Skip List

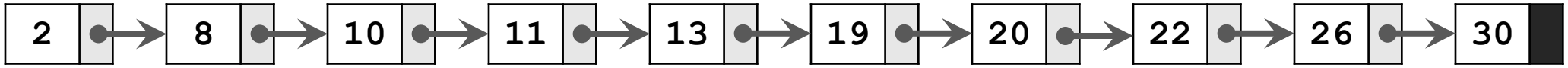
Shusen Wang

Why skip list?

- Linked list does not support binary search.
- Skip list allows fast search and insertion.
- **Search:** $O(\log n)$ time complexity on average.
- **Insertion:** $O(\log n)$ time complexity on average.

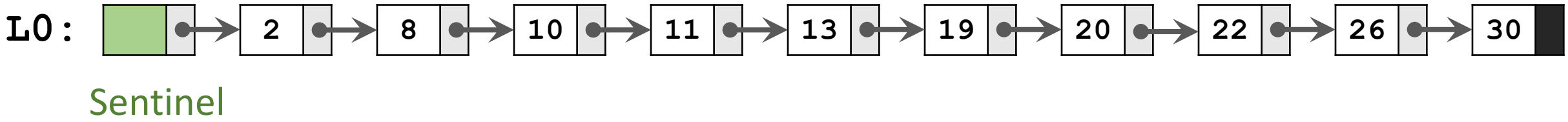
Build a Skip List

Initial State

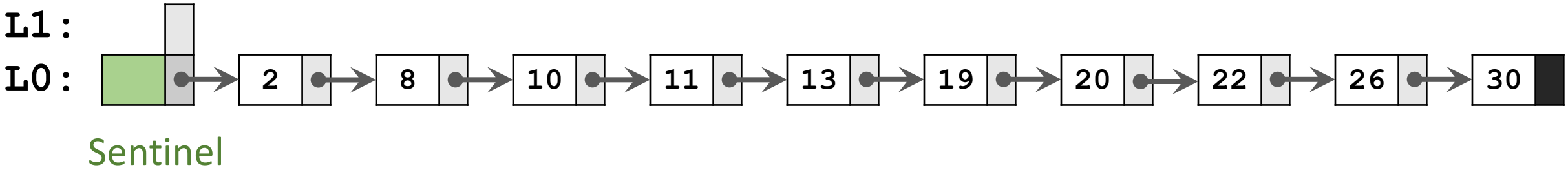


- Initially, we have a linked list containing n numbers in ascending order.

Add sentinel in the front

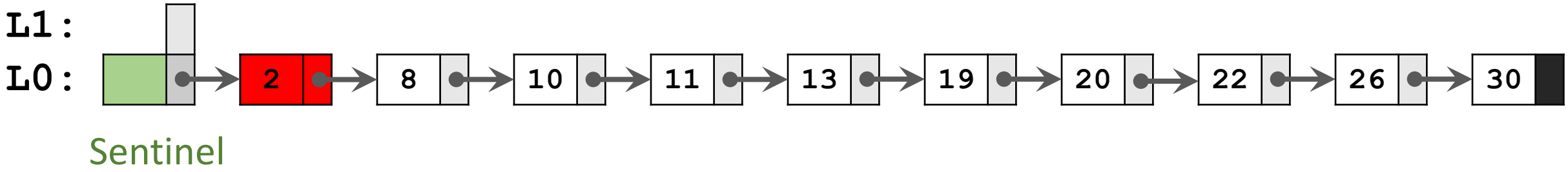


Iteration 1



- Build the L1 linked list.

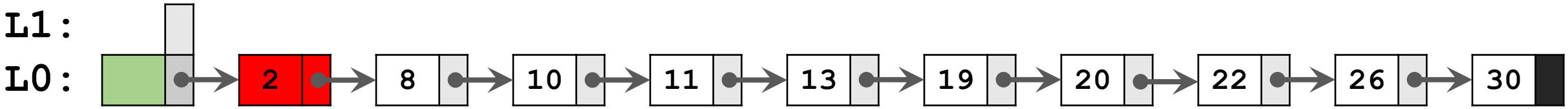
Iteration 1(A)



- Decide on whether to grow this node's height.

Iteration 1(A)

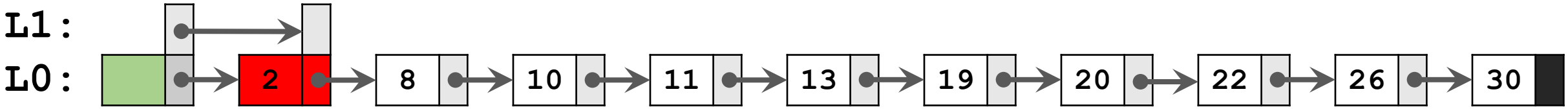
Flip a coin.



Sentinel

Iteration 1(A)

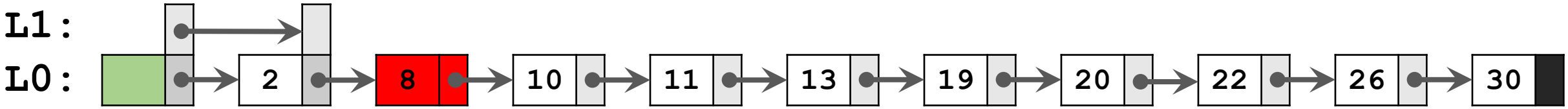
Flip a coin.



Sentinel

Iteration 1(B)

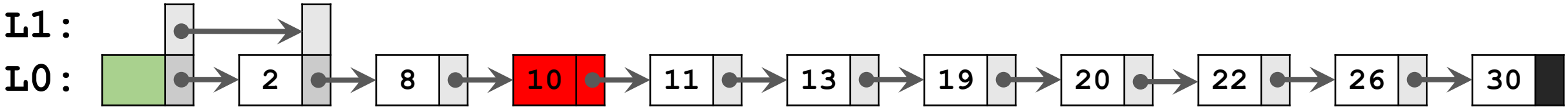
Flip a coin.



Sentinel

Iteration 1(C)

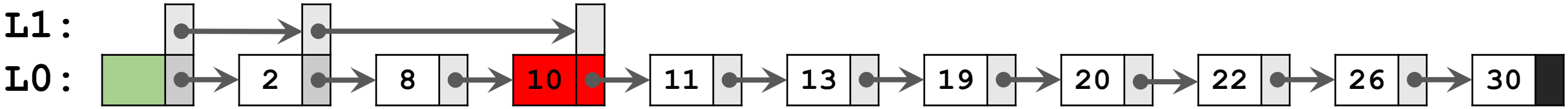
Flip a coin.



Sentinel

Iteration 1(C)

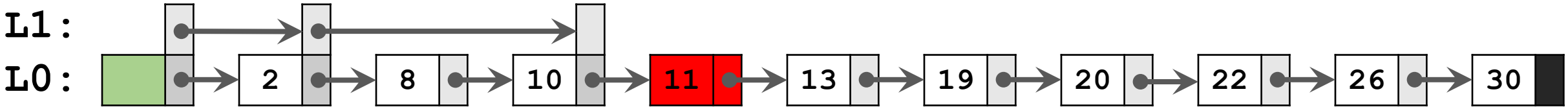
Flip a coin.



Sentinel

Iteration 1(D)

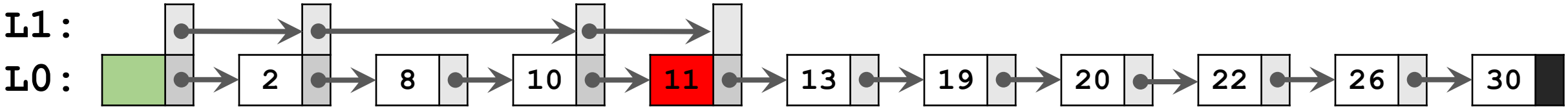
Flip a coin.



Sentinel

Iteration 1(D)

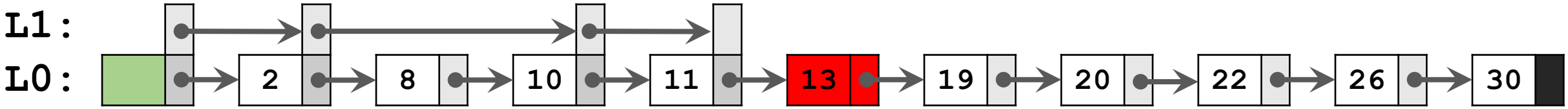
Flip a coin.



Sentinel

Iteration 1(E)

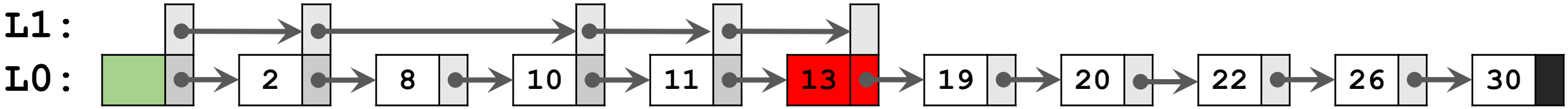
Flip a coin.



Sentinel

Iteration 1(E)

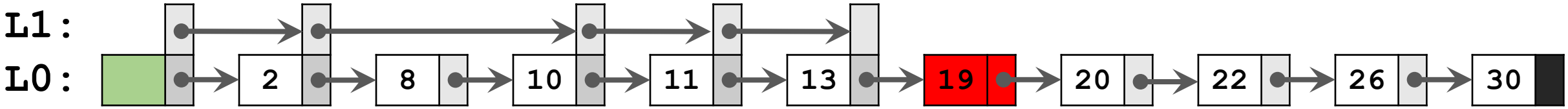
Flip a coin.



Sentinel

Iteration 1(F)

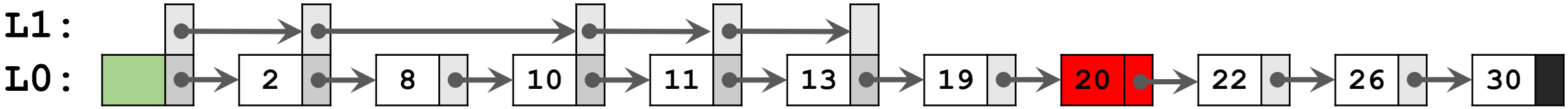
Flip a coin.



Sentinel

Iteration 1(G)

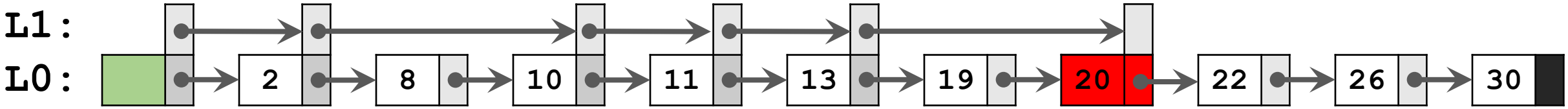
Flip a coin.



Sentinel

Iteration 1(G)

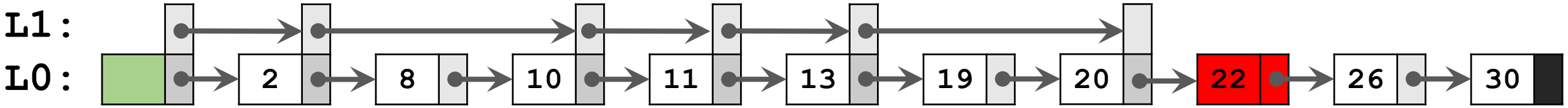
Flip a coin.



Sentinel

Iteration 1(H)

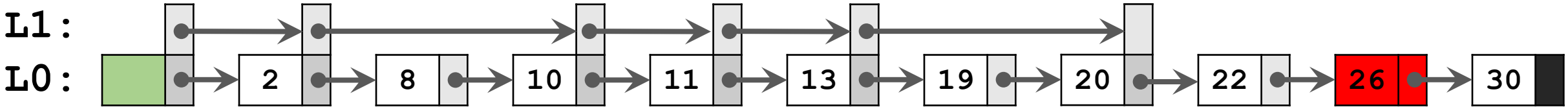
Flip a coin.



Sentinel

Iteration 1(I)

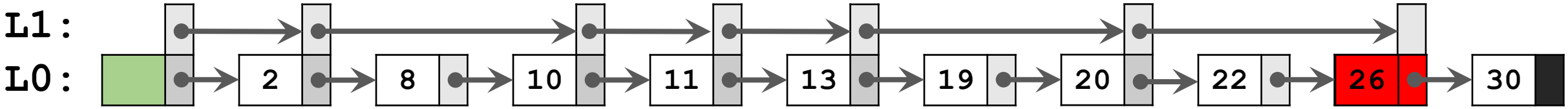
Flip a coin.



Sentinel

Iteration 1(I)

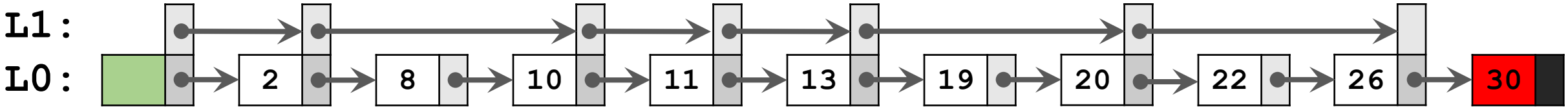
Flip a coin.



Sentinel

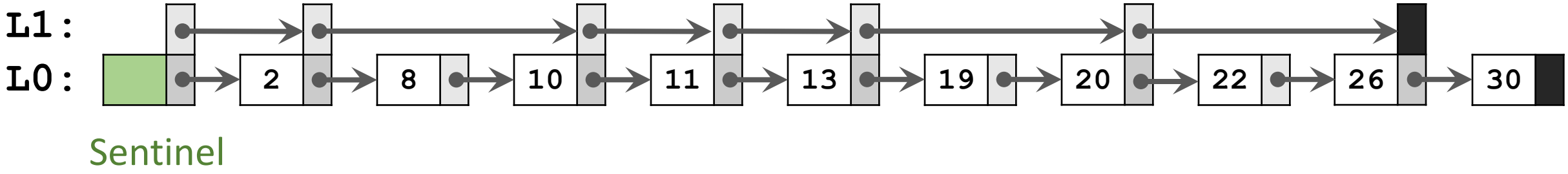
Iteration 1(J)

Flip a coin.

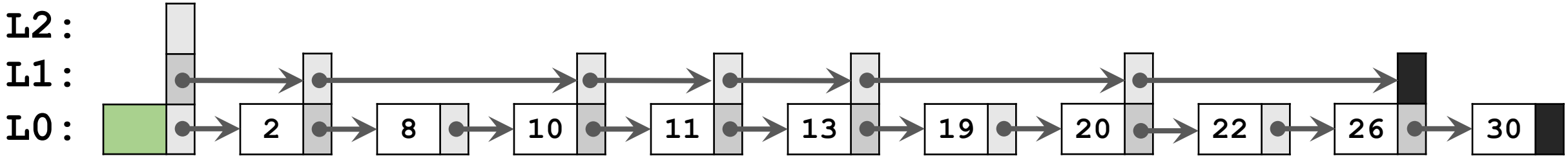


Sentinel

Iteration 1(End)



Iteration 2

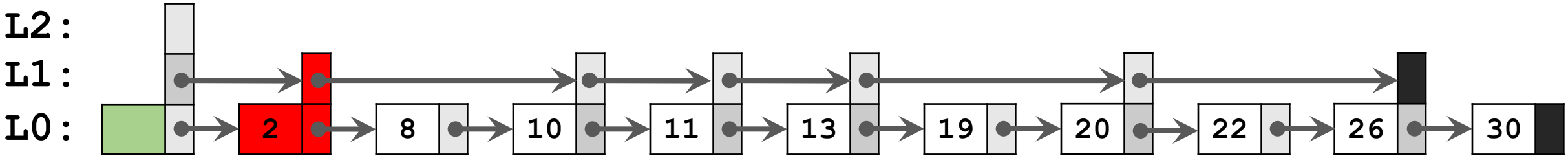


Sentinel

- Build the L2 linked list.

Iteration 2(A)

Flip a coin.

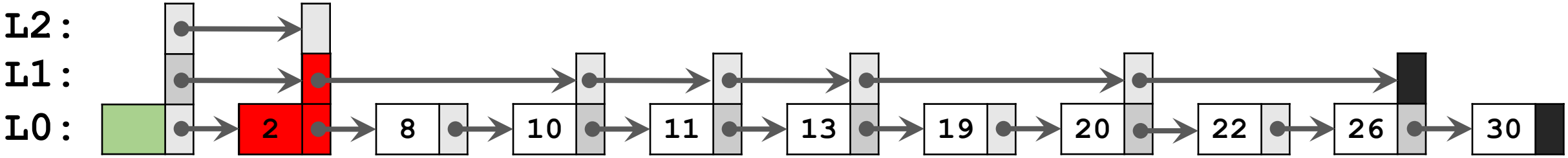


Sentinel

- Decide on whether to increase this node's height.

Iteration 2(A)

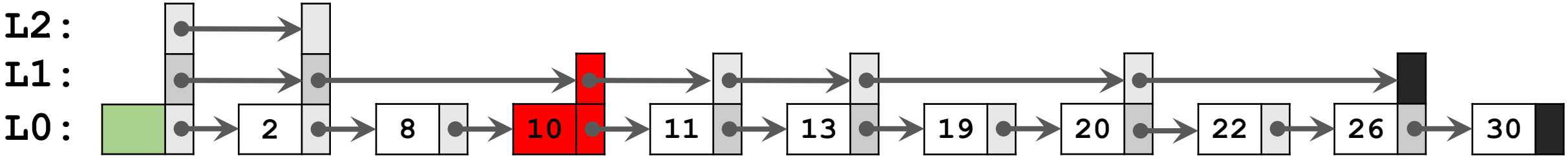
Flip a coin.



Sentinel

Iteration 2(B)

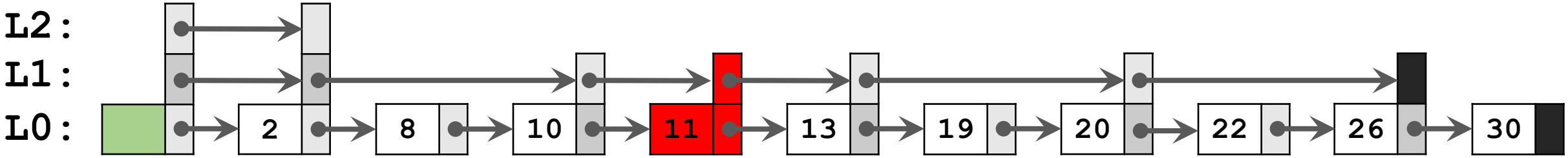
Flip a coin.



Sentinel

Iteration 2(C)

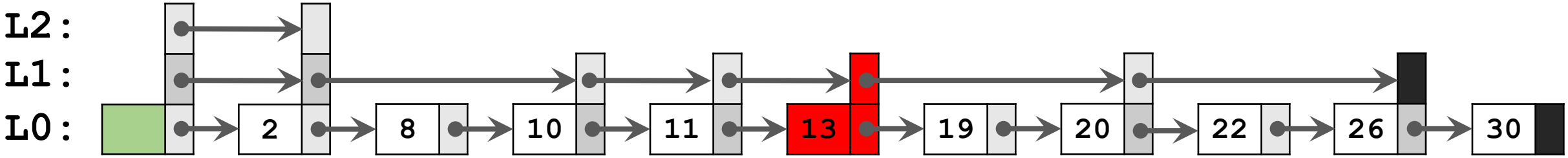
Flip a coin.



Sentinel

Iteration 2(D)

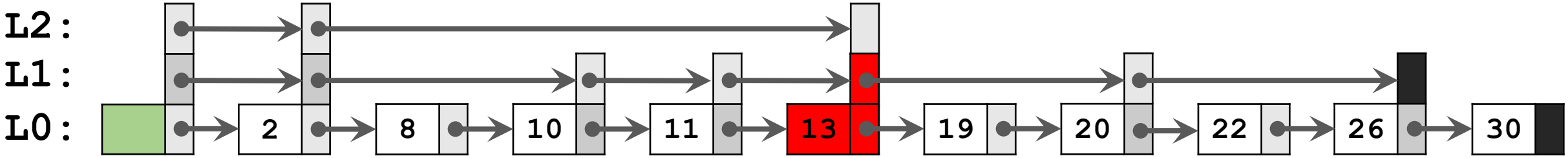
Flip a coin.



Sentinel

Iteration 2(D)

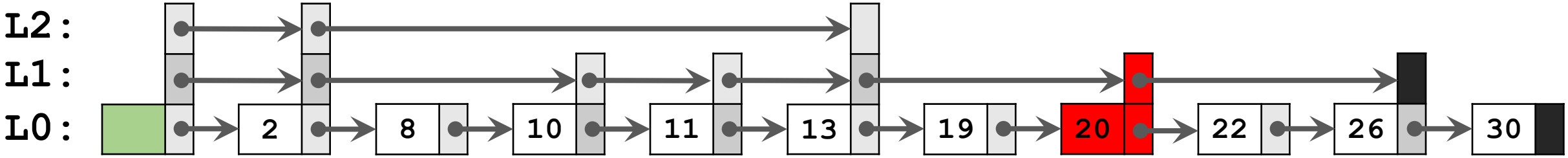
Flip a coin.



Sentinel

Iteration 2(E)

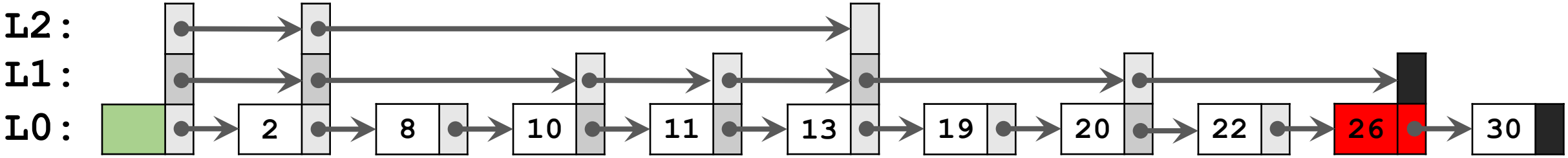
Flip a coin.



Sentinel

Iteration 2(F)

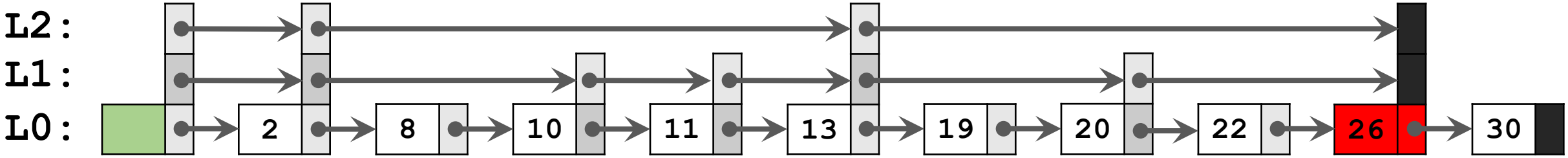
Flip a coin.



Sentinel

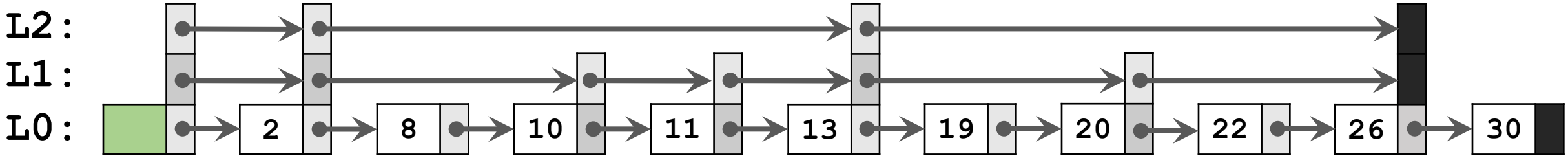
Iteration 2(F)

Flip a coin.



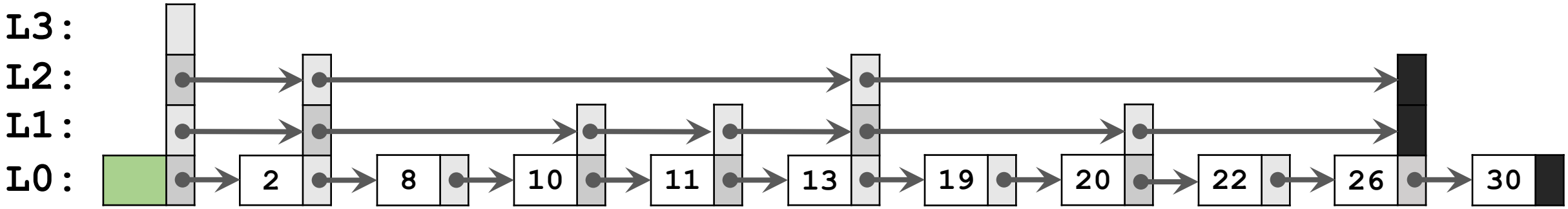
Sentinel

Iteration 2(End)



Sentinel

Iteration 3

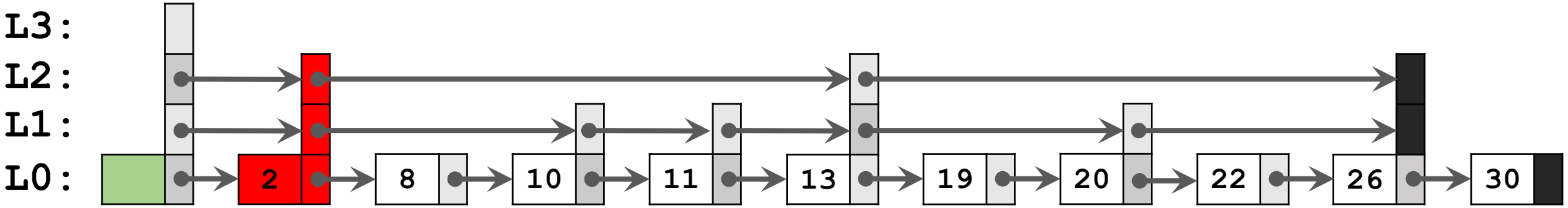


Sentinel

- Build the L3 linked list.

Iteration 3(A)

Flip a coin.

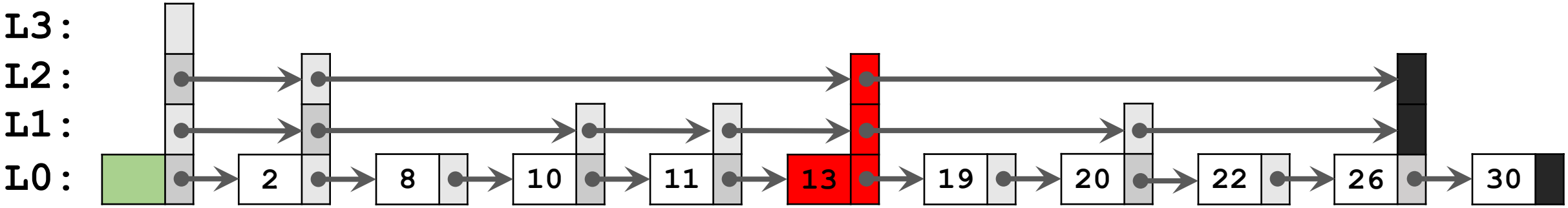


Sentinel

- Decide on whether to increase this node's height.

Iteration 3(B)

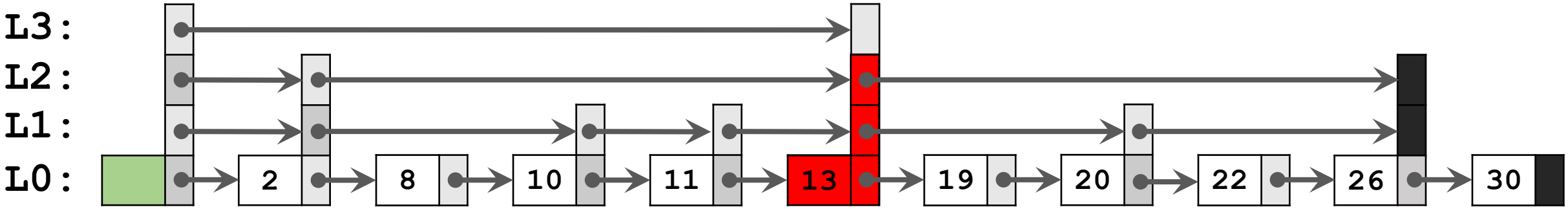
Flip a coin.



Sentinel

Iteration 3(B)

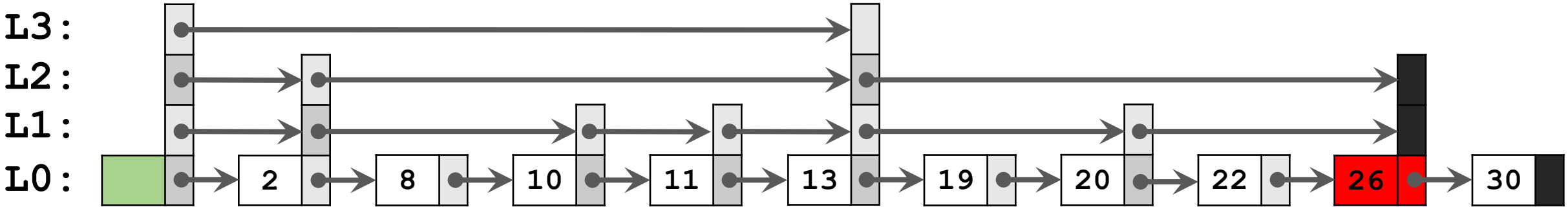
Flip a coin.



Sentinel

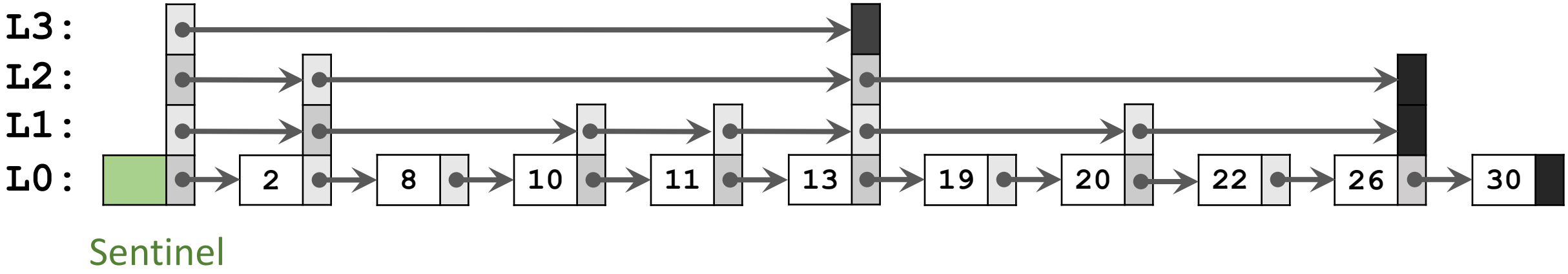
Iteration 3(C)

Flip a coin.

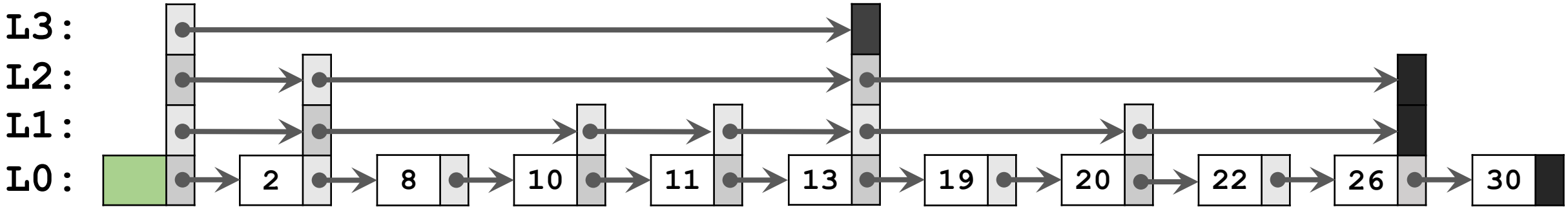


Sentinel

Iteration 3(End)



End of Procedure

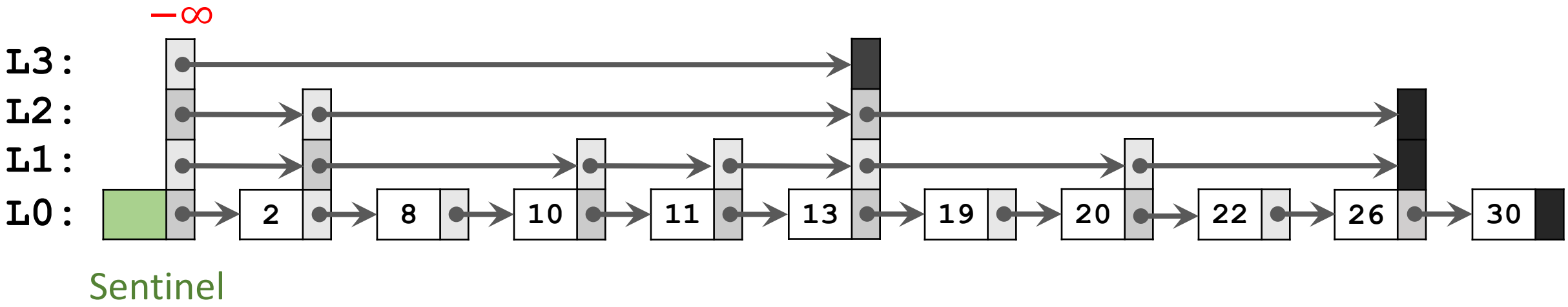


Sentinel

- Number of levels is up to the user.
- We use a total of 4 layers.

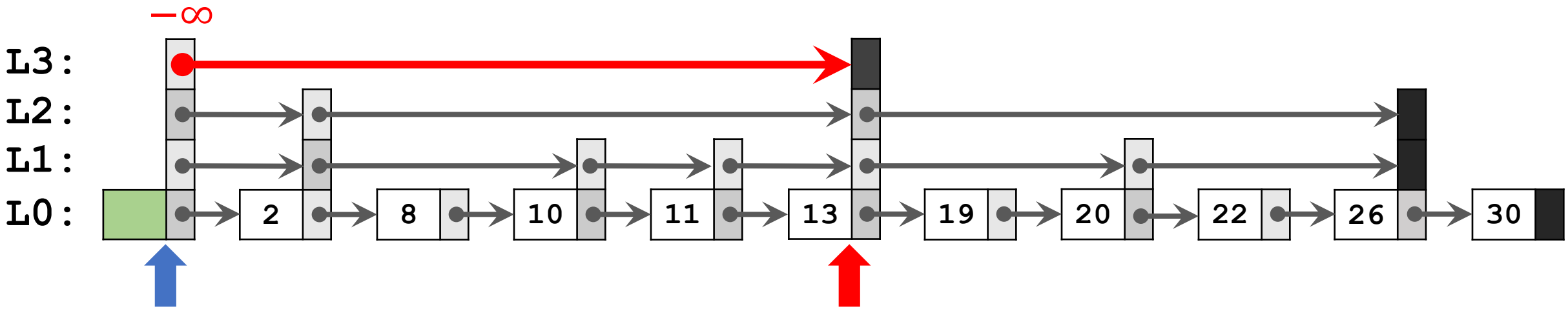
Search

Search: key=13



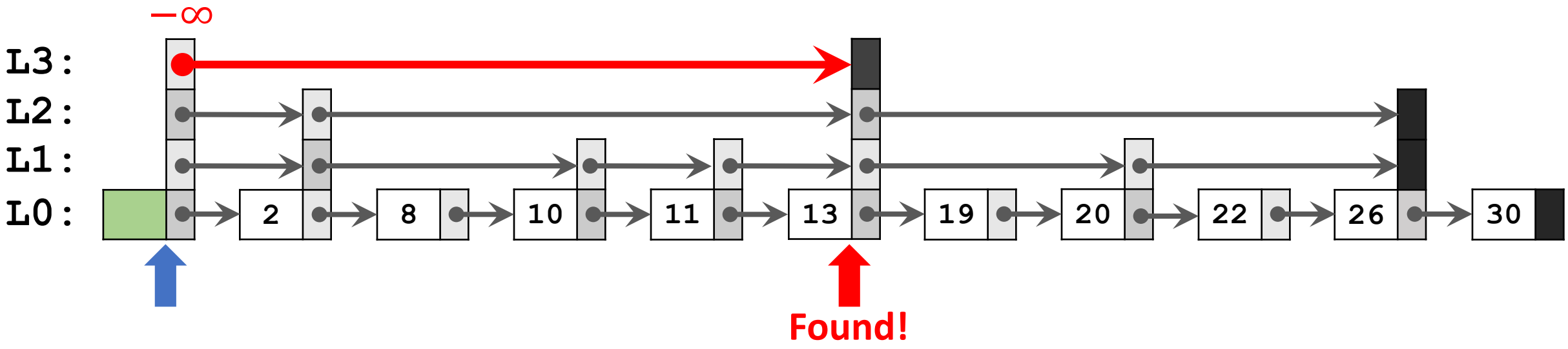
Start at the sentinel and traverse along the top level.

Search: key=13

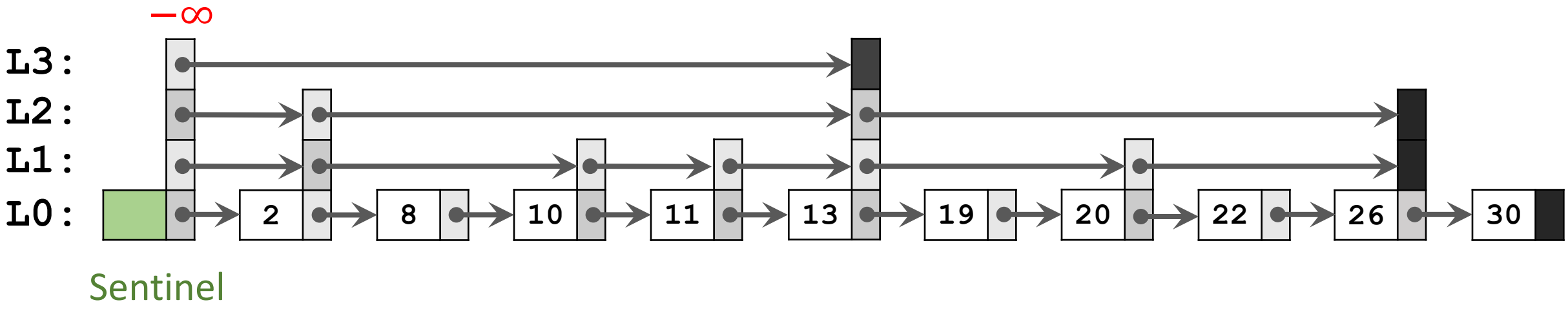


Start at the sentinel and traverse along the top level.

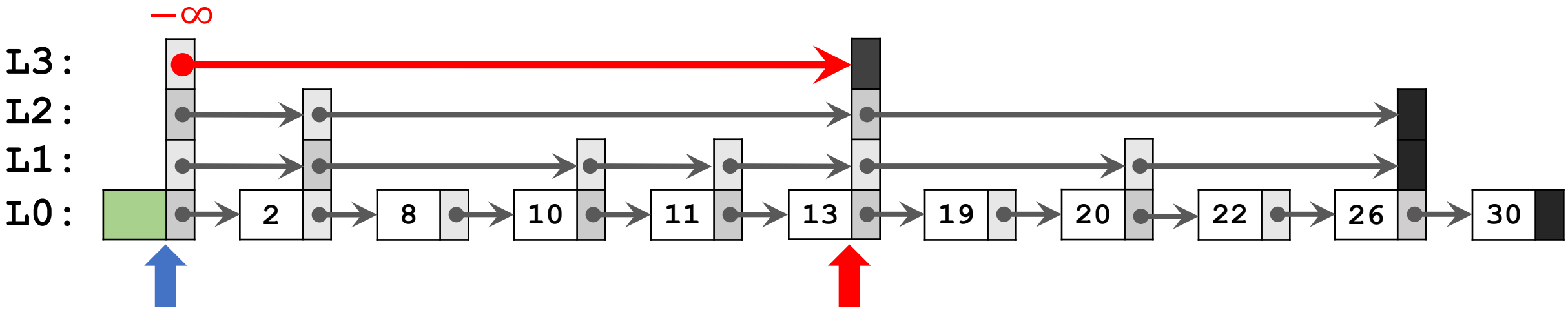
Search: key=13



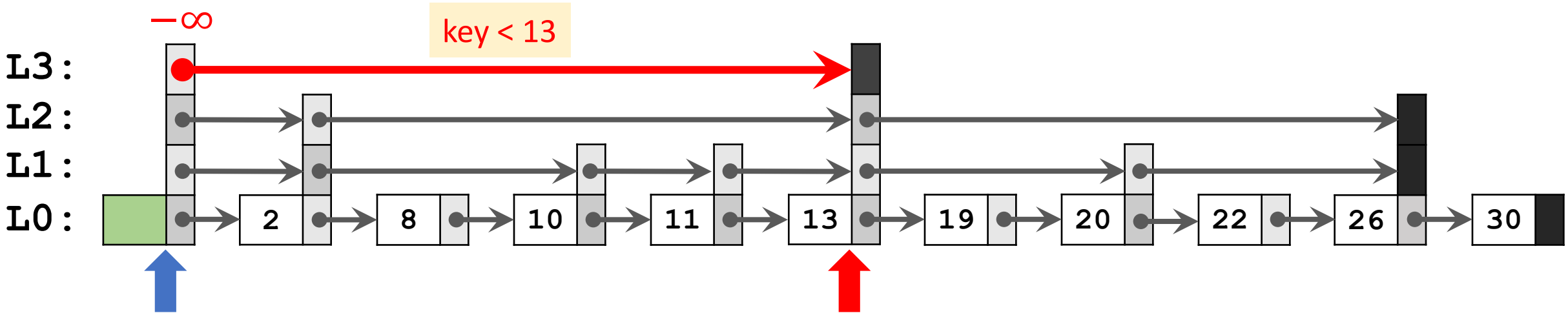
Search: key=8



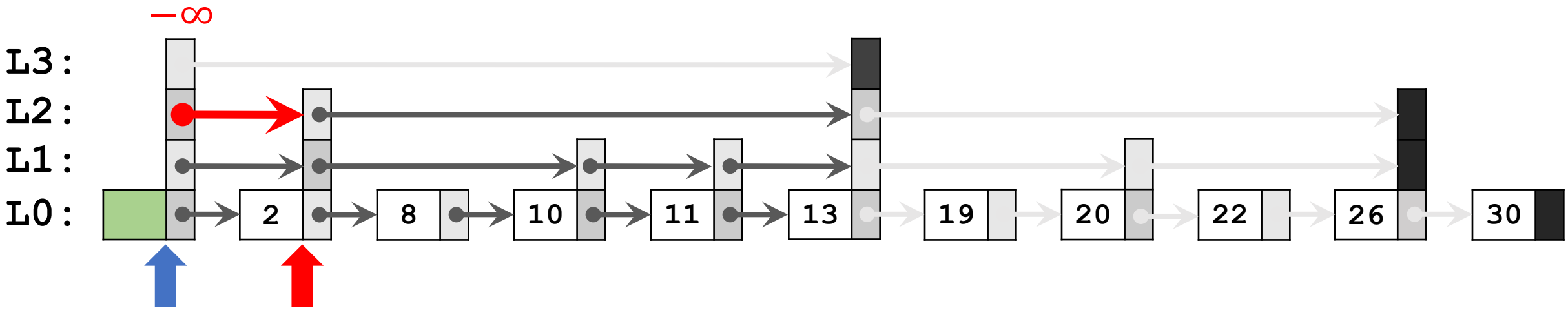
Search: key=8



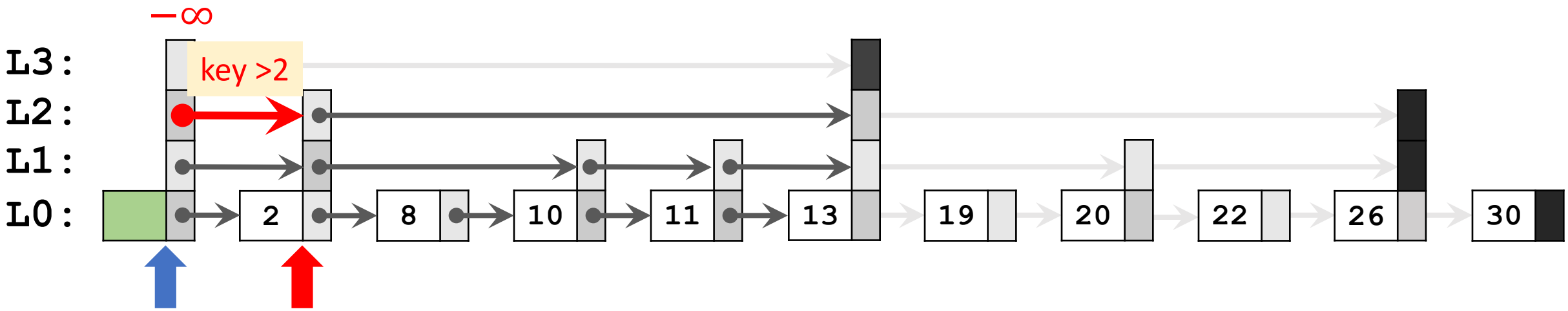
Search: key=8



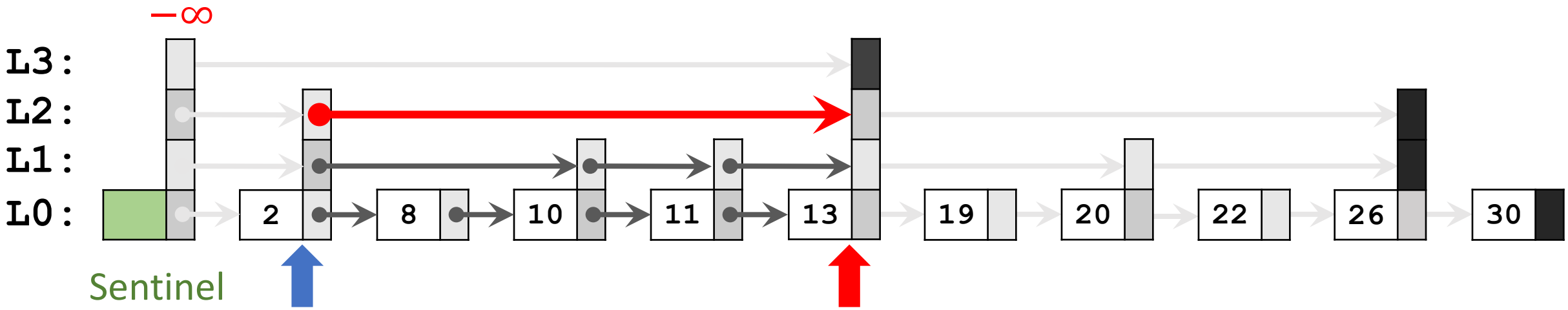
Search: key=8



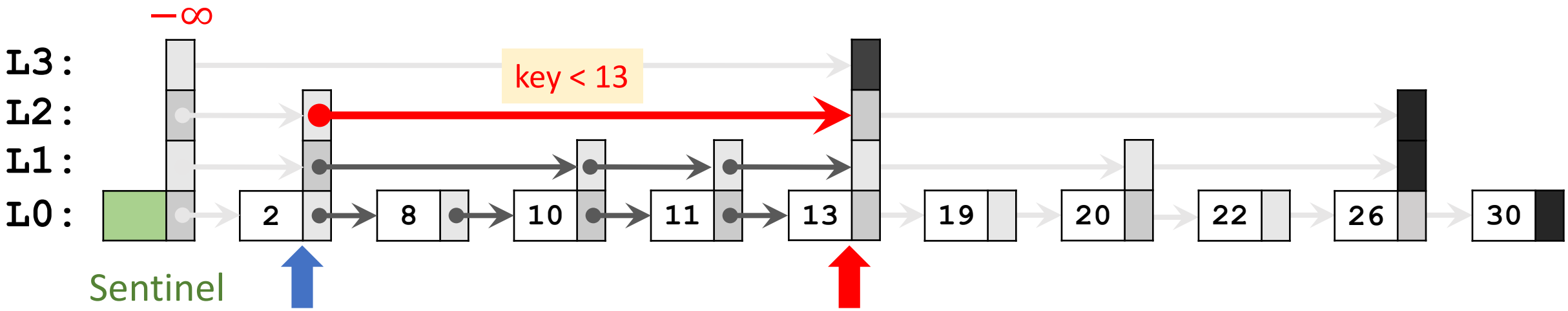
Search: key=8



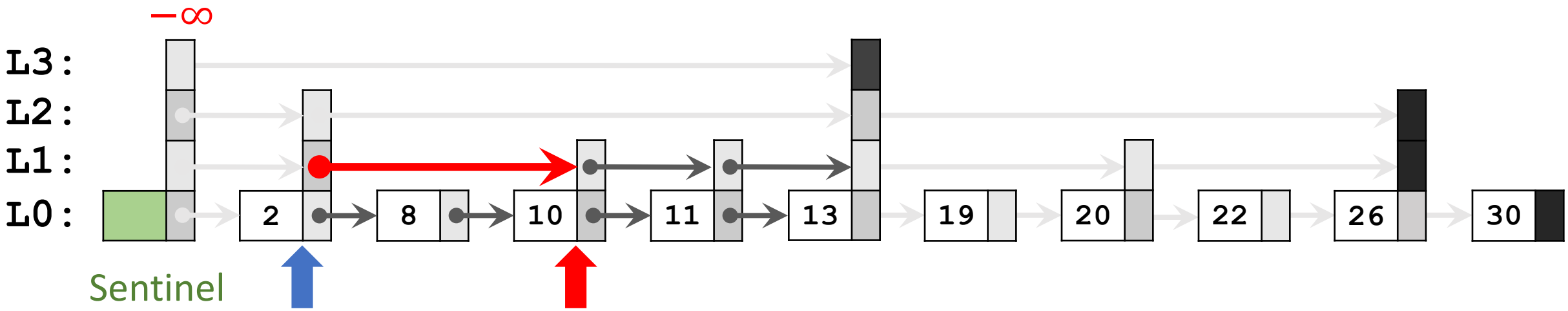
Search: key=8



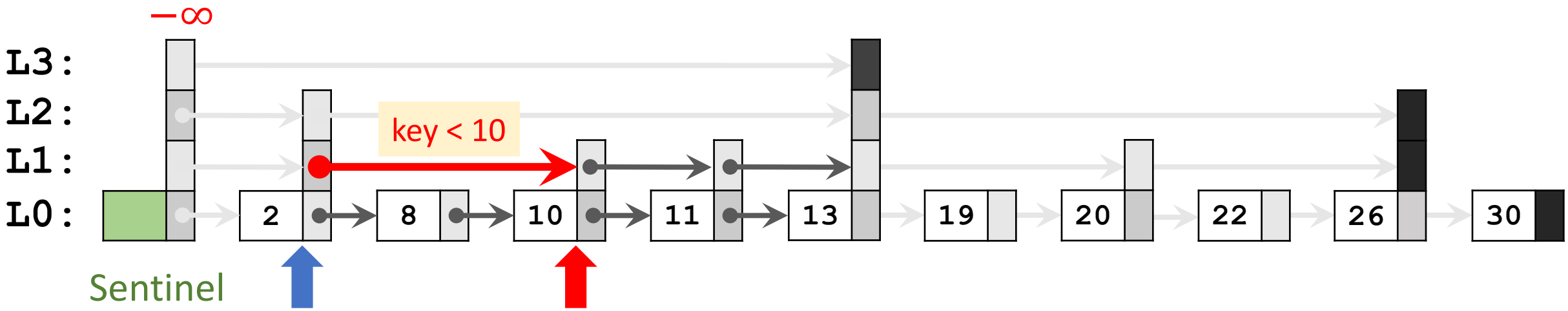
Search: key=8



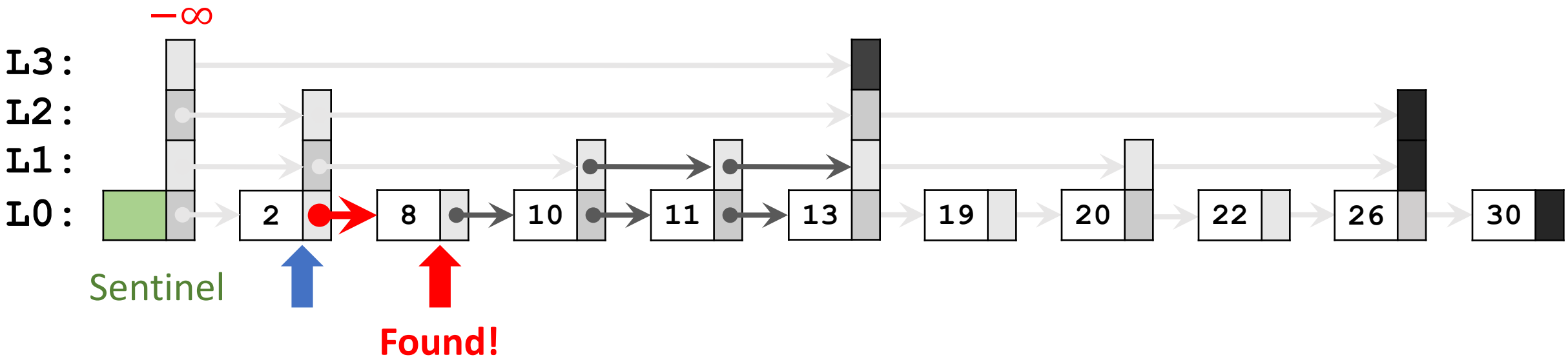
Search: key=8



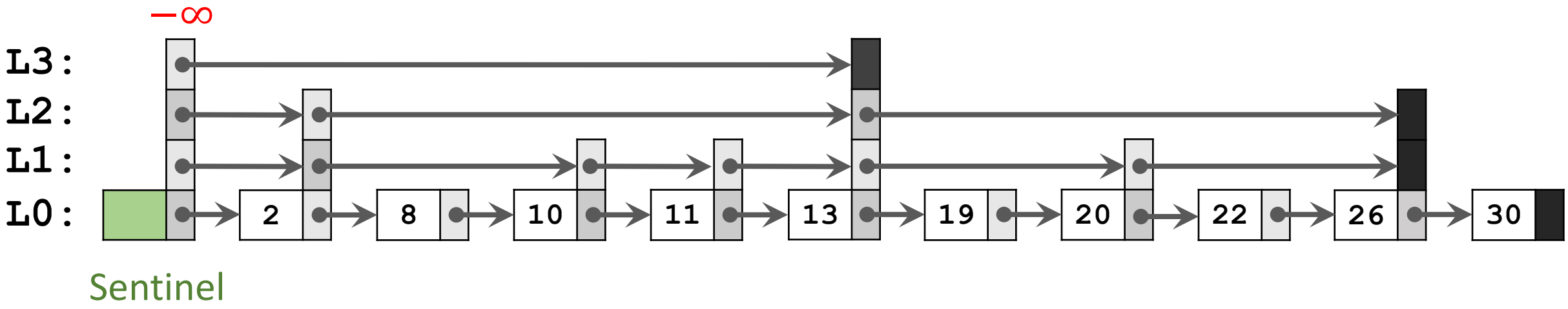
Search: key=8



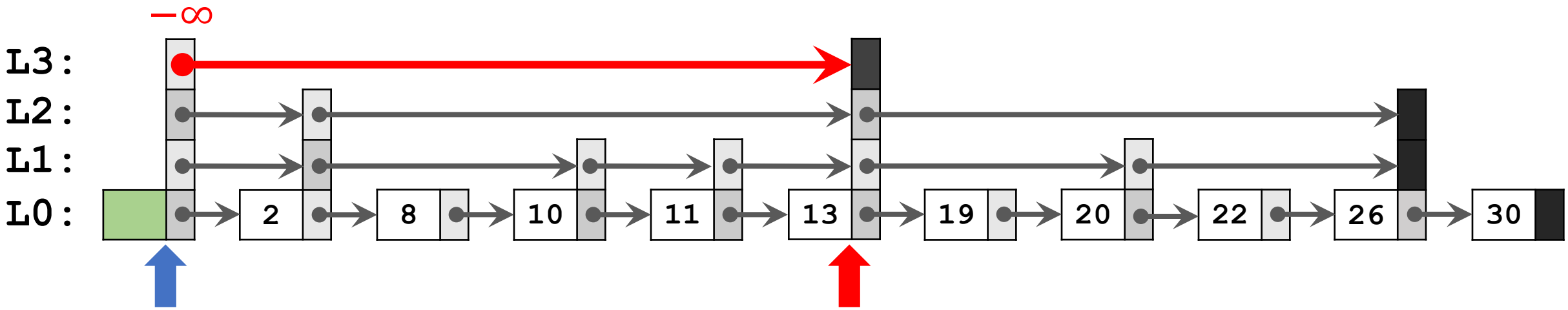
Search: key=8



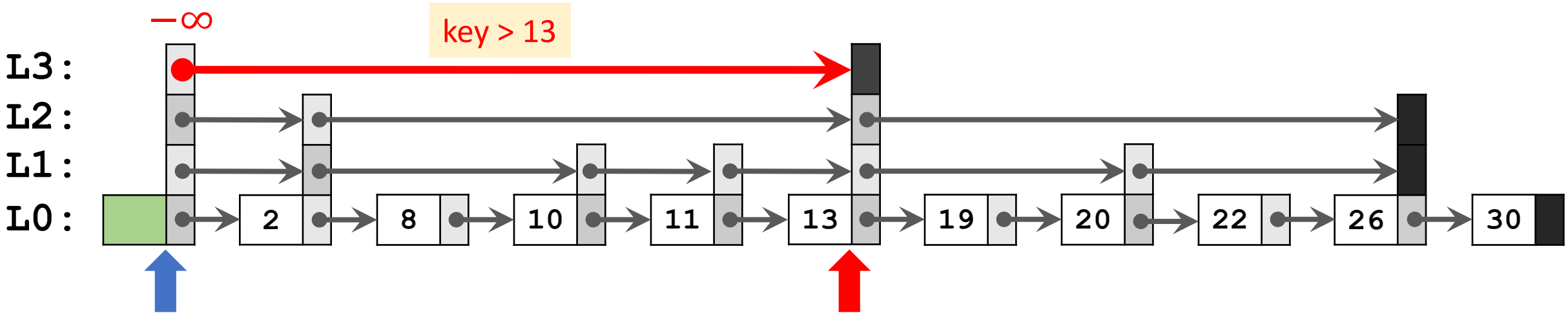
Search: key=20



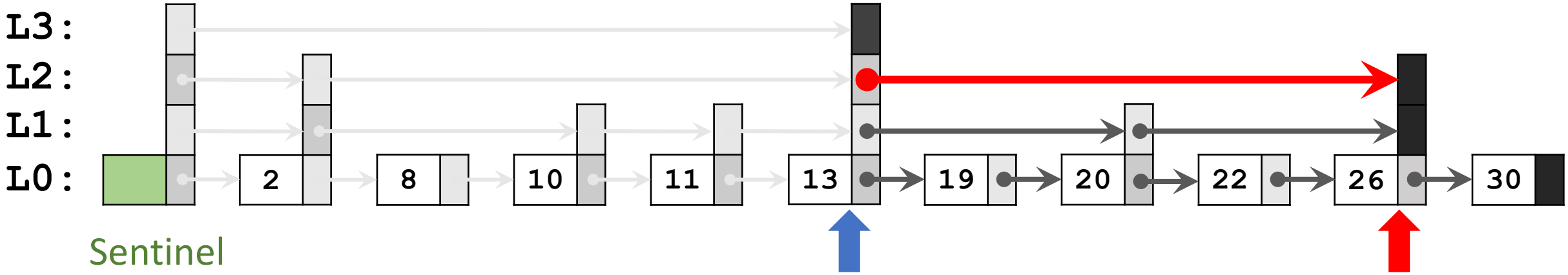
Search: key=20



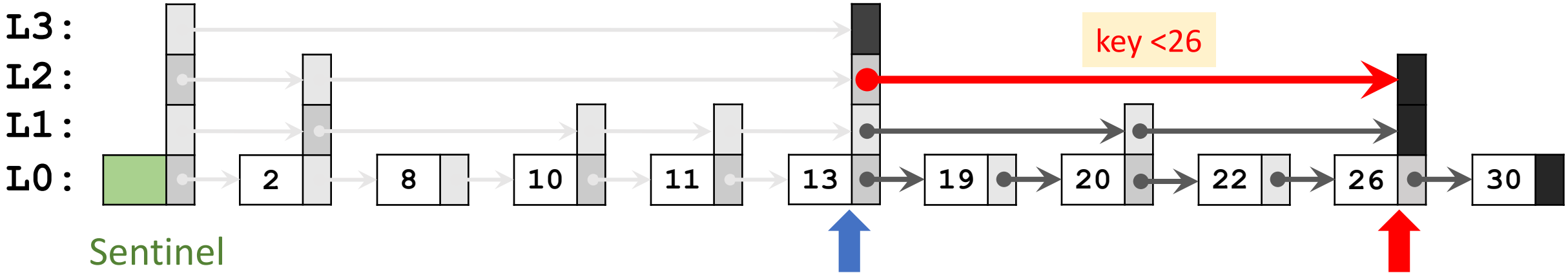
Search: key=20



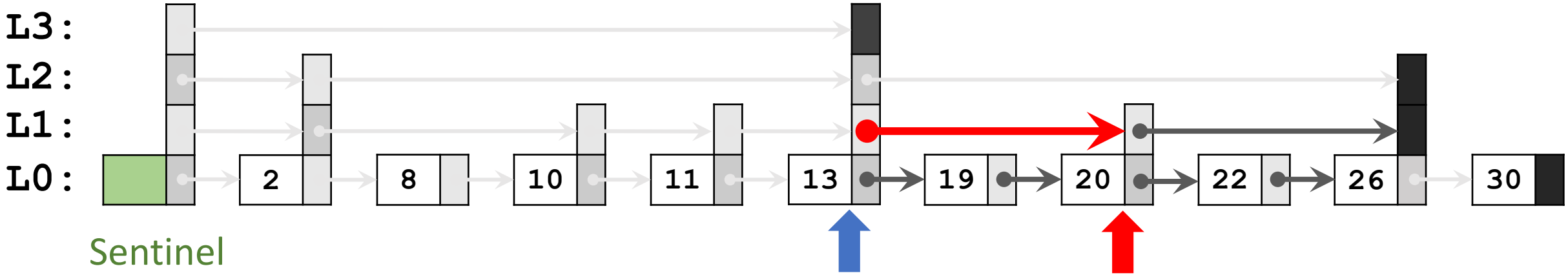
Search: key=20



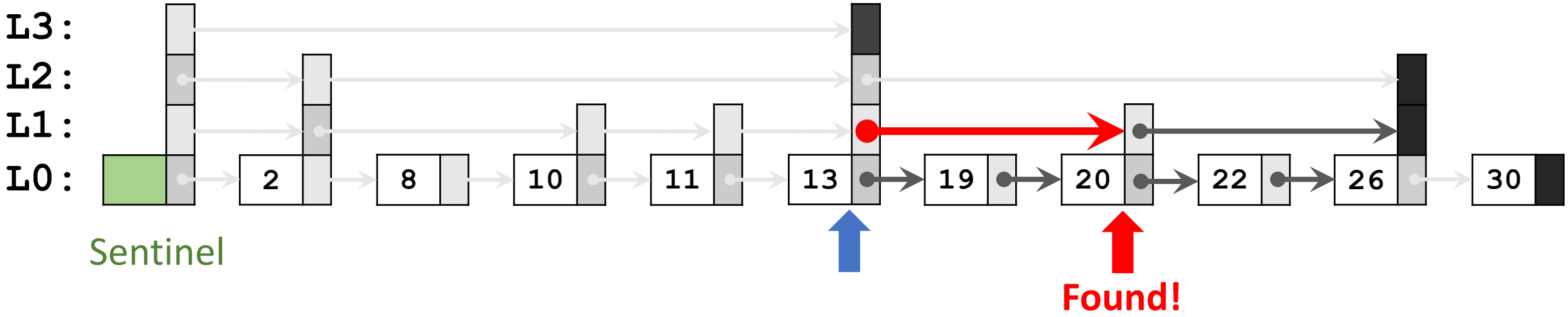
Search: key=20



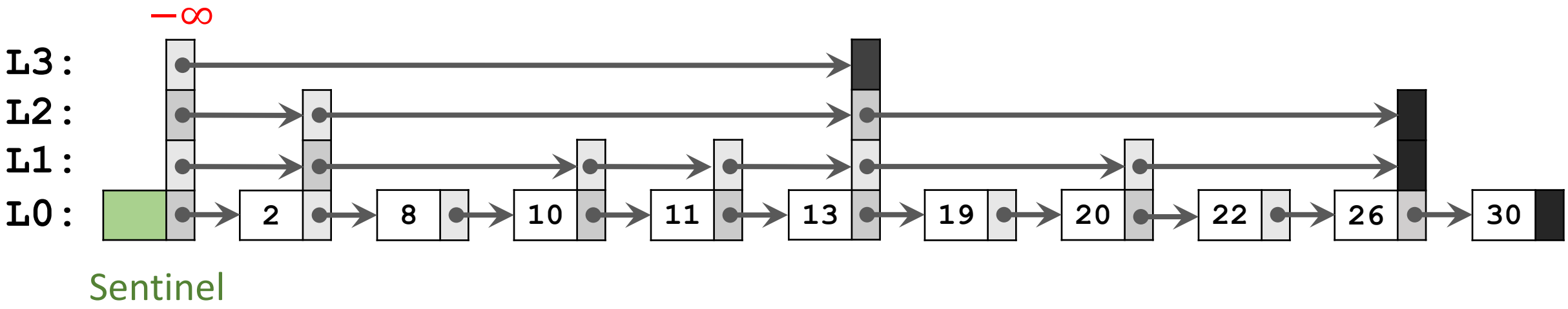
Search: key=20



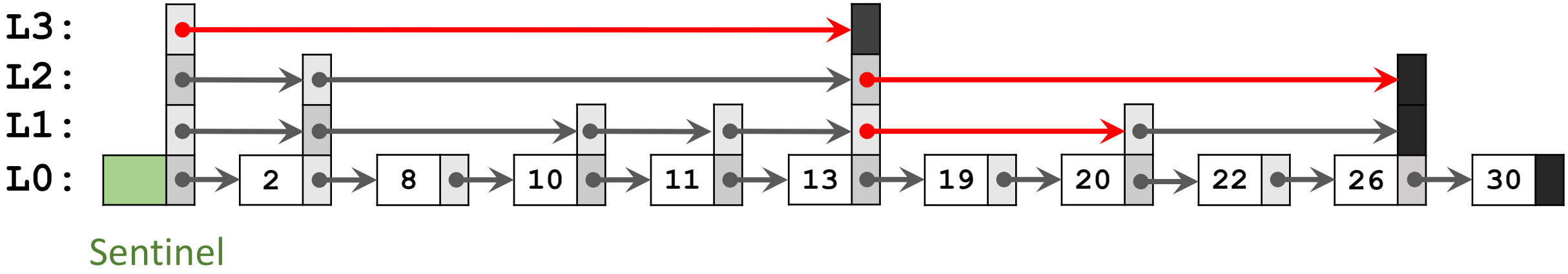
Search: key=20



Search: key=21

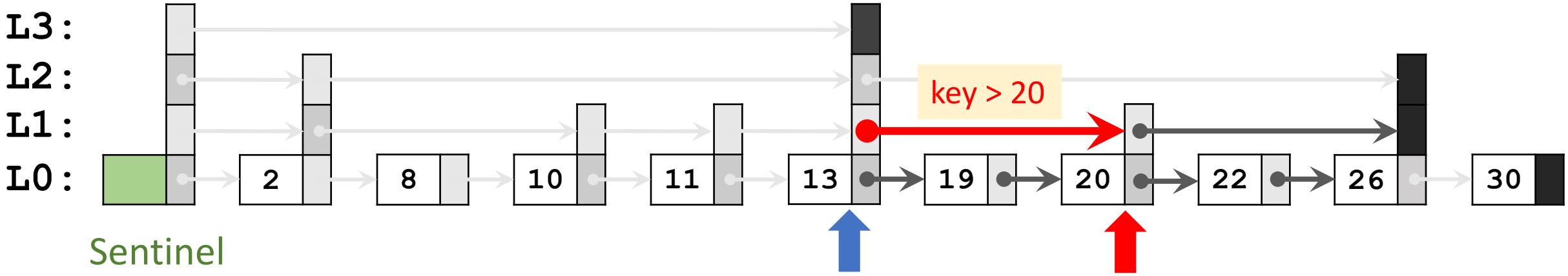


Search: key=21

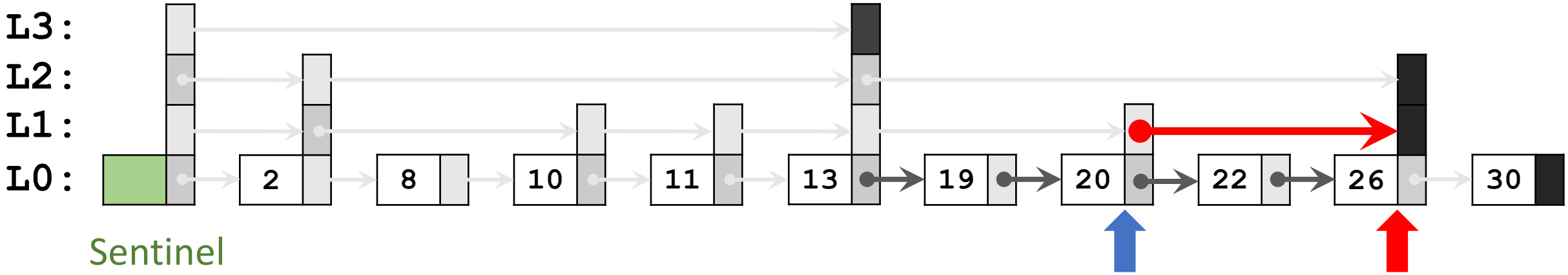


Do the same as before: traverse L3, L2, and then L1.

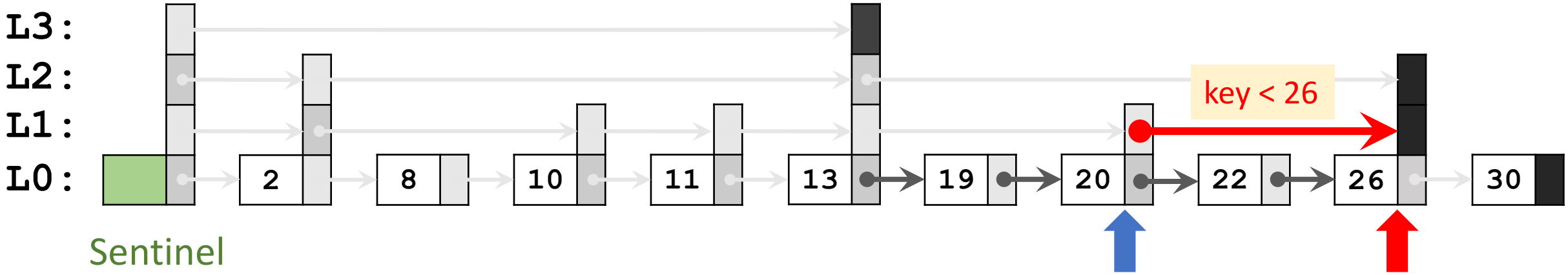
Search: key=21



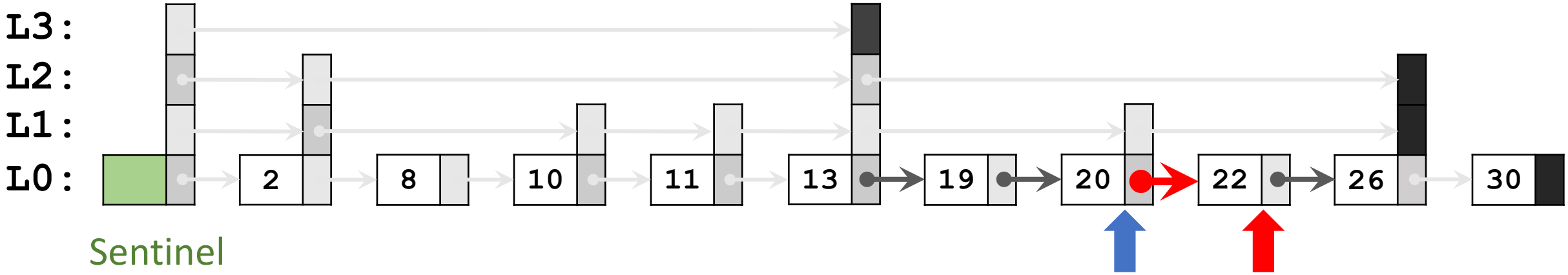
Search: key=21



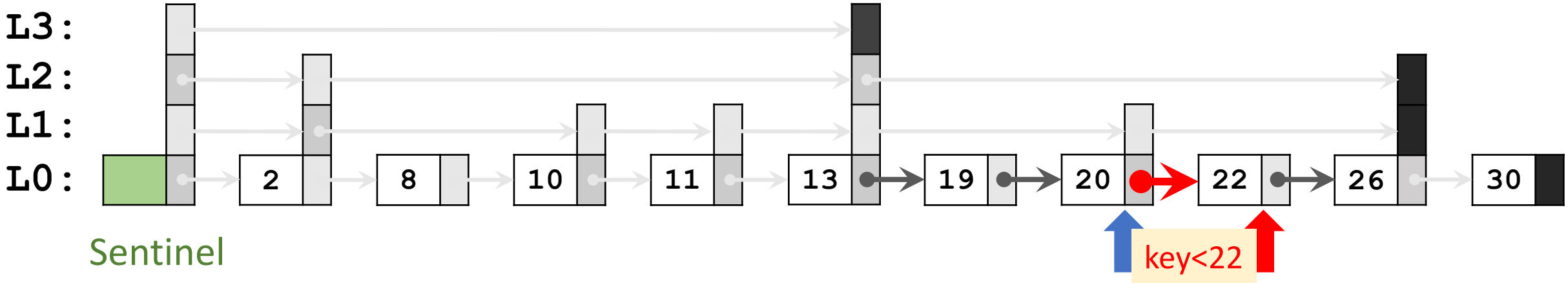
Search: key=21



Search: key=21



Search: key=21

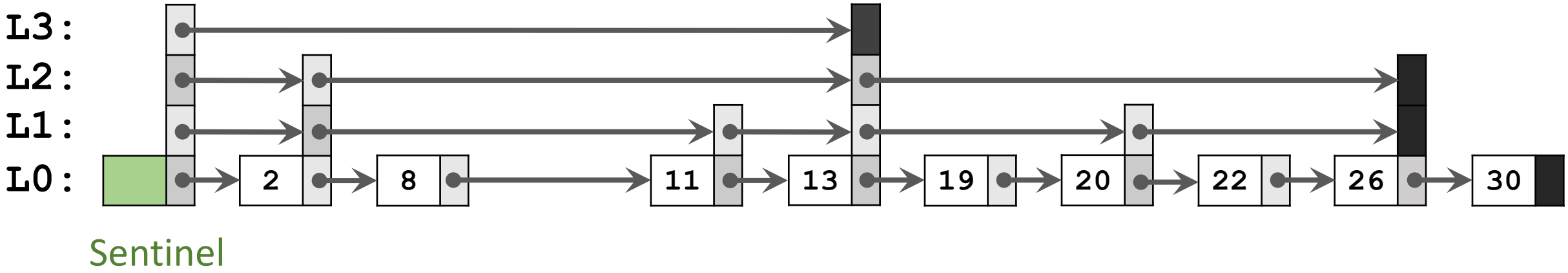


Key=21 is not found!

Insertion

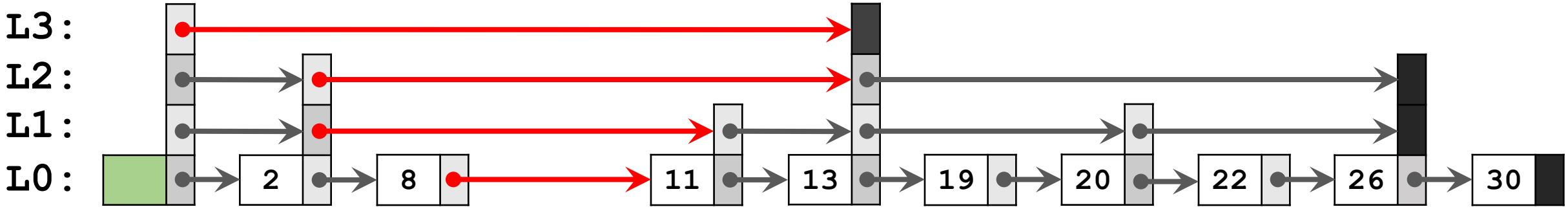
Insert (key=9)

First, search key=9 and record the path.



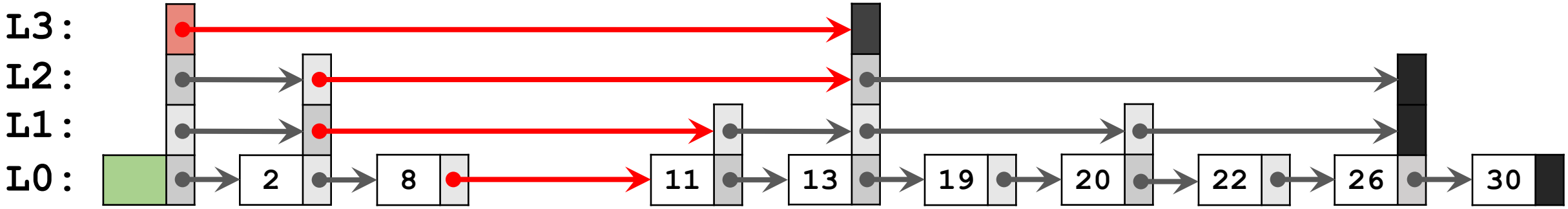
Insert (key=9)

First, search key=9 and record the path.



Insert (key=9)

First, search key=9 and record the path.

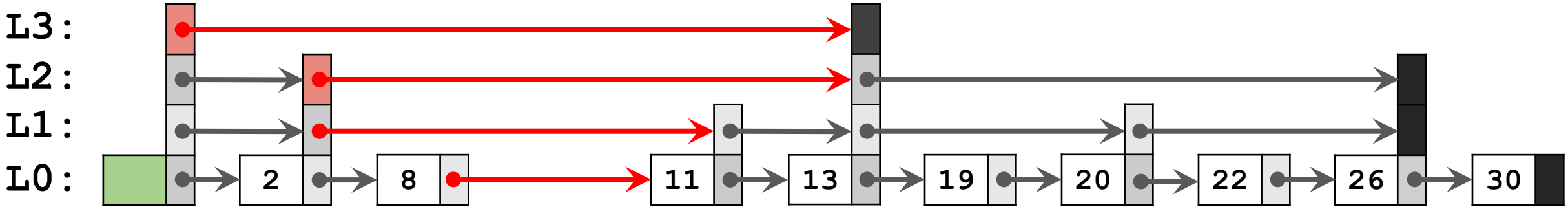


Start from:

L3:	Sentinel
L2:	
L1:	
L0:	

Insert (key=9)

First, search key=9 and record the path.

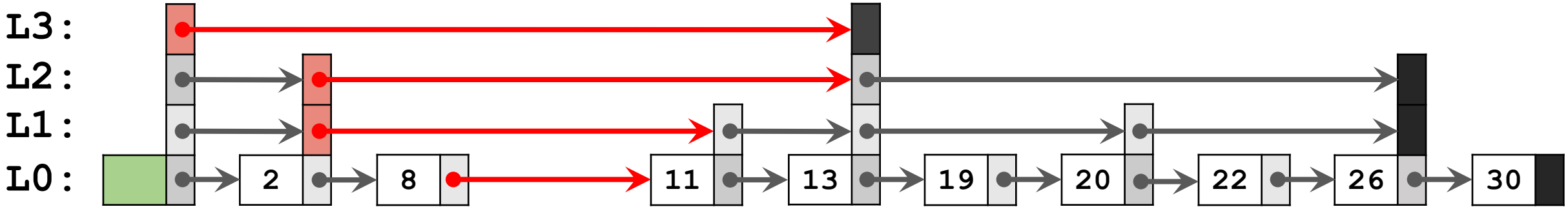


Start from:

L3:	Sentinel
L2:	Node 2
L1:	
L0:	

Insert (key=9)

First, search key=9 and record the path.

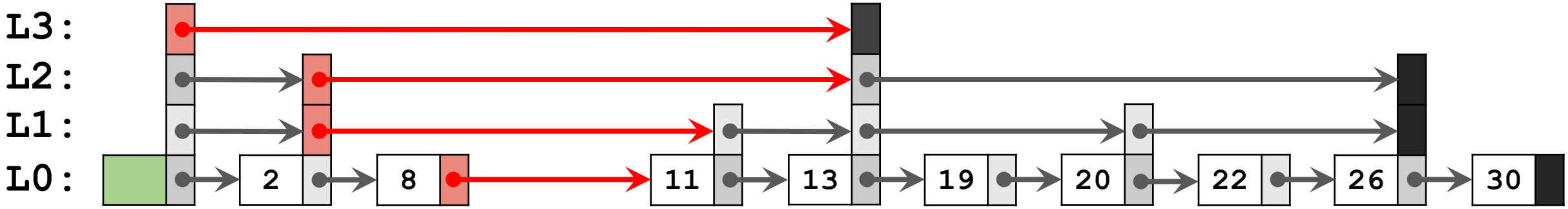


Start from:

L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	

Insert (key=9)

First, search key=9 and record the path.

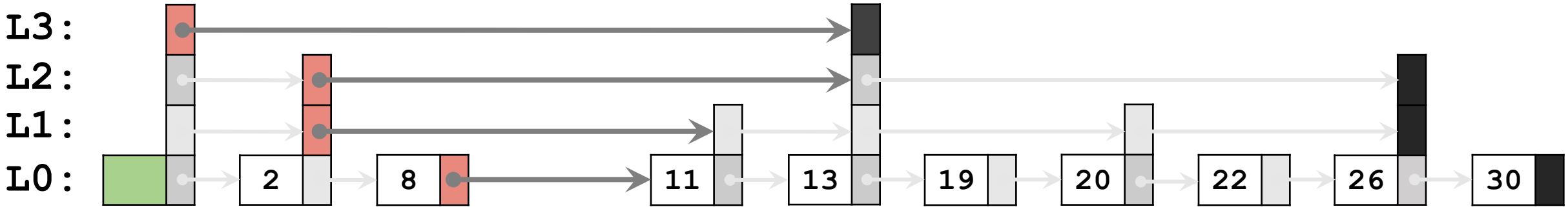


Start from:

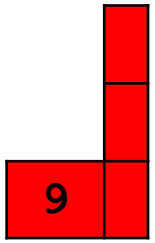
L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	Node 8

Insert (key=9)

Second, create a node whose level is random, e.g, height=2.



Create a node (height=2):

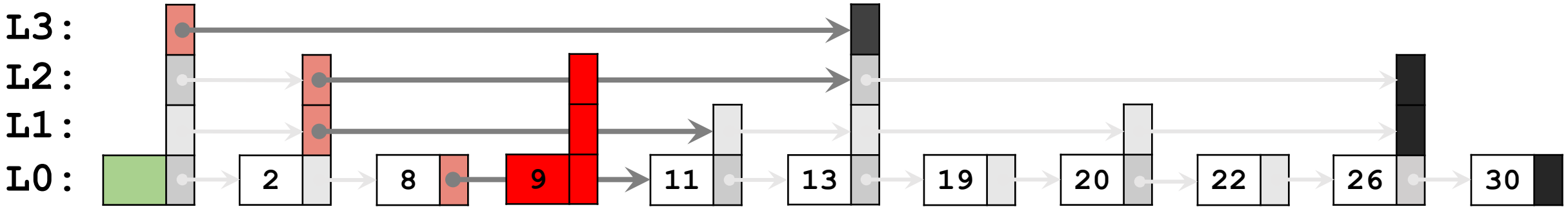


Start from:

L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	Node 8

Insert (key=9)

Third, link the new node to the skip list.

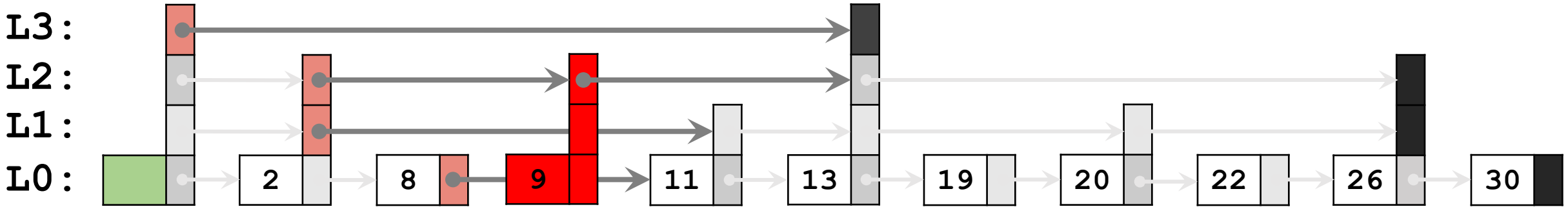


Start from:

L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	Node 8

Insert (key=9)

Third, link the new node to the skip list.

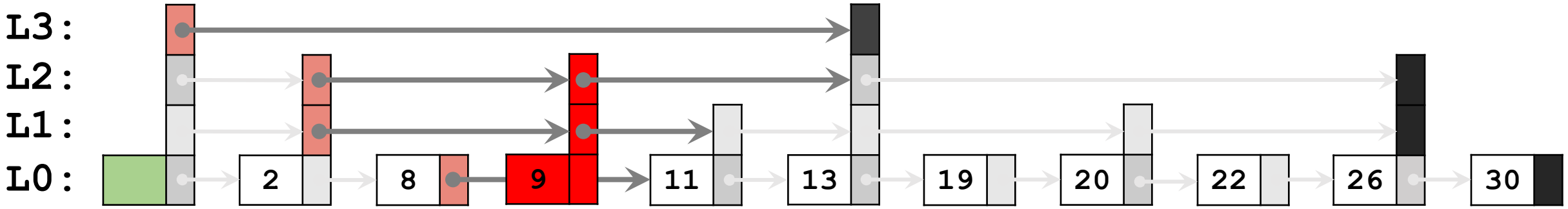


Start from:

L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	Node 8

Insert (key=9)

Third, link the new node to the skip list.

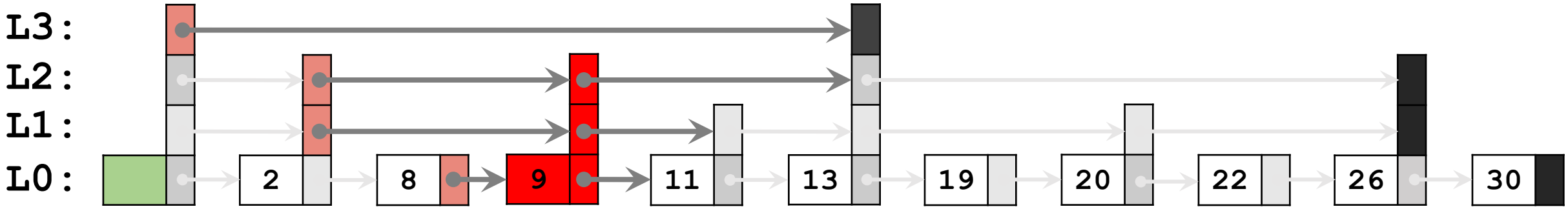


Start from:

L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	Node 8

Insert (key=9)

Third, link the new node to the skip list.



Start from:

L3:	Sentinel
L2:	Node 2
L1:	Node 2
L0:	Node 8

Thank You!