Quicksort

Shusen Wang

Quicksort: 3 Steps

1. Selecting a pivot.
2. Partition.
3. Recursion.

Step 1: Selecting Pivot

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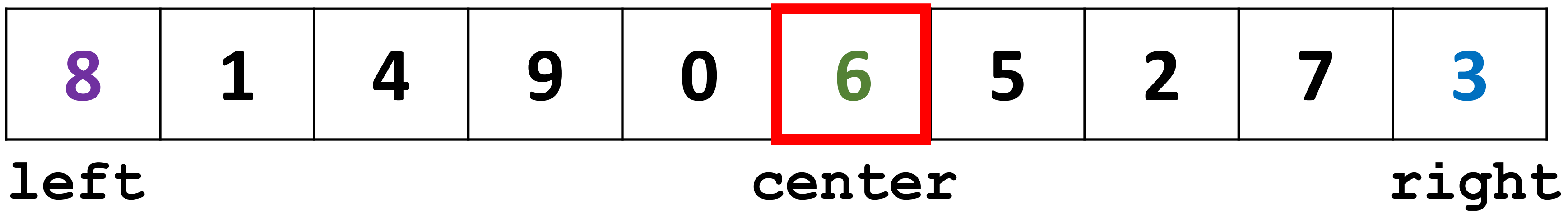
8	**1**	**4**	**9**	**0**	**6**	**5**	**2**	**7**	**3**

- Select one element from the array as the pivot.
- The pivot should be close to the median.

Step 1: Selecting Pivot

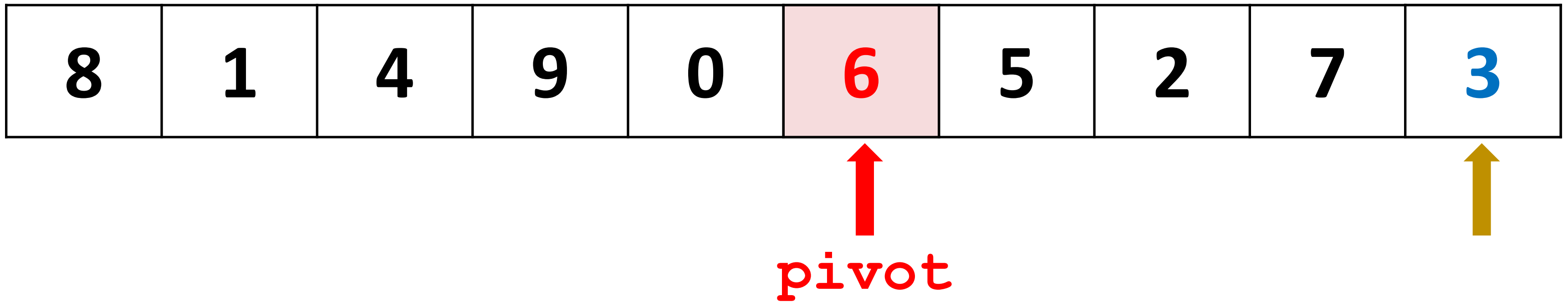


Pick it as pivot!

- Select one element from the array as the pivot.
- The pivot should be close to the median. (Why?)
- Heuristic:

$$\text{pivot} = \text{median}(\text{left}, \text{center}, \text{right})$$

Step 1: Selecting Pivot



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$$\text{pivot} = \text{median}(\text{left}, \text{center}, \text{right})$$
- Put the pivot at the end.

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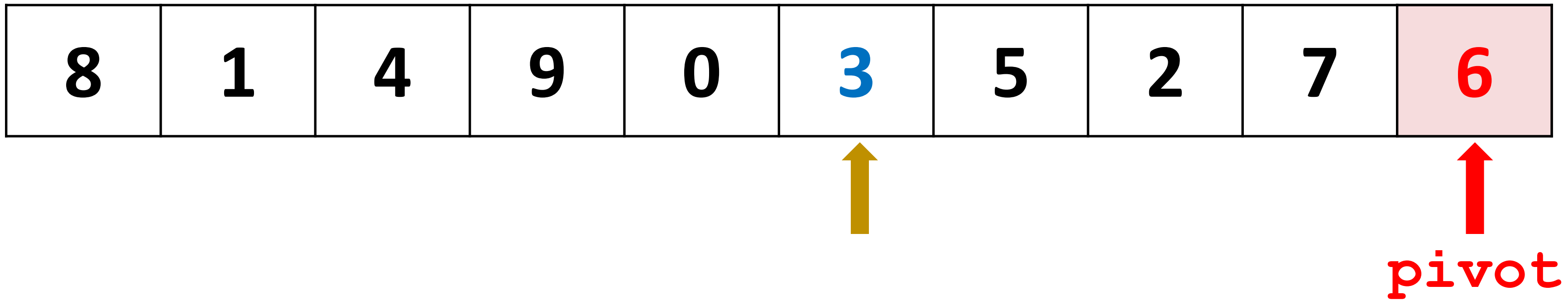

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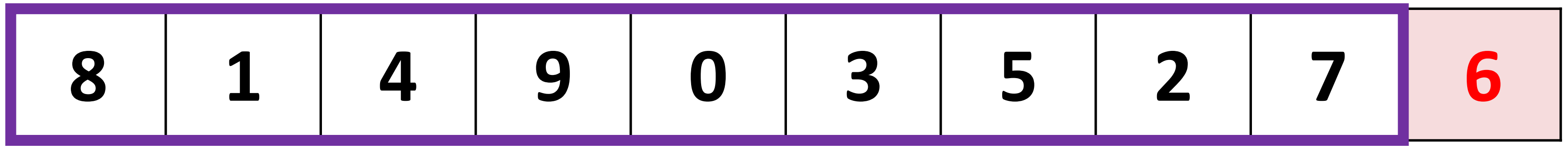
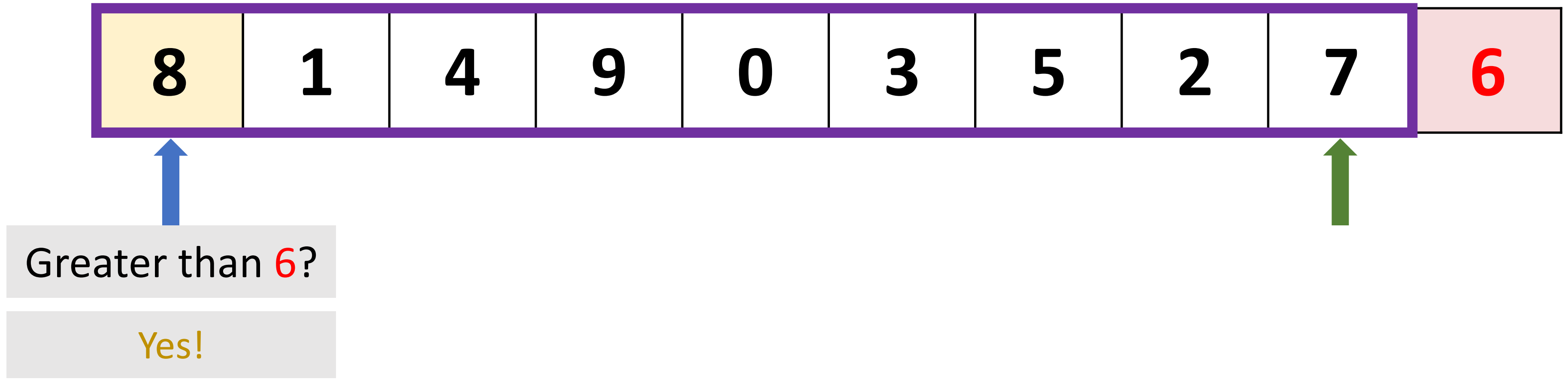


- Partition the first $n - 1$ elements to 2 groups.
- Group 1: $\{x \mid x \leq 6\}$.
- Group 2: $\{x \mid x \geq 6\}$.

Step 2: Partition



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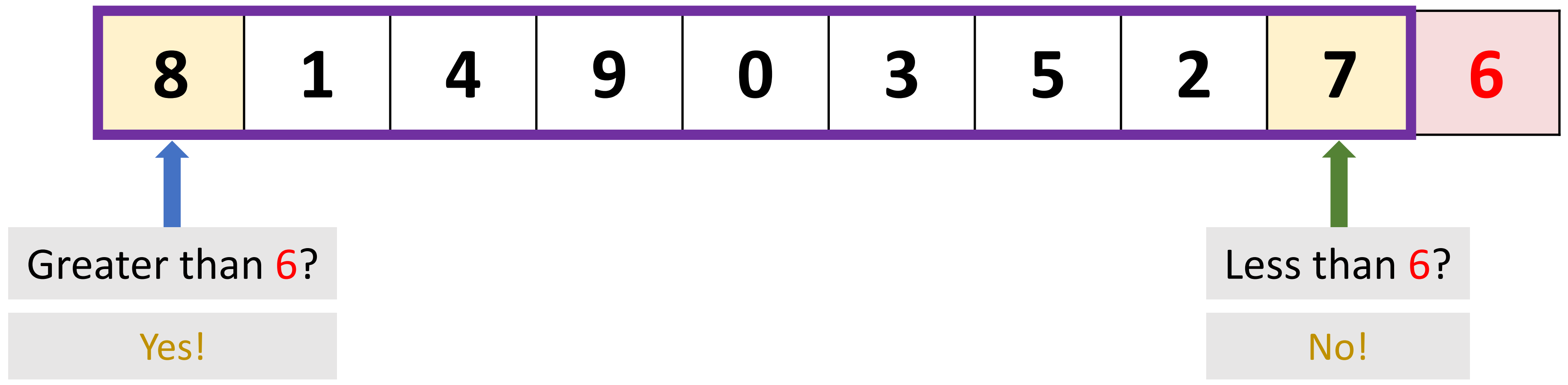

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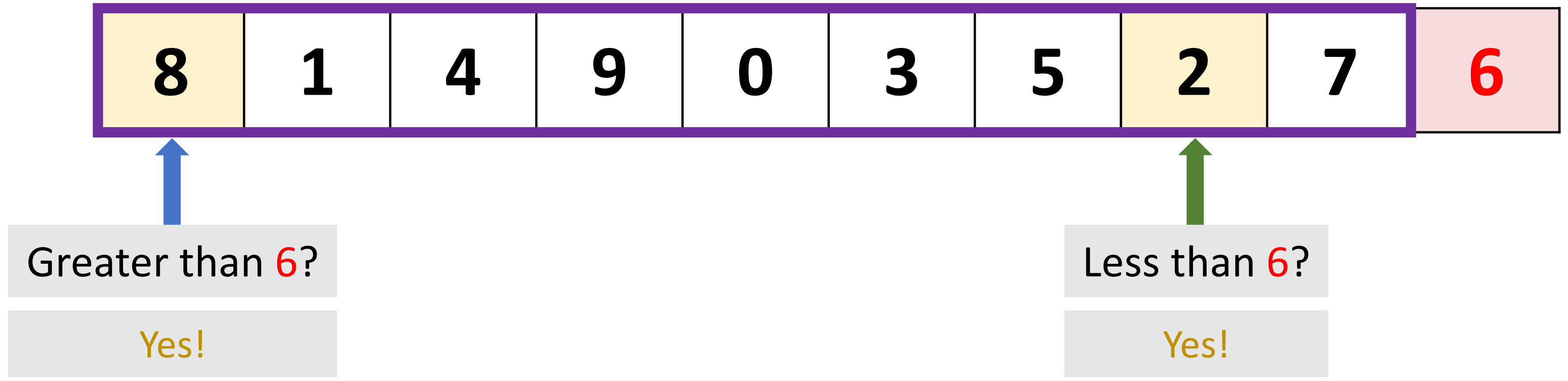

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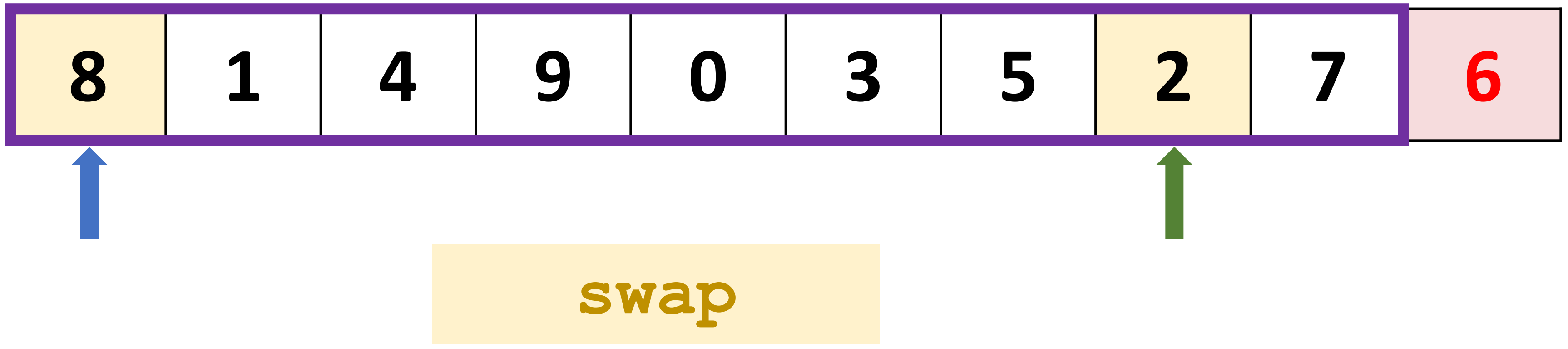

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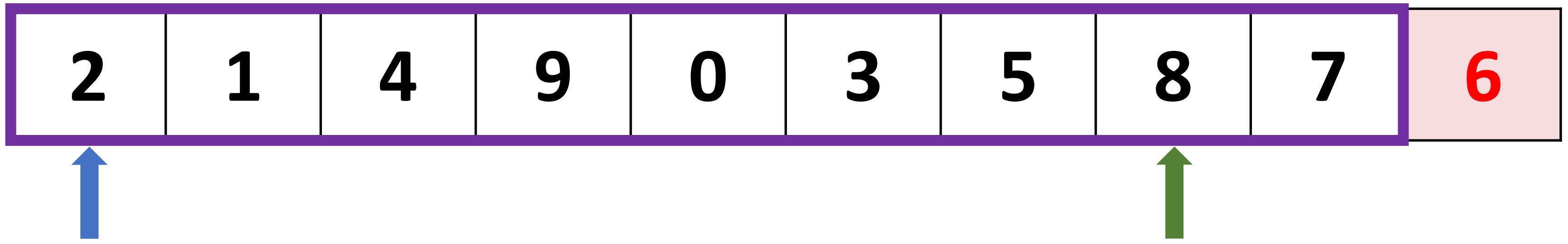

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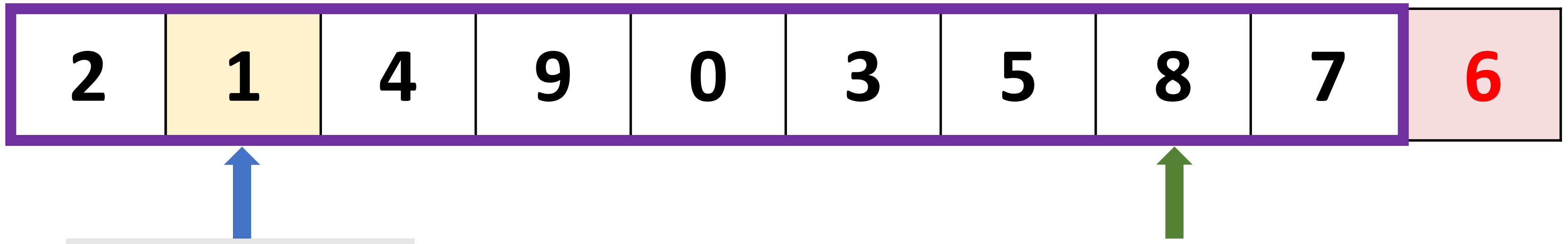
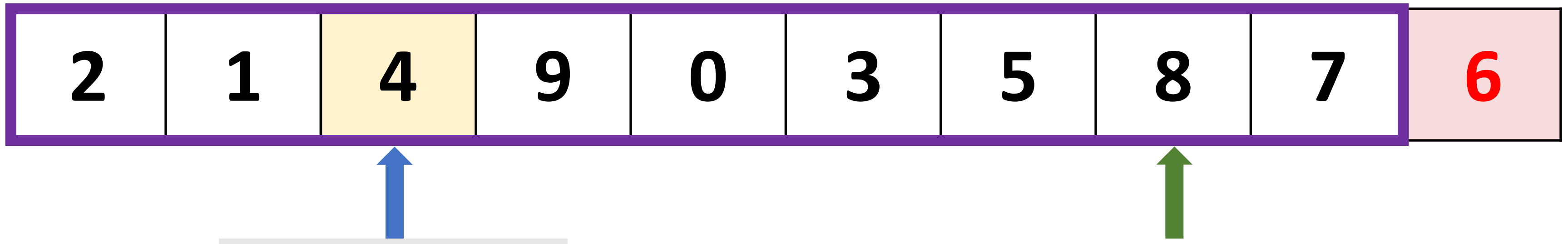


Greater than 6?

No!

Step 2: Partition



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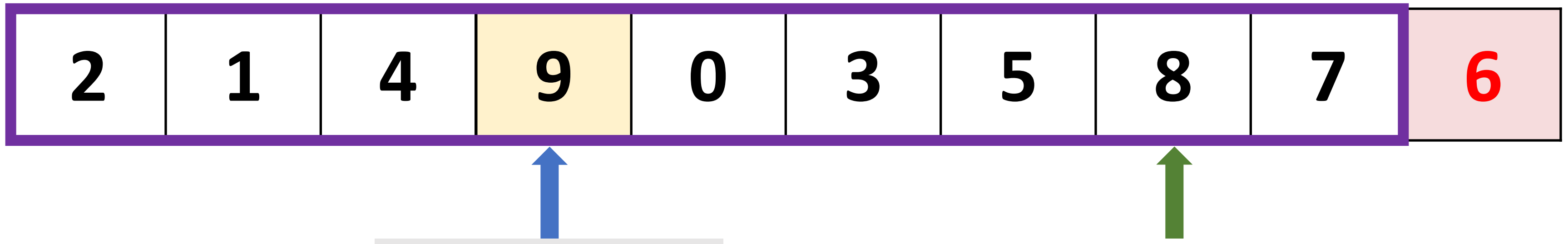

Greater than 6?

Yes!

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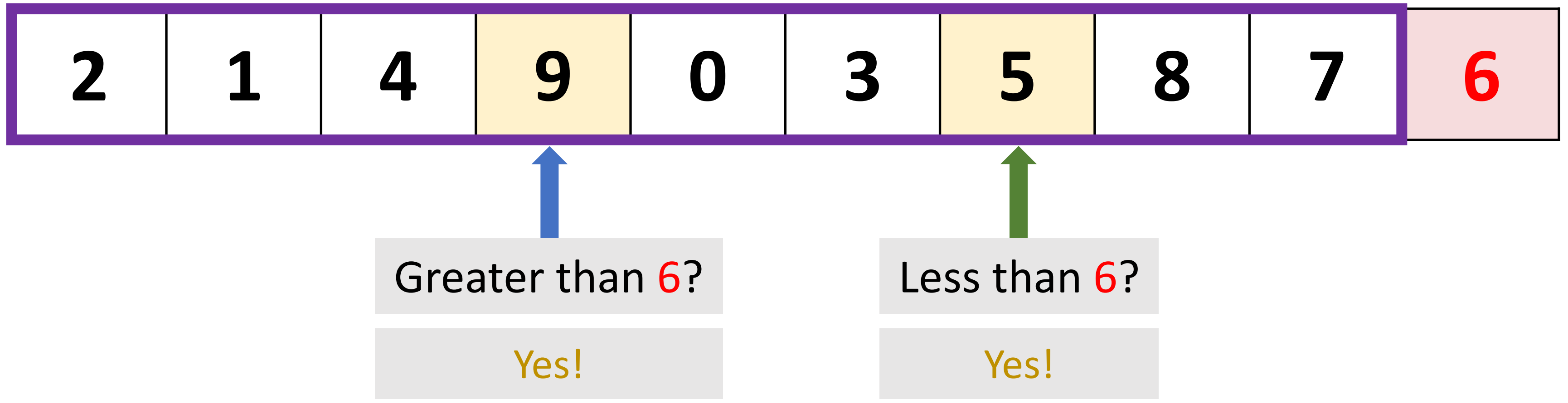

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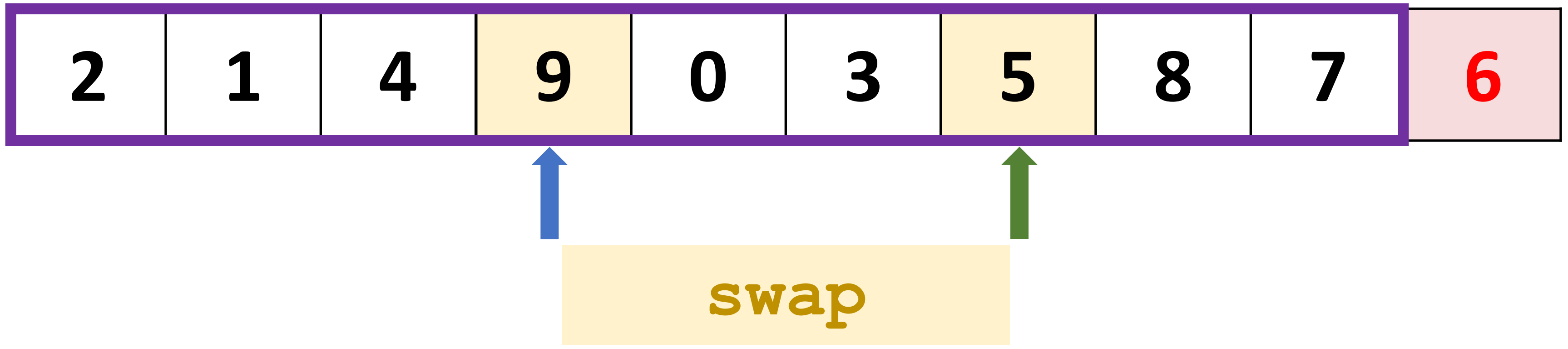

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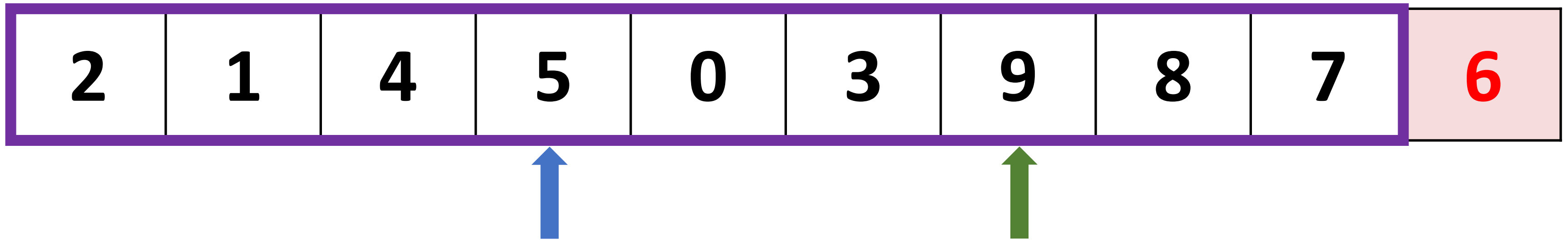

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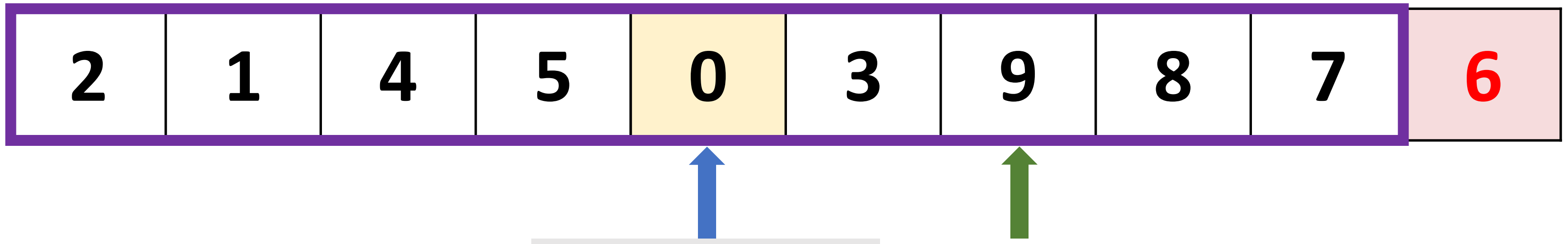

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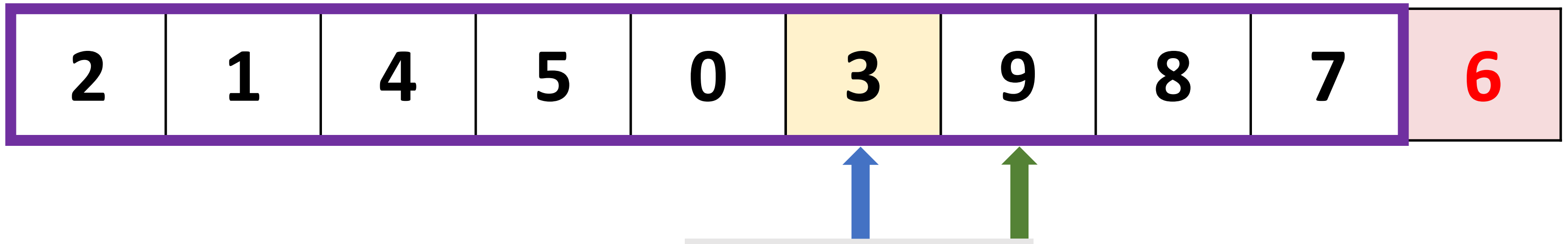

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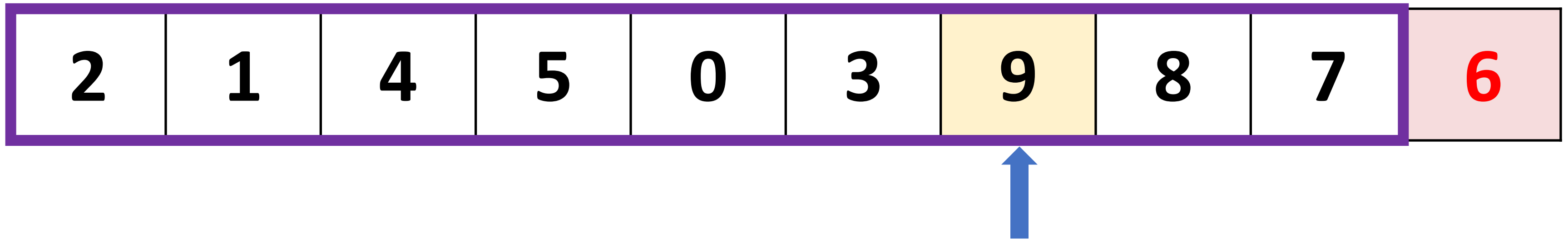
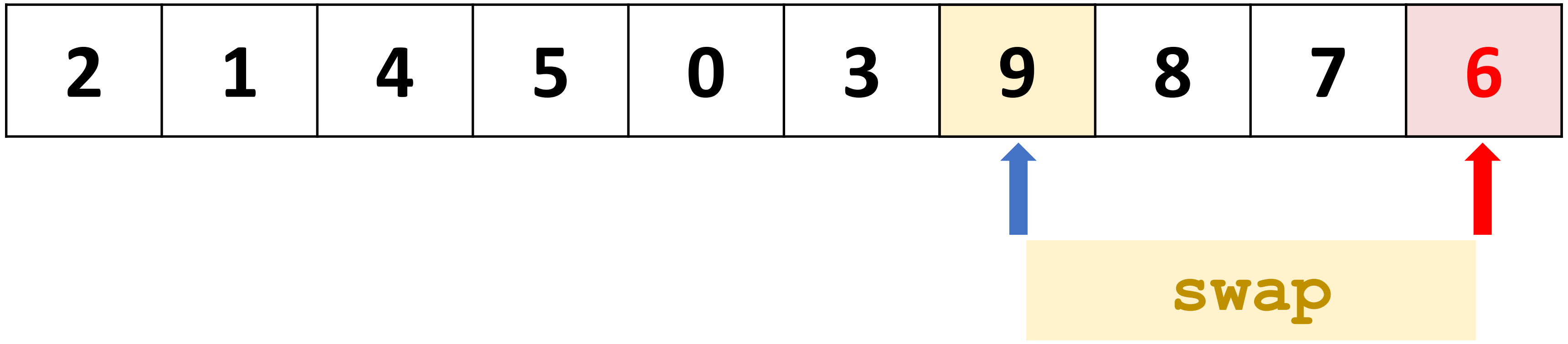


Stop when the two pointers are equal!

Step 2: Partition



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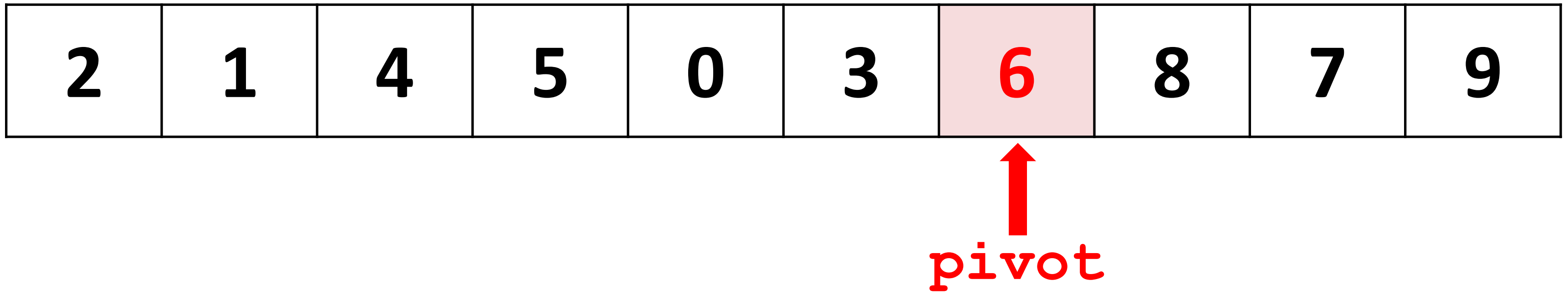

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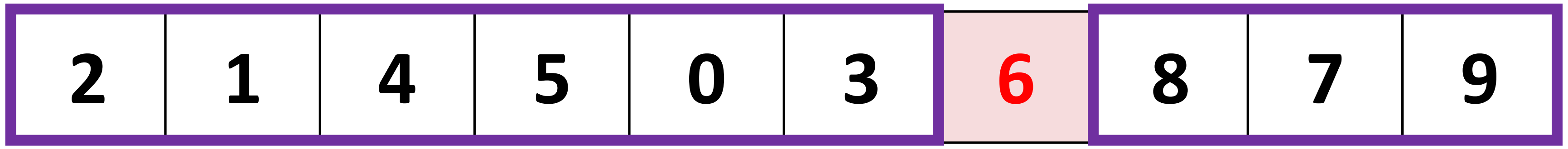

Group 1: $\{x \mid x \leq 6\}$.

Group 2: $\{x \mid x \geq 6\}$.

```
int partition(int arr[], int left, int right) {
     int ptr1 = left;
     int ptr2 = right-1;
     int pivot = arr[right];
     while (true) {
          while (arr[ptr1] < pivot) ptr1++;
          while (ptr2 > 0 && arr[ptr2] > pivot) ptr2--;
          if (ptr1 < ptr2)
               swap(arr, ptr1, prt2);
          else
                break;
     }
     return ptr1;
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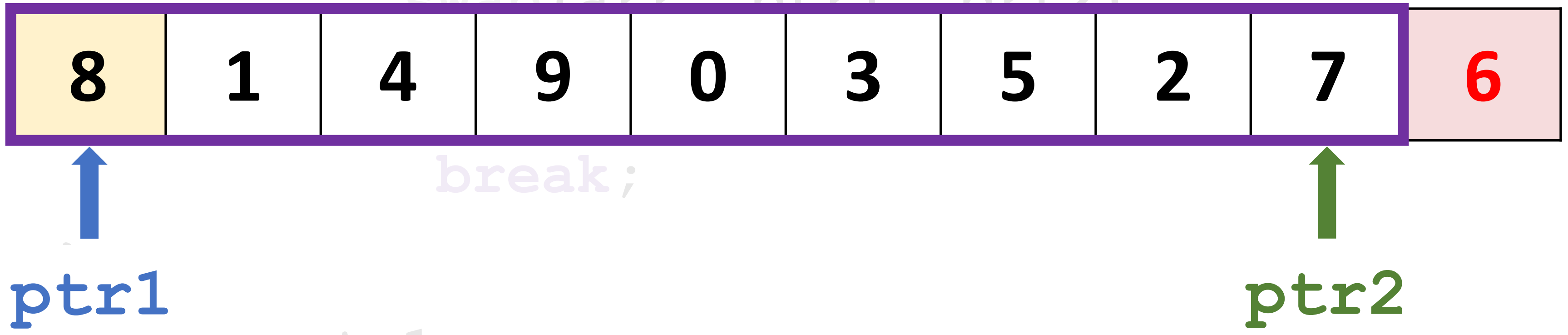

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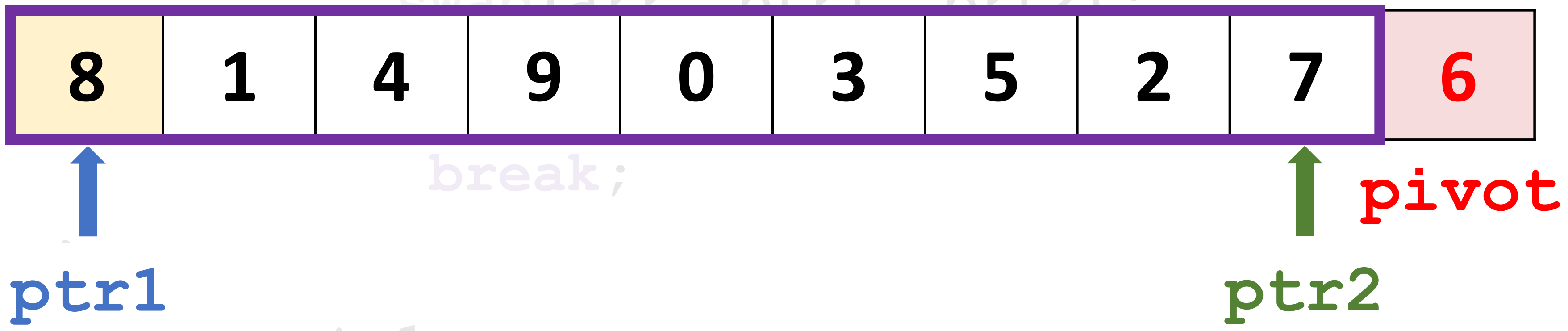

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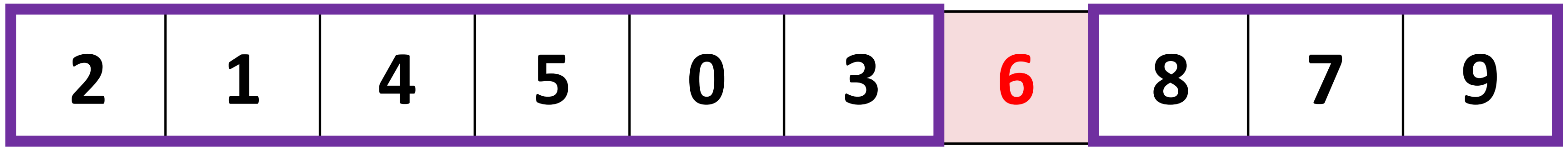

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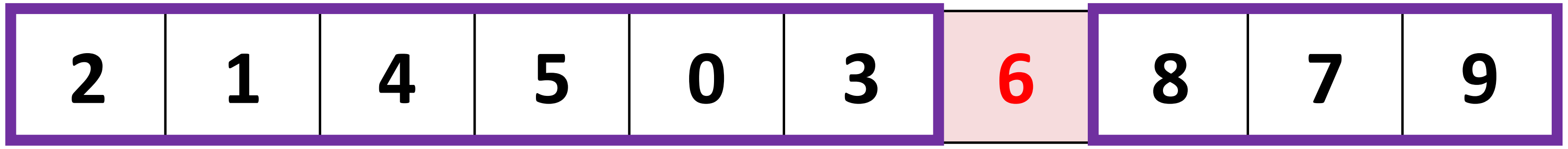
Step 3: Recursion



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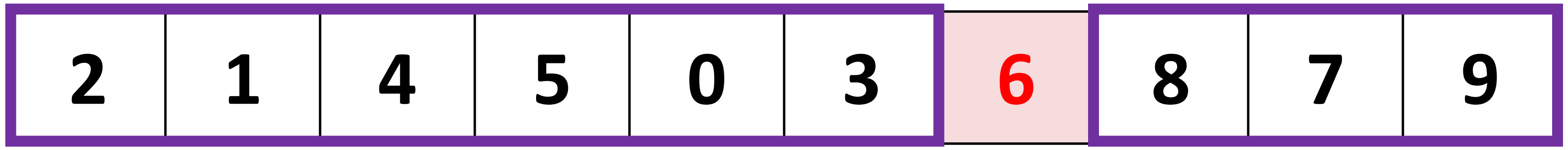
- **Recursion:**
  - Quicksort(Group 1).
  - Quicksort(Group 2).

Step 3: Recursion



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- **Recursion:**
  - $\text{Quicksort}(\text{Group 1})$.
  - $\text{Quicksort}(\text{Group 2})$.
- **Cut-Off:**
  - If array is small (e.g., $n < 10$), Quicksort is not quick.
  - In this case, use insertion sort instead.

```
void quickSort(int arr[], int left, int right) {
    if (left+10 > right) { // for short array
        insertsort(arr, left, right);
    }
    else { // for long array
        int p = selectpivot(arr, left, right);// pivot position
        swap(arr, p, right-1); //put pivot in the end
        int i = partition(arr, left, right);
        swap(arr, i, right-1); // restore pivot
        // recursively sort the two sub-arrays
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Time Complexity

Best-Case Time Complexity

Suppose pivot is the median (best-case).

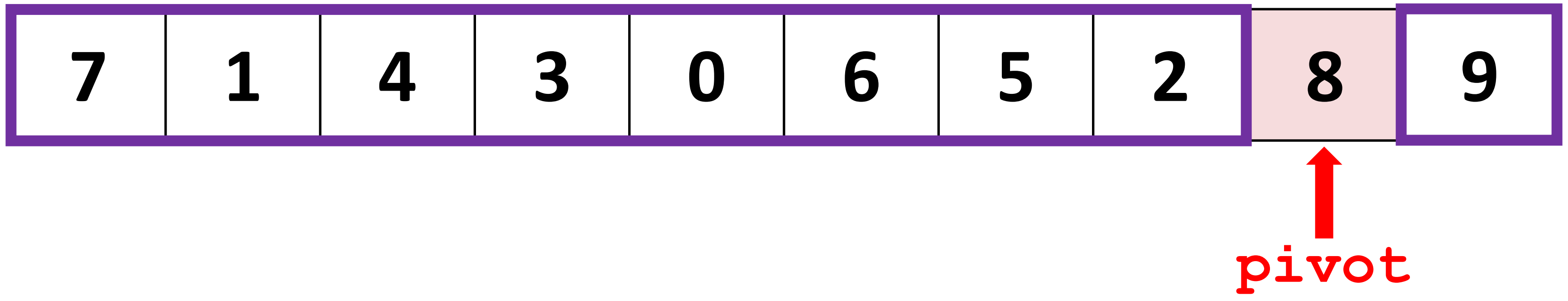
- $T(n)$: Time complexity of sorting size-n array.
- Sizes of the two subarrays are both $\frac{n}{2}$.
- Time complexity:

$$T(n) \;=\; 2\,T(n/2) \;+\; c\,n.$$

- ➔ $T(n) = O(n \log n)$.

Worst-Case Time Complexity

Pivot can be the second largest (worst-case).



- Group 1 has $n - 2$ elements.
- Group 2 has only one element.

Worst-Case Time Complexity

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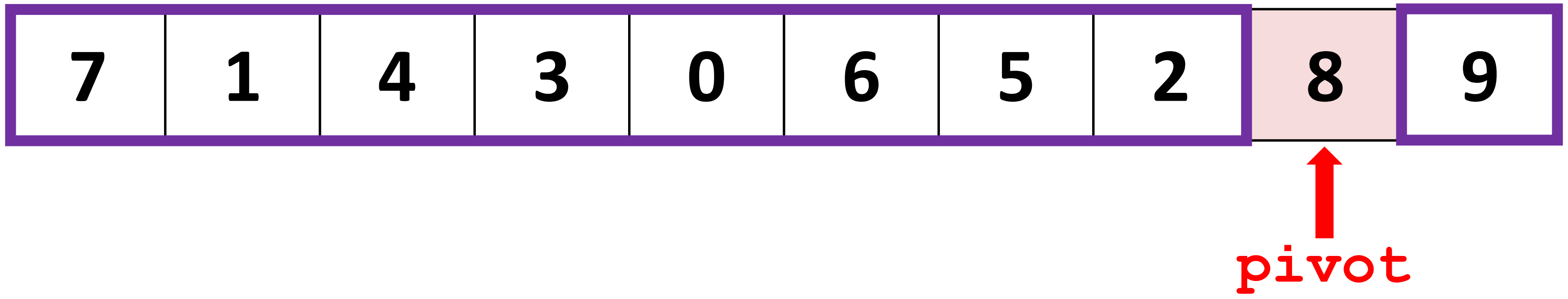

- Time complexity:

$$T(n) = T(n-2) + T(1) + cn.$$

- ➔ $T(n) = O(n^2)$.

Time Complexity: Comparisons

- **Best case:**
  - Sizes of the two subarrays are both $n/2$.
  - Time complexity: $O(n \log n)$.

- **Worst case:**
  - Size of Group 1: $n-2$
  - Size of Group 2: 1.
  - Time complexity: $O(n^2)$.

This is why we hope pivot is close to the median!

Time Complexity: Comparisons

- **Average case:**
  - Assume the data is randomly shuffled.
  - All the elements are equally likely to be the pivot.
  - Expected time complexity: $O(n \log n)$.
  - Algorithm is non-random; data is random.

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- **Random pivot:**
  - The position of the pivot is random.
  - Expected time complexity: $O(n \log n)$.
  - Algorithm is random; data is non-random.

Thank You!

Best-Case Time Complexity

Suppose pivot is the median (best-case).

- $\frac{T(n)}{n} = \frac{T(n/2)}{n\,/\,2} + c$.
- $\frac{T(n/2)}{n/2} = \frac{T(n/4)}{n/4} + c$.
- $\frac{T(n/4)}{n/4} = \frac{T(n/8)}{n/8} + c$.

$$\vdots$$

- $\frac{T(2)}{2} = \frac{T(1)}{1} + c$.

$$\begin{aligned}
\frac{T(n)}{n} &= \frac{T(n/2)}{n\,/\,2} + c \\
&= \frac{T(n/4)}{n/4} + 2c \\
&= \frac{T(n/8)}{n/8} + 3c \\
&= \cdots \\
&= \frac{T(1)}{1} + c\log_2 n
\end{aligned}$$

Worst-Case Time Complexity

Pivot can be the second largest (worst-case).

- $T(n) \ = \ T(n-2) + cn.$
- $T(n-2) \ = \ T(n-4) + c(n-2).$
- $T(n-4) \ = \ T(n-6) + c(n-4).$

$$\vdots$$

- $T(4) \ = \ T(2) + 4c.$
- $T(2) \ = 2c.$
- Thus $T(n) \ = c \cdot [2c + 4c + \cdots + (n-4) + (n-2) + n]$

$$= O(n^2)$$