Graphs

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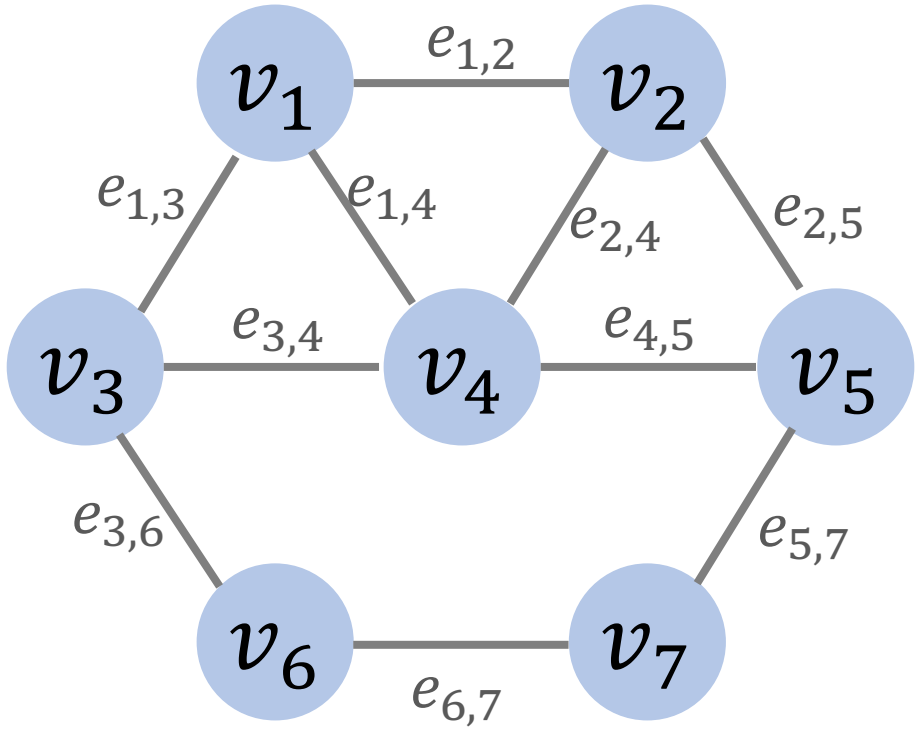
What is graph?



Definitions

- Set of vertices:

$$\mathcal{V} = \{v_1, v_2, \cdots, v_7\}.$$

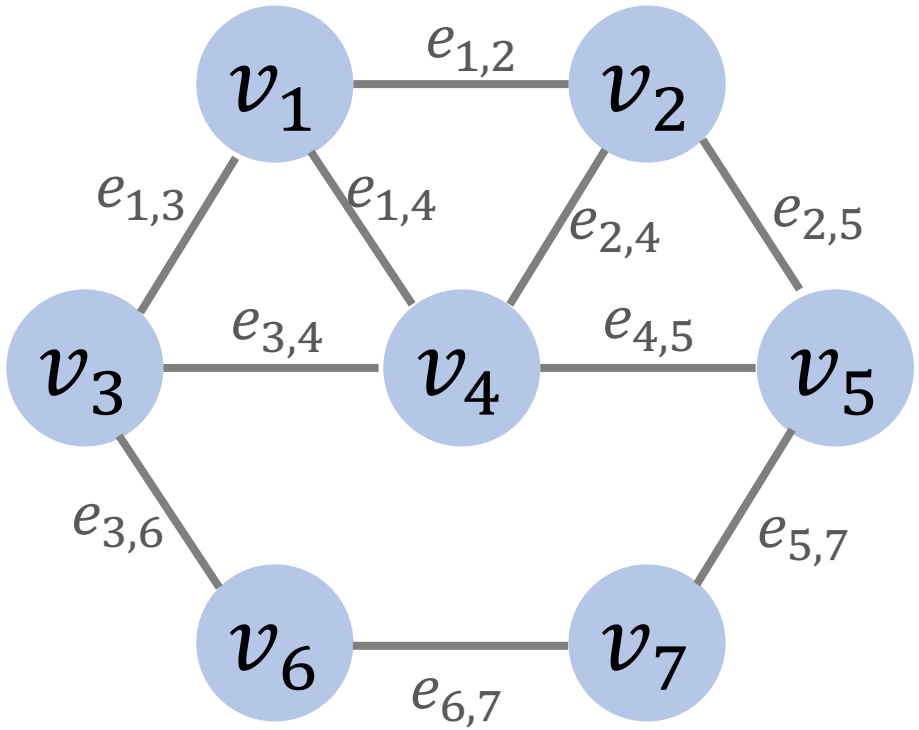
- Set of edges:

$$\mathcal{E} = \{e_{1,2}, e_{1,3}, e_{1,4}, e_{2,4}, \cdots, e_{6,7}\}.$$

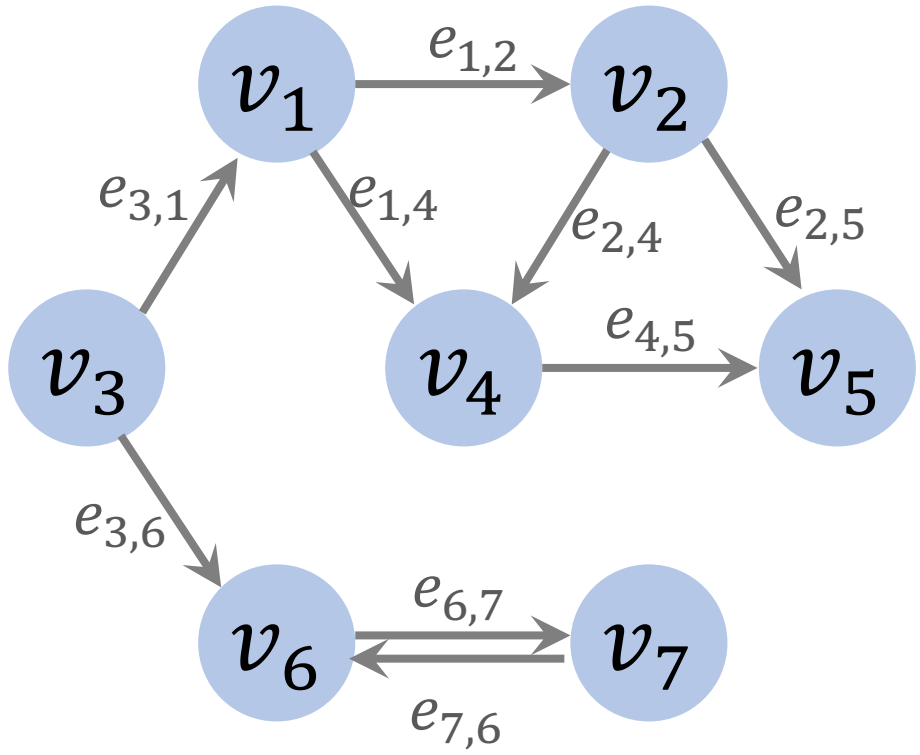
- Graph: $\mathcal{G} = (\mathcal{V}, \mathcal{E})$.

Undirected vs Directed

Undirected Graph

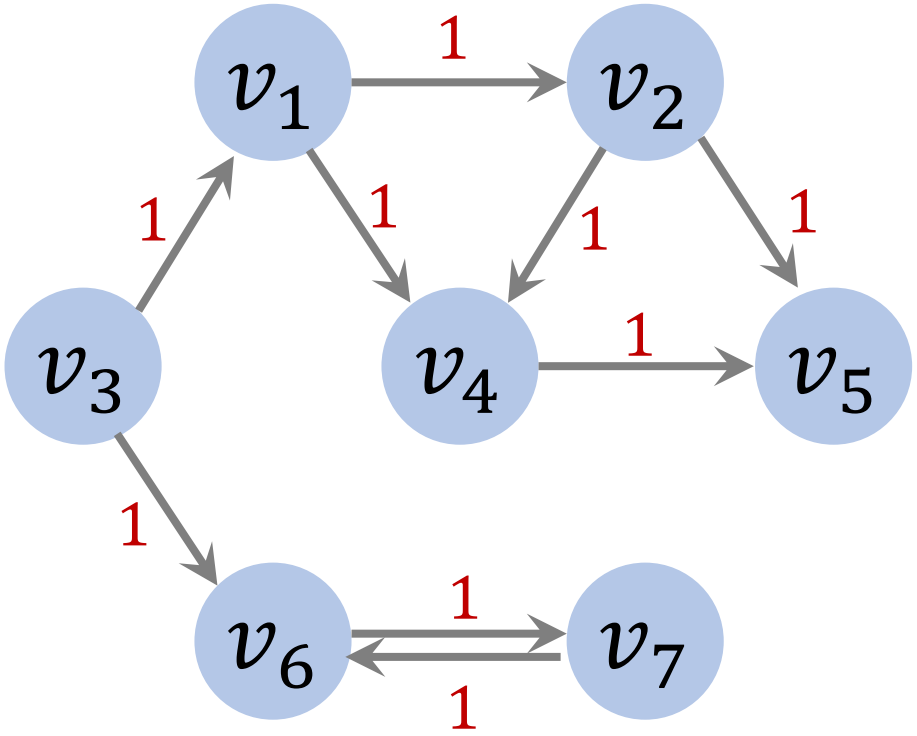


Directed Graph

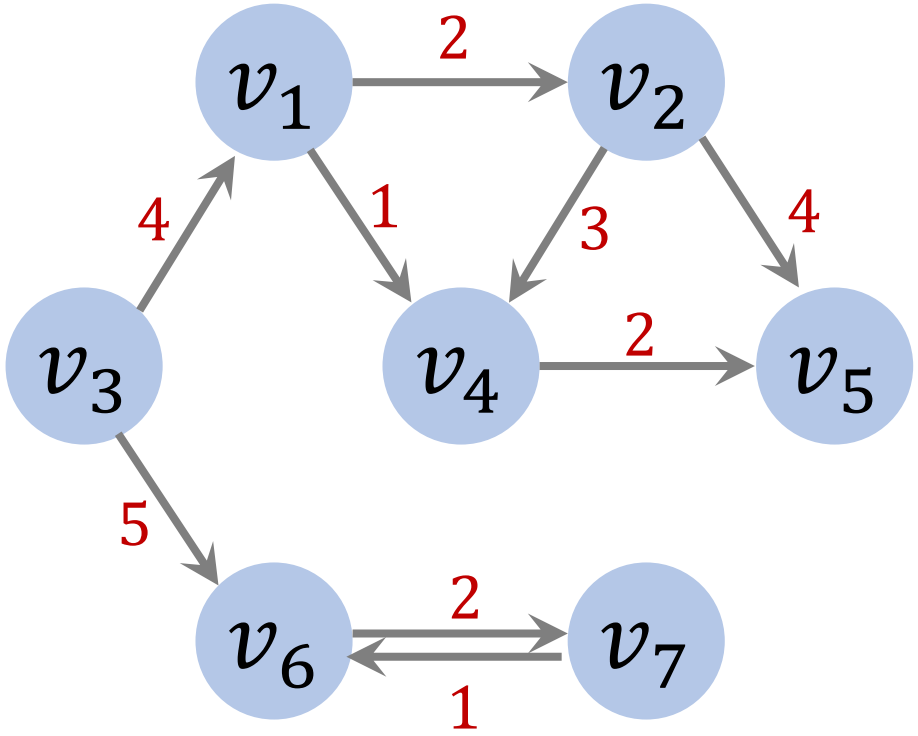


Unweighted vs Weighted

Unweighted Graph

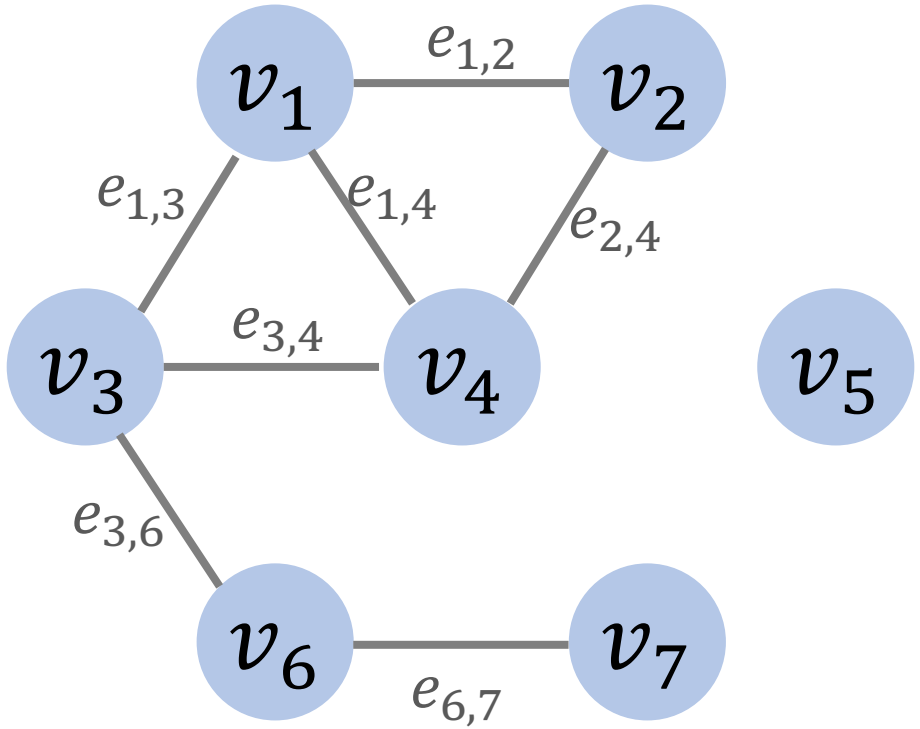


Weighted Graph



Undirected Unweighted Graphs

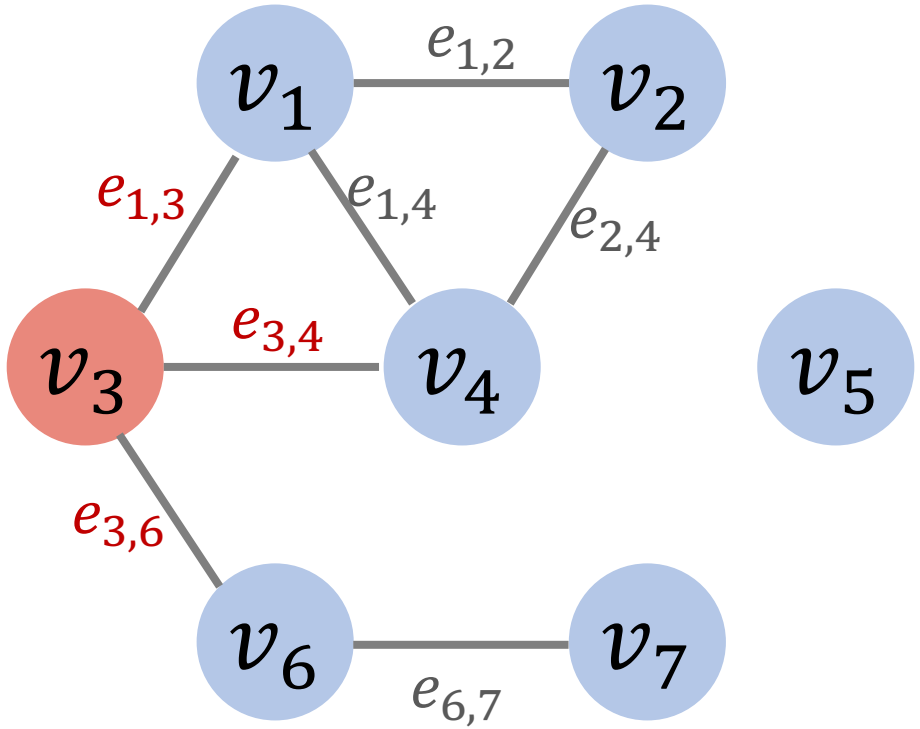
Undirected Unweighted Graph



Adjacency list:

Vertex	Neighbors
1	2, 3, 4
2	1, 4
3	1, 4, 6
4	1, 2, 3
5	empty
6	3, 7
7	6

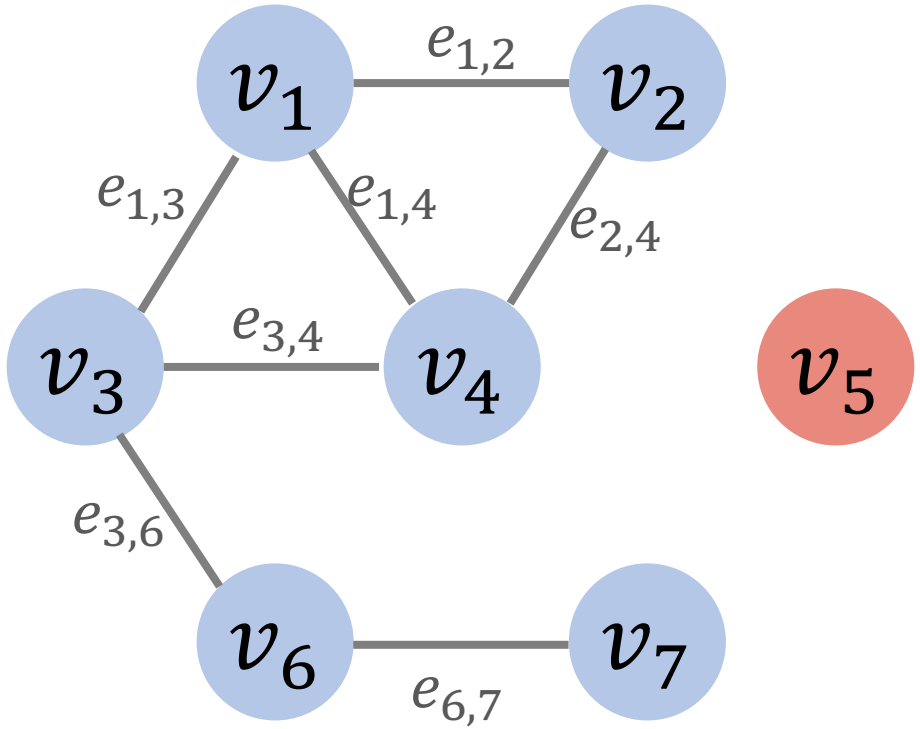
Undirected Unweighted Graph



Adjacency list:

Vertex	Neighbors
1	2, 3, 4
2	1, 4
3	**1, 4, 6**
4	1, 2, 3
5	empty
6	3, 7
7	6

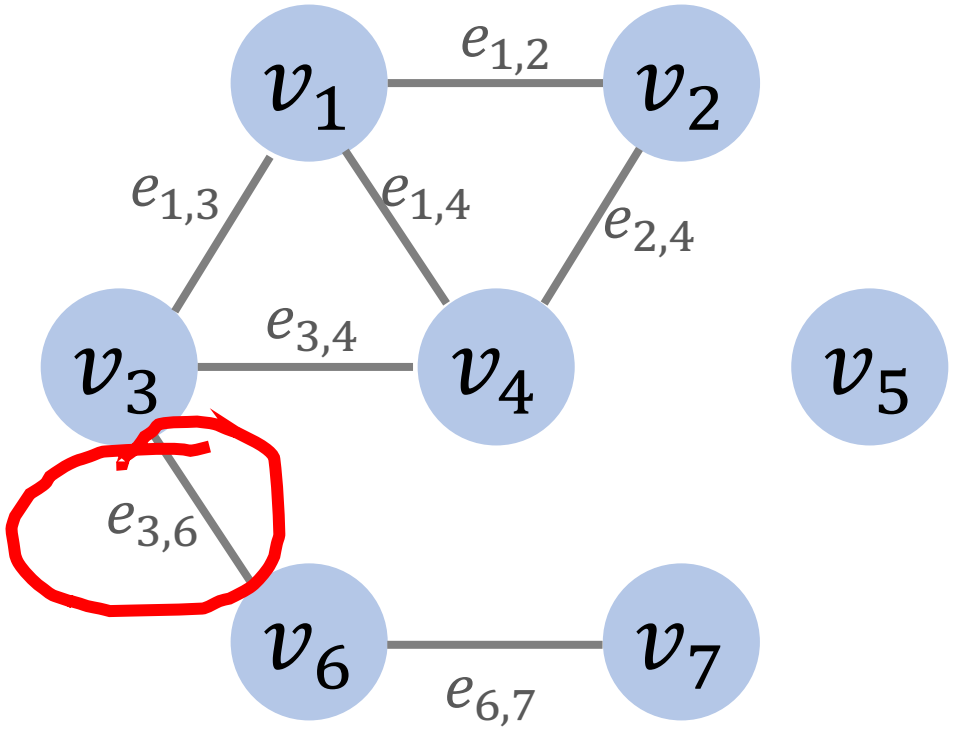
Undirected Unweighted Graph



Adjacency list:

Vertex	Neighbors
1	2, 3, 4
2	1, 4
3	1, 4, 6
4	1, 2, 3
5	**empty**
6	3, 7
7	6

Undirected Unweighted Graph

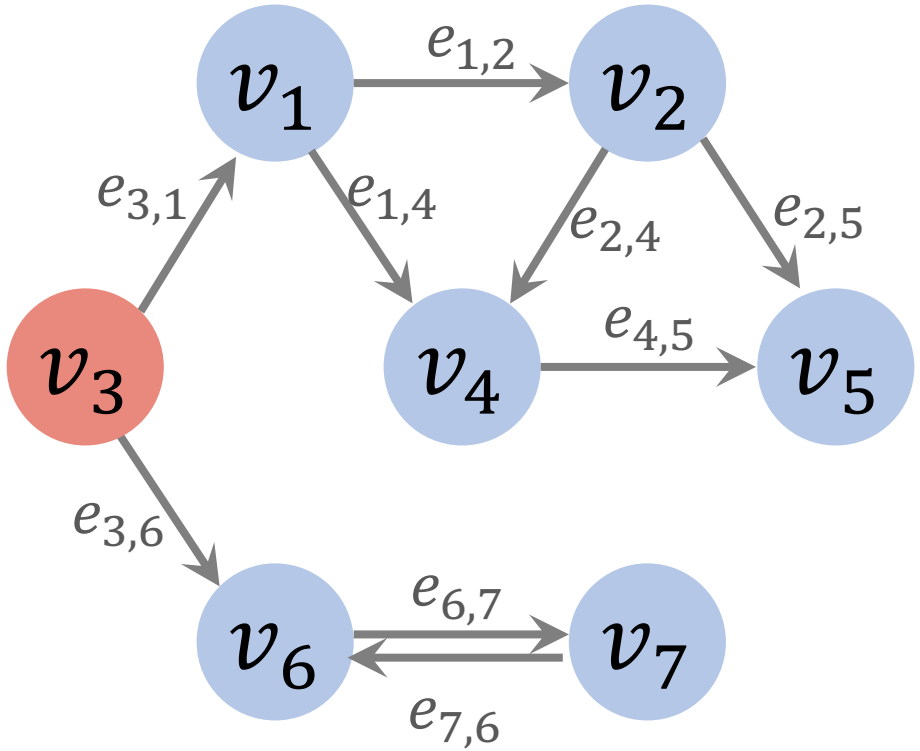


Adjacency matrix:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	1	1	1	0	0	0
v_2	1	0	0	1	0	0	0
v_3	1	0	0	1	0	1	0
v_4	1	1	1	0	0	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	1	0	0	0	1
v_7	0	0	0	0	0	1	0

Directed Unweighted Graph

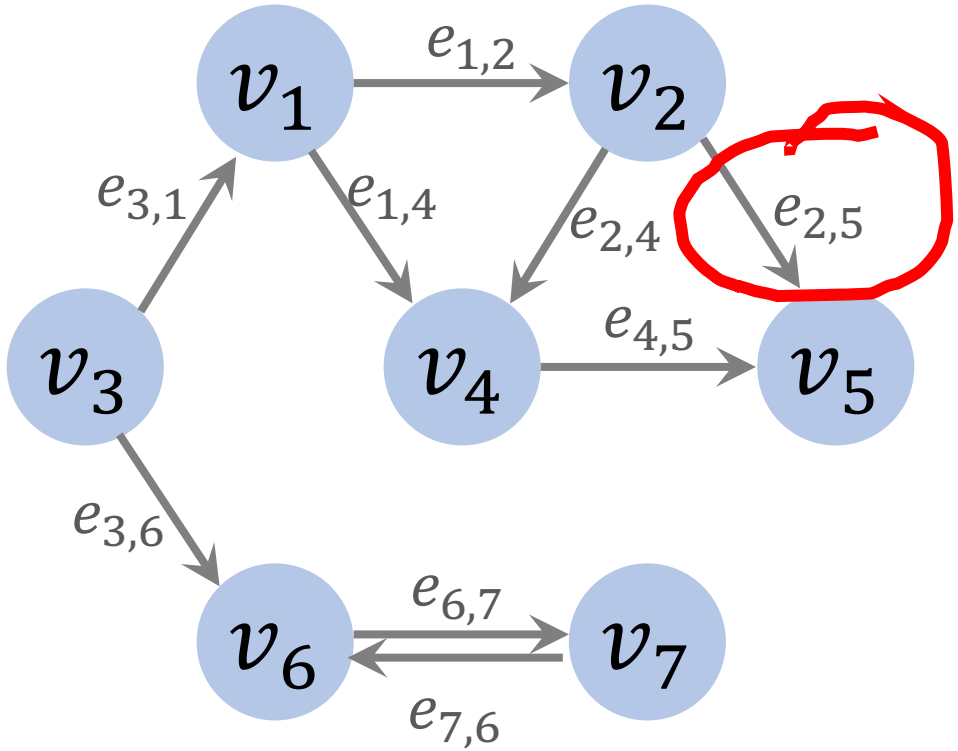
Directed Unweighted Graph



Adjacency list:

From	To
1	2, 4
2	4, 5
3	**1, 6**
4	5
5	empty
6	7
7	6

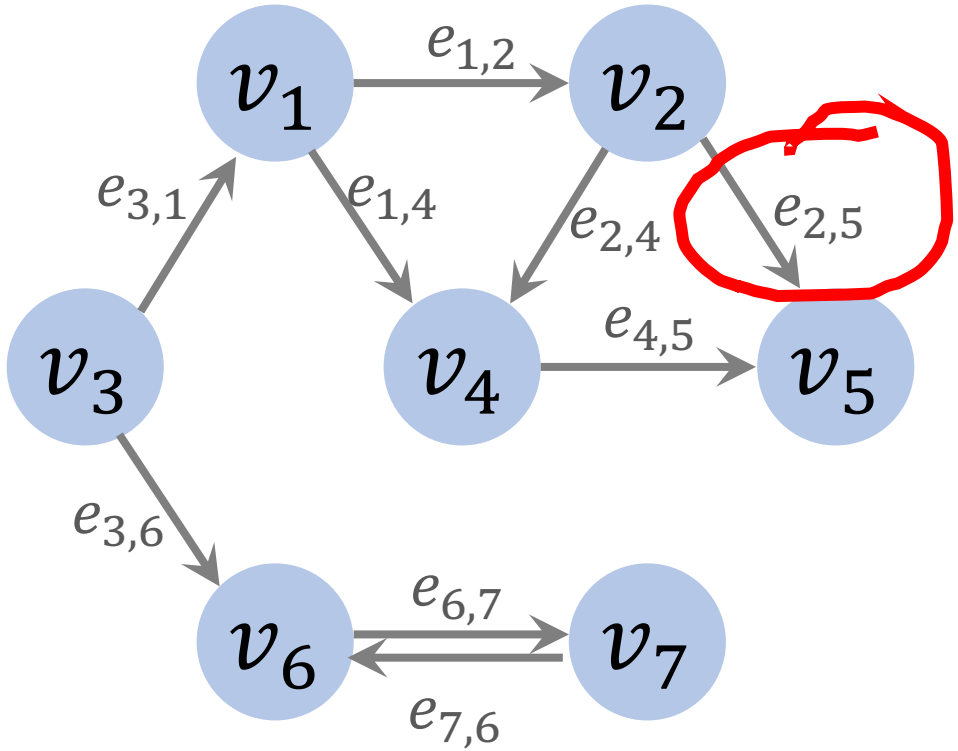
Directed Unweighted Graph



Adjacency matrix:

From \ To	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**1**	0	**1**	0	0	0
v_2	0	0	0	**1**	**1**	0	0
v_3	**1**	0	0	0	0	**1**	0
v_4	0	0	0	0	**1**	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	0	0	0	0	**1**
v_7	0	0	0	0	0	**1**	0

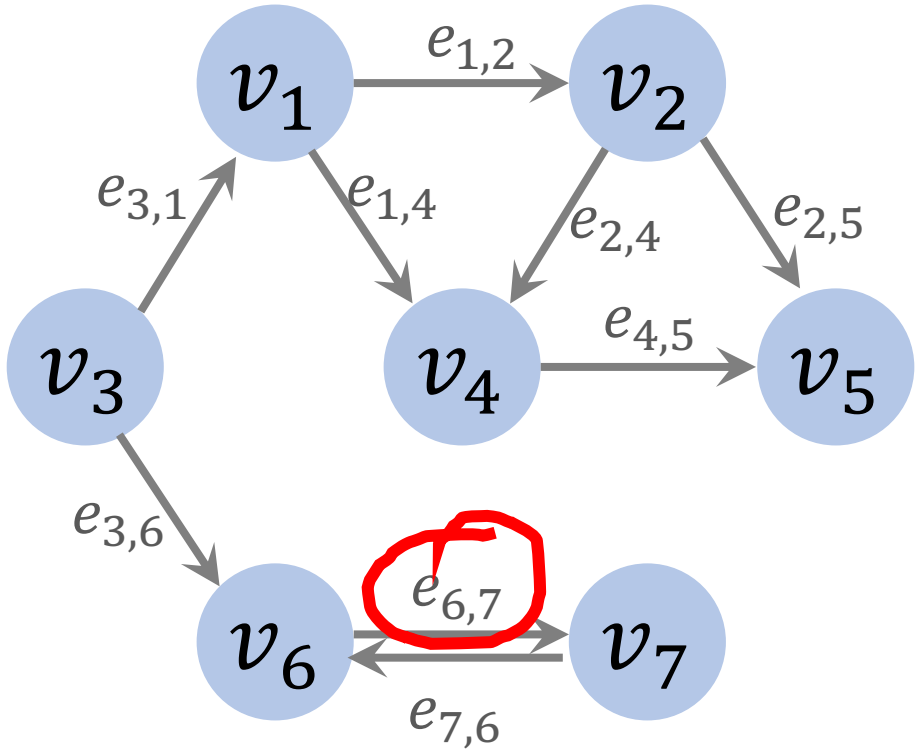
Directed Unweighted Graph



Adjacency matrix:

From \ To	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**1**	0	**1**	0	0	0
v_2	0	0	0	**1**	**1**	0	0
v_3	**1**	0	0	0	0	**1**	0
v_4	0	0	0	0	**1**	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	0	0	0	0	**1**
v_7	0	0	0	0	0	**1**	0

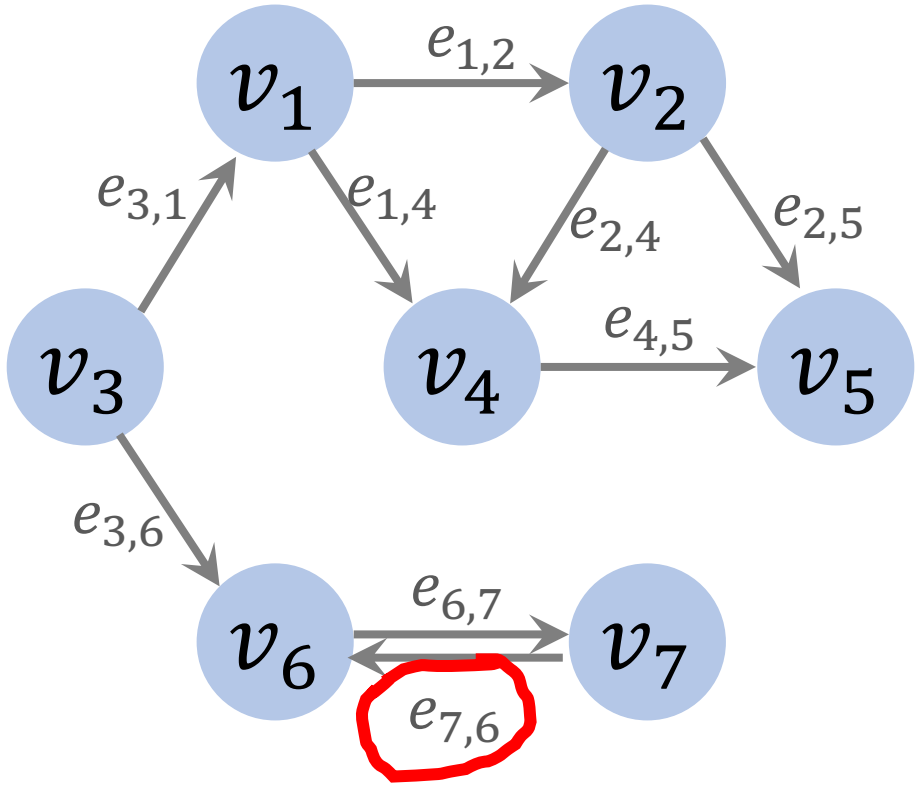
Directed Unweighted Graph



Adjacency matrix:

From \ To	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**1**	0	**1**	0	0	0
v_2	0	0	0	**1**	**1**	0	0
v_3	**1**	0	0	0	0	**1**	0
v_4	0	0	0	0	**1**	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	0	0	0	0	**1**
v_7	0	0	0	0	0	**1**	0

Directed Unweighted Graph

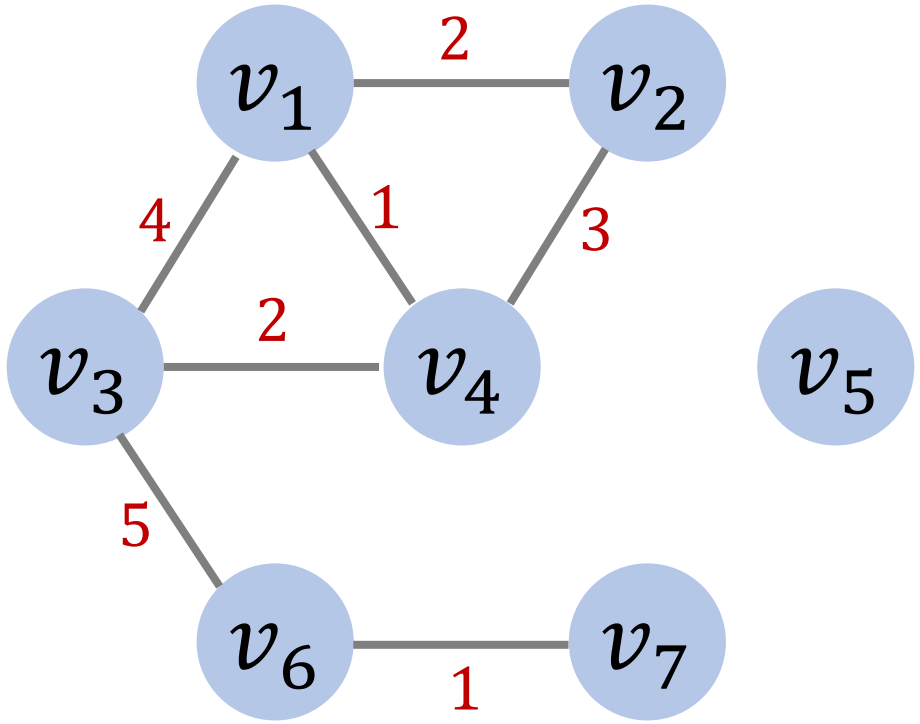


Adjacency matrix:

From \ To	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**1**	0	**1**	0	0	0
v_2	0	0	0	**1**	**1**	0	0
v_3	**1**	0	0	0	0	**1**	0
v_4	0	0	0	0	**1**	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	0	0	0	0	**1**
v_7	0	0	0	0	0	**1**	0

Weighted Graphs

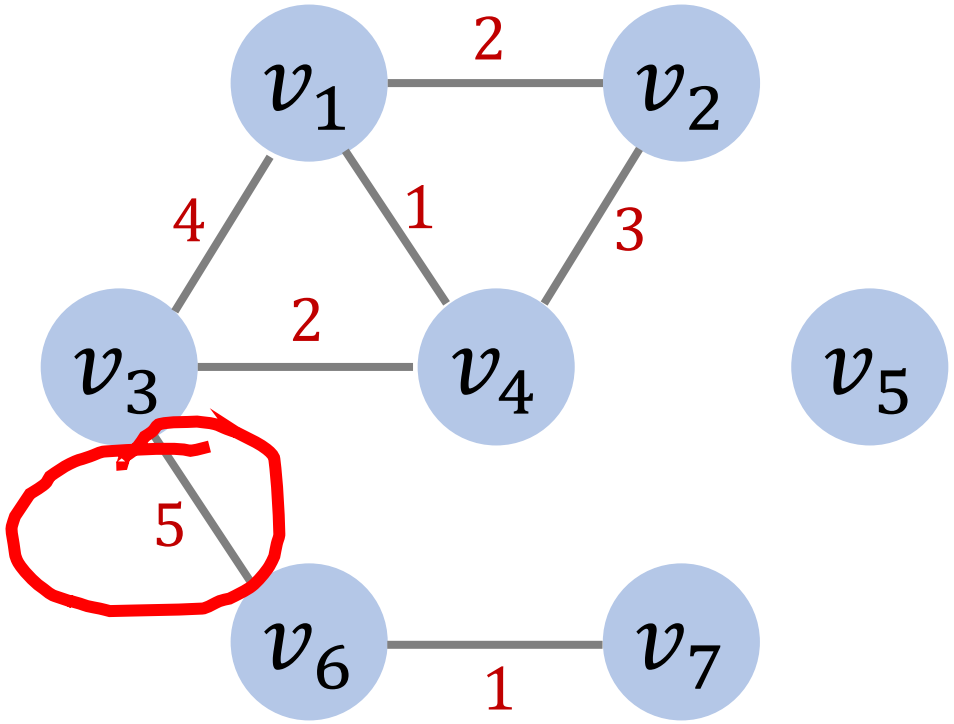
Undirected Weighted Graph



Adjacency matrix:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**2**	**4**	**1**	0	0	0
v_2	**2**	0	0	**3**	0	0	0
v_3	**4**	0	0	**2**	0	**5**	0
v_4	**1**	**3**	**2**	0	0	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	**5**	0	0	0	**1**
v_7	0	0	0	0	0	**1**	0

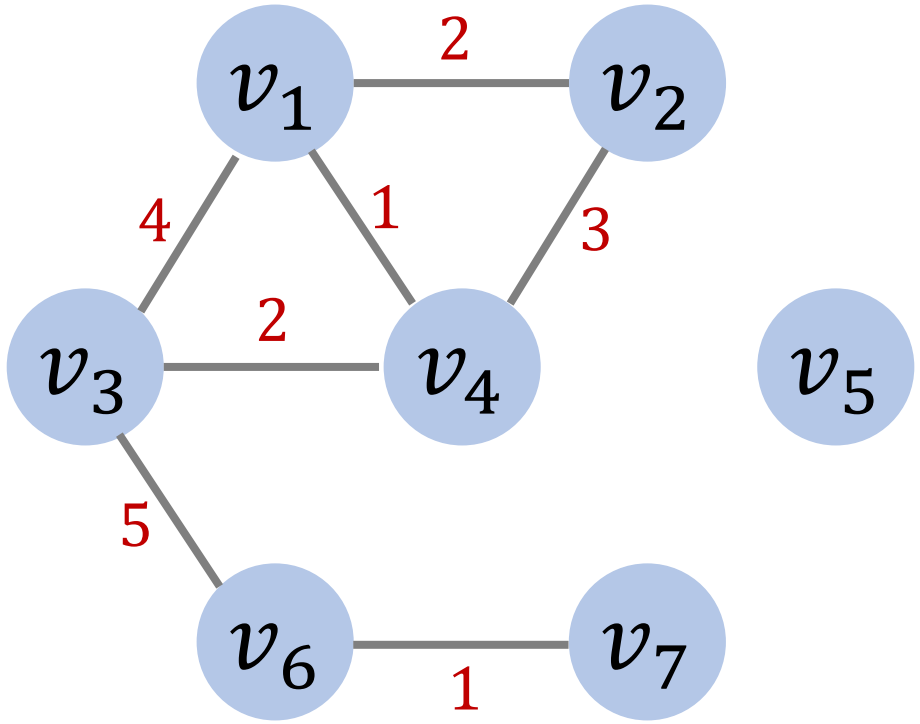
Undirected Weighted Graph



Adjacency matrix:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**2**	**4**	**1**	0	0	0
v_2	**2**	0	0	**3**	0	0	0
v_3	**4**	0	0	**2**	0	**5**	0
v_4	**1**	**3**	**2**	0	0	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	**5**	0	0	0	**1**
v_7	0	0	0	0	0	**1**	0

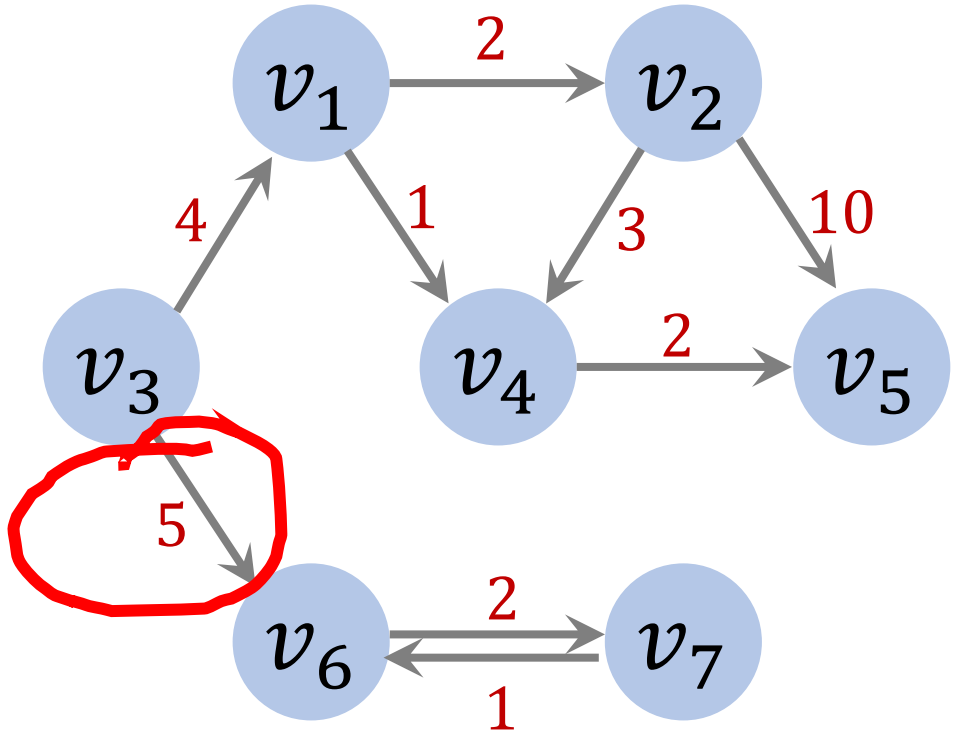
Undirected Weighted Graph



Adjacency matrix:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	∞	**2**	**4**	**1**	∞	∞	∞
v_2	**2**	∞	∞	**3**	∞	∞	∞
v_3	**4**	∞	∞	**2**	∞	**5**	∞
v_4	**1**	**3**	**2**	∞	∞	∞	∞
v_5	∞	∞	∞	∞	∞	∞	∞
v_6	∞	∞	**5**	∞	∞	∞	**1**
v_7	∞	∞	∞	∞	∞	**1**	∞

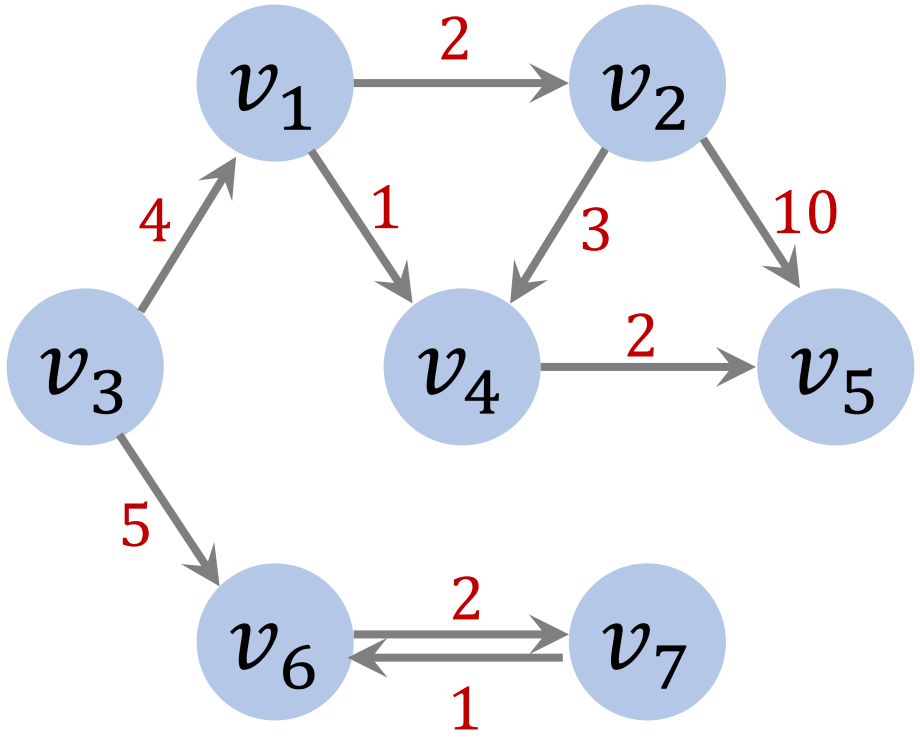
Directed Weighted Graph



Adjacency matrix:

From \ To	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	0	**2**	0	**1**	0	0	0
v_2	0	0	0	**3**	**10**	0	0
v_3	**4**	0	0	0	0	**5**	0
v_4	0	0	0	0	**2**	0	0
v_5	0	0	0	0	0	0	0
v_6	0	0	0	0	0	0	**2**
v_7	0	0	0	0	0	**1**	0

Directed Weighted Graph

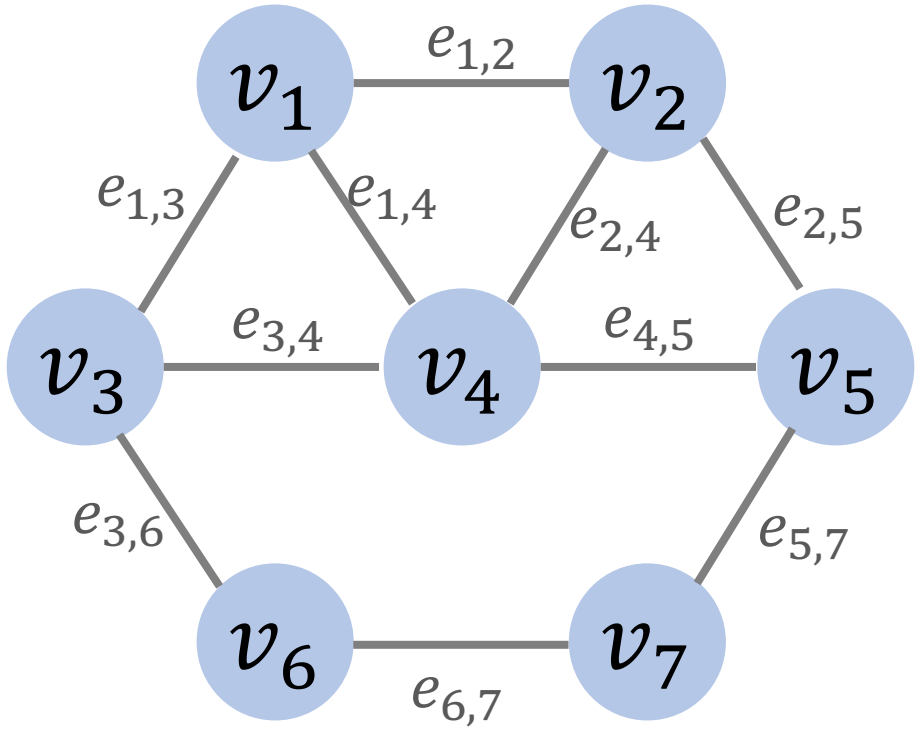


Adjacency matrix:

From \ To	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1	∞	**2**	∞	**1**	∞	∞	∞
v_2	∞	∞	∞	**3**	**10**	∞	∞
v_3	**4**	∞	∞	∞	∞	**5**	∞
v_4	∞	∞	∞	∞	**2**	∞	∞
v_5	∞	∞	∞	∞	∞	∞	∞
v_6	∞	∞	∞	∞	∞	∞	**2**
v_7	∞	∞	∞	∞	∞	**1**	∞

Questions

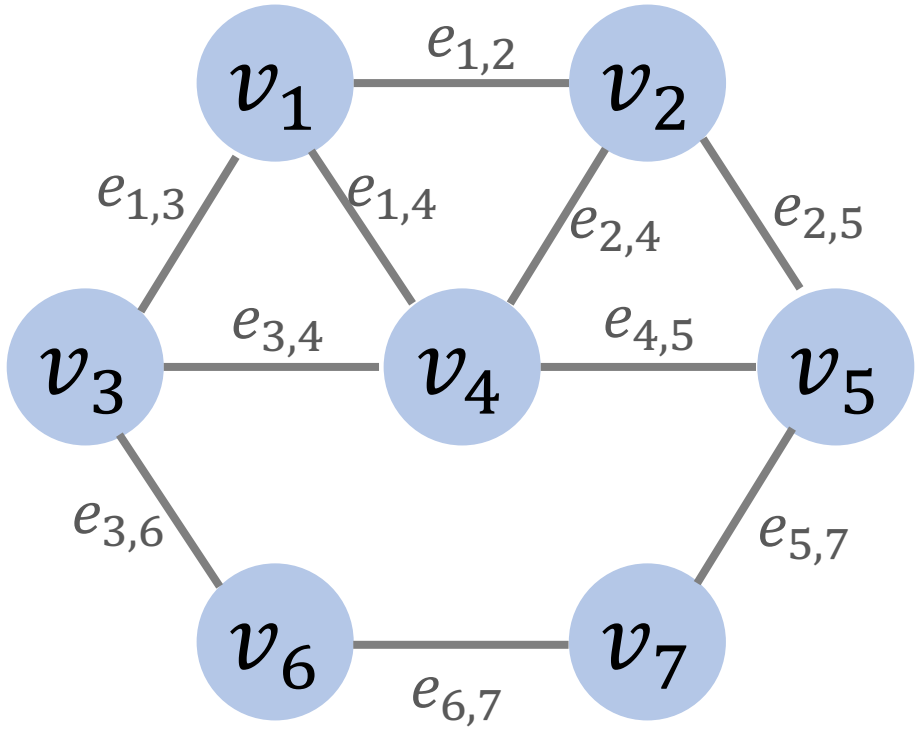
Question 1: Fill in the adjacency list



Adjacency list:

Vertex	Neighbors
1	
2	
3	
4	
5	
6	
7	

Question 2: Fill in the adjacency matrix



Adjacency matrix:

	v_1	v_2	v_3	v_4	v_5	v_6	v_7
v_1							
v_2							
v_3							
v_4							
v_5							
v_6							
v_7							

Thank You!