

Longest Common Substring

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Longest Common Substring

- Example inputs:
 - `x = "IntroductionToAlgorithms".`
 - `y = "AdvancedAlgorithmDesignImplementation".`

Longest Common Substring

- Example inputs:
 - $x = \text{"IntroductionToAlgorithms"}$.
 - $y = \text{"AdvancedAlgorithmDesignImplementation"}$.

Longest Common Substring

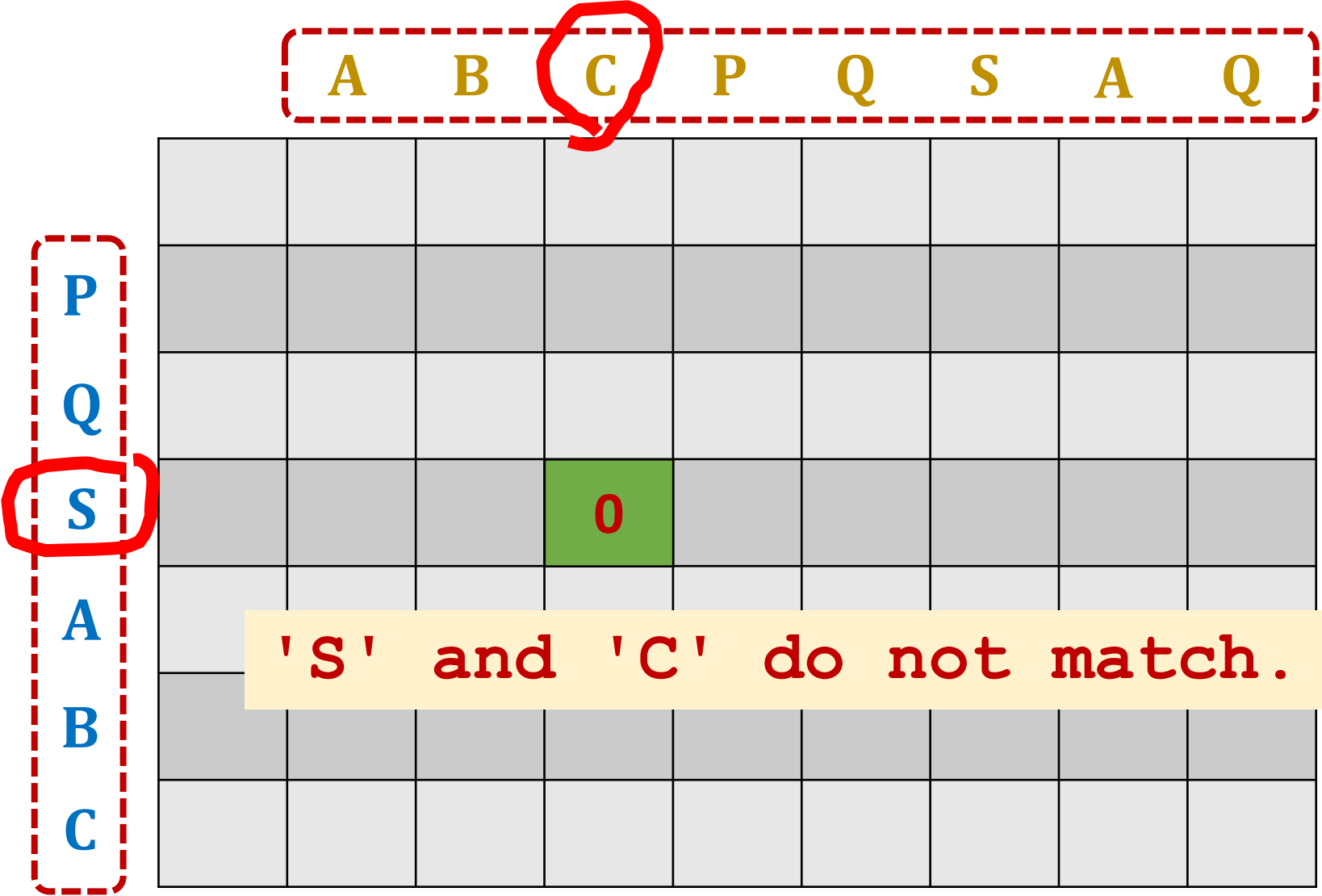
- Example inputs:
 - $x = \text{"IntroductionToAlgorithm\textcolor{red}{s"}}.$
 - $y = \text{"AdvancedAlgorithmDesignImplementation"}.$
- Example outputs:
 - Longest common substring: $\text{"Algorithm"}.$
 - Length of the substring: 9.

Optimal Substructure

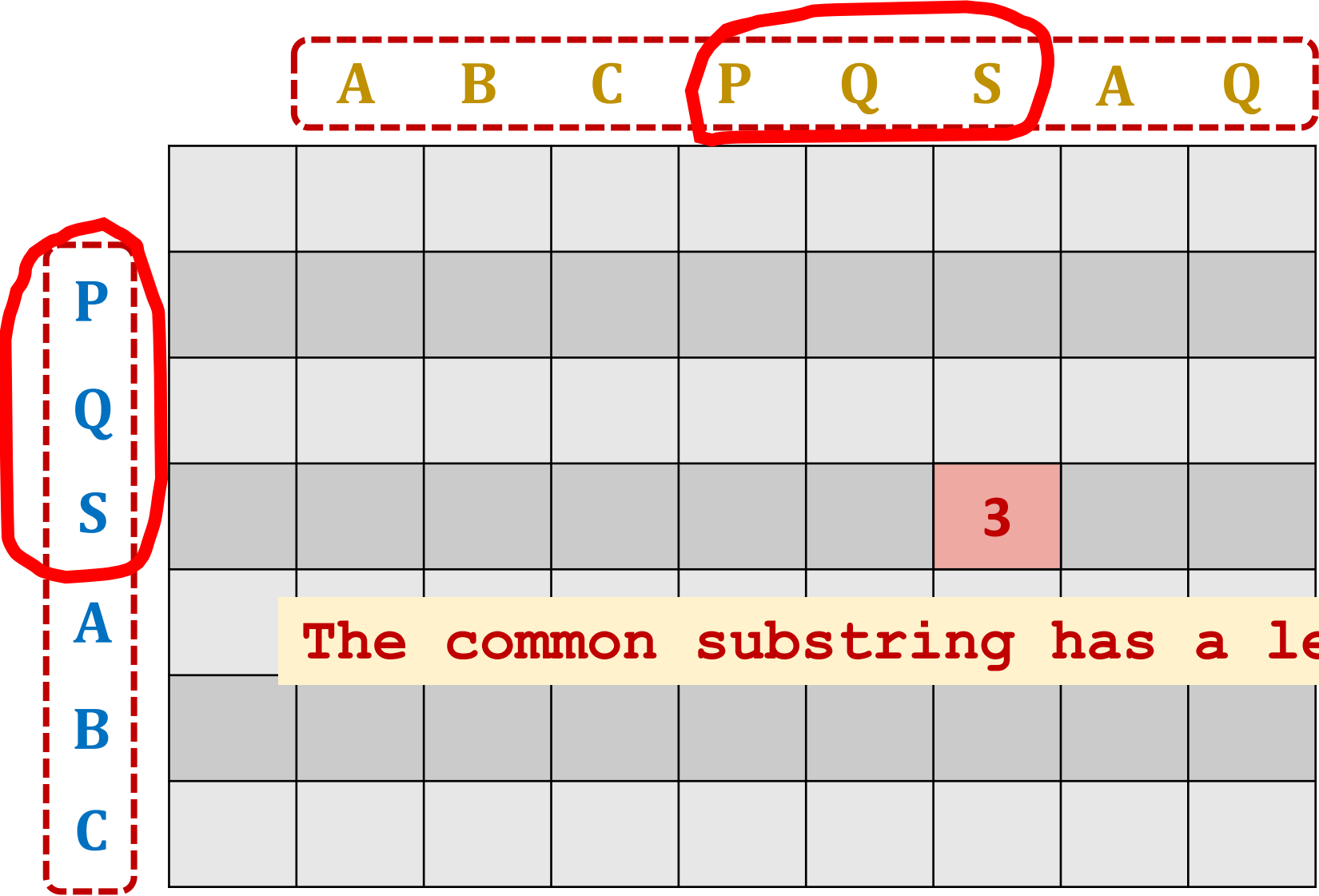
Basic Idea

	A	B	C	P	Q	S	A	Q
P								
Q								
S								
A								
B								
C								

Basic Idea

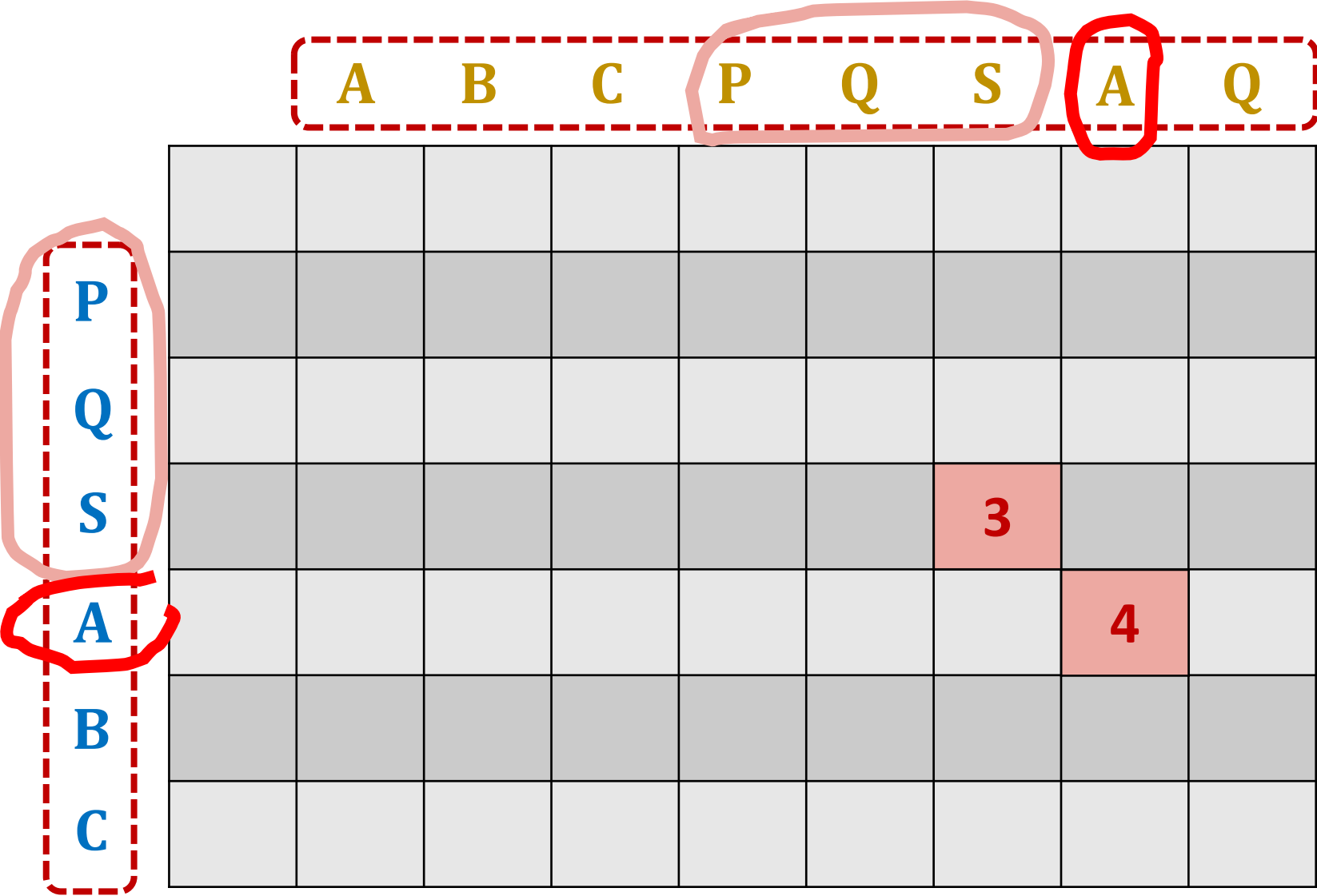


Basic Idea



The common substring has a length of 3.

Basic Idea



Basic Idea

<div><div>A</div><div>B</div><div>C</div><div>P</div><div>Q</div><div>S</div><div>A</div><div>Q</div></div>								
P								
Q								
S								
A						3		
B							4	
C								0

Optimal Substructure

- Inputs: strings x and y .
- Build a table L for storing the lengths of common substrings.
- For indices i and j : compare $x[i]$ and $y[j]$.

Optimal Substructure

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- Build a table L for storing the lengths of common substrings.
- For indices i and j : compare $x[i]$ and $y[j]$.
- If $x[i]$ and $y[j]$ are the same, then

$$L[i][j] = 1 + L[i-1][j-1].$$

- If $x[i]$ and $y[j]$ are different, then

$$L[i][j] = 0.$$

Optimal Substructure

- Inputs: strings x and y .
- Build a table L for storing the lengths of common substrings.
- For indices i and j : compare $x[i]$ and $y[j]$.
- If $x[i]$ and $y[j]$ are the same, then
- If $x[i]$ and $y[j]$ are different, then

$$L[i][j] = 1 + L[i-1][j-1]$$

$$L[i][j] = 0.$$

Dynamic Programming

Inputs: two strings

A B C P Q S A Q

P

Q

S

A

B

C

Base Case: $L[0][:] = 0$

	A	B	C	P	Q	S	A	Q
NULL	0	0	0	0	0	0	0	0
P								
Q								
S								
A								
B								
C								

Base Case: $L[:,0] = 0$

	NULL	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0	0
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[1]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0							
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[2]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0					
Q	0							
S	0							
A	0							
B	0							
C	0							

$x[1] \neq y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0				
S	0							
A	0							
B	0							
C	0							

$x[1] = y[4]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0					
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] = y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0				
S	0							
A	0							
B	0							
C	0							

$L[1][4] = 1 + L[0][3]$

$x[1] = y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1			
S	0							
A	0							
B	0							
C	0							

$L[1][4] = 1 + L[0][3]$

$x[1] \neq y[5]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0			
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0		
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[7]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[1] \neq y[8]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0								
S	0								
A	0								
B	0								
C	0								

$x[2] \neq y[1]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0		1	0	0	0	0
S	0							
A	0							
B	0							
C	0							

$x[2] \neq y[2]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0							
A	0							
B	0							
C	0							

$x[2] \neq y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0							
A	0							
B	0							
C	0							

$x[2] \neq y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0							
A	0							
B	0							
C	0							

$x[2] = y[5]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0			
A	0							
B	0							
C	0							

$x[2] = y[5]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0				
S	0							
A	0							
B	0							
C	0							

$L[2][5] = 1 + L[1][4]$

$x[2] = y[5]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2		
S	0							
A	0							
B	0							
C	0							

$L[2][5] = 1 + L[1][4]$

$x[2] \neq y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0		
S	0								
A	0								
B	0								
C	0								

$x[2] \neq y[7]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	
S	0							
A	0							
B	0							
C	0							

$x[2] = y[8]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	
S	0								
A	0								
B	0								
C	0								

$x[2] = y[8]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	2	0	0	
S	0							
A	0							
B	0							
C	0							

$L[2][8] = 1 + L[1][7]$

$x[2] = y[8]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0							
A	0							
B	0							
C	0							

$L[2][8] = 1 + L[1][7]$

$x[3] \neq y[1]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0							
B	0							
C	0							

$x[3] \neq y[2]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0							
B	0							
C	0							

$x[3] \neq y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0							
B	0							
C	0							

$x[3] \neq y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0							
B	0							
C	0							

$x[3] \neq y[5]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0							
B	0							
C	0							

$x[3] = y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0			
A	0								
B	0								
C	0								

$x[3] = y[6]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0			
A	0							
B	0							
C	0							

$L[3][6] = 1 + L[2][5]$

$x[3] = y[6]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	3		
A	0							
B	0							
C	0							

$L[3][6] = 1 + L[2][5]$

$x[3] \neq y[7]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	3	0	
A	0							
B	0							
C	0							

$x[3] \neq y[7]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0								
B	0								
C	0								

$x[4] = y[1]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0								
B	0								
C	0								

$x[4] = y[1]$

	<div>A</div>	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	0	3	0
<div>A</div>	0							
B	0							
C	0							

$L[4][1] = 1 + L[3][0]$

$x[4] = y[1]$

	<div>A</div>	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	0	3	0
<div>A</div>	0	1						
B	0							
C	0							

$L[4][1] = 1 + L[3][0]$

$x[4] \neq y[2]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0					
C	0							

$x[4] \neq y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	1	0	0				
B	0							
C	0							

$x[4] \neq y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	1	0	0				
B	0							
C	0							

$x[4] \neq y[5]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	1	0	0	0	0
Q	0	0	0	0	2	0	0	1
S	0	0	0	0	0	3	0	0
A	0	1	0	0	0			
B	0							
C	0							

$x[4] \neq y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0		
B	0								
C	0								

$x[4] = y[7]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	0	3	0
A	0	1	0	0	0	0		
B	0							
C	0							

$x[4] = y[7]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0		
C	0							

$L[4][7] = 1 + L[3][6]$

$x[4] = y[7]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	4	
C	0							

$L[4][7] = 1 + L[3][6]$

$x[4] \neq y[8]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0								
C	0								

$x[5] \neq y[1]$

		A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0	0
S	0	0	0	0	0	2	0	0	1
A	0	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0	0	4	0
C	0	0							

$x[5] = y[2]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	4
C	0	0	0	0	0	0	0	0

$x[5] = y[2]$

	A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0
Q	0	0	0	0	0	2	0	1
S	0	0	0	0	0	0	3	0
A	0	1	0	0	0	0	4	0
B	0	0						
C	0		$L[5][2] = 1 + L[4][1]$					

$x[5] \neq y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	1	0	0	0	0	4	0
B	0	0	2	0				
C	0							

$x[5] \neq y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	1	0	0	0	0	4	0
B	0	0	2	0				
C	0							

$x[5] \neq y[5]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	4
C	0	0	2	0	0	0	0	0

$x[5] \neq y[6]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	2	0	0	1
S	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	4	0
B	0	0	2	0	0	0		
C	0							

$x[5] \neq y[7]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	1	0	0	0	0	4	0
B	0	0	2	0	0	0	0	
C	0							

$x[5] \neq y[8]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0	4	0
C	0	0	2	0	0	0	0	0
	0							

$x[6] \neq y[1]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0							

$x[6] \neq y[2]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	4
C	0	0	2	0	0	0	0	0
	0	0	0					

$x[6] = y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	4
C	0	0	2	0	0	0	0	0
	0	0	0					

$x[6] = y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	4
C	0	0	2	0	0	0	0	0

$L[6][3] = 1 + L[5][2]$

$x[6] = y[3]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	0
C	0	0	2	0	0	0	0	0
	0	0	0	3				

$L[6][3] = 1 + L[5][2]$

$x[6] \neq y[4]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0	4	0
C	0	0	2	0	0	0	0	0
	0	0	0	3	0			

$x[6] \neq y[5]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0
S	0	0	0	0	0	2	0	1
A	0	0	0	0	0	0	3	0
B	0	1	0	0	0	0	0	4
C	0	0	2	0	0	0	0	0
	0	0	0	3	0			

$x[6] \neq y[6]$

		A	B	C	P	Q	S	A	Q
	0	0	0	0	0	0	0	0	0
P	0	0	0	0	1	0	0	0	0
Q	0	0	0	0	0	2	0	0	1
S	0	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0	0
C	0	0	0	3	0	0	0		

$x[6] \neq y[7]$

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0	4	0
C	0	0	2	0	0	0	0	0
	0	0	0	3	0	0	0	

$x[6] \neq y[8]$

		A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0	0
Q	0	0	0	0	1	0	0	0	0
S	0	0	0	0	0	2	0	0	1
A	0	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0	0	4	0
C	0	0	2	0	0	0	0	0	0
	0	0	0	3	0	0	0	0	0

Argmax L[i][j]

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	0	2	0	0	1
S	0	0	0	0	0	3	0	0
A	0	1	0	0	0	0	4	0
B	0	0	2	0	0	0	0	0
C	0	0	0	3	0	0	0	0

Argmax $L[i][j]$

		A	B	C	P	Q	S	A	Q
		0	0	0	0	0	0	0	0
P		0	0	0	0	1	0	0	0
Q		0	0	0	0	0	2	0	1
S		0	0	0	0	0	0	3	0
A		0	1	0	0	0	0	4	0
B		0	0	2	0	0	0	0	0
C		0	0	0	3	0	0	0	0

Output: " PQSA "

	A	B	C	P	Q	S	A	Q
P	0	0	0	0	0	0	0	0
Q	0	0	0	1	0	0	0	0
S	0	0	0	0	2	0	0	1
A	0	0	0	0	0	3	0	0
B	0	1	0	0	0	0	4	0
C	0	0	2	0	0	0	0	0
	0	0	0	3	0	0	0	0

Time Complexity

- Let the lengths of the input strings be m and n .
- The size of the table is $(m + 1) \times (n + 1)$.
- It takes $O(1)$ time to fill each entry.
- Thus, the overall time complexity is $O(mn)$.

Thank You!