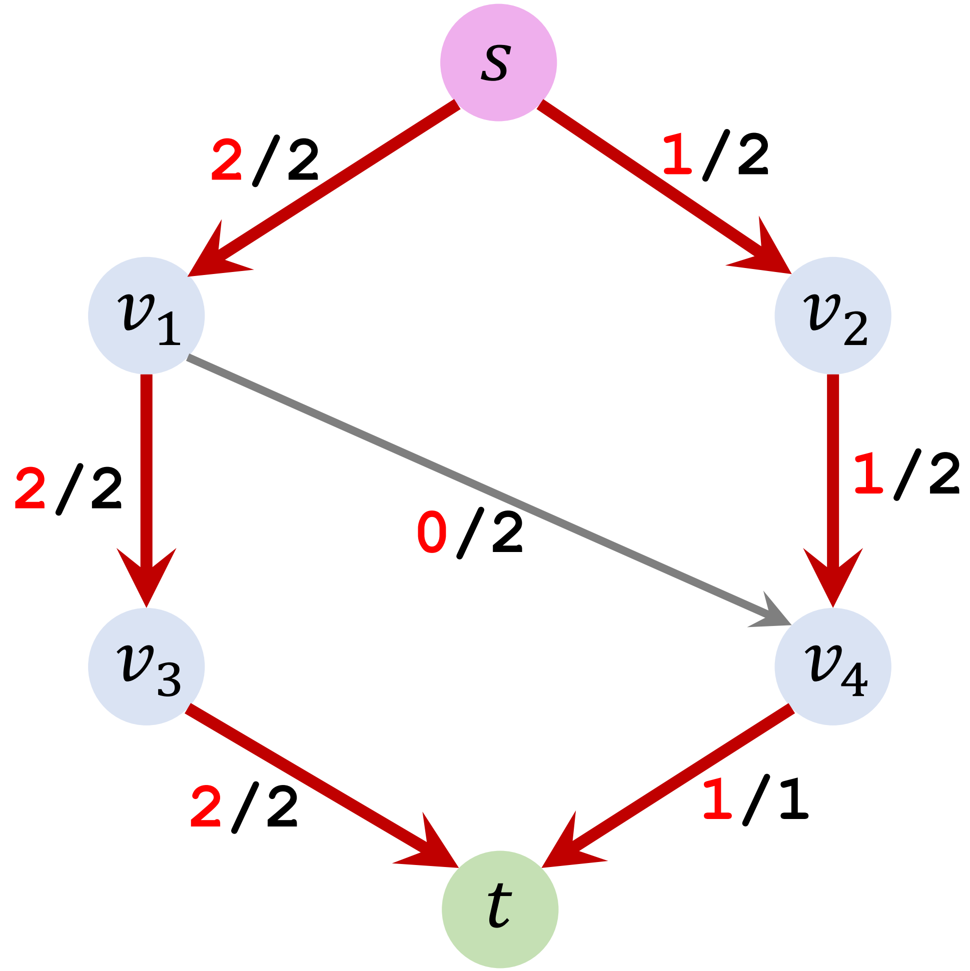


- A flow is **blocking flow** if no more flow from source to sink can be found.
- Max flow is blocking flow; blocking flow may not be max flow.
- Blocking flow can be found by the naïve algorithm.

Blocking Flow

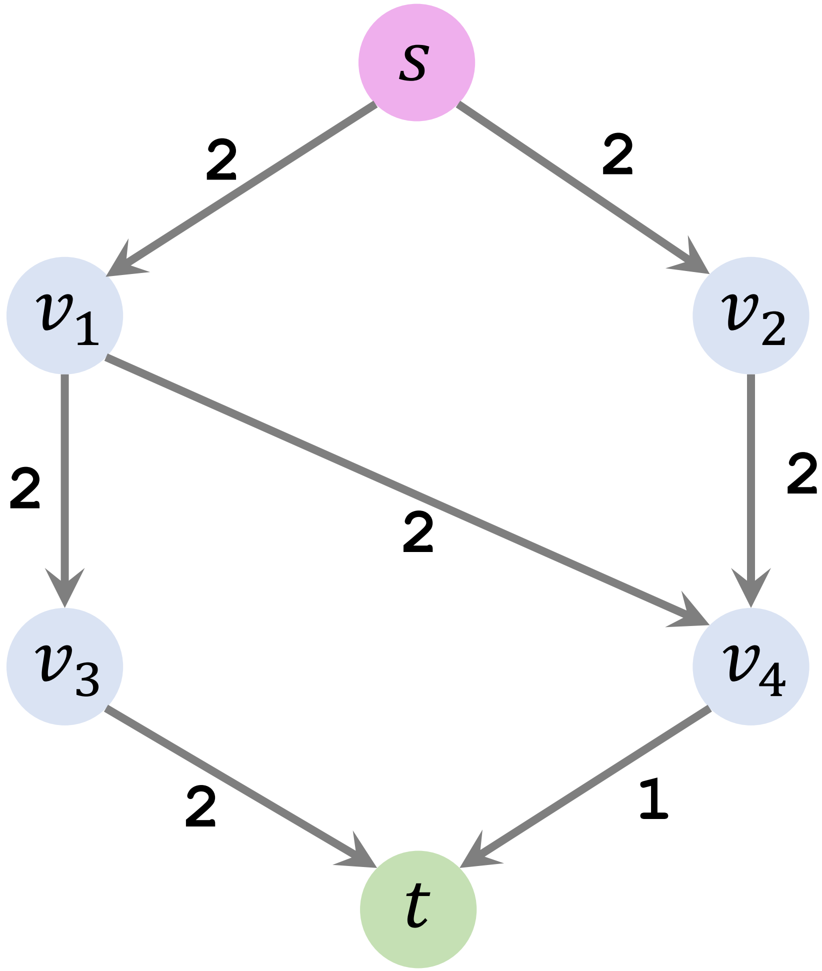


Original Graph

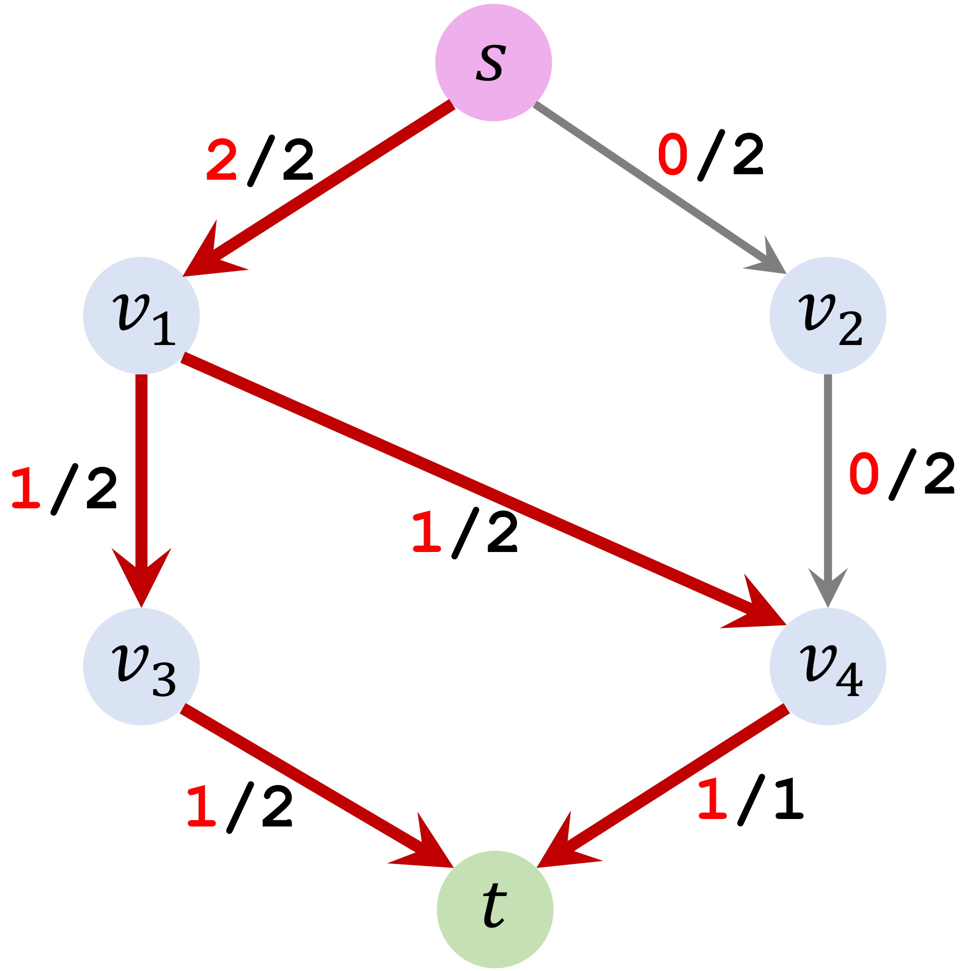


A Blocking Flow.

Blocking Flow



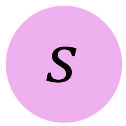
Original Graph



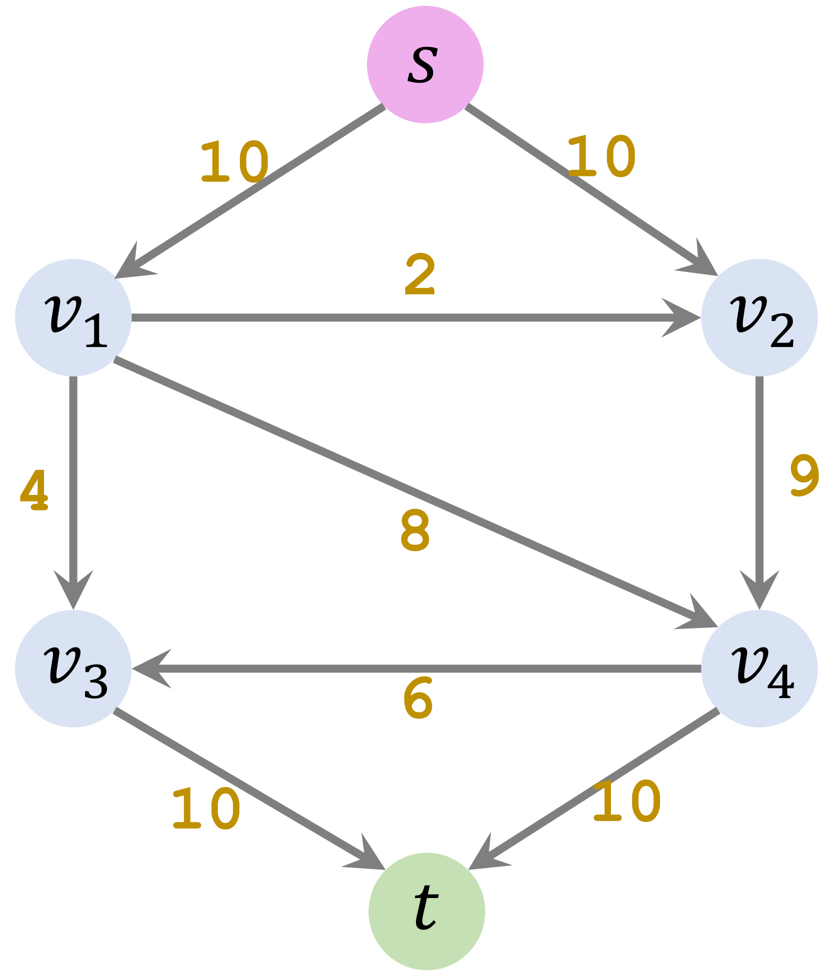
Another Blocking Flow.

Key Concept: Level Graph

Level Graph: Example 1

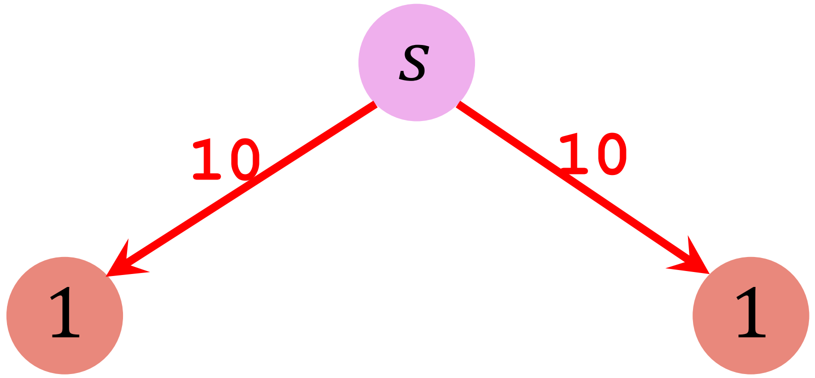


Level Graph

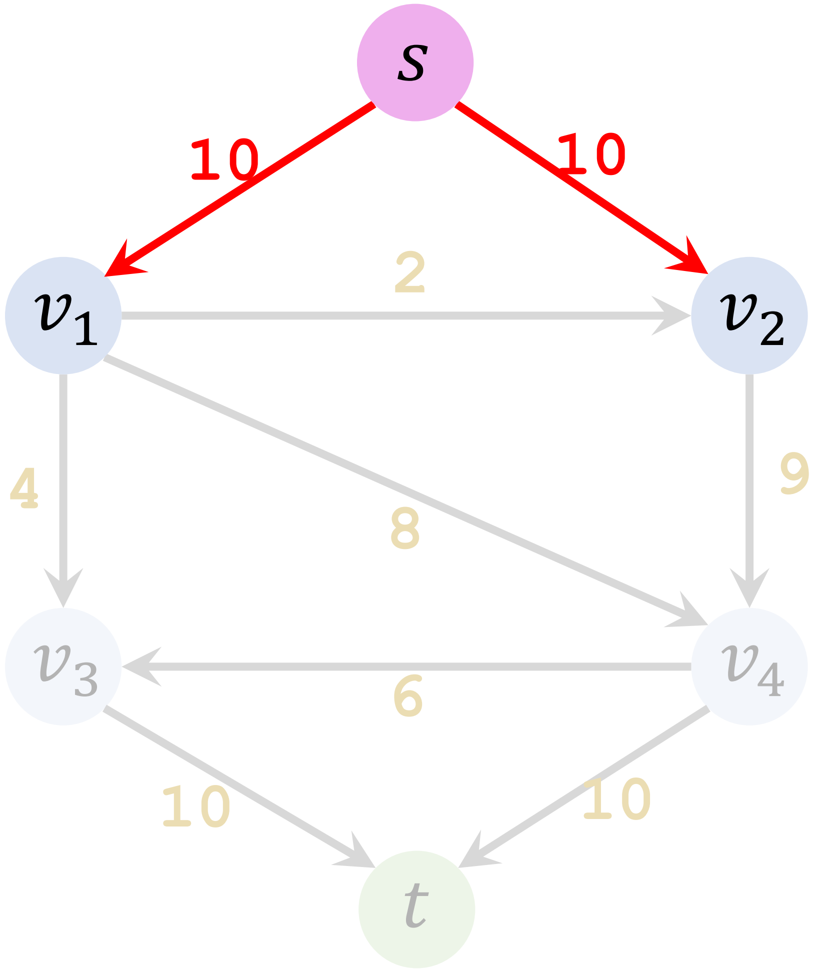


Original Graph

Level Graph: Example 1

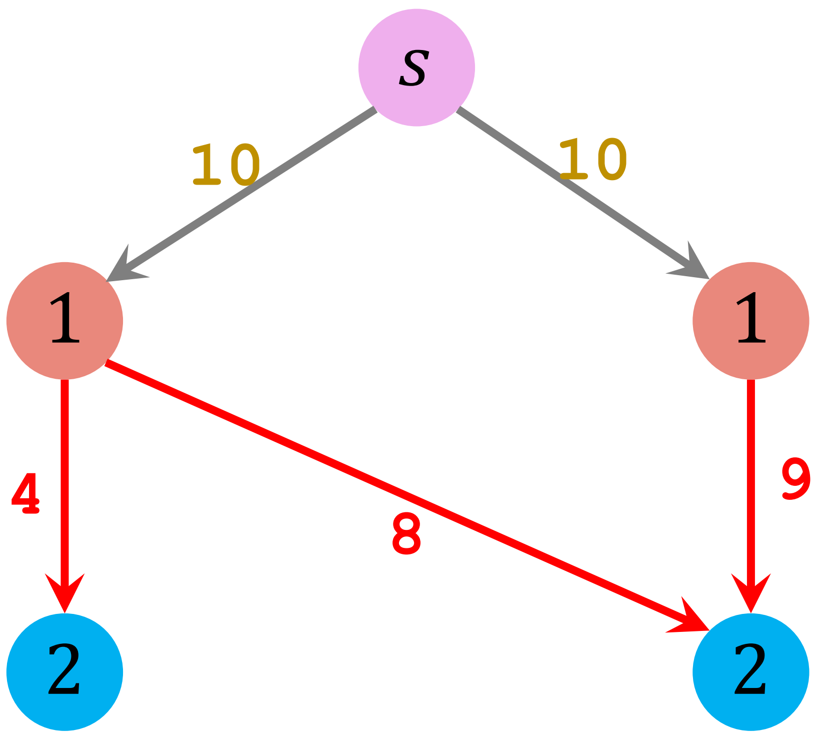


Level Graph

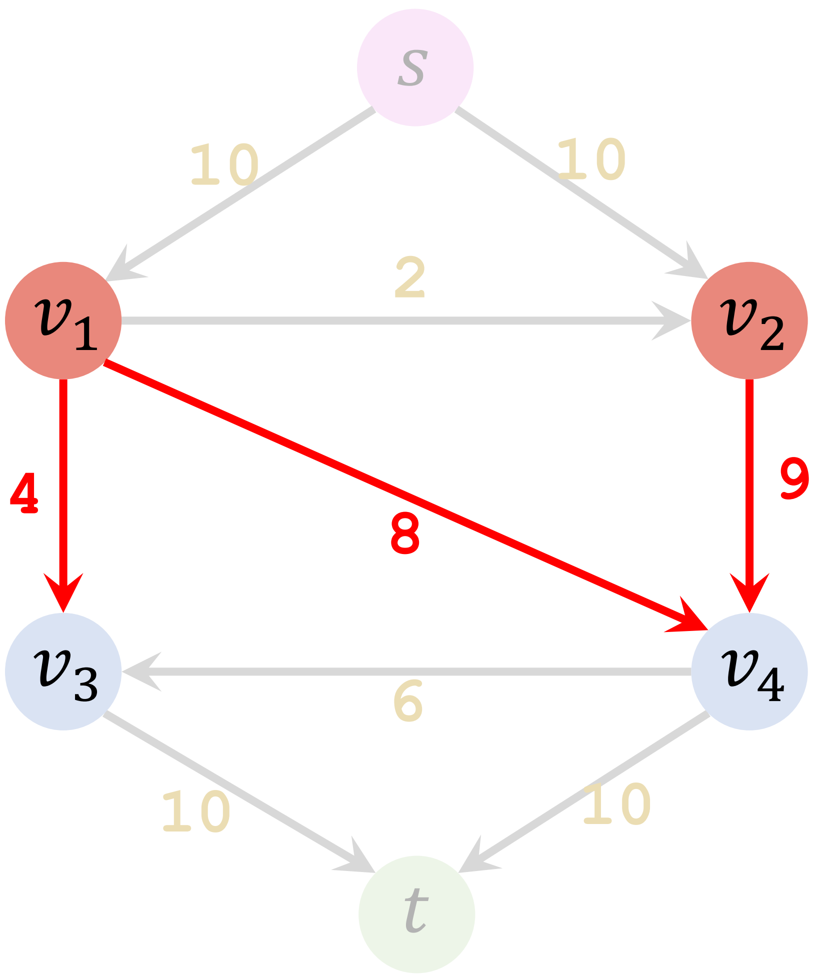


Original Graph

Level Graph: Example 1

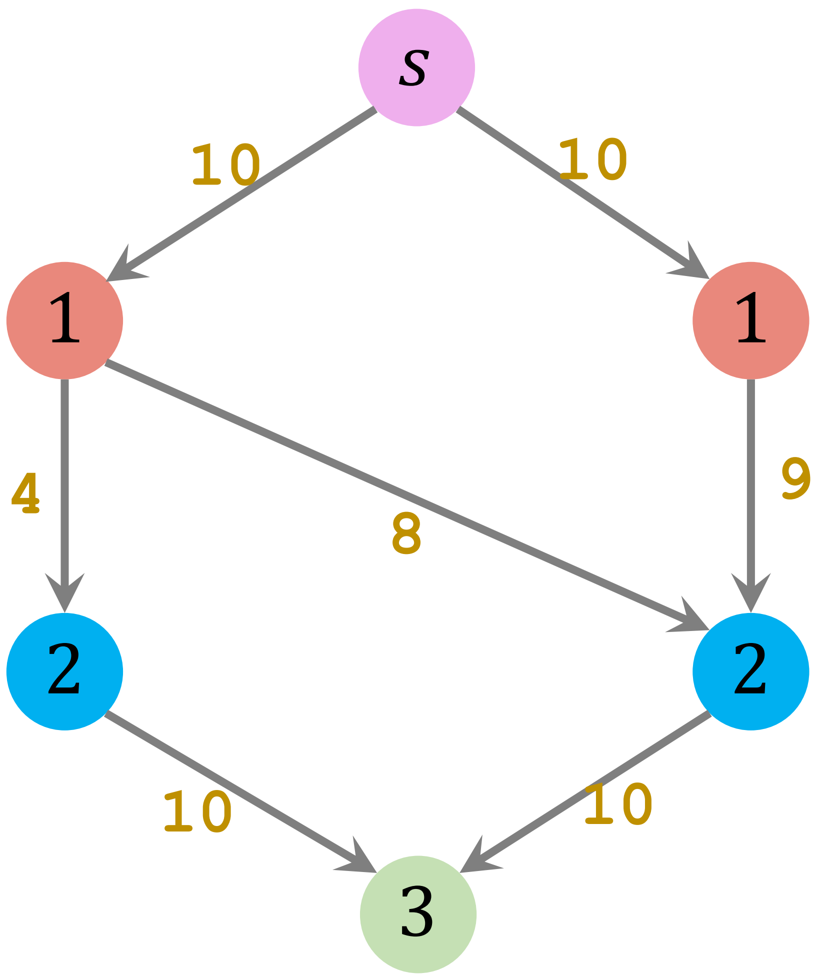


Level Graph

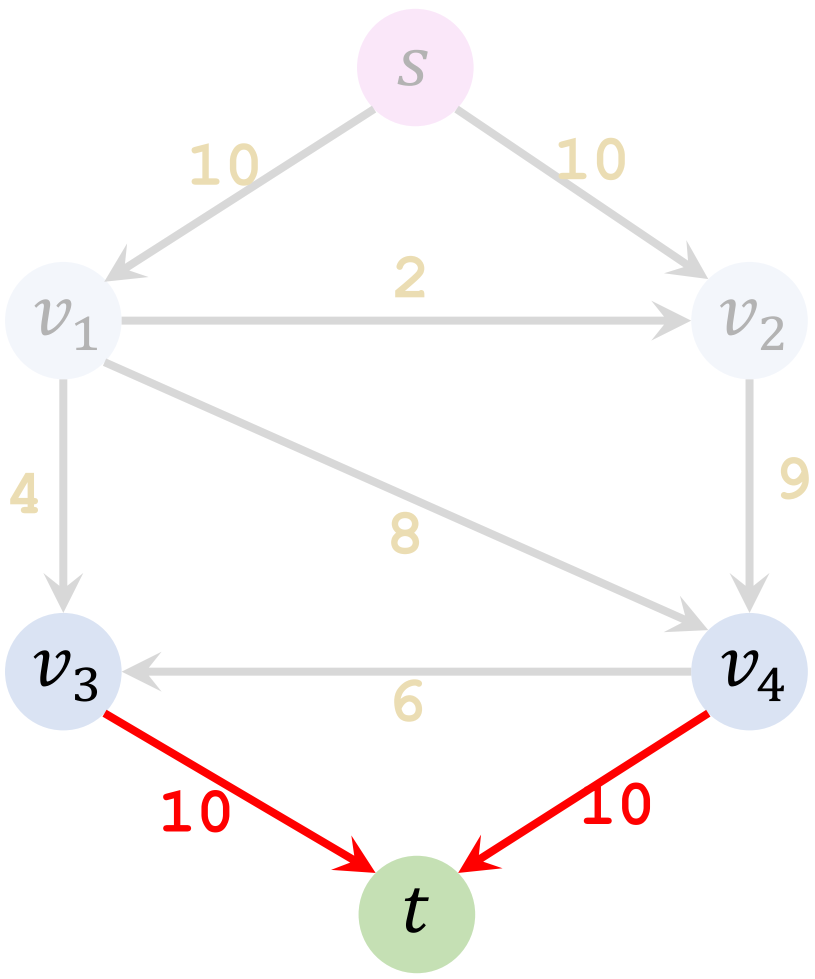


Original Graph

Level Graph: Example 1

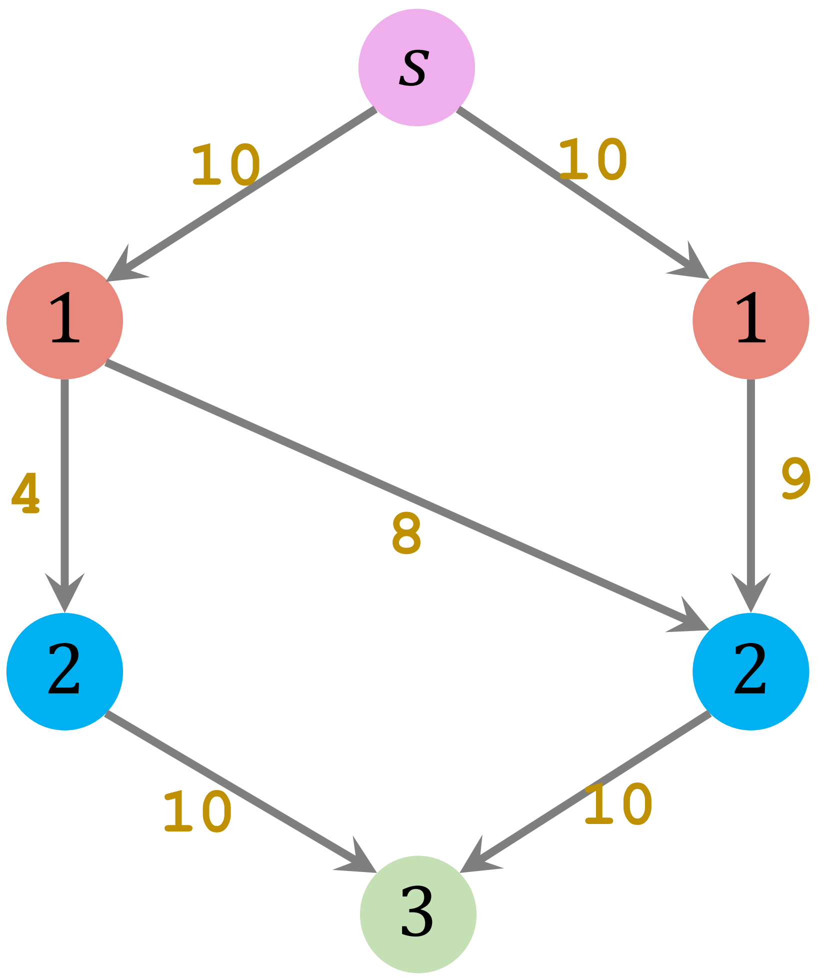


Level Graph

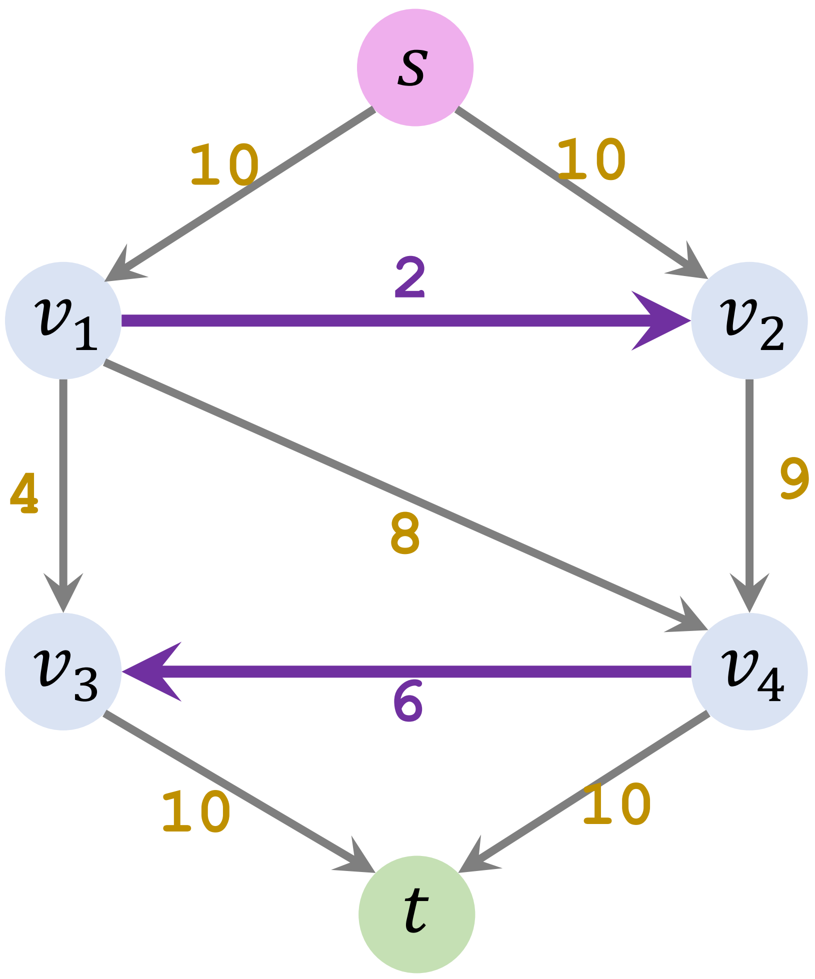


Original Graph

Level Graph: Example 1

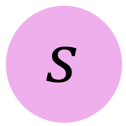


Level Graph

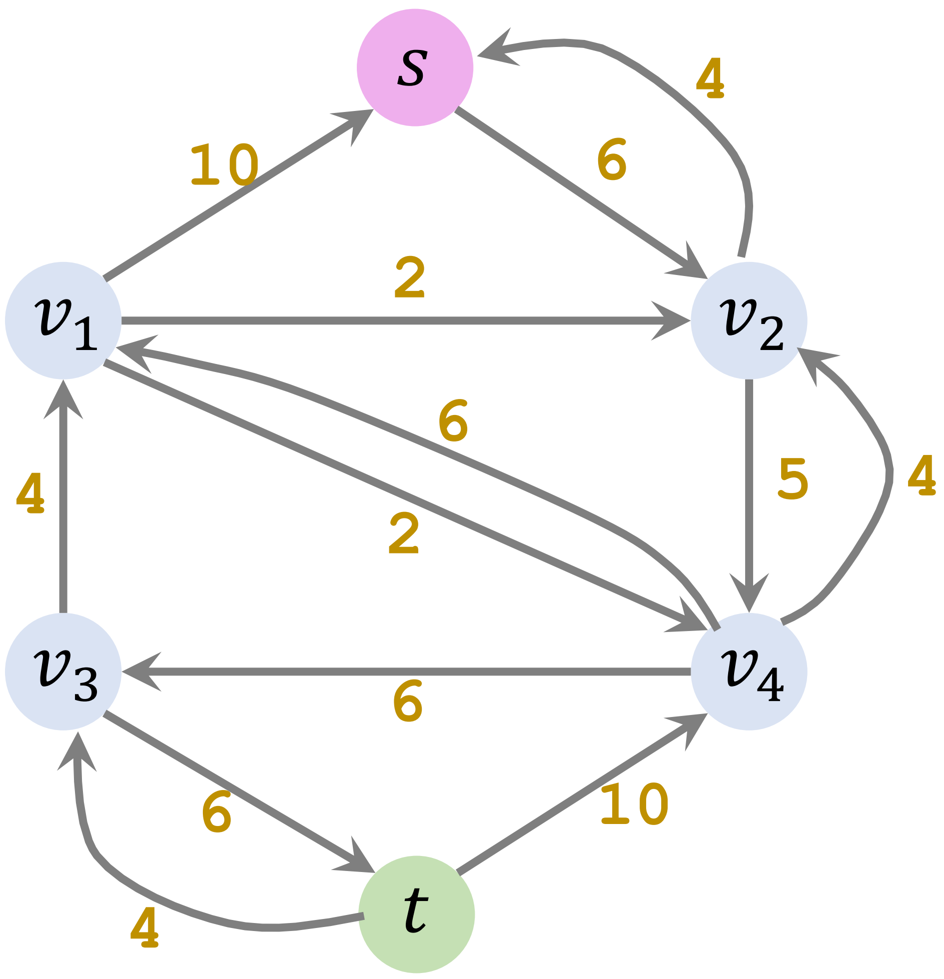


Original Graph

Level Graph: Example 2

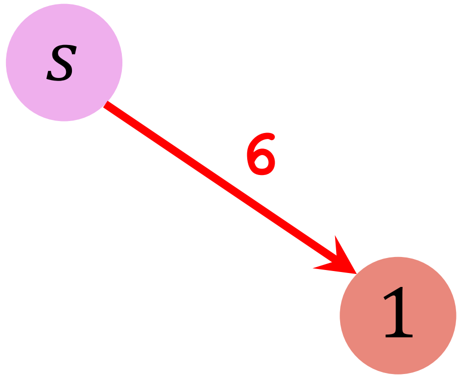


Level Graph

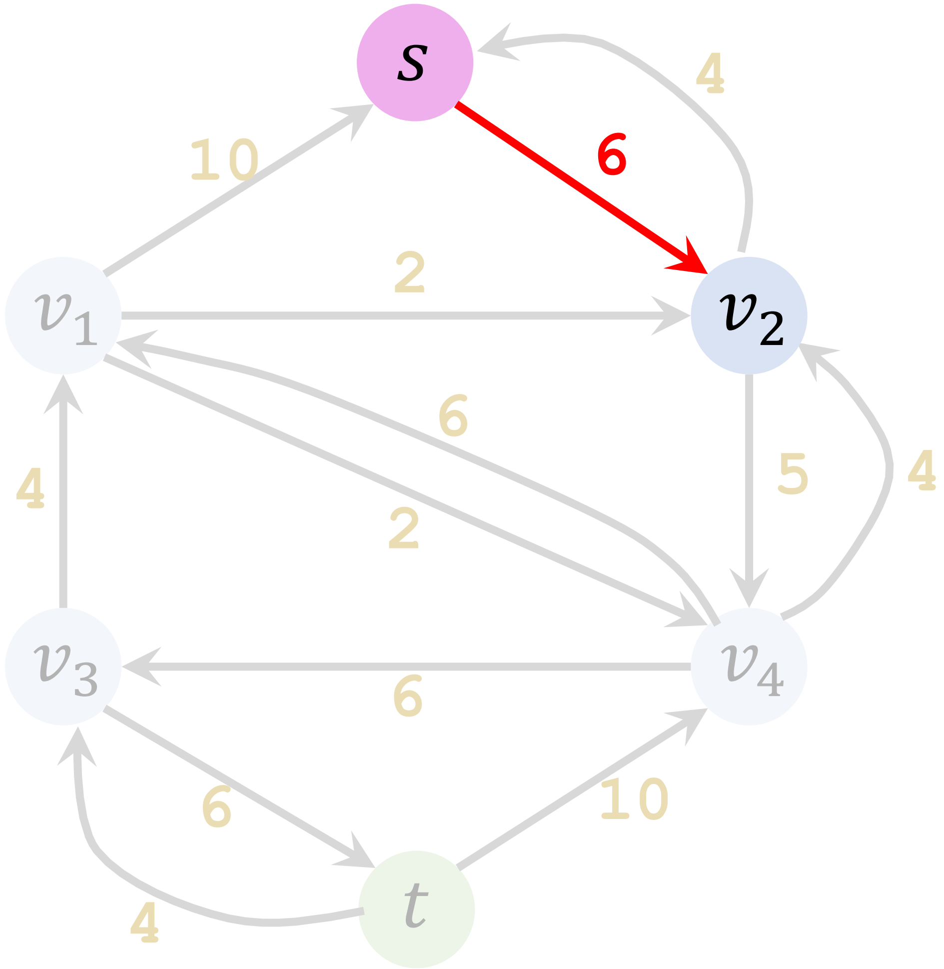


Original Graph

Level Graph: Example 2

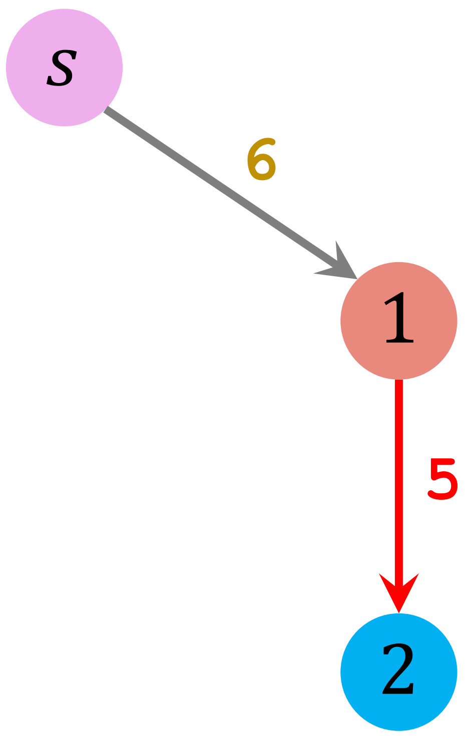


Level Graph

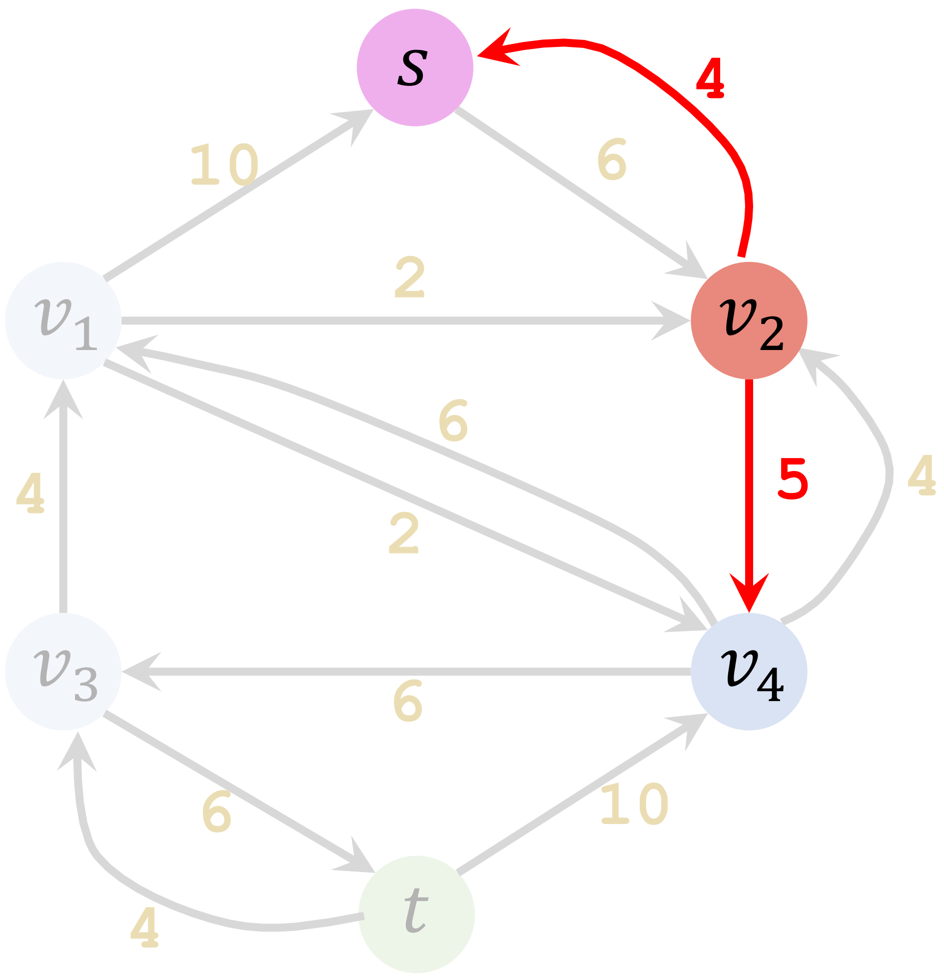


Original Graph

Level Graph: Example 2

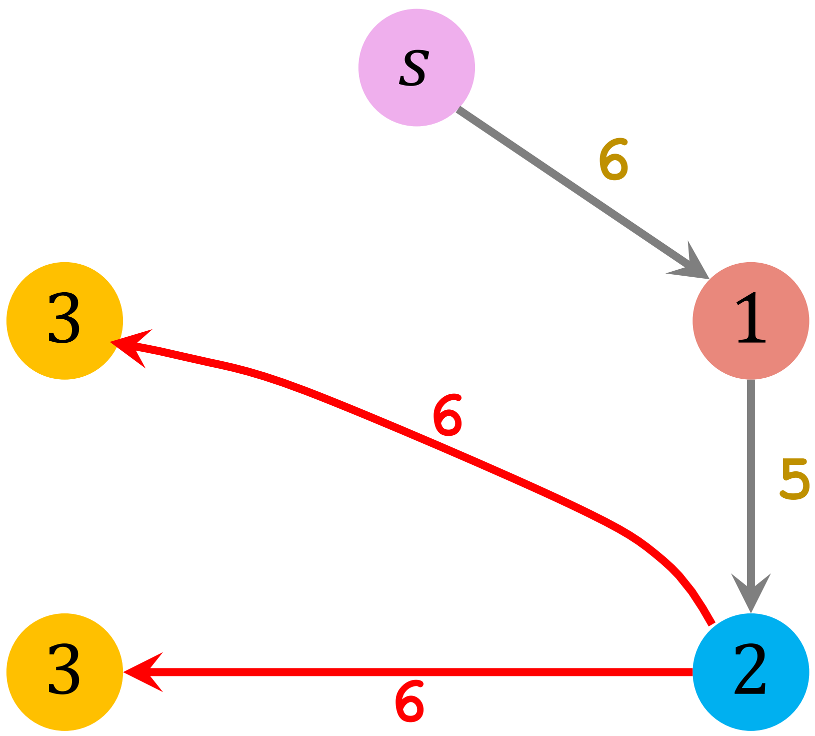


Level Graph

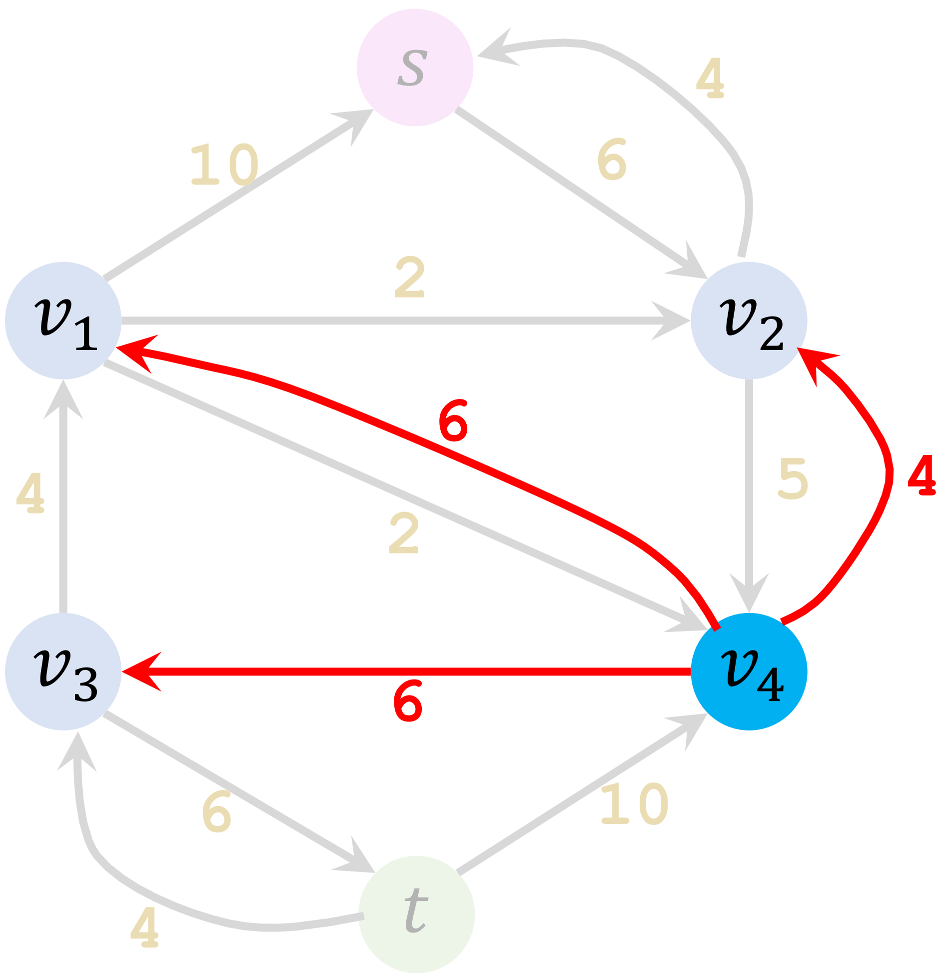


Original Graph

Level Graph: Example 2

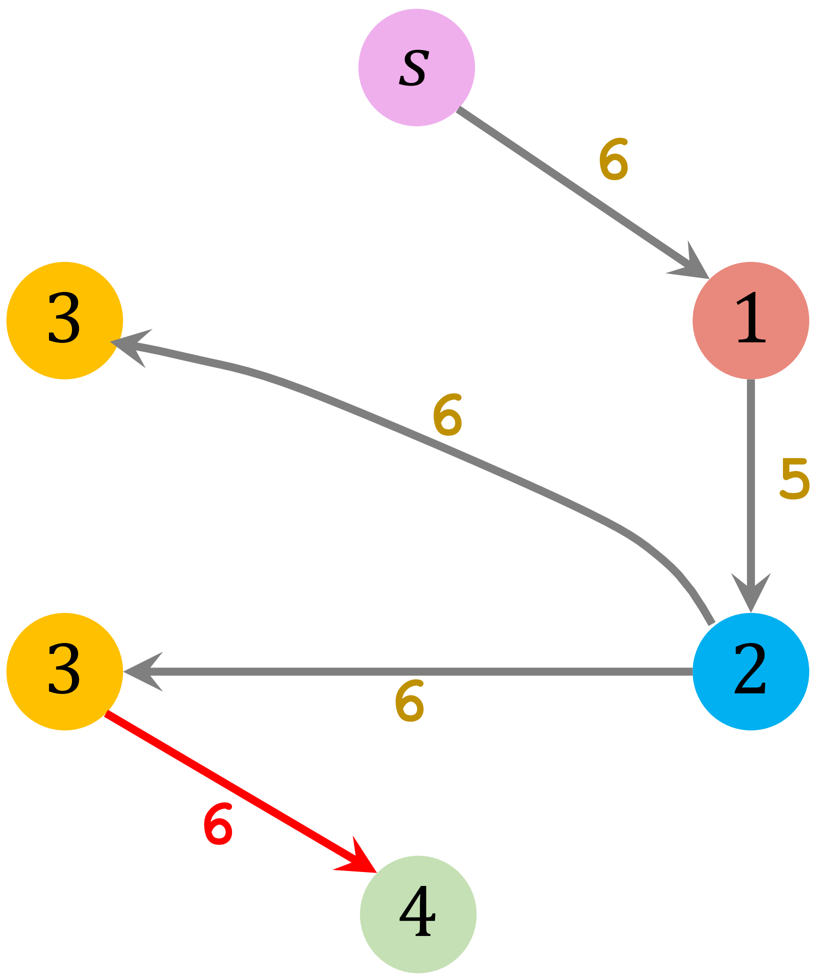


Level Graph

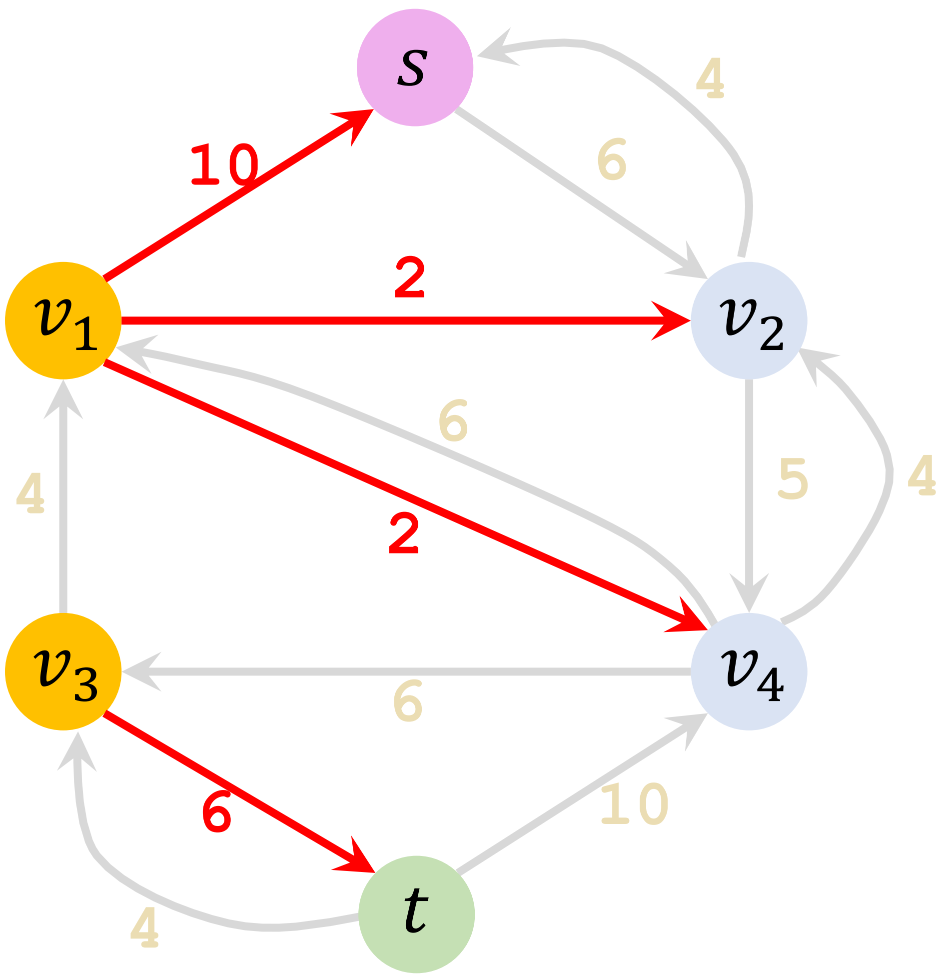


Original Graph

Level Graph: Example 2



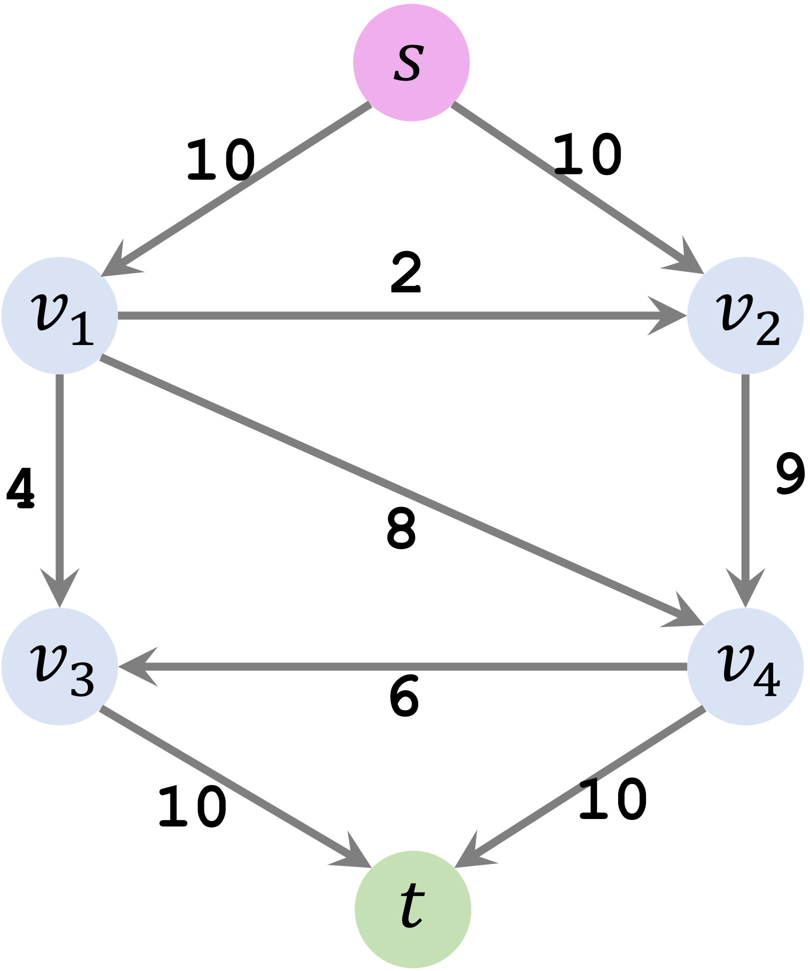
Level Graph



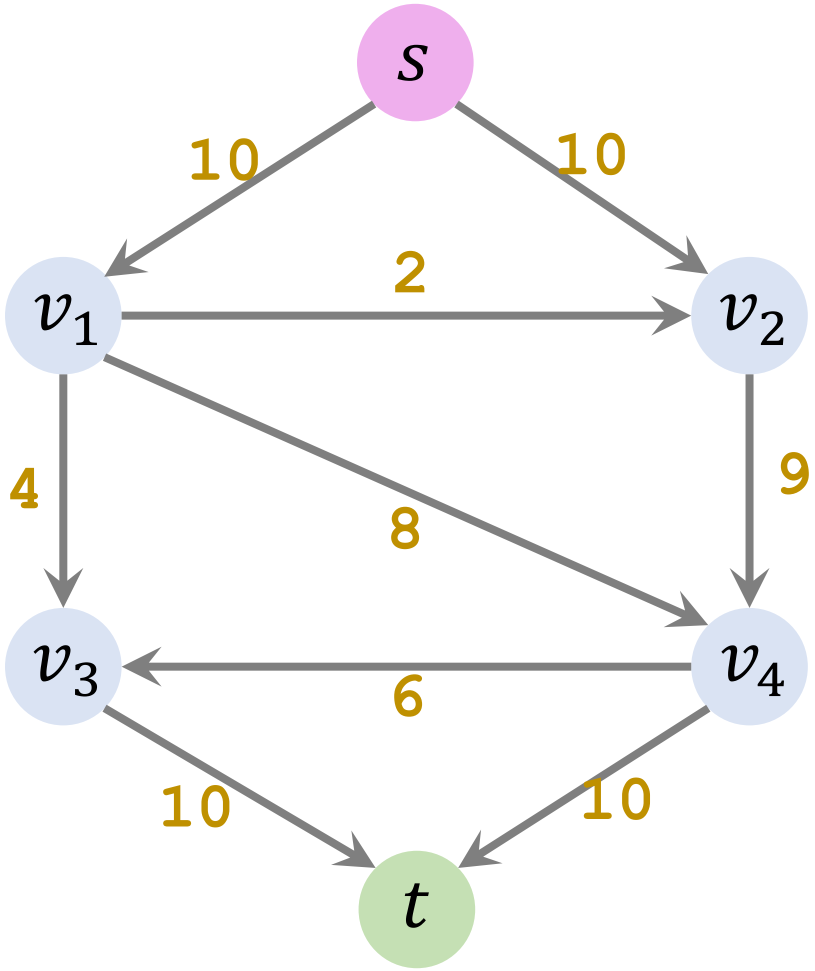
Original Graph

Dinic's Algorithm

Initialization

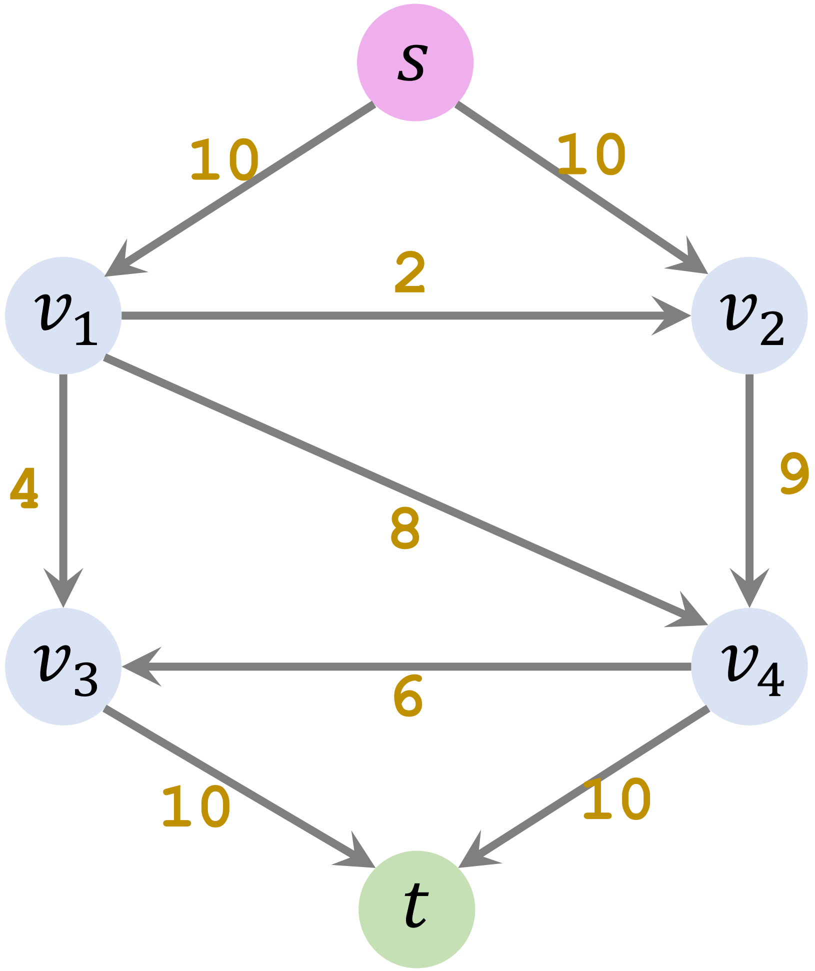


Original Graph



Residual Graph

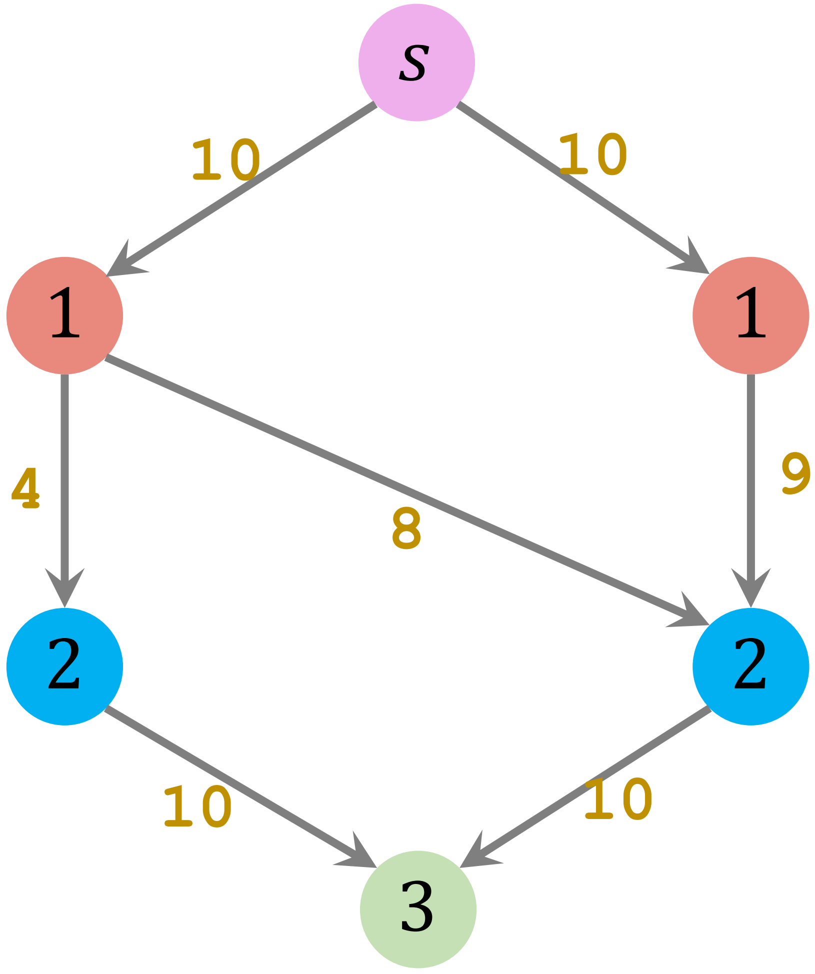
Iteration 1: Construct level graph



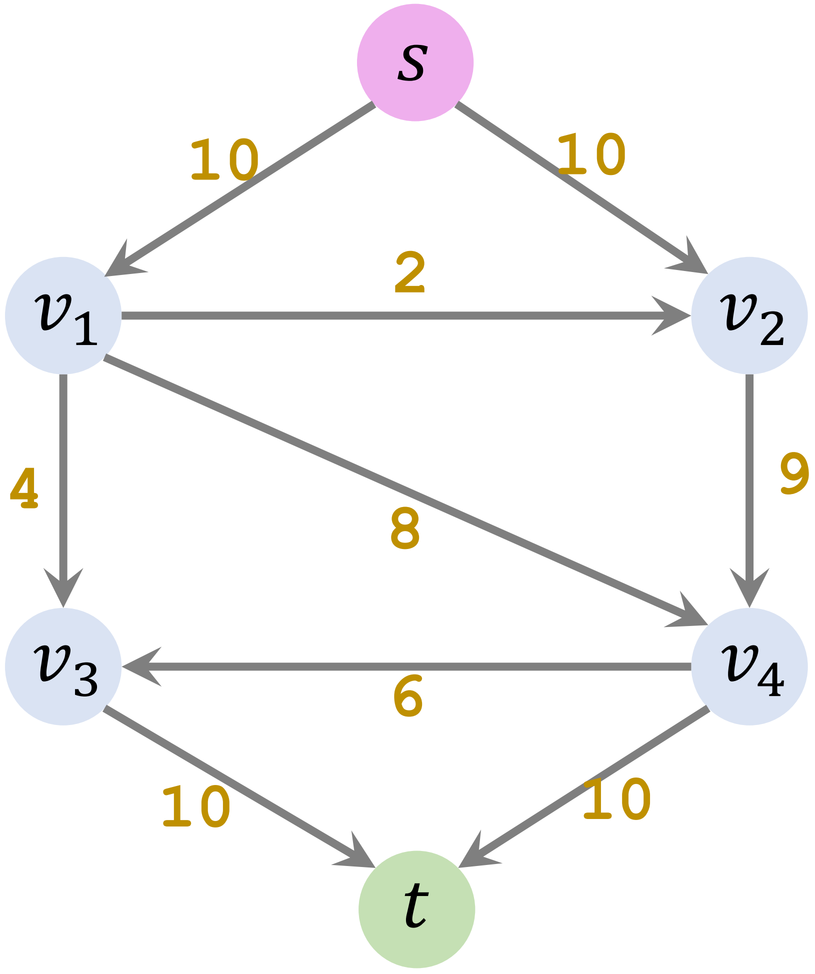
Level Graph

Residual Graph

Iteration 1: Construct **level graph**

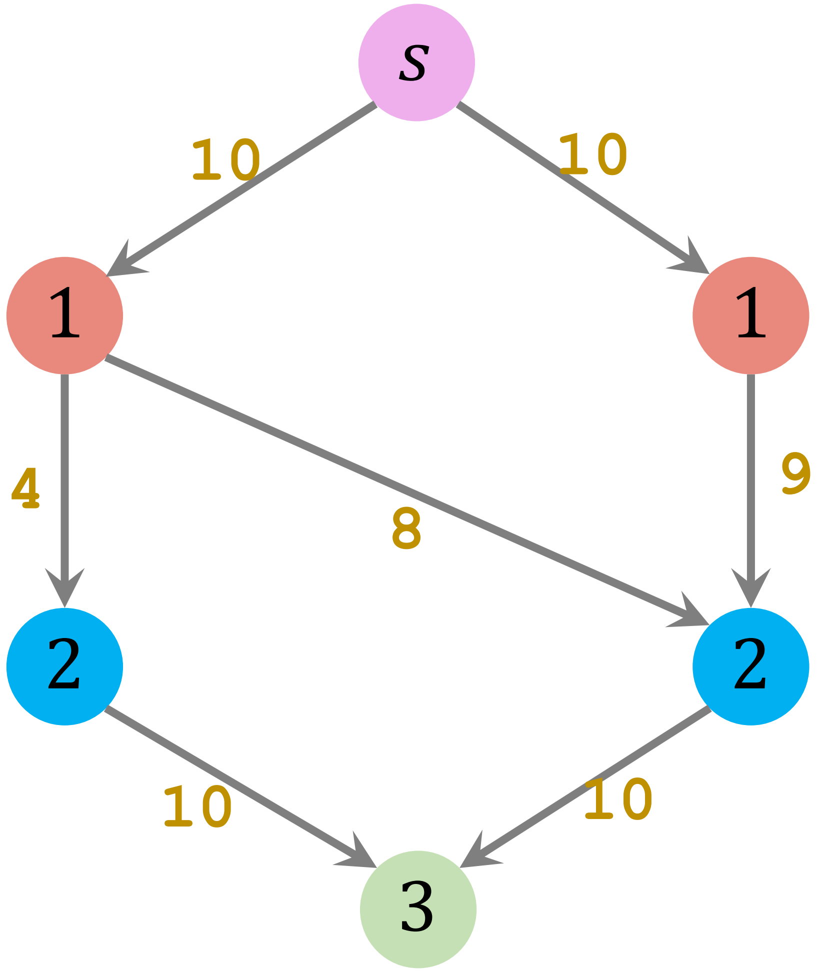


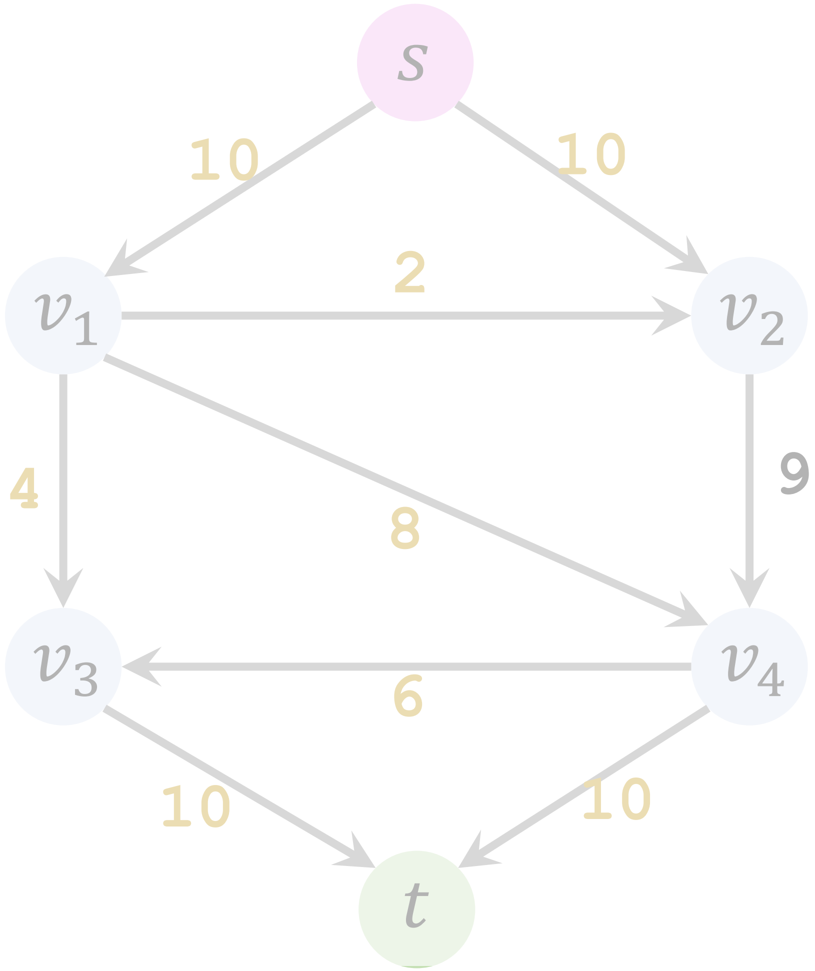
Level Graph



Residual Graph

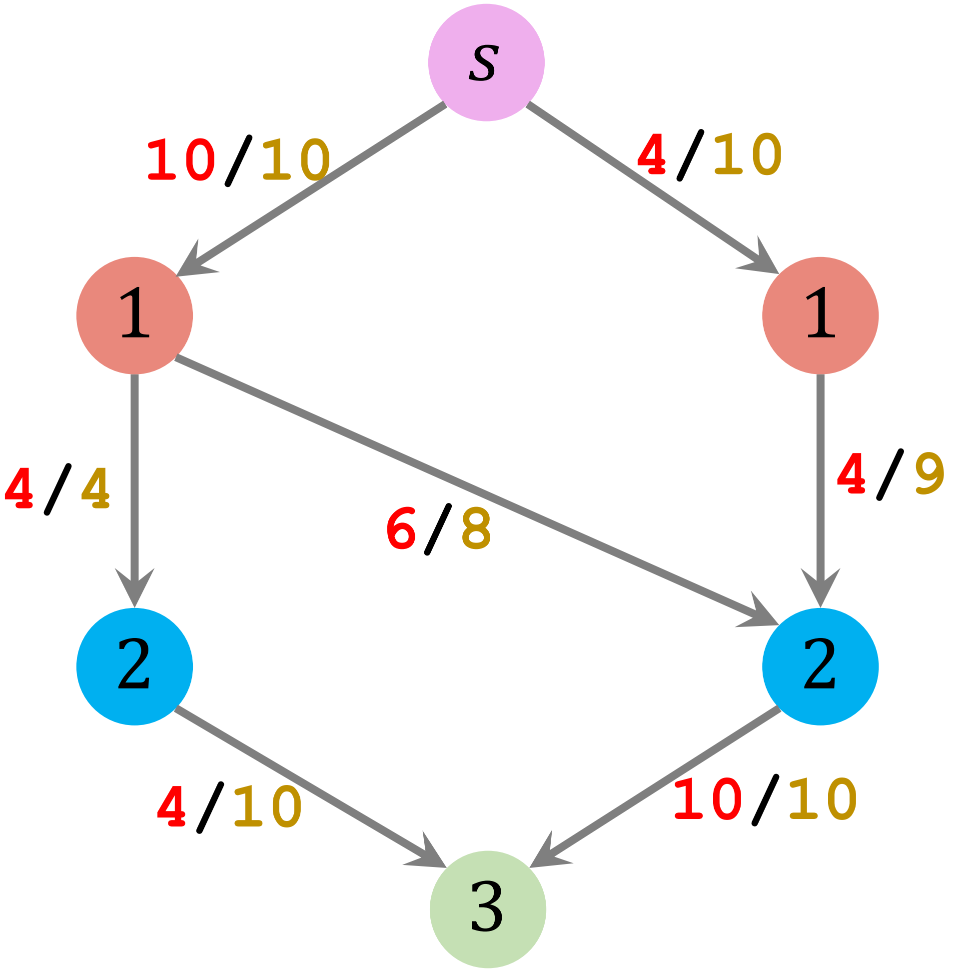
Iteration 1: Find **blocking flow** in **level graph**

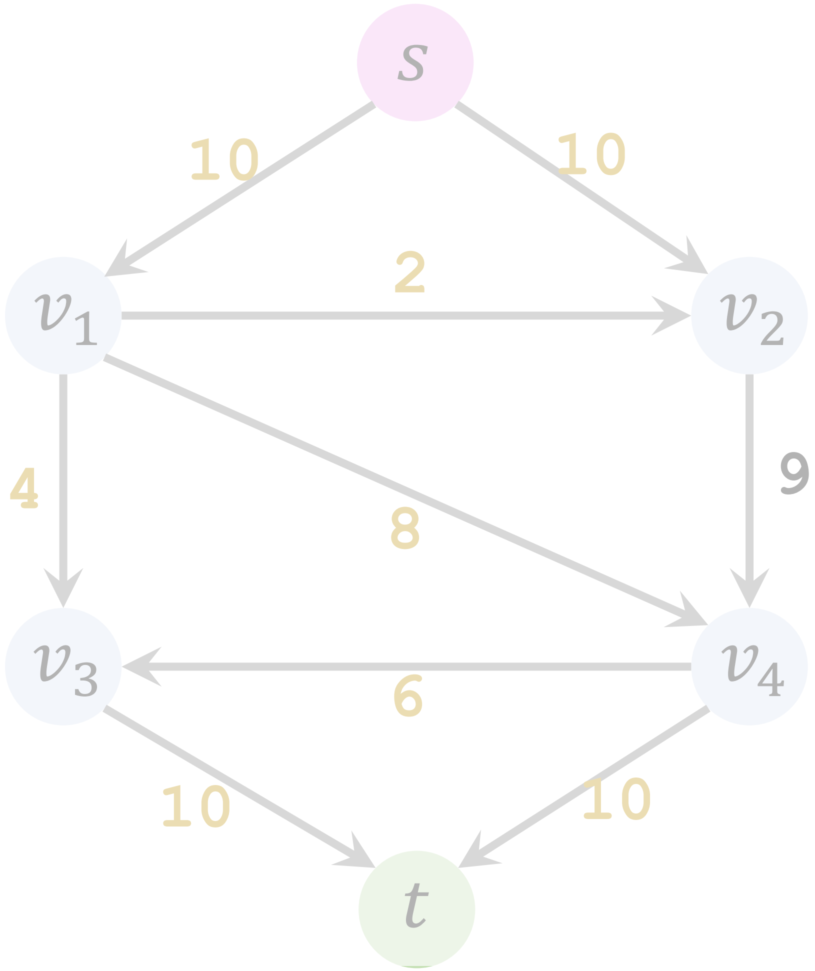




A flow is **blocking flow** if no more flow from source to sink can be found.

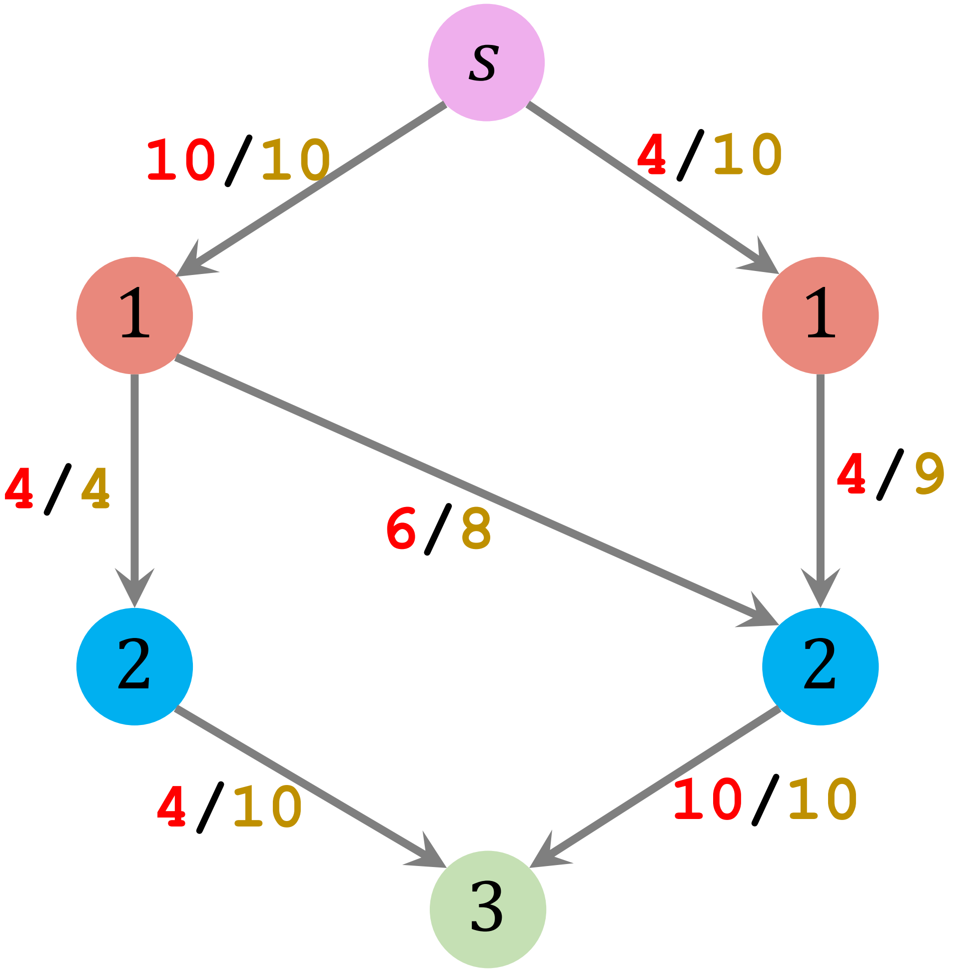
Iteration 1: Find blocking flow in level graph



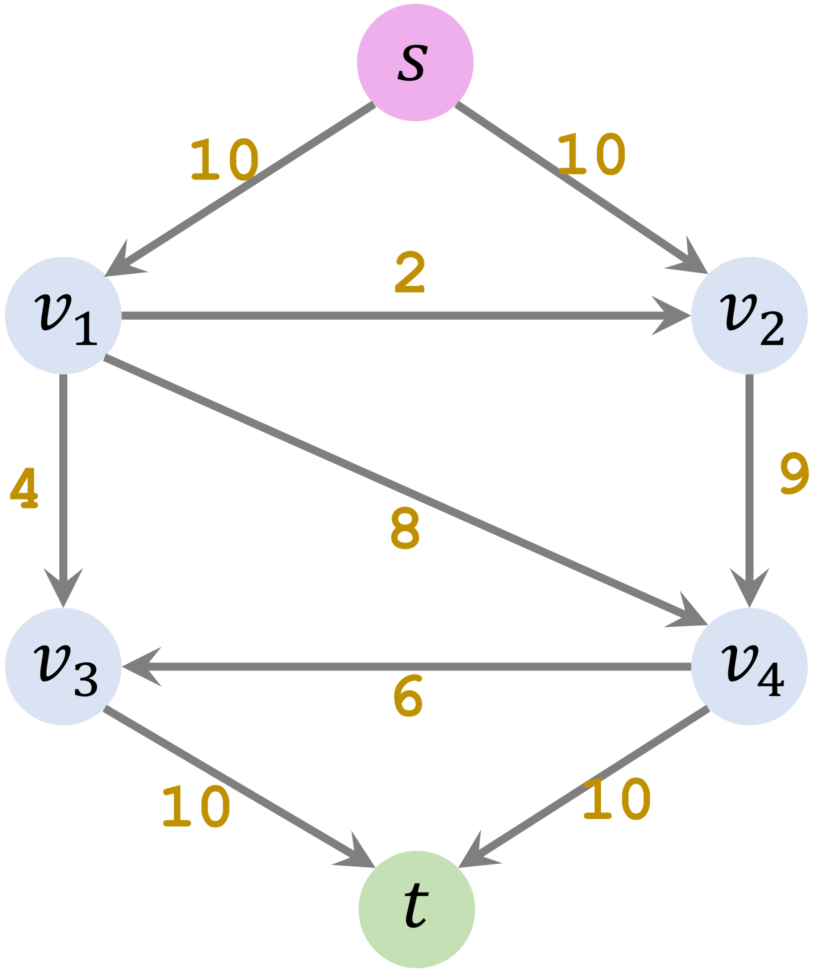


Blocking flow can be found using the naïve algorithm.

Iteration 1: Update the residual graph

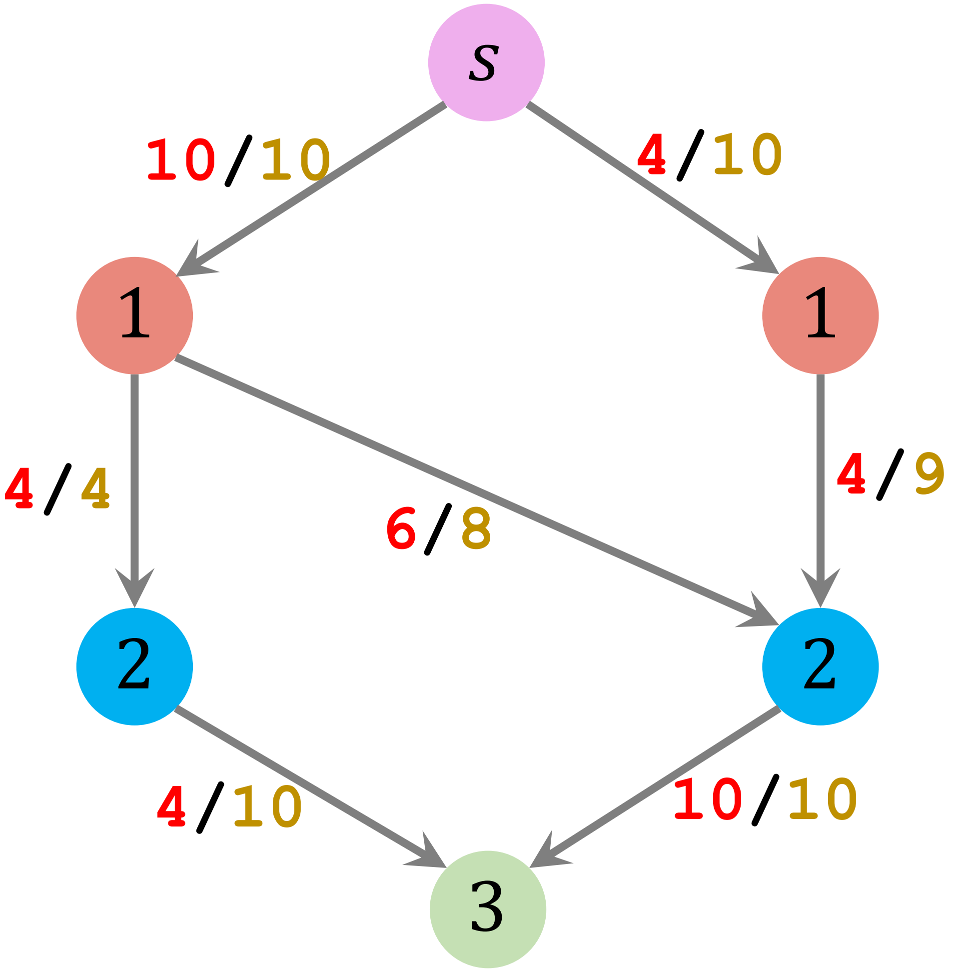


Level Graph

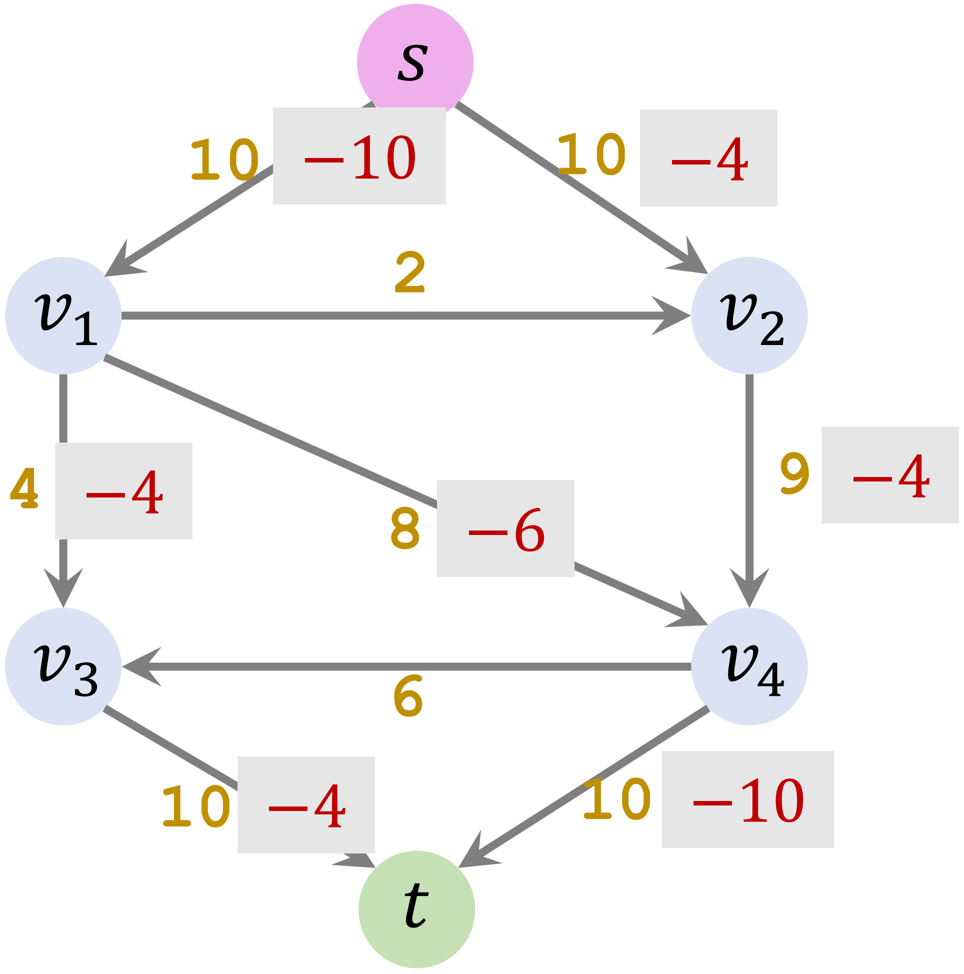


Old Residual Graph

Iteration 1: Update the residual graph

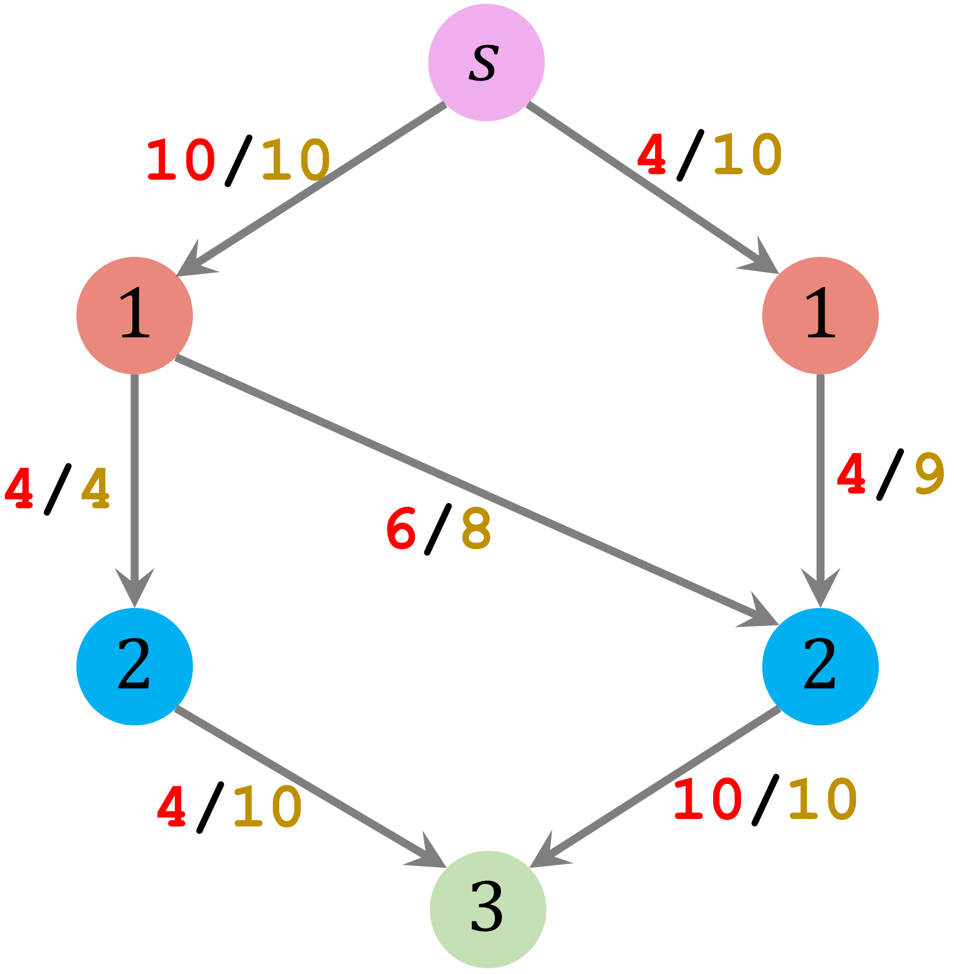


Level Graph

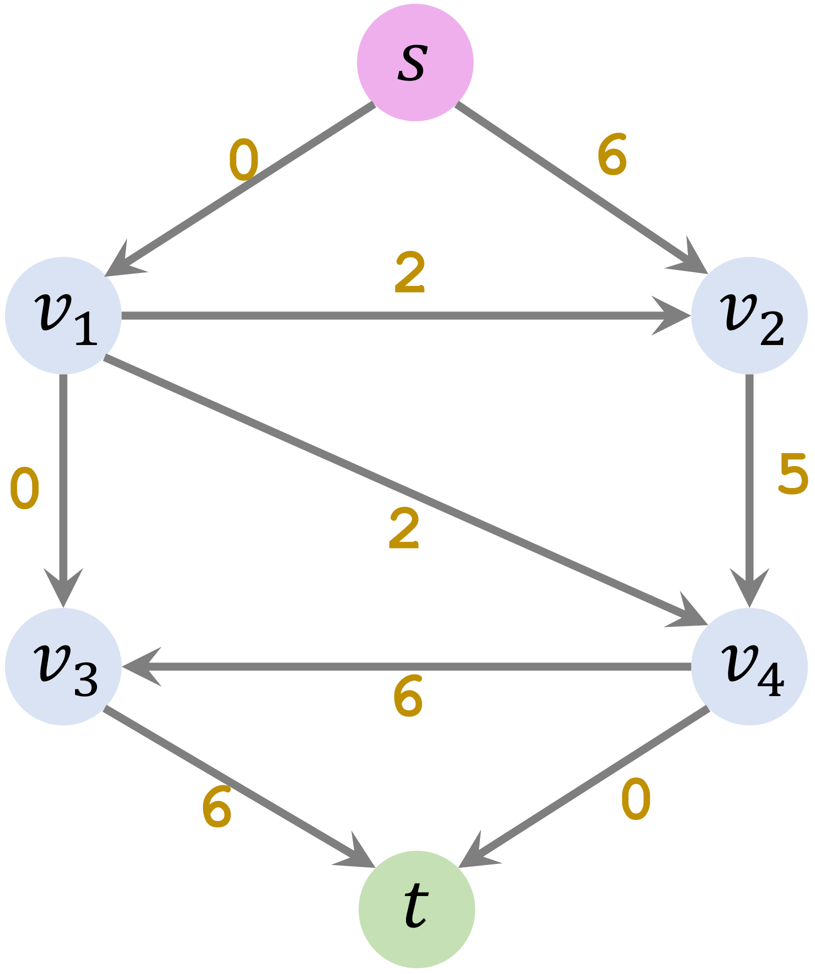


Old Residual Graph

Iteration 1: Update the residual graph

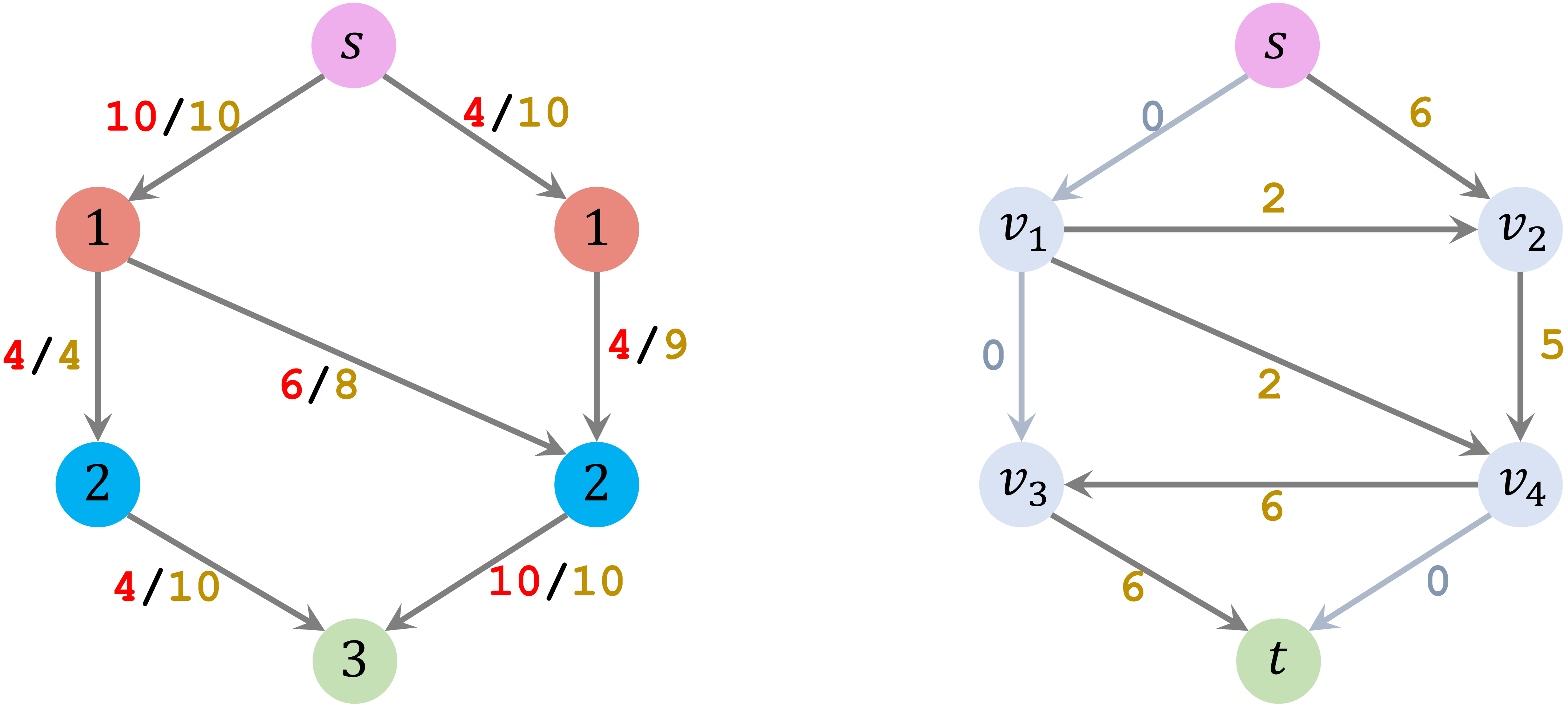


Level Graph



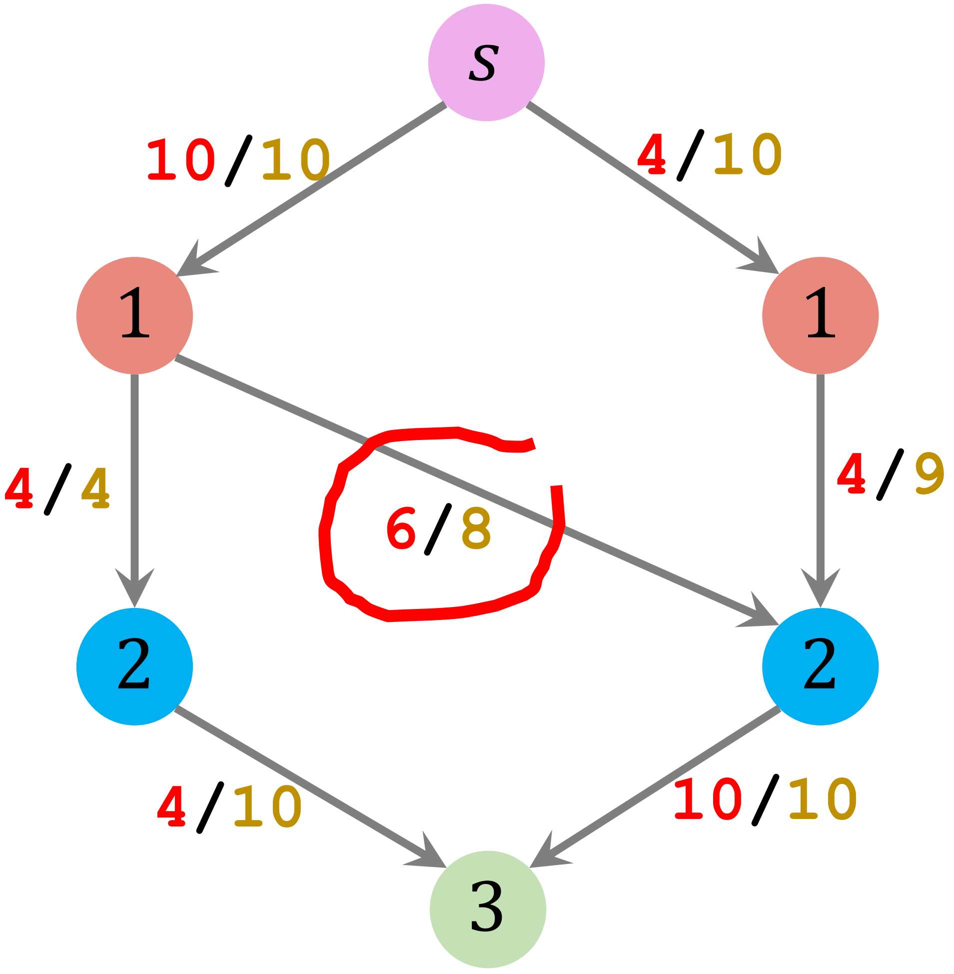
New Residual Graph

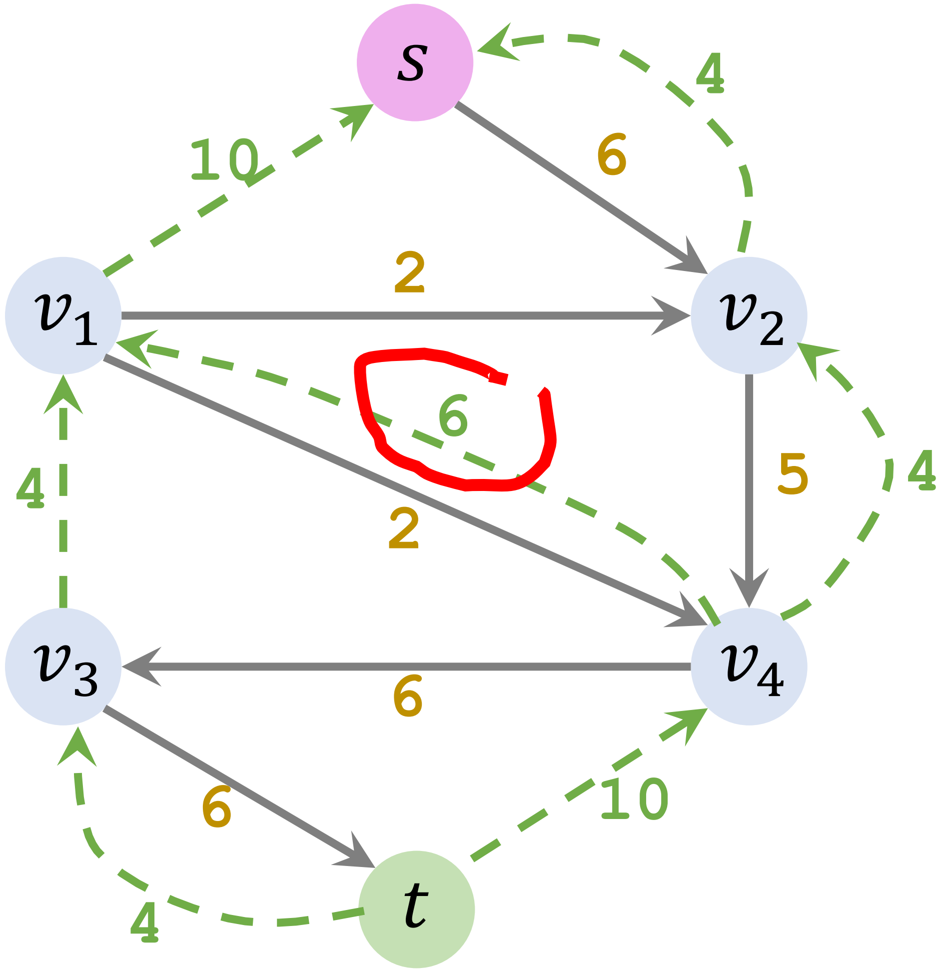
Iteration 1: Update the residual graph



Removed saturated edges from residual graph.

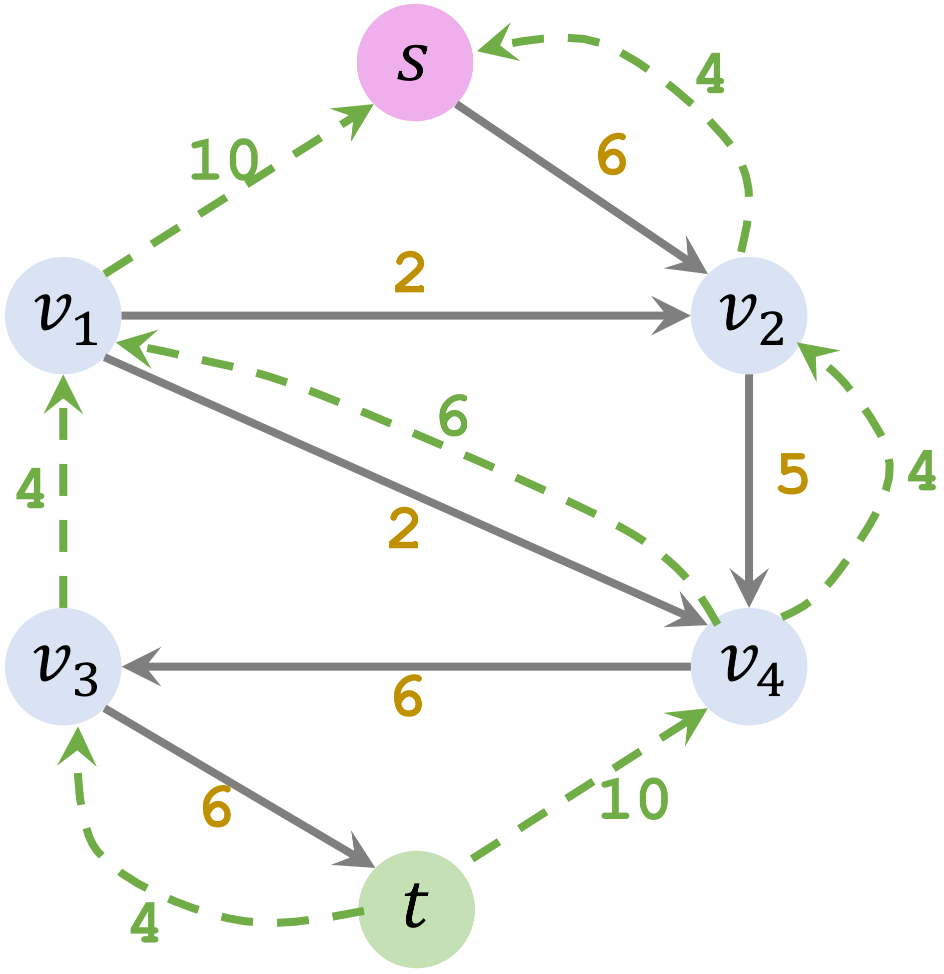
Iteration 1: Update the residual graph





Add flows to the residual graph as backward paths.

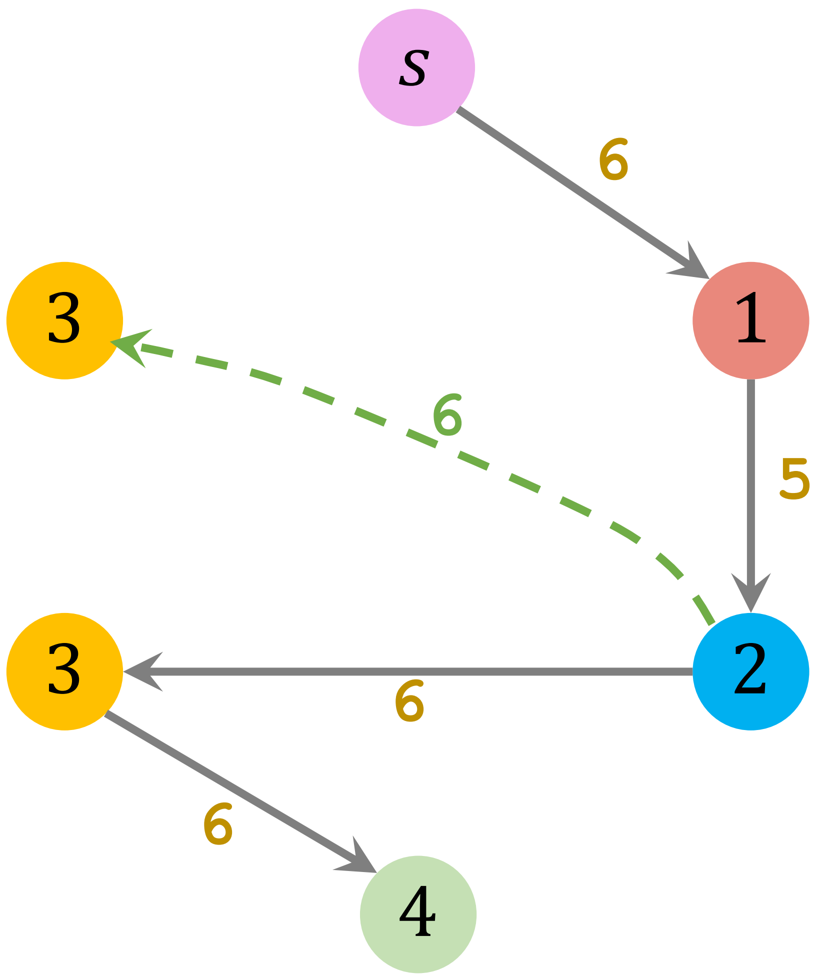
Iteration 2: Construct level graph



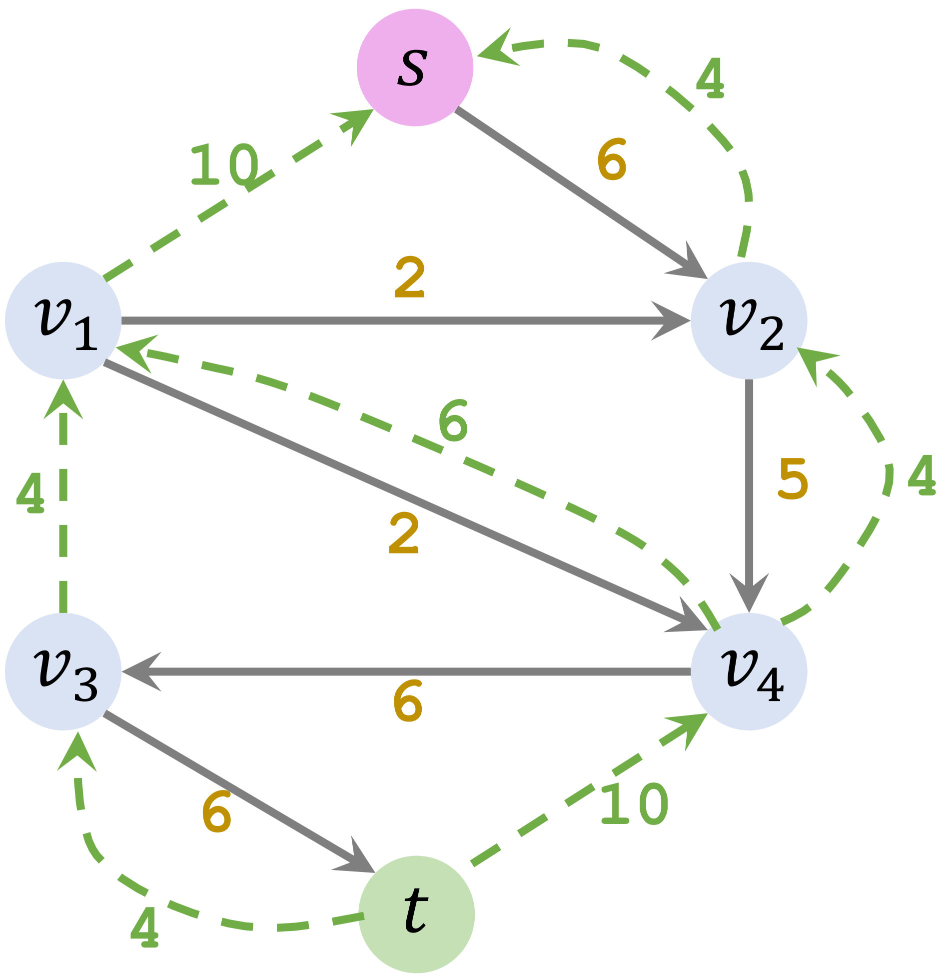
Level Graph

Residual Graph

Iteration 2: Construct level graph

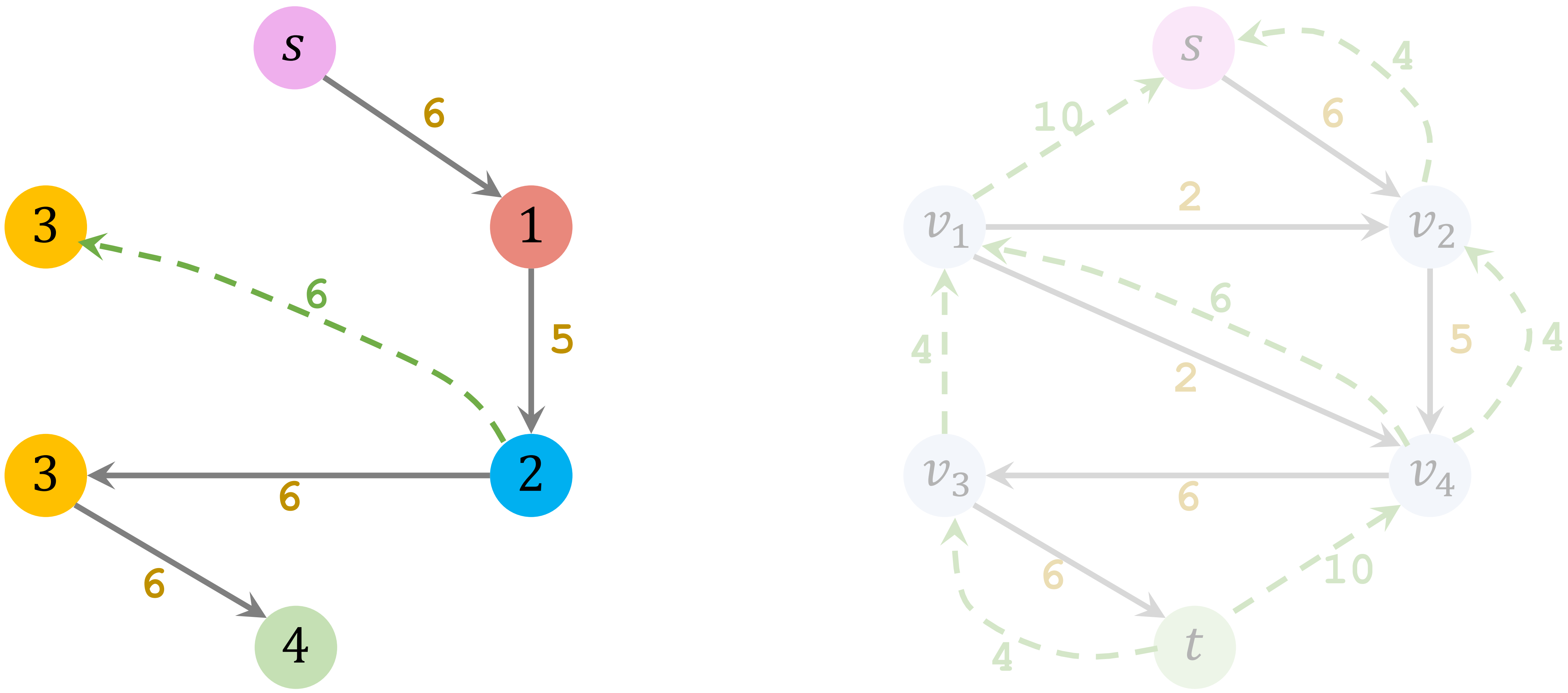


Level Graph



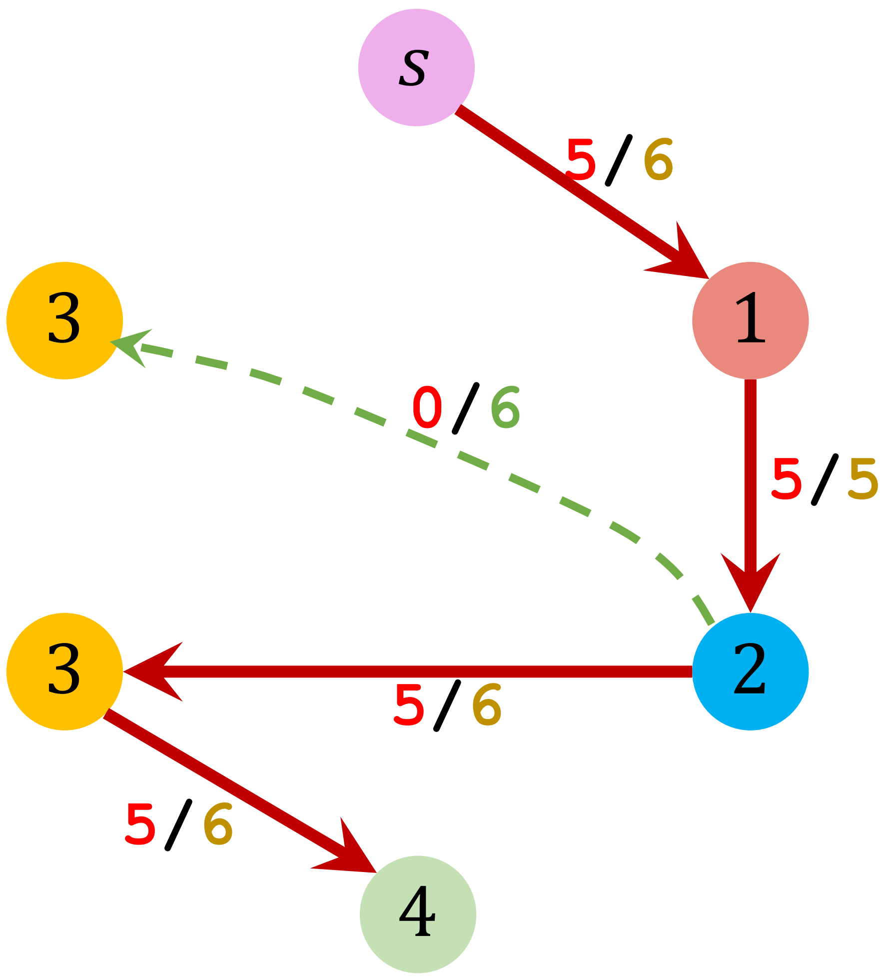
Residual Graph

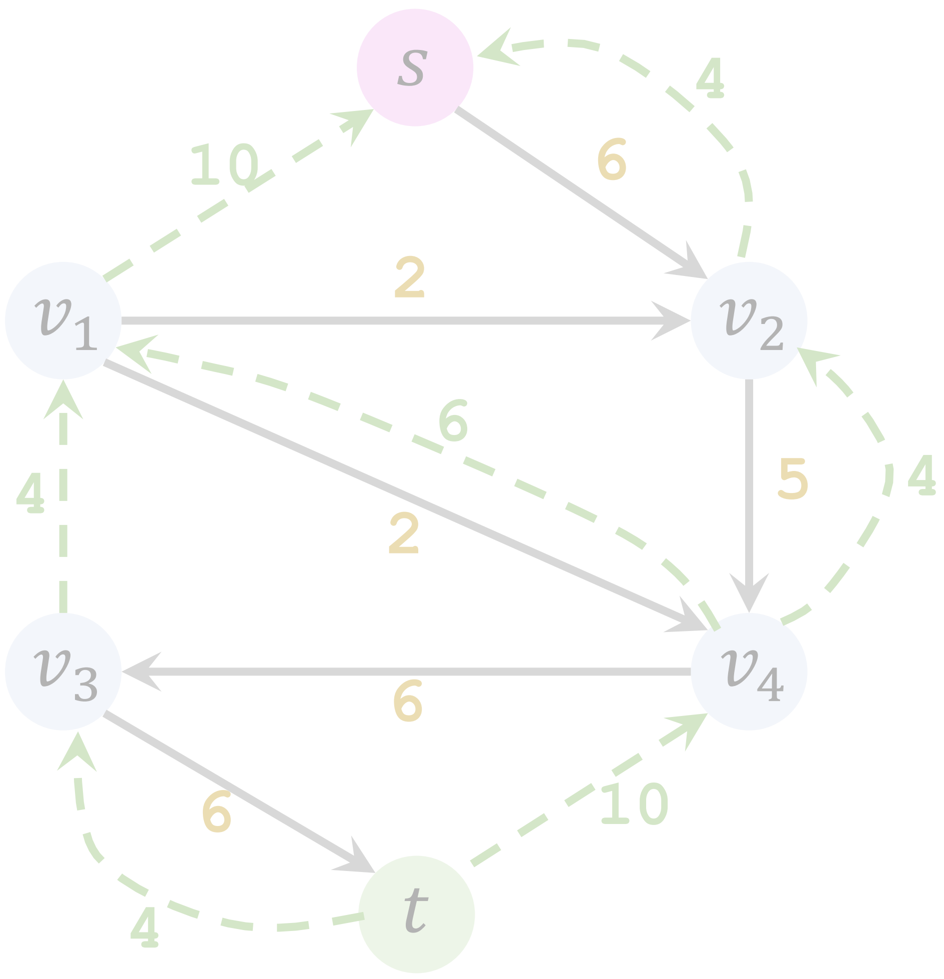
Iteration 2: Find blocking flow in level graph



A flow is **blocking flow** if no more flow from source to sink can be found.

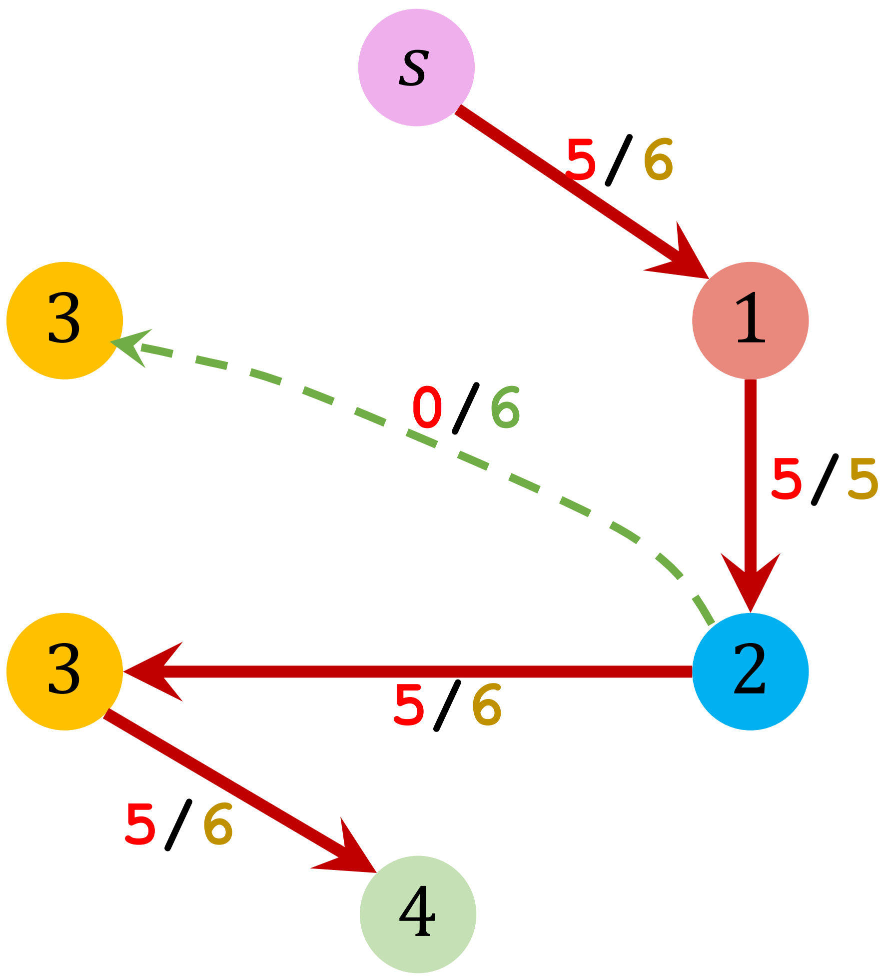
Iteration 2: Find blocking flow in level graph



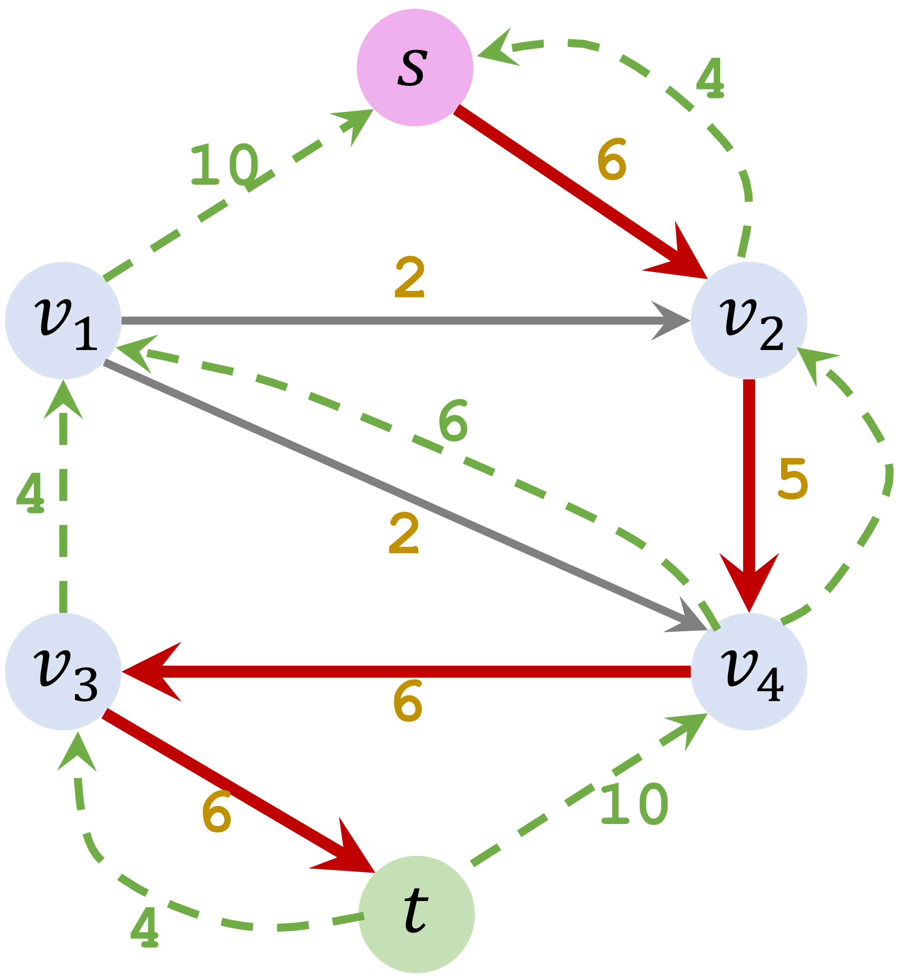


Blocking flow can be found using the naïve algorithm.

Iteration 2: Update the **residual graph**

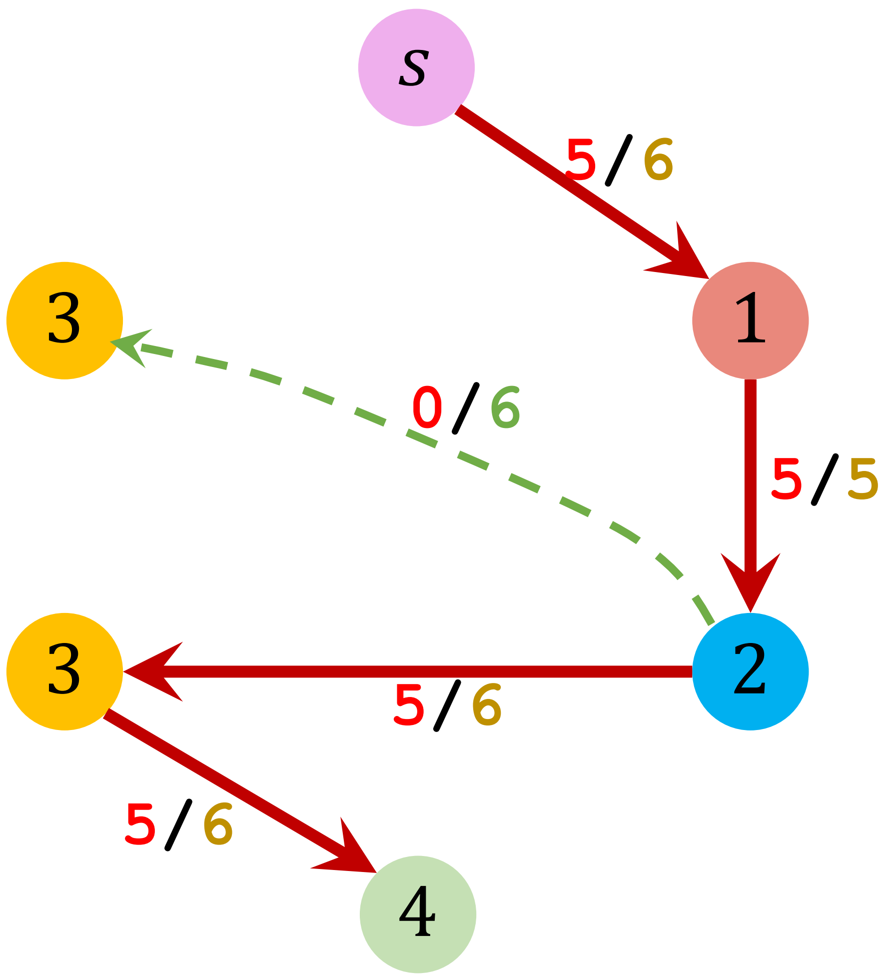


Blocking Flow

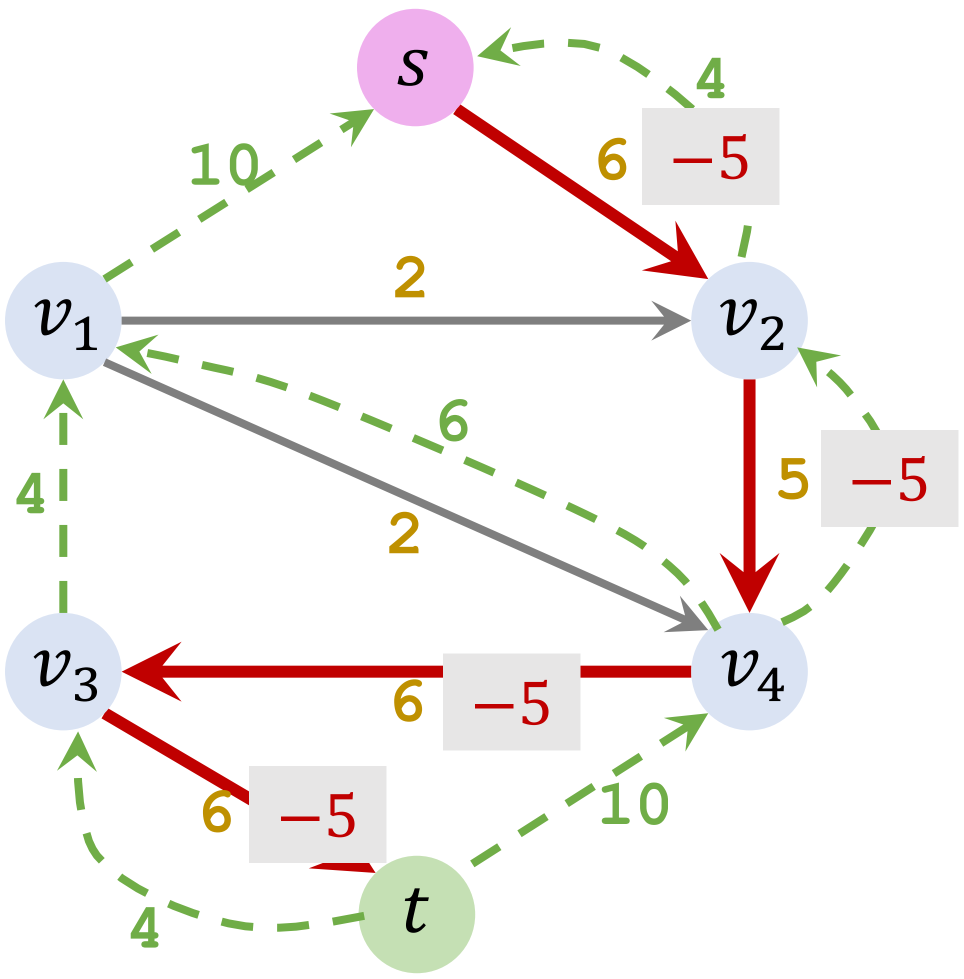


Old Residual Graph

Iteration 2: Update the residual graph

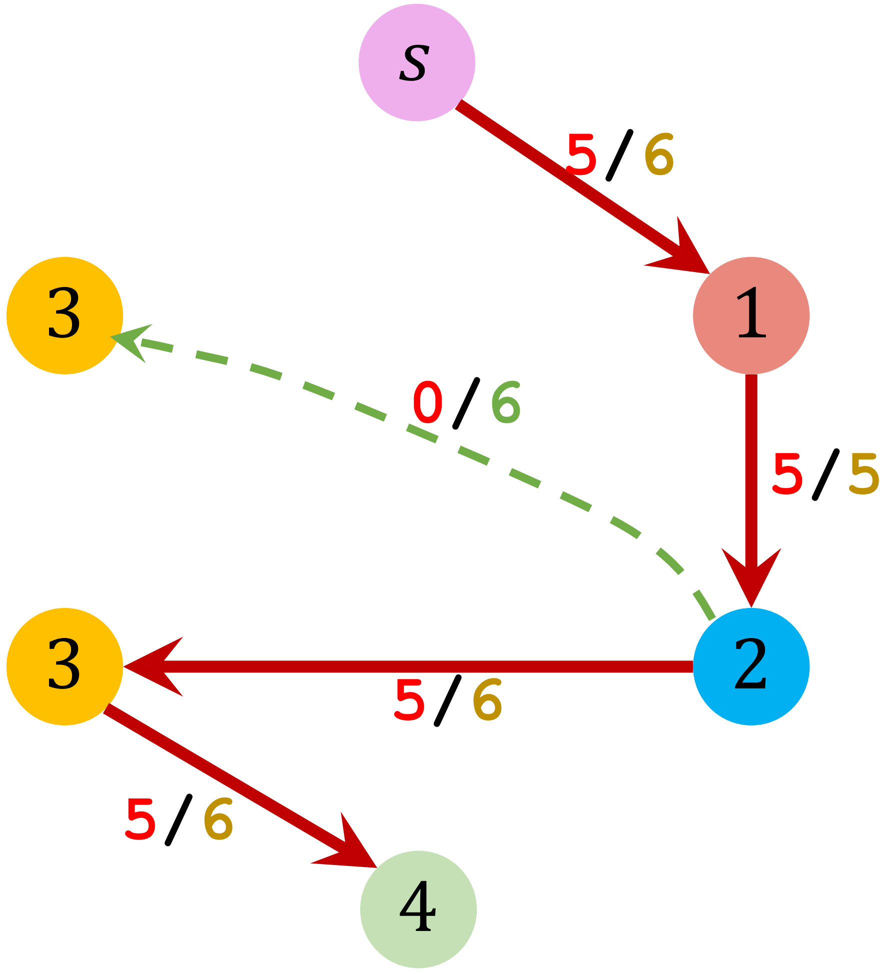


Blocking Flow

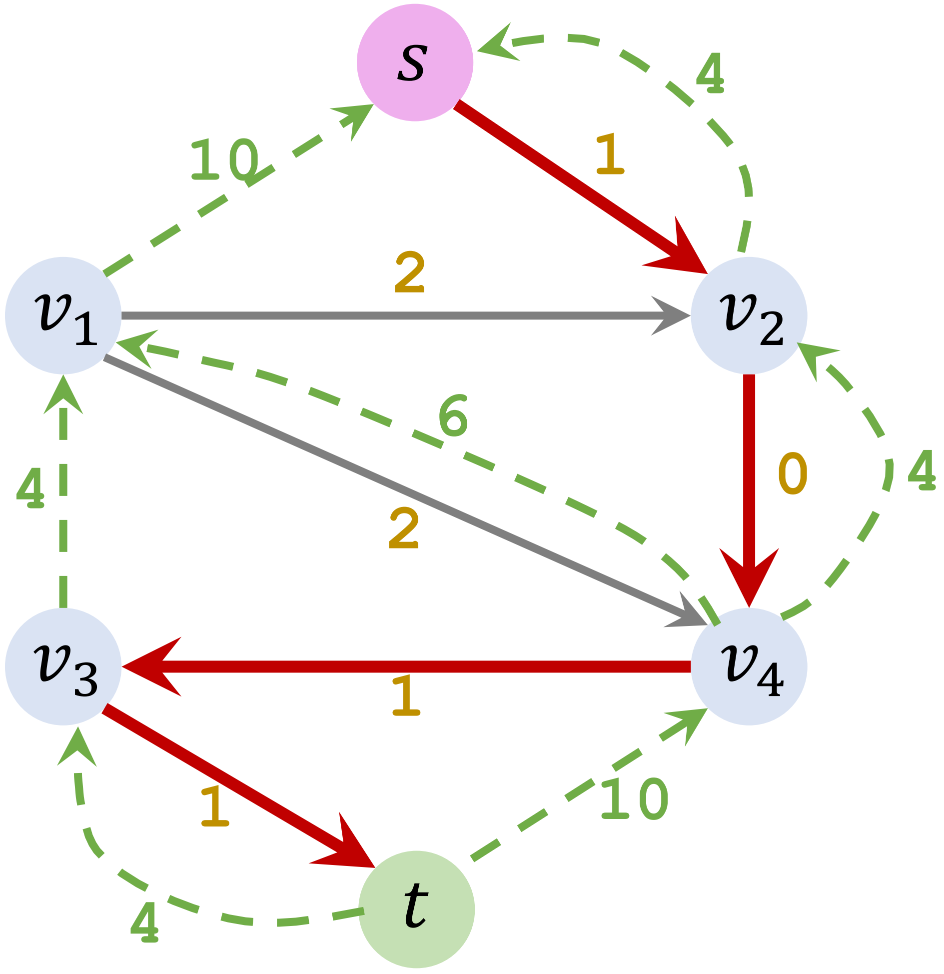


Old Residual Graph

Iteration 2: Update the residual graph

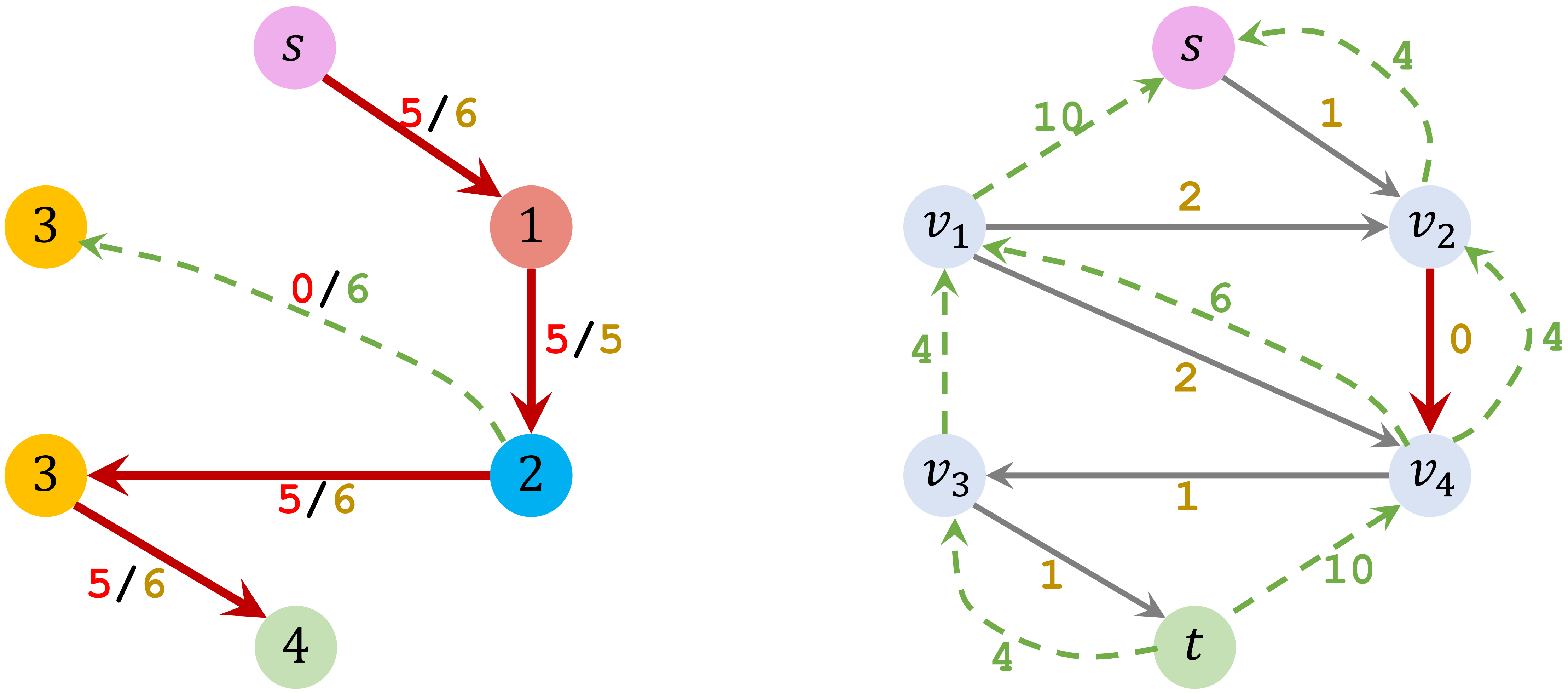


Blocking Flow



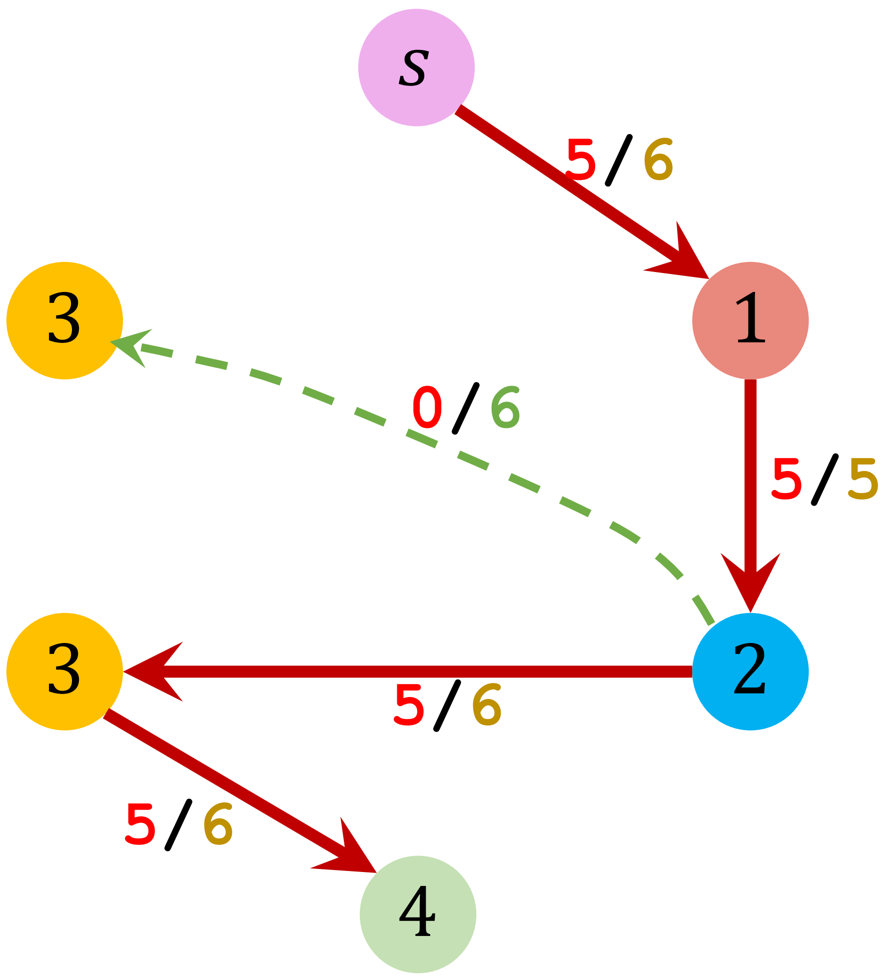
New Residual Graph

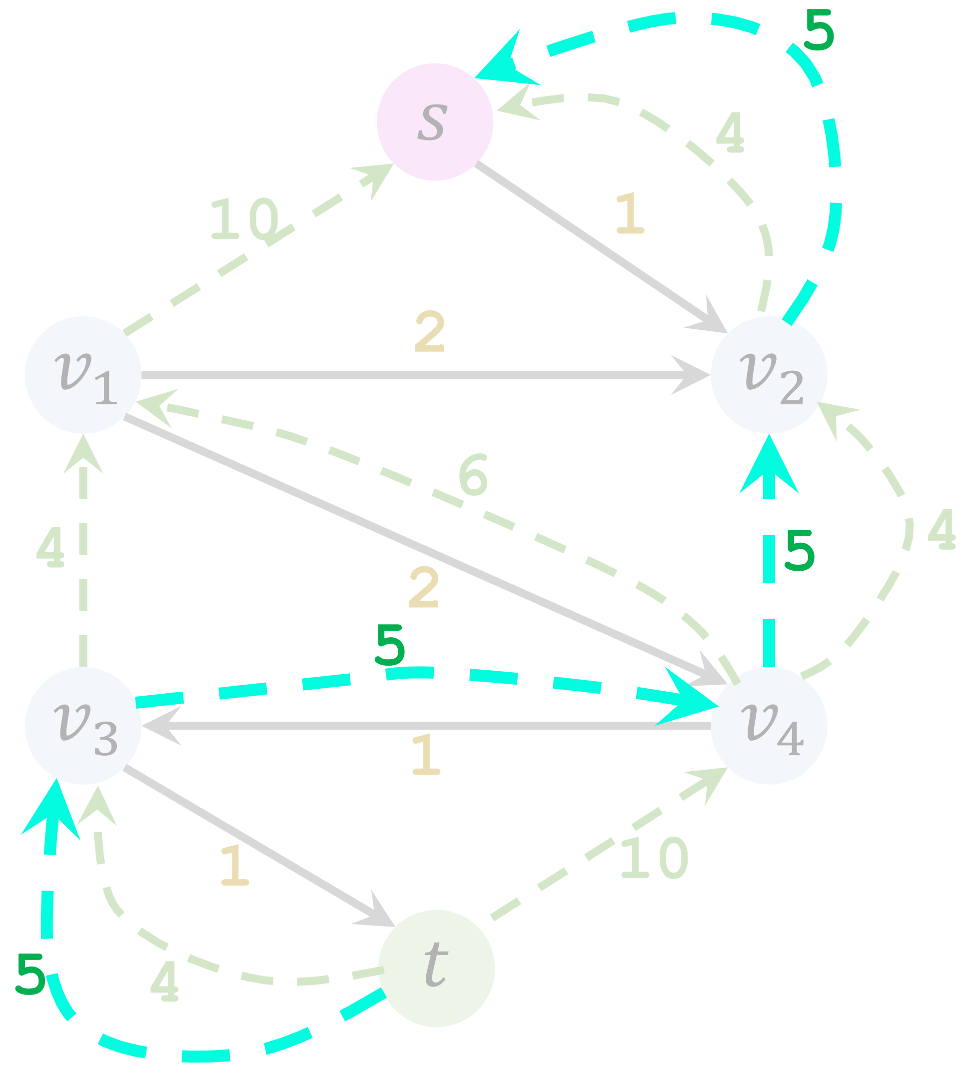
Iteration 2: Update the residual graph



Removed saturated edges from residual graph.

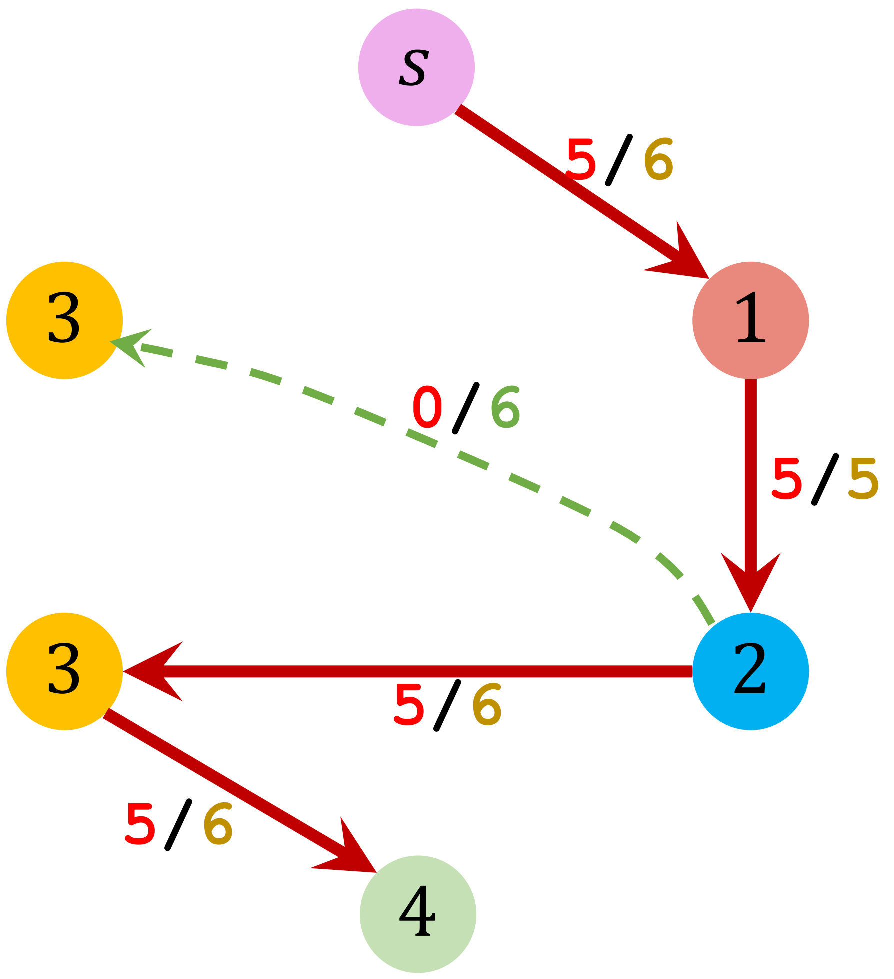
Iteration 2: Update the residual graph

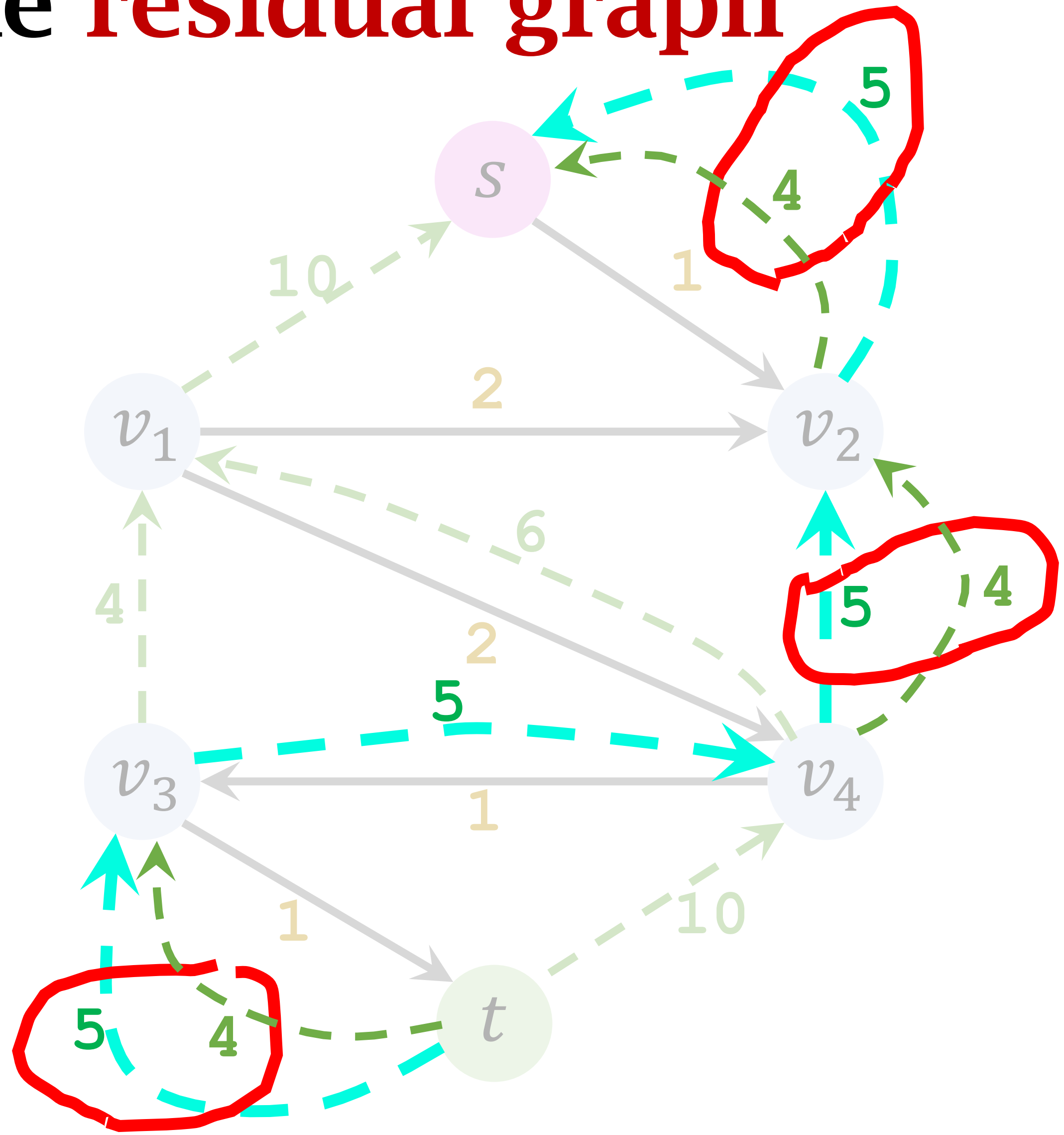




Add flows to the residual graph as backward paths.

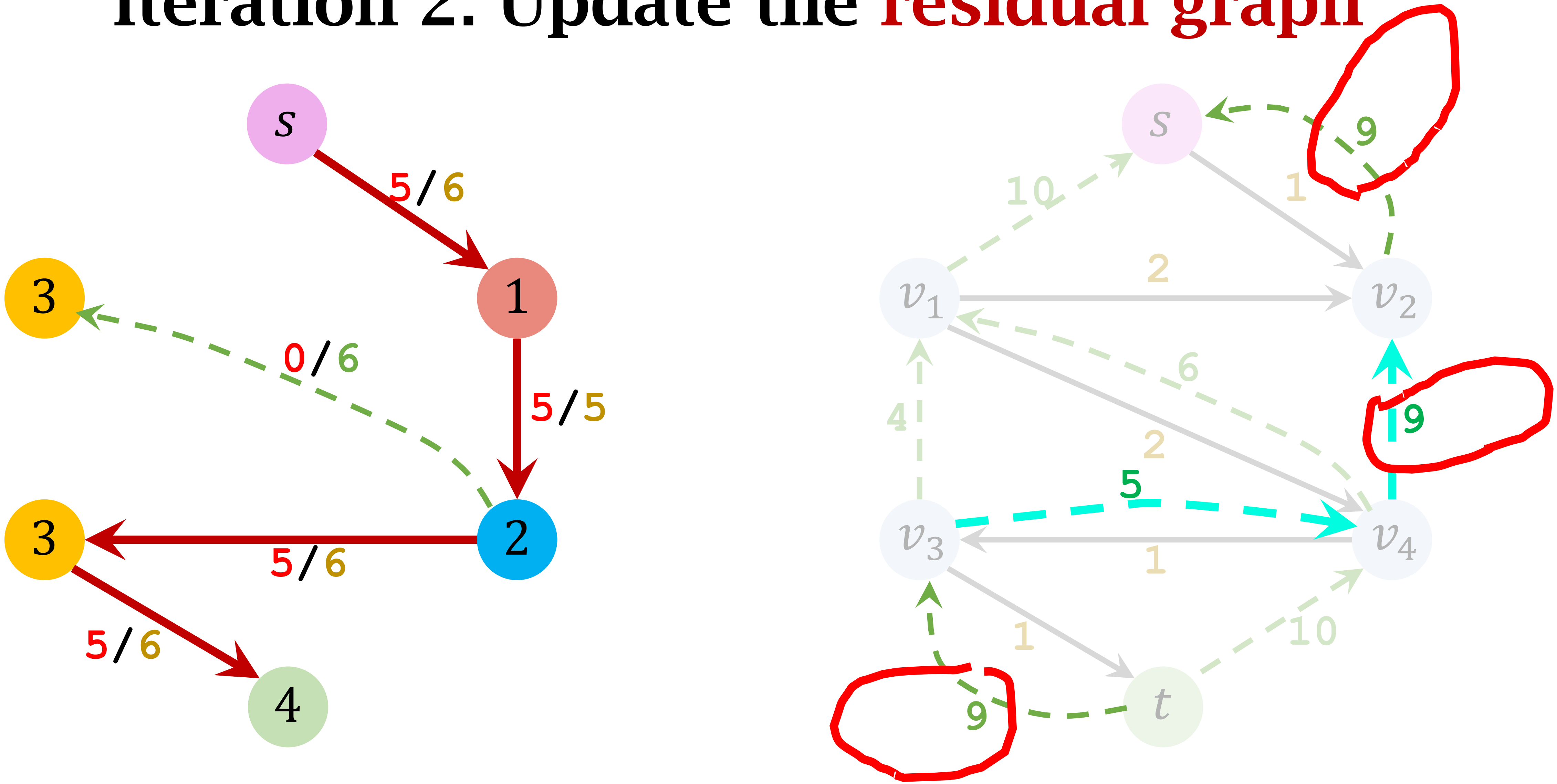
Iteration 2: Update the residual graph





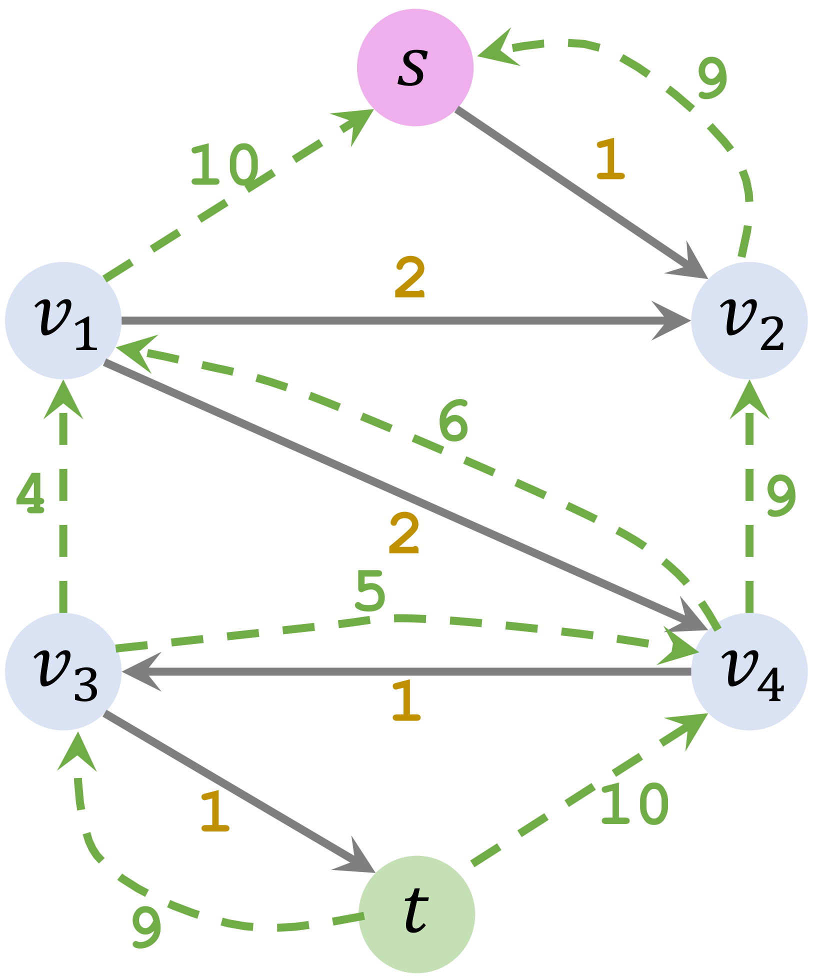
Merge Edges

Iteration 2: Update the residual graph



Merge Edges

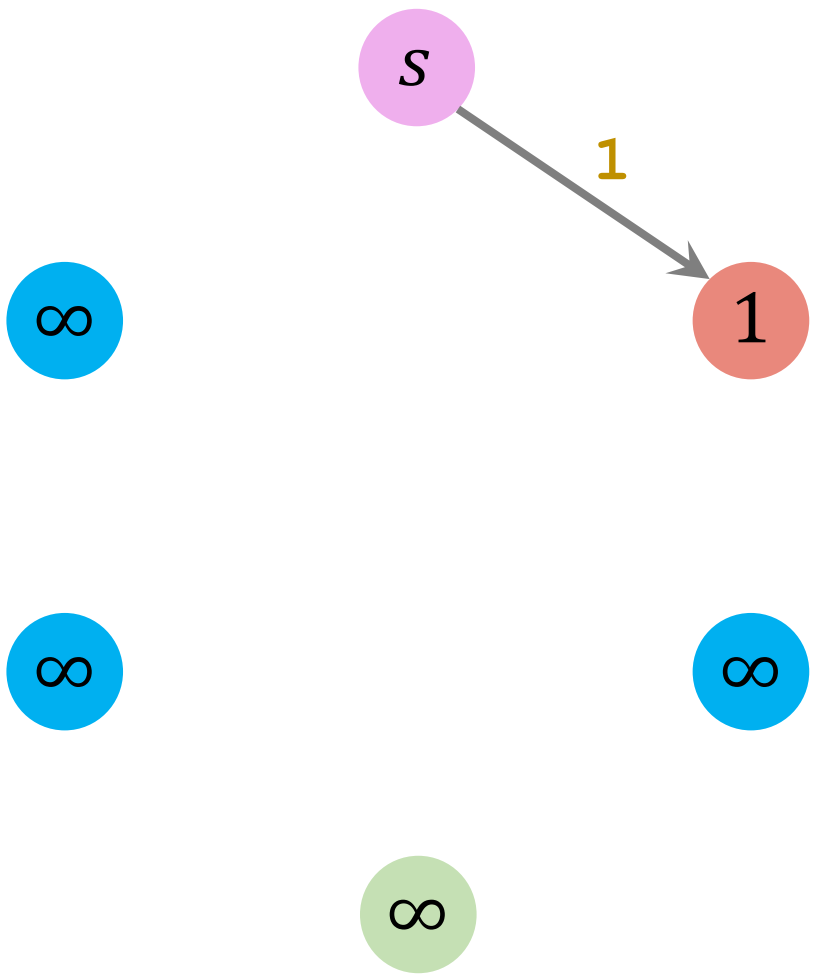
Iteration 3: Construct level graph



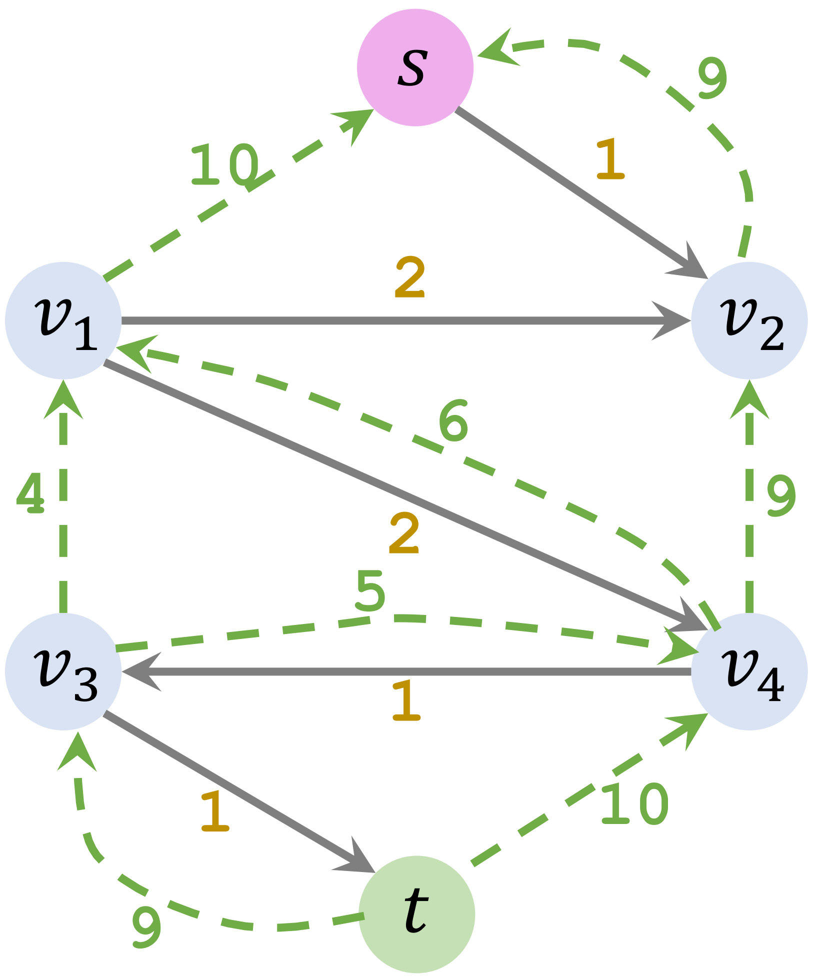
Level Graph

Residual Graph

Iteration 3: Construct level graph

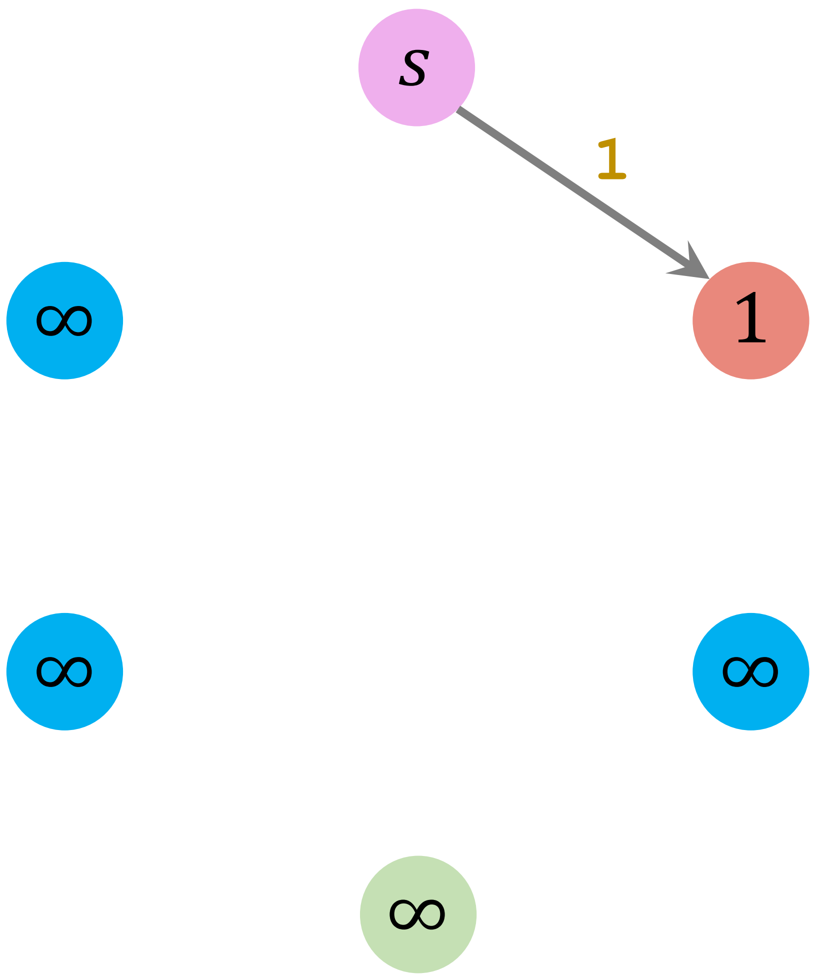


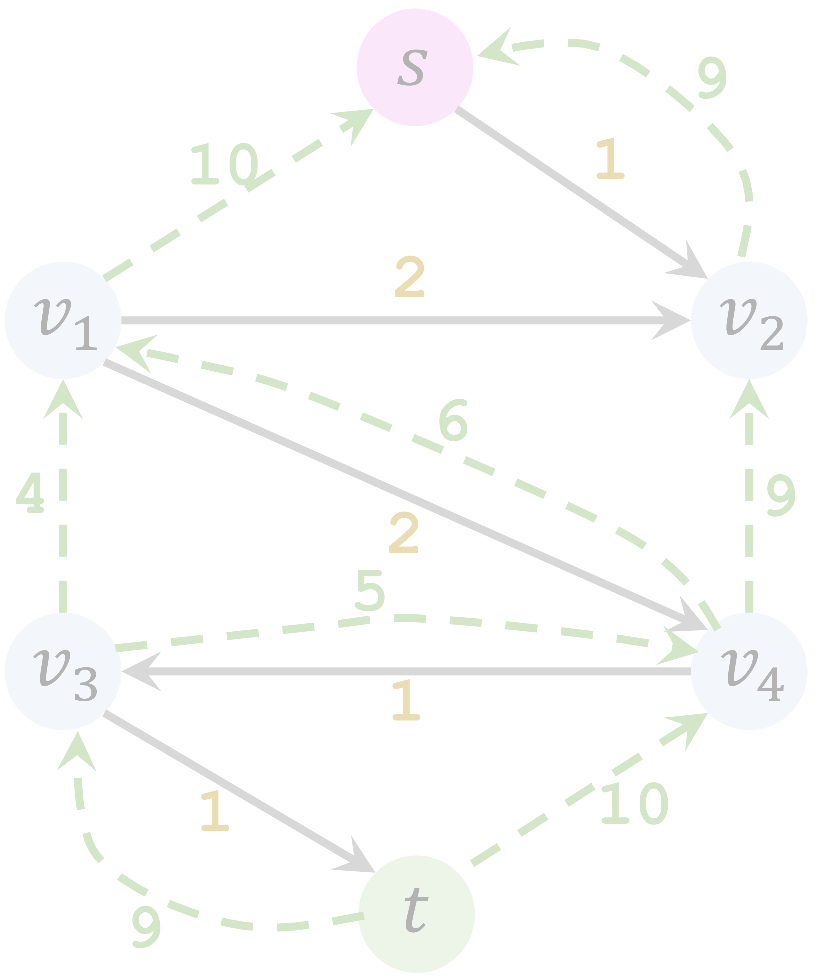
Level Graph



Residual Graph

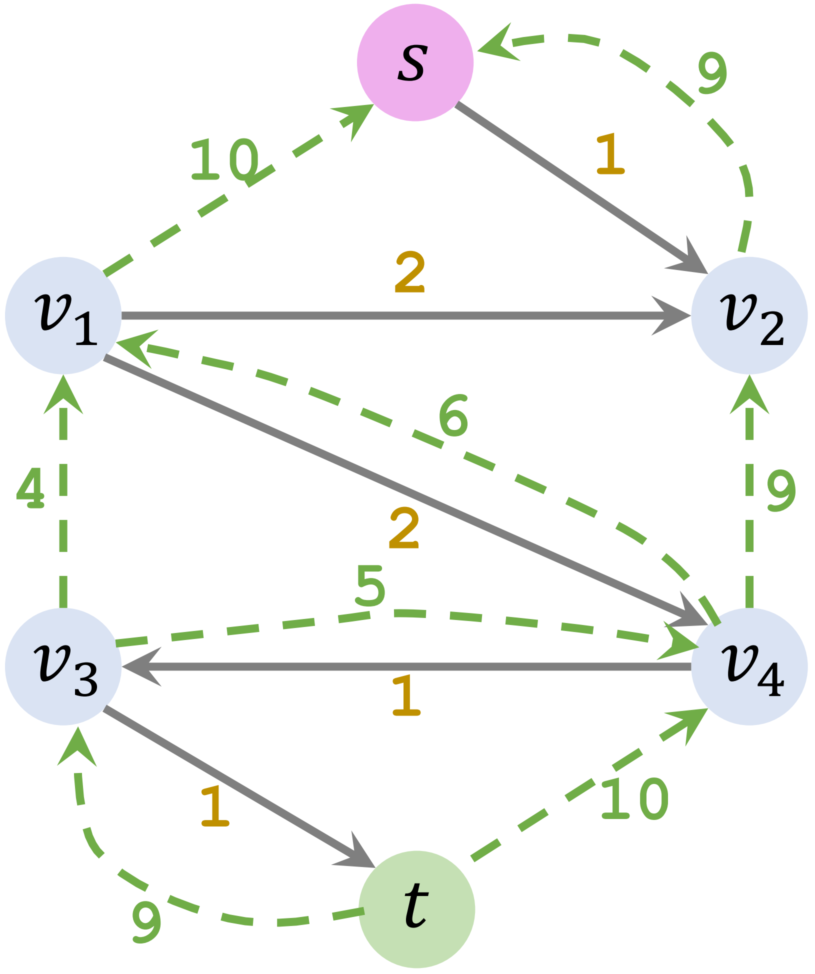
Iteration 3: Find blocking flow in level graph





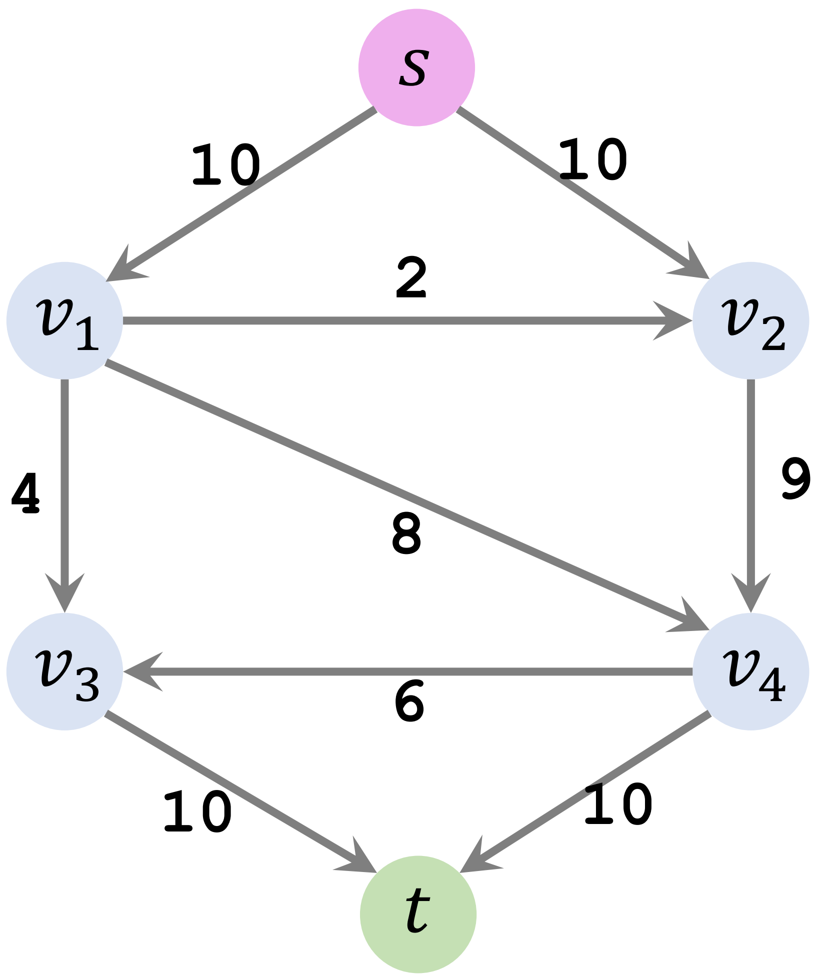
No flow can be found in the level graph.

End of Procedure

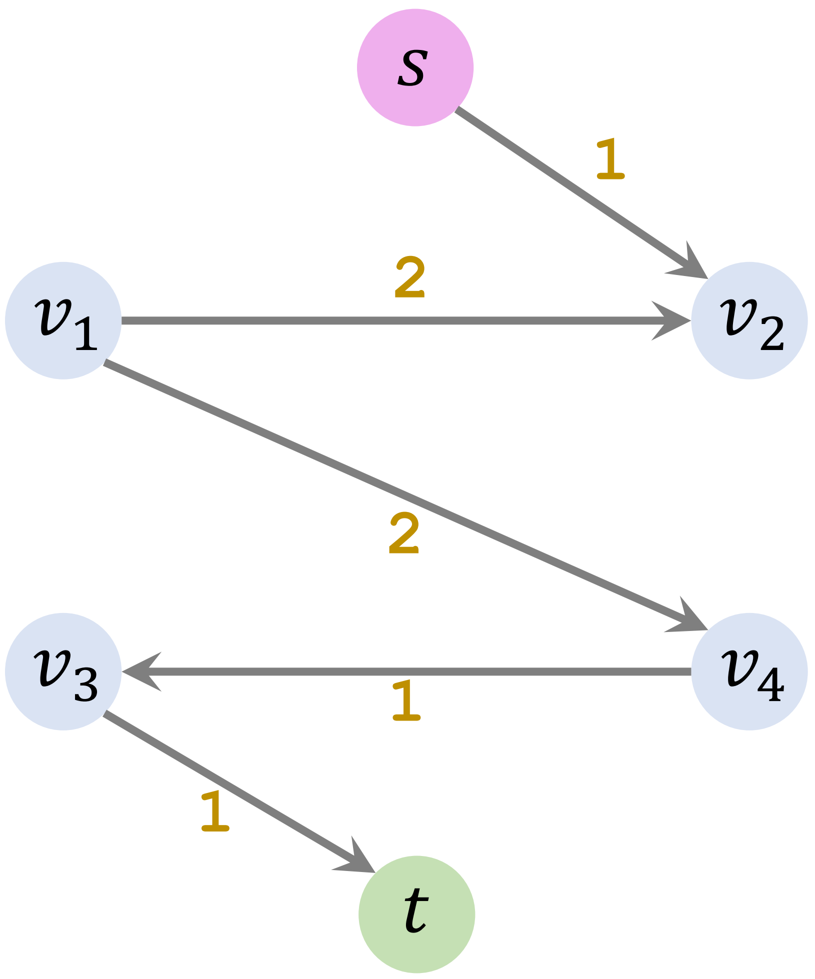


Residual Graph

End of Procedure

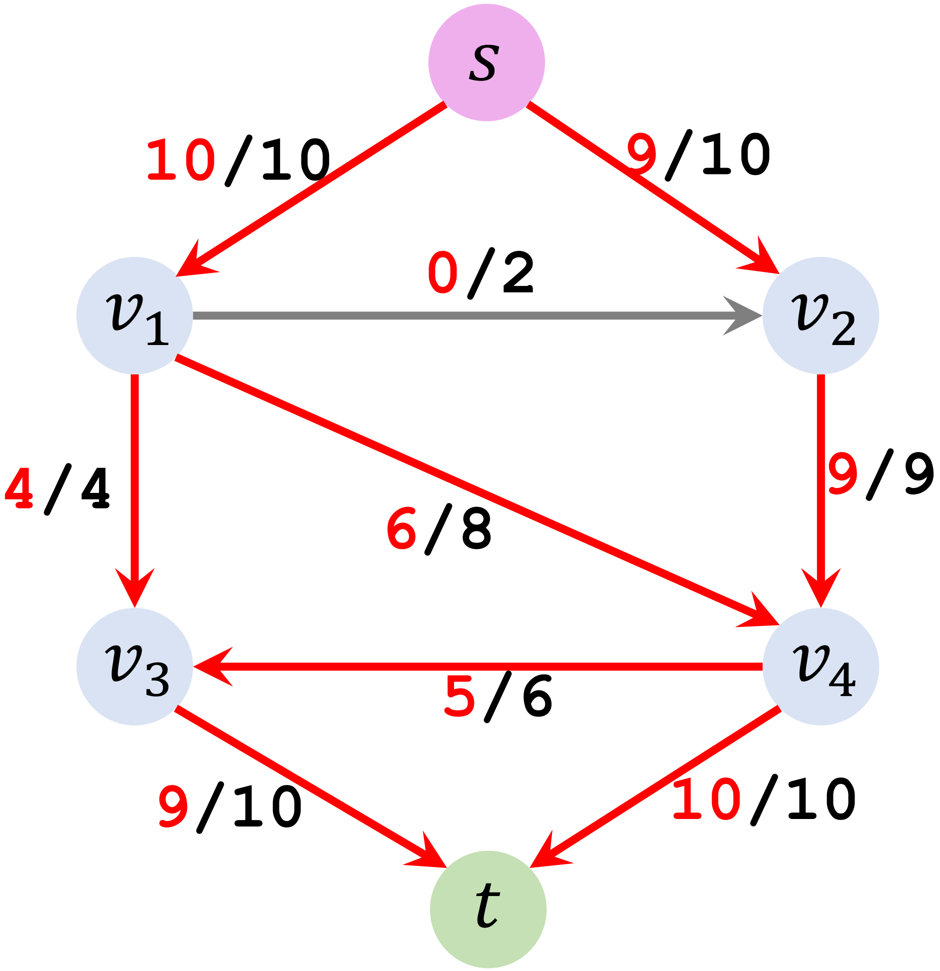


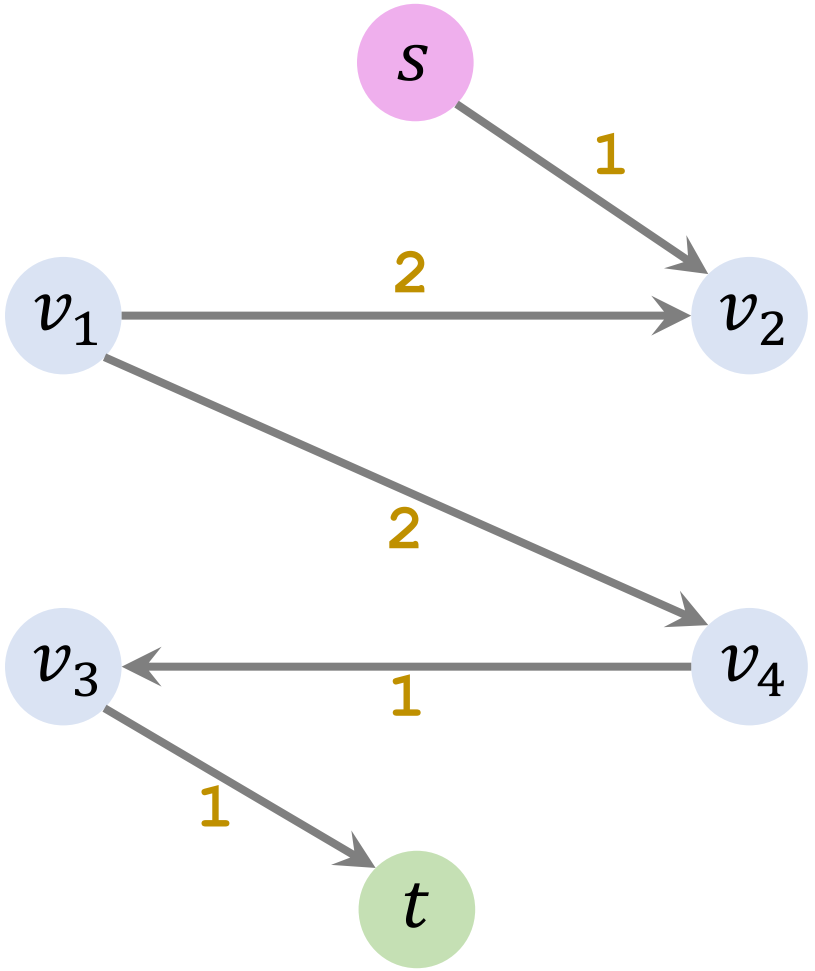
Original Graph



Residual Graph

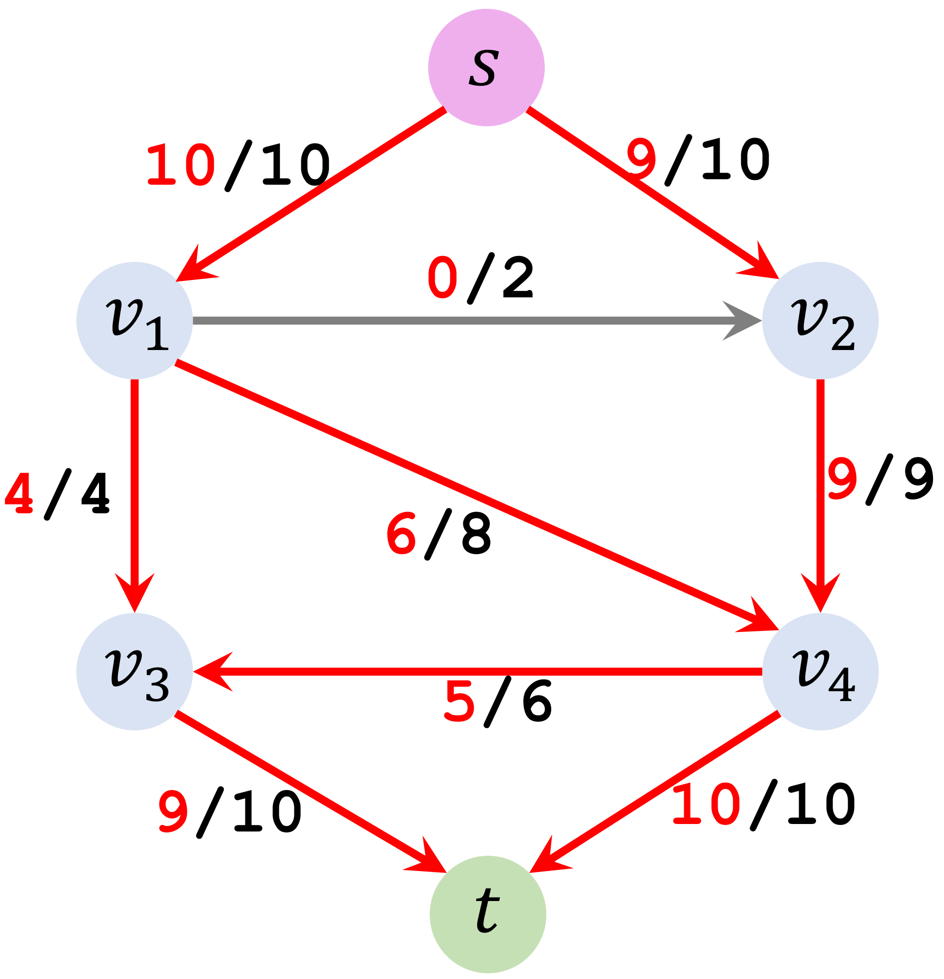
End of Procedure

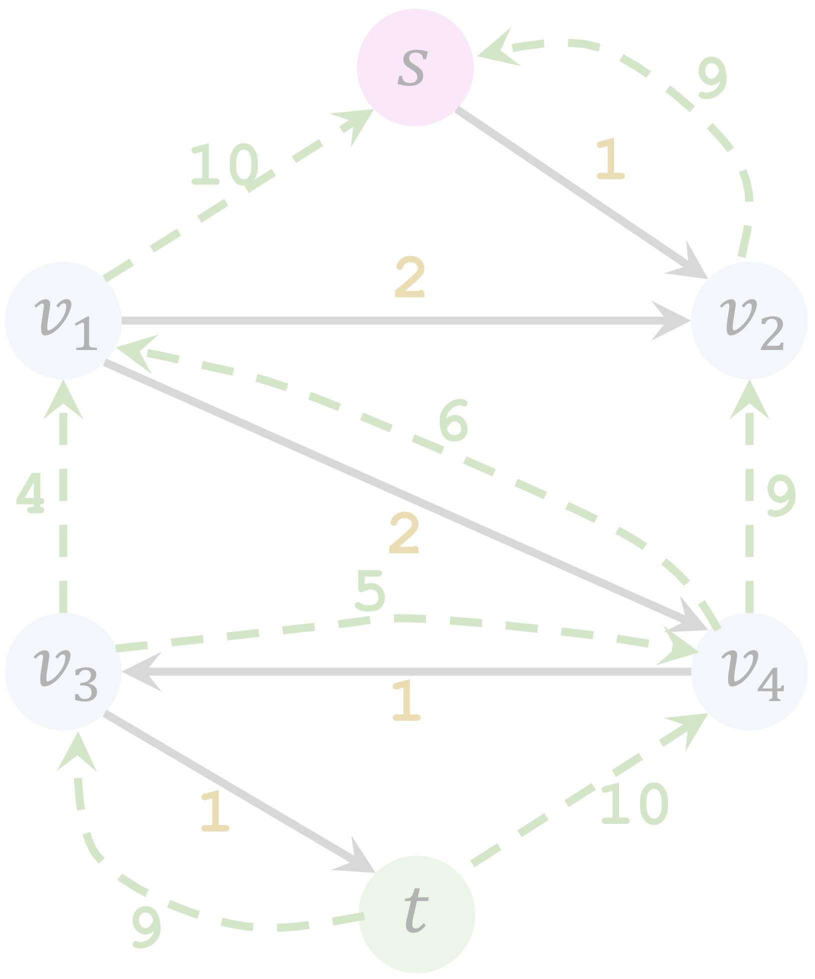




Flow = Capacity − Residual.

End of Procedure





Max Flow = 19. (Why? The flow leaving the source sum to 19.)

Dinic's Algorithm

1. Build the residual graph which is a copy of the original graph.

Dinic’s Algorithm

1. Build the residual graph which is a copy of the original graph.
2. Repeat:
    a. Construct the level graph of the residual graph.
    b. Break if the level graph has no path from source to sink.
    c. Find a blocking flow on the level graph.
    d. Update the residual graph (update the weights, remove saturated edges, and add backward edges.)

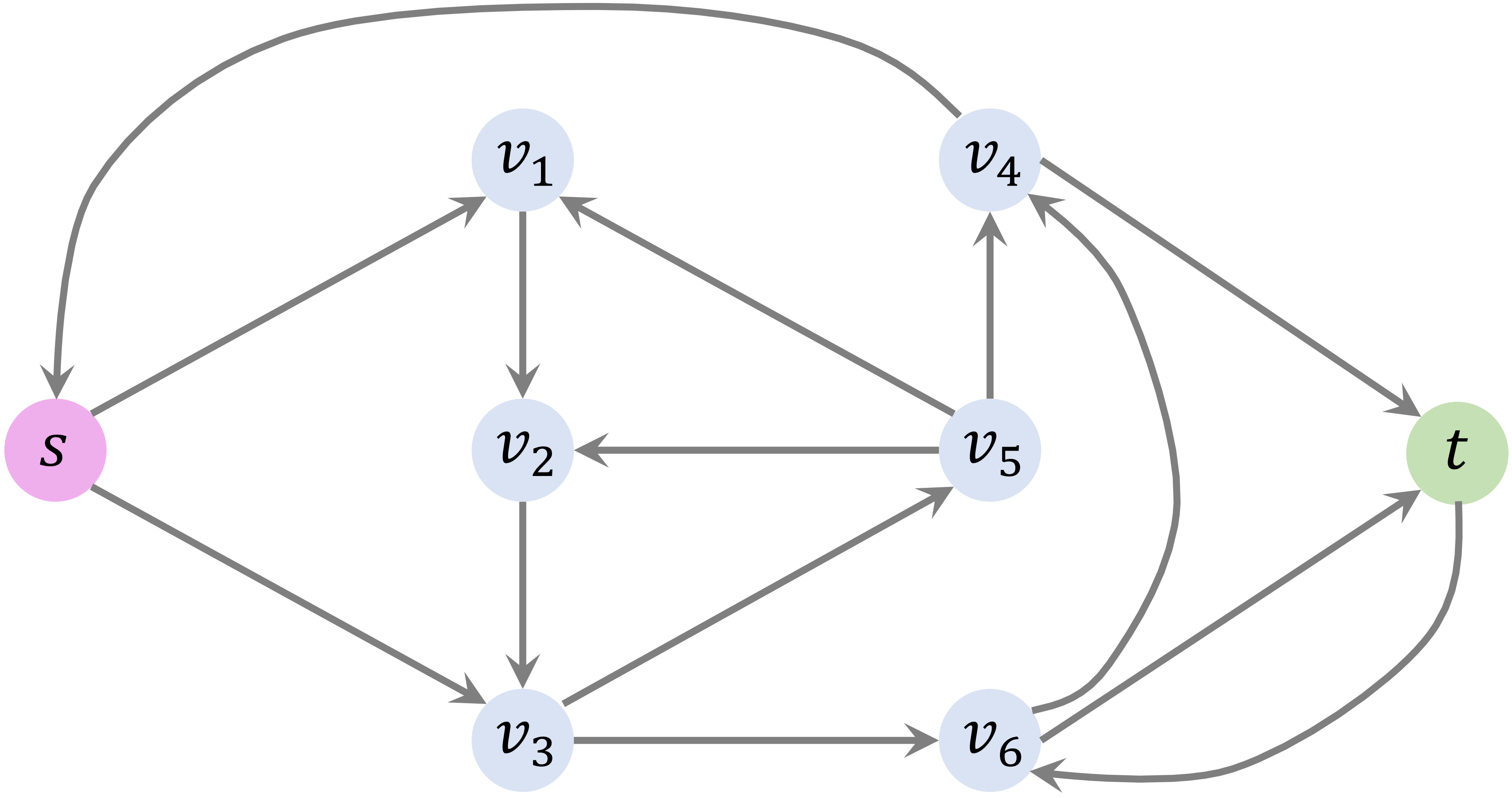
Time Complexity

Time complexity: $O(m \cdot n^2)$. (m is #edges; n is #vertices.)

- Dinic's algorithm has at most $n - 1$ iterations.
- Per-iteration time complexity is $O(mn)$.

Questions

Q1: What is the level graph?



Thank You!