Shusen Wang

- Let x to y be two strings.
- Convert x to y by insertion, deletion, and replacement.
- Edit distance is the minimum number of operations.

• Example inputs:

```
• x = "algorithm".
```

• y = "algorithms".

• Example output:

• Distance = 1. (Inserted 's' to x.)

• Example inputs:

```
• x = "algorithm".
```

• y = "alorithm".

• Example output:

• Distance = 1. (Deleted 'g'.)

• Example inputs:

```
• x = "algorithm".
```

```
• y = "blforithm".
```

• Example output:

• Distance = 1. (Replaced 'a' by 'b'.)

• Example inputs:

```
• x = "give".
```

• y = "giving".

• Example input:

• Distance = 3. (Replaced 'e' by 'i'; inserted 'n' and 'g'.)

Applications

- Application 1: Spelling corrections.
 - Find a word in the dictionary that minimizes the edit distance.
 - E.g., "algorithn" ==> "algorithm".

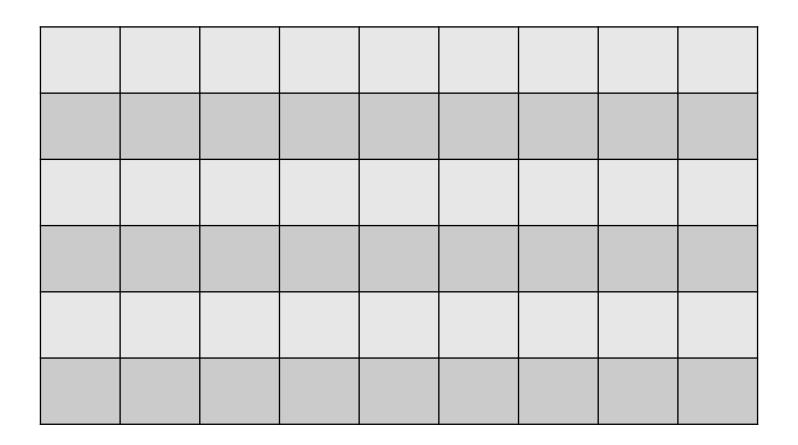
Applications

- Application 1: Spelling corrections.
 - Find a word in the dictionary that minimizes the edit distance.
 - E.g., "algorithn" ==> "algorithm".

- Application 2: Quantify the difference between two DNA sequences.
 - **DNA1** = "ATTAGCCCAAGGTAAACCCAGT...".
 - **DNA2** = "ATCAGCTCAAGGTAGACTCAAT...".

Optimal Substructure

D[i][j]: edit distance between x[1:i] and y[1:j].



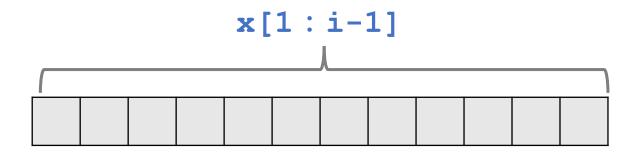
D[i][j]: edit distance between x[1:i] and y[1:j].

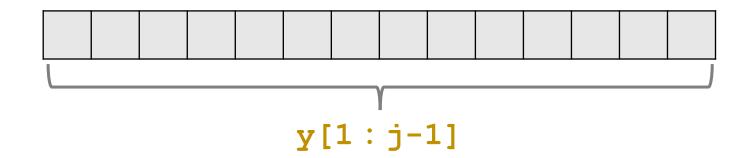
•
$$x = \text{"PQZABCDEFG"}$$
. • $x[1:4] = \text{"PQZA"}$.

•
$$y = "PQBCDEFG"$$
. • $y[1:2] = "PQ"$.

• What is D[4][2]? • Thus D[4][2] = 2.

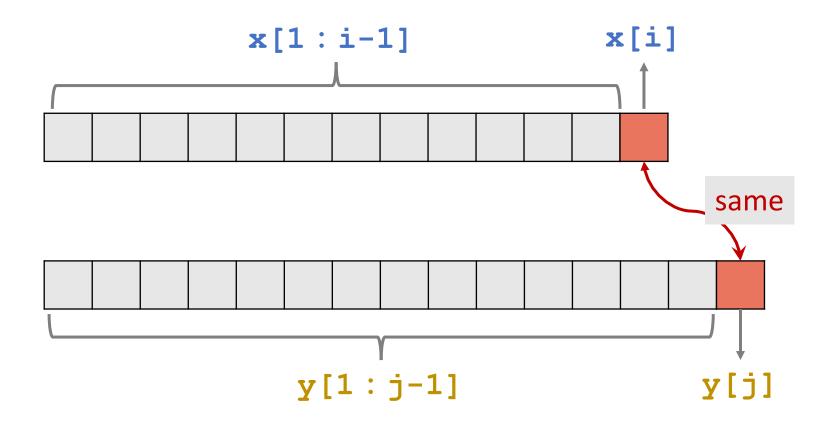
Case 1: x[i] = y[j]



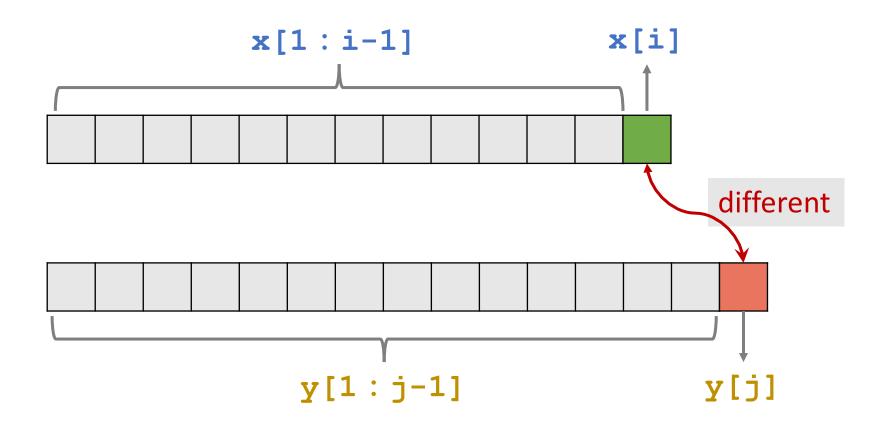


Suppose we already know D[i-1][j-1].

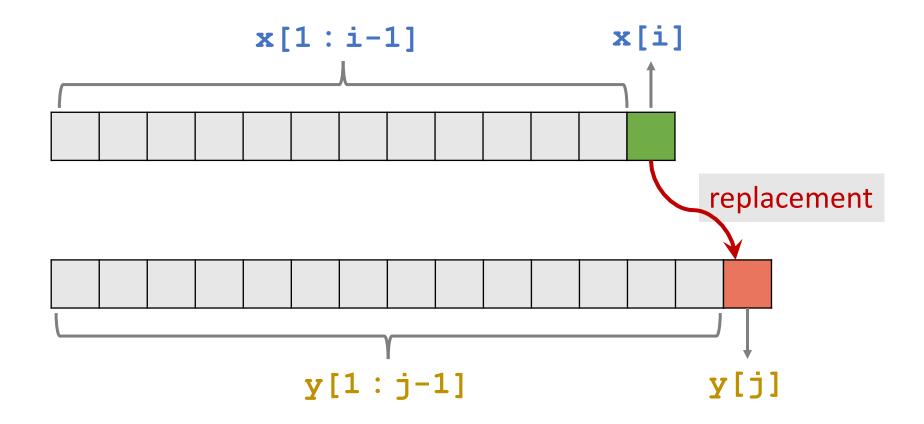
Case 1: x[i] = y[j]



If x[i] = y[j], then D[i][j] = D[i-1][j-1].

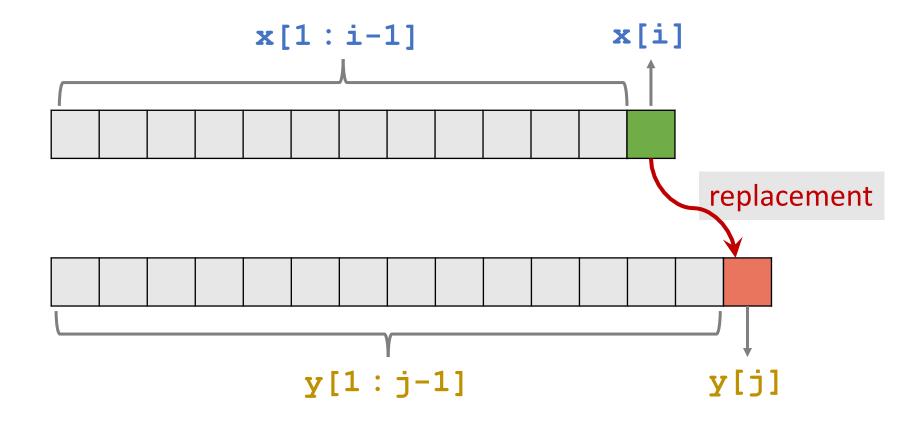


Case 2(A): Replacement



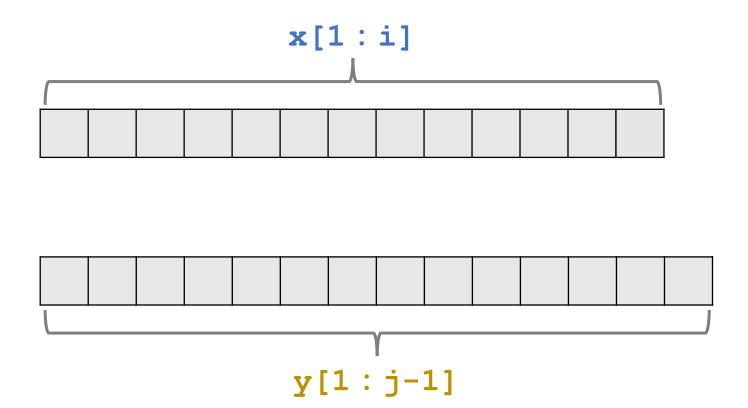
Case 2(A): y[j] is obtained by replacing x[i].

Case 2(A): Replacement



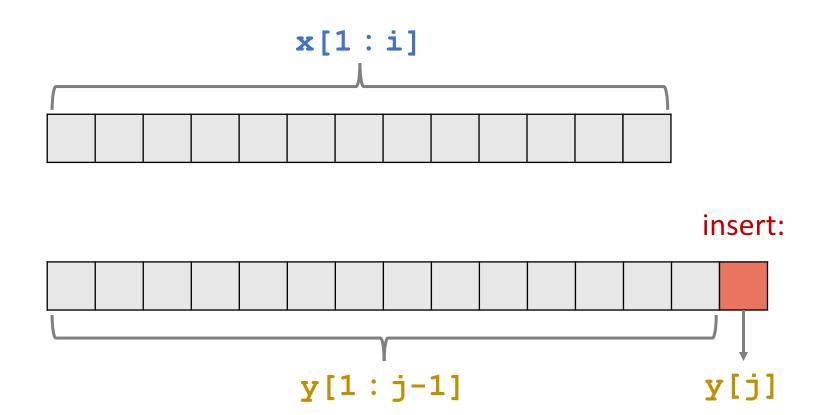
In the case of replacement, D[i][j] = D[i-1][j-1] + 1.

Case 2(B): Insertion



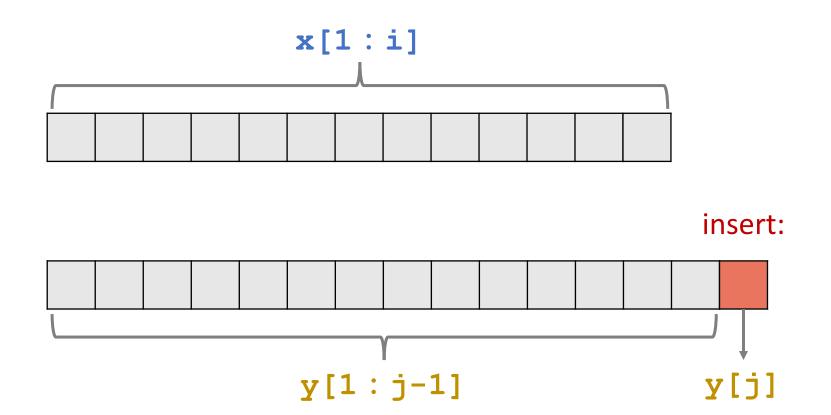
Case 2(B): y[j] is obtained by insertion.

Case 2(B): Insertion



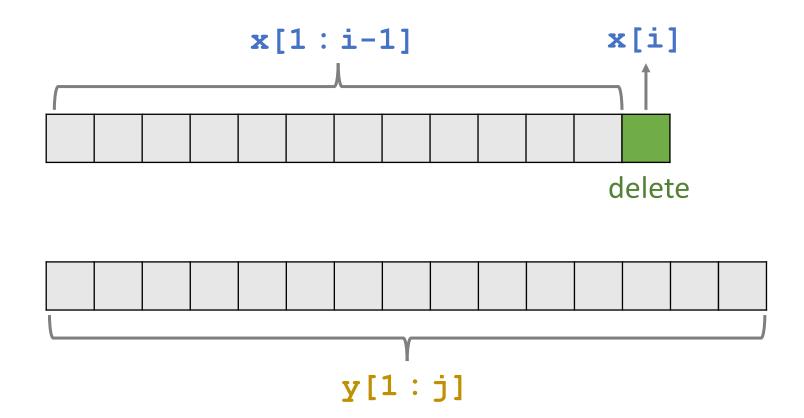
Case 2(B): y[j] is obtained by insertion.

Case 2(B): Insertion



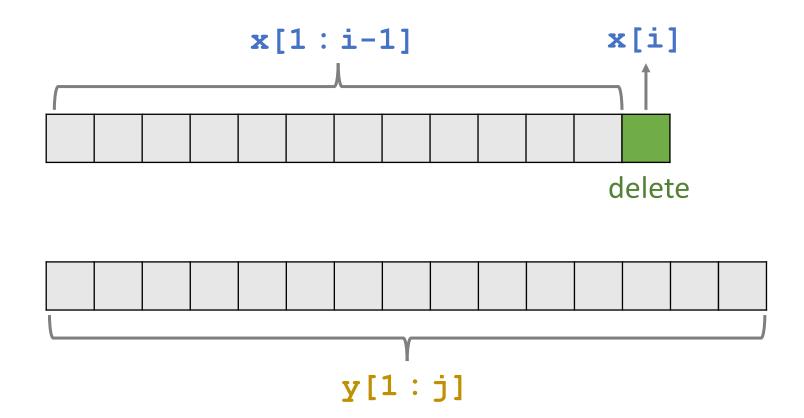
In the case of insertion, D[i][j] = D[i][j-1] + 1.

Case 2(C): Deletion



Case 2(C): y[1:j] is obtained by deleting x[i].

Case 2(C): Deletion



In the case of deletion, D[i][j] = D[i-1][j] + 1.

```
In the case of replacement, D[i][j] = D[i-1][j-1] + 1.

In the case of insertion, D[i][j] = D[i][j-1] + 1.

In the case of deletion, D[i][j] = D[i-1][j] + 1.
```

```
In the case of replacement, D[i][j] = D[i-1][j-1] + 1.

In the case of insertion, D[i][j] = D[i][j-1] + 1.

In the case of deletion, D[i][j] = D[i-1][j] + 1.
```

In the case of replacement, D[i][j] = D[i-1][j-1] + 1.

In the case of insertion, D[i][j] = D[i][j-1] + 1.

In the case of deletion, D[i][j] = D[i-1][j] + 1.

• In sum, if
$$\mathbf{x}[i] \neq \mathbf{y}[j]$$
 then
$$D[i][j] = 1 + \min \begin{cases} D[i-1][j-1] \\ D[i][j-1] \\ D[i-1][j] \end{cases}.$$

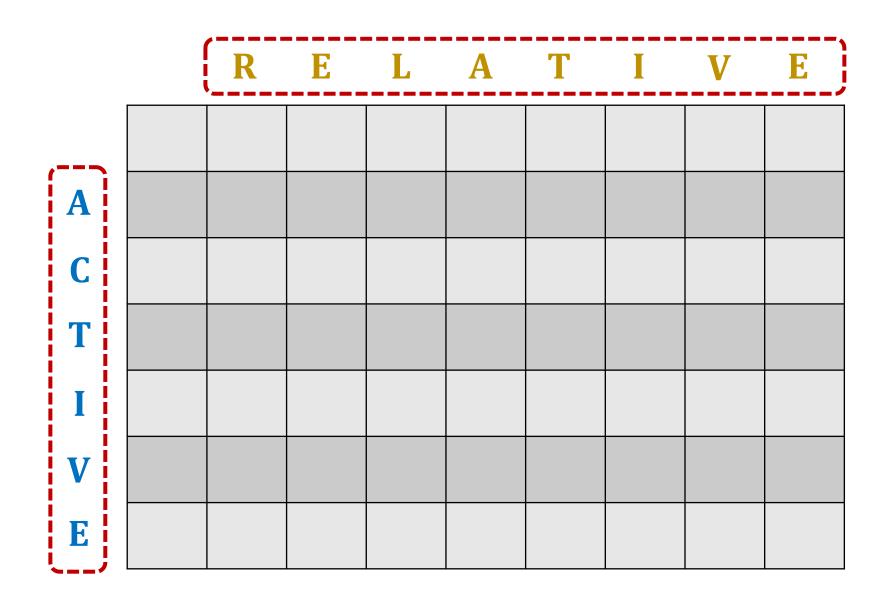
Optimal Substructure

• Case 1: If x[i] = y[j], then D[i][j] = D[i-1][j-1].

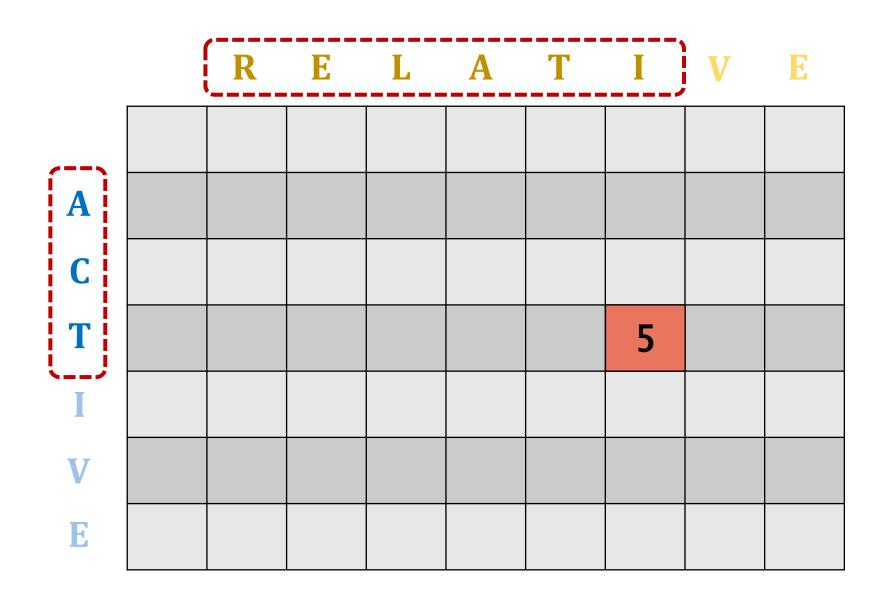
• Case 2: If $x[i] \neq y[j]$, then

$$D[i][j] = 1 + \min \begin{cases} D[i-1][j-1] \\ D[i][j-1] \\ D[i-1][j] \end{cases}.$$

Example



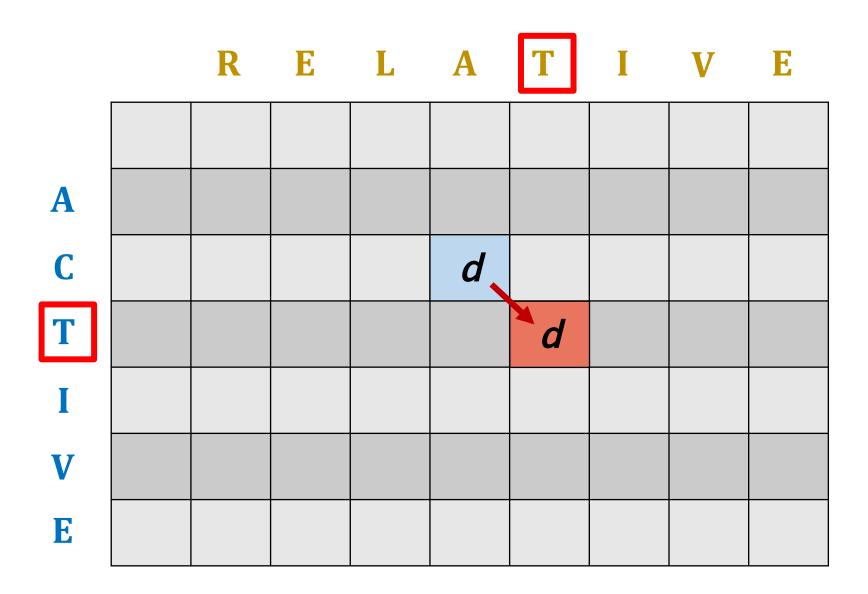
Example



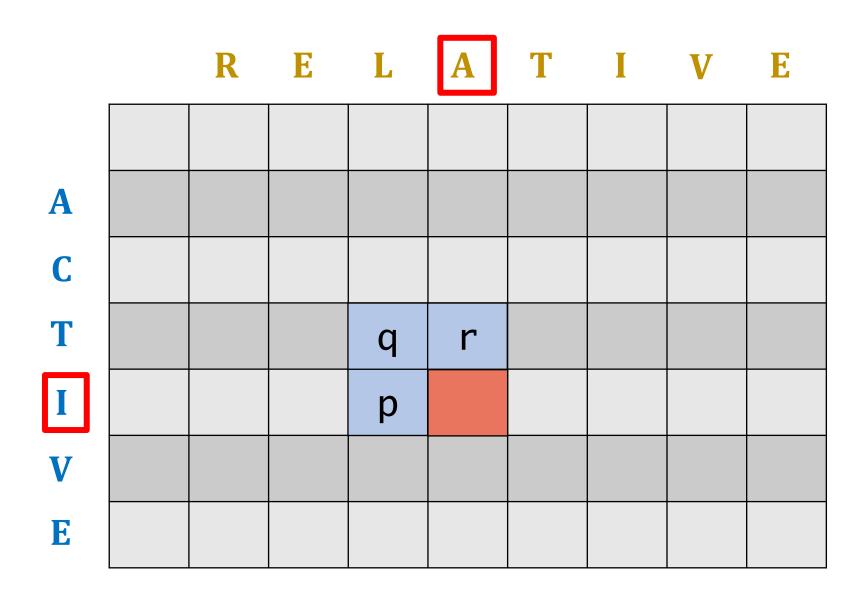
Case 1: x[i] = y[j]

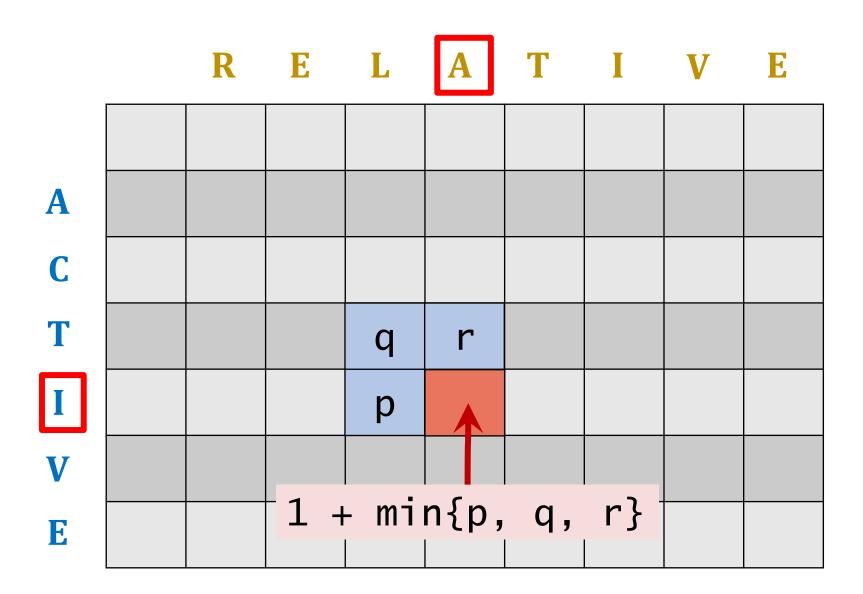
	R	E	L	A	T	I	V	E
A								
C				d				
T								
I								
V								
E								

Case 1: x[i] = y[j]



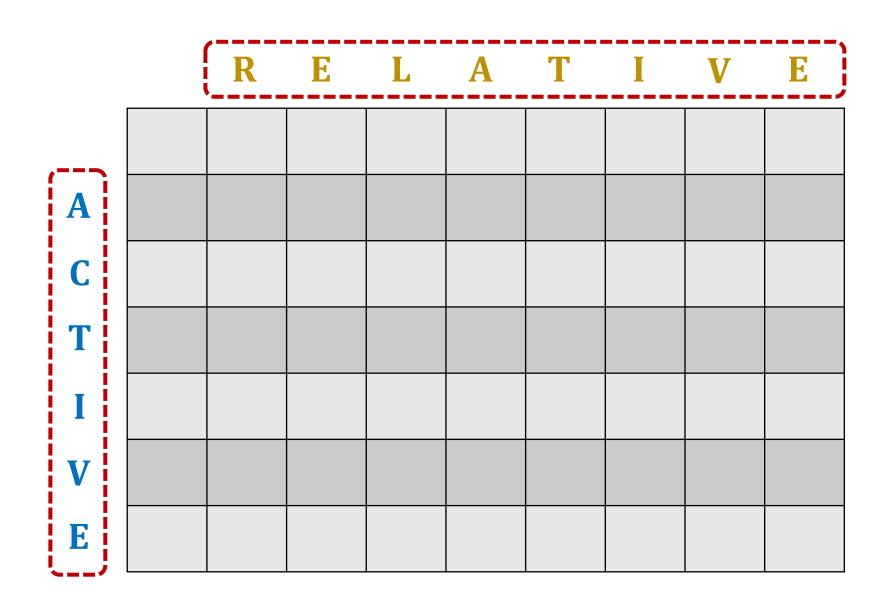
Case 2: $x[i] \neq y[j]$





Dynamic Programming

Inputs: two strings



Base Case

NULL R			E	L	A	T	I	V	E
NULL									
A									
C									
T									
I									
V									
E									

Base Case

NULL R			E	L	A	T	I	V	E	
NULL	0									
A										
C										
T										
I										
V										
E										

Base Case

NULL R			E	L	A	T	I	V	E	
NULL	0	1	2	3	4	5	6	7	8	
A										
C										
T										
I										
V										
E										

Base Case

	NULL	R	E	L	A	T	I	V	E
NULI	0	1	2	3	4	5	6	7	8
A									
C									
T									
I									
V									
E									

Base Case

	NULL	R	E	L	A	T	I	V	E
NULI	0	1	2	3	4	5	6	7	8
A	1								
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	?							
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	?							
C	2 1	min	{1,(0,1	+	1			
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	?						
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1-	→?						
C	2	m	in{	1,1	,2}	+ 1	L		
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[3]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2-	→ 3					
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] = y[4]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	?				
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] = y[4]$$

		R	E	L	A	T	I	V	E	
	0	1	2	3	4	5	6	7	8	
A	1	1	2	3	?					
C	2				L[1] [4]] =	L[()][3	3]
T	3									
I	4									
V	5									
E	6									

$$x[1] \neq y[5]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3 –	→ 4			
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[6]$$

		R	E	L	A	T	Ι	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4-	→ 5		
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[7]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5 –	→ 6	
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[1] \neq y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6 –	→ 7
C	2								
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2							
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2						
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[3]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2-	→ 3					
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[4]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3 –	4				
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[5]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4			
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[6]$$

		R	E	L	A	T	Ι	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4 –	→ 5		
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[7]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5 -	6	
T	3								
I	4								
V	5								
E	6								

$$x[2] \neq y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6 –	→ 7
T	3								
I	4								
V	5								
E	6								

$$x[3] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3							
I	4								
V	5								
E	6								

$$x[3] \neq y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3						
I	4								
V	5								
E	6								

$$x[3] \neq y[3]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3					
I	4								
V	5								
E	6								

$$x[3] \neq y[4]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3 -	4				
I	4								
V	5								
E	6								

$$x[3] = y[5]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4				
I	4								
V	5								
E	6								

$$x[3] = y[5]$$

		R	E	L	A	T	I	V	E	
	0	1	2	3	4	5	6	7	8	
A	1	1	2	3	3	4	5	6	7	
C	2	2	2	3	4	4	5	6	7	
T	3	3	3	3	4	?				
I	4					L[3] [5]] =	L[2	2][4]
V	5									
E	6									

$$x[3] \neq y[6]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4-	5		
I	4								
V	5								
E	6								

$$x[3] \neq y[7]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5 –	→ 6	
I	4								
V	5								
E	6								

$$x[3] \neq y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6 -	→ 7
I	4								
V	5								
E	6								

$$x[4] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	—	3	3	4	4	5	6	7
I	4	4							
V	5								
E	6								

$$x[4] \neq y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	—	3	4	4	5	6	7
I	4	4	4						
V	5								
E	6								

$$x[4] \neq y[3]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3_	4	4	5	6	7
I	4	4	4	4					
V	5								
E	6								

$$x[4] \neq y[4]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4				
V	5								
E	6								

$$x[4] \neq y[5]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5			
V	5								
E	6								

$$x[4] = y[6]$$

		R	E	L	A	T	Ι	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5			
V	5								
E	6								

$$x[4] = y[6]$$

		R	E	L	A	T	I	V	E	
	0	1	2	3	4	5	6	7	8	
A	1	1	2	3	3	4	5	6	7	
C	2	2	2	3	4	4	5	6	7	
T	3	3	3	3	4	4	5	6	7	
I	4	4	4	4	4	5	?			
V	5					•	L[4]	[6]	=	L[3][5]
E	6									

$$x[4] \neq y[7]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4-	5	
V	5								
E	6								

$$x[4] \neq y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5 –	→ 6
V	5								
E	6								

$$x[5] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4_	4	4	4	5	4	5	6
V	5	5							
E	6								

$$x[5] \neq y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4_	4	4	5	4	5	6
V	5	5	5						
E	6								

$$x[5] \neq y[3]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5					
E	6								

$$x[5] \neq y[4]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4_	5	4	5	6
V	5	5	5	5	5				
E	6								

$$x[5] \neq y[5]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5			
E	6								

$x[5] \neq y[6]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5		
E	6								

$$x[5] = y[7]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	? ·	
E	6								

$$x[5] = y[7]$$

		R	E	L	A	T	I	V	E	
	0	1	2	3	4	5	6	7	8	
A	1	1	2	3	3	4	5	6	7	
C	2	2	2	3	4	4	5	6	7	
T	3	3	3	3	4	4	5	6	7	
I	4	4	4	4	4	5	4	5	6	
V	5	5	5	5	5	5	5	*?		
E	6						L[5] [7]] =	L[4][6]

$$x[5] \neq y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4 –	→ 5
E	6								

$$x[6] \neq y[1]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5_	5	5	5	5	5	4	5
E	6	6							

$$x[6] = y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	?:						

$$x[6] = y[2]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	?	L	[6]	[2]	=]	L[5]	[1]

$$x[6] \neq y[3]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5 –	→ 6					

$$x[6] \neq y[4]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5_	5	5	4	5
E	6	6	5	6	6				

$$x[6] \neq y[5]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5_	5	4	5
E	6	6	5	6	6	6			

$$x[6] \neq y[6]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5_	4	5
E	6	6	5	6	6	6	6		

$$x[6] \neq y[7]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	

$$x[6] = y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	?

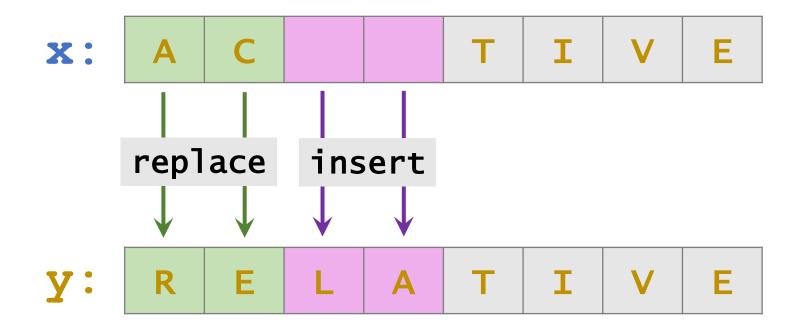
$$x[6] = y[8]$$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	L[6	5] [8	8] =	: L[5] [7] -	?

End of Procedure

		R	E	L	A	T	I	V	E
.===	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
Ι	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

Edit Distance = 4



Summary

Optimal Substructure

```
Case 1: x[i] = y[j].
```

• Recursion: D[i][j] = D[i-1][j-1].

Optimal Substructure

Case 1:
$$x[i] = y[j]$$
.

• Recursion: D[i][j] = D[i-1][j-1].

Case 2:
$$x[i] \neq y[j]$$
.

• Recursion: D[i][j] =
$$1 + \min \begin{cases} D[i-1][j-1] & \text{replacement} \\ D[i][j-1] & \text{insertion} \\ D[i-1][j] & \text{deletion} \end{cases}$$

Time Complexity

- Inputs:
 - String \mathbf{x} . (The length is m.)
 - String y. (The length is n.)
- Size of the table is $(m + 1) \times (n + 1)$.
- O(1) time for filling each entry of the table.
- Thus, the overall time complexity is O(mn).

Thank You!

Find the edits from the table

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

		R	E	L	V	E			
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	a	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

insert										
R E L A T I V E										
	0	1	2	3	4	5	6	7	8	
A	1	1	2	3	3	4	5	6	7	
C	2	2	2-	→ 3	4	4	5	6	7	
T	3	3	3	3	4	4	5	6	7	
I	4	4	4	4	4	5	4	5	6	
V	5	5	5	5	5	5	5	4	5	
E	6	6	5	6	6	6	6	5	4	

	replace insert								
		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
replace C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

			repla	ace	inse	ert				
			R	E	L	A	T	I	V	E
		0	1	2	3	4	5	6	7	8
replace	A	1	1	2	3	3	4	5	6	7
replace	C	2	2	2	3	4	4	5	6	7
	T	3	3	3	3	4	4	5	6	7
	I	4	4	4	4	4	5	4	5	6
	V	5	5	5	5	5	5	5	4	5
	E	6	6	5	6	6	6	6	5	4