

Edit Distance

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Edit Distance

- Let x to y be two strings.
- Convert x to y by insertion, deletion, and replacement.
- **Edit distance** is the minimum number of operations.

Edit Distance

- Example inputs:
 - **x** = "algorithm".
 - **y** = "algorithm**s**".
- Example output:
 - Distance = 1. (Inserted '**s**' to **x**.)

Edit Distance

- Example inputs:
 - x = "algorithms".
 - y = "alorithm".
- Example output:
 - Distance = 1. (Deleted 'g'.)

Edit Distance

- Example inputs:

- $x = \text{"algorithms"}$.

- $y = \text{"blgorithms"}$.

- Example output:

- Distance = 1. (Replaced '**a**' by '**b**'.)

Edit Distance

- Example inputs:

- $x = \text{"give"}.$

- $y = \text{"giving"}.$

- Example input:

- Distance = 3. (Replaced ' e ' by ' i '; inserted ' n ' and ' g '.)

Applications

- **Application 1:** Spelling corrections.
 - Find a word in the dictionary that minimizes the edit distance.
 - E.g., “algorithmⁿ” ==> “algorithm^m”.

Applications

- **Application 1:** Spelling corrections.
 - Find a word in the dictionary that minimizes the edit distance.
 - E.g., “algorithmⁿ” ==> “algorithm^m”.
- **Application 2:** Quantify the difference between two DNA sequences.
 - **DNA1** = “ATTAGCCCAAGGTAAACCCAGT...”.
 - **DNA2** = “ATCAGCTCAAGGTAGACTCAAT...”.

Optimal Substructure

$D[i][j]$: edit distance between $x[1:i]$ and $y[1:j]$.

[illegible]

$D[i][j]$: edit distance between $x[1:i]$ and $y[1:j]$.

- $x = \text{"PQZABCDEFGG"}$.

- $x[1:4] = \text{"PQZA"}$.

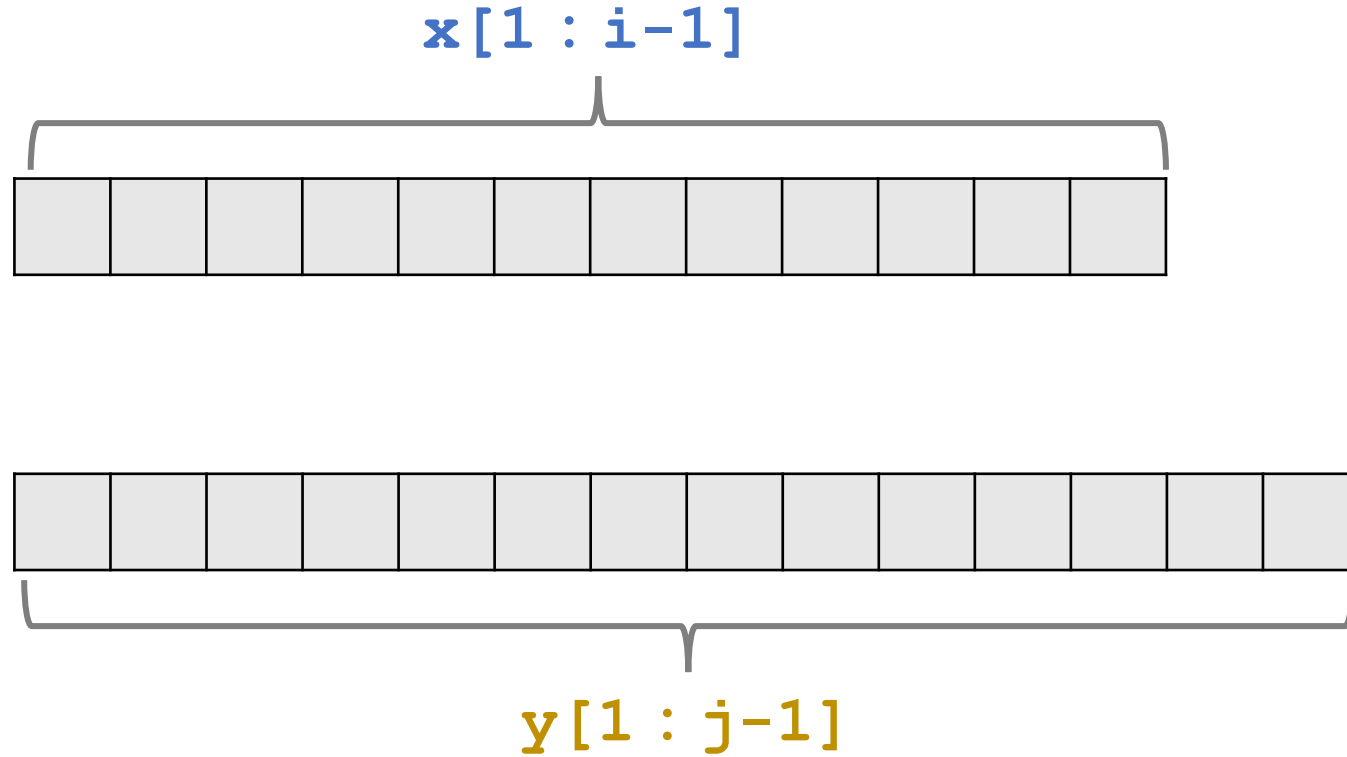
- $y = \text{"PQBCDEFGG"}$.

- $y[1:2] = \text{"PQ"}$.

- What is $D[4][2]$?

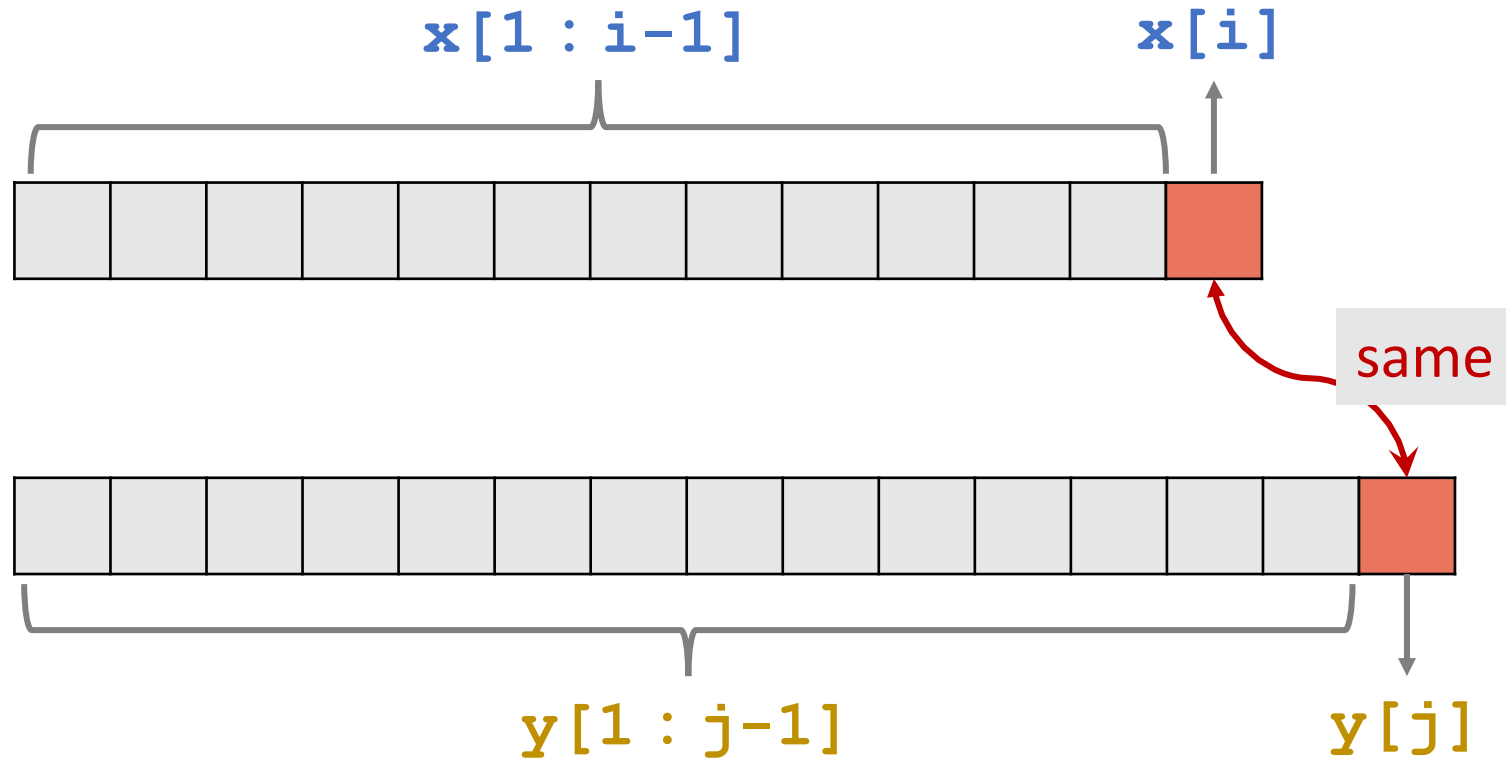
- Thus $D[4][2] = 2$.

Case 1: $x[i] = y[j]$



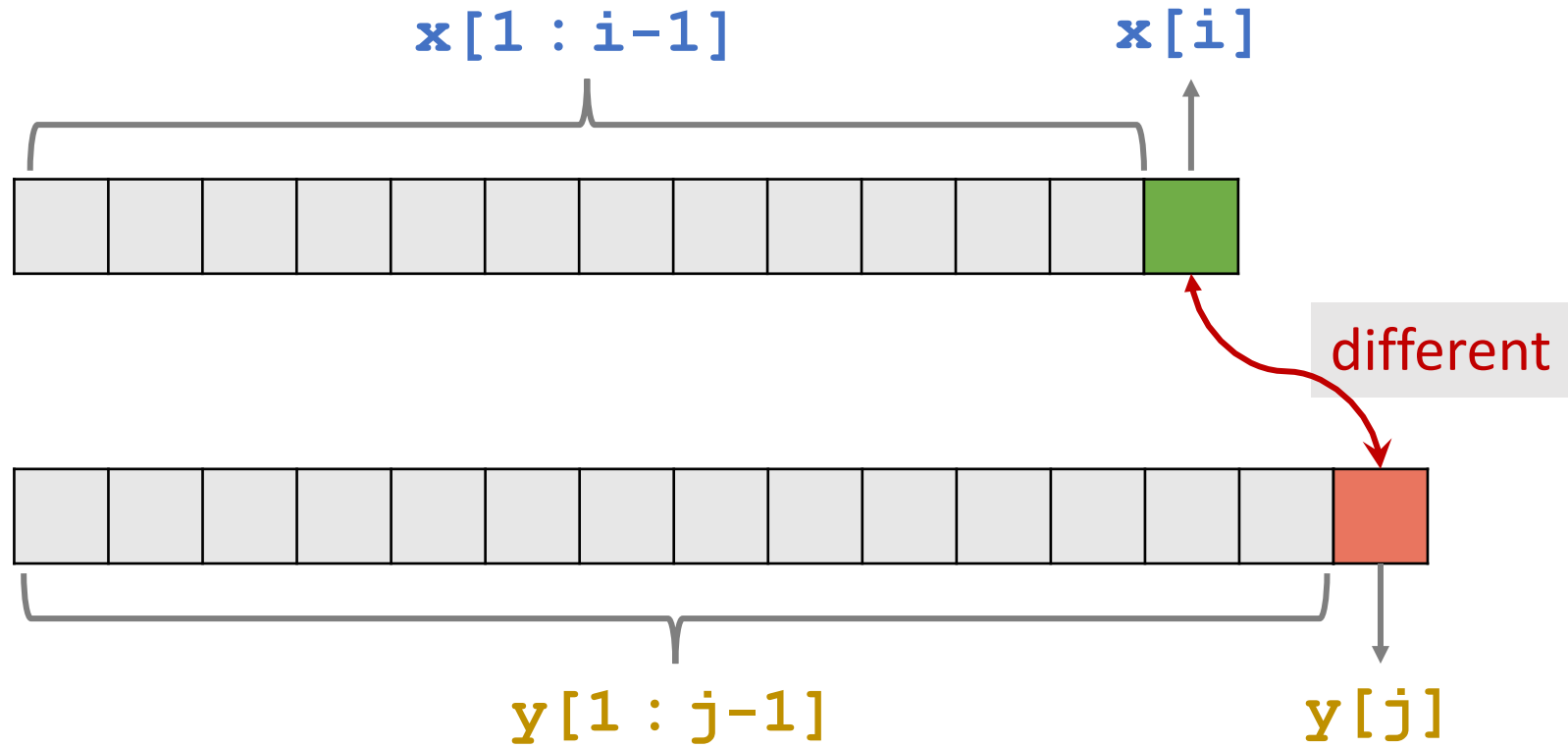
Suppose we already know $D[i-1][j-1]$.

Case 1: $x[i] = y[j]$

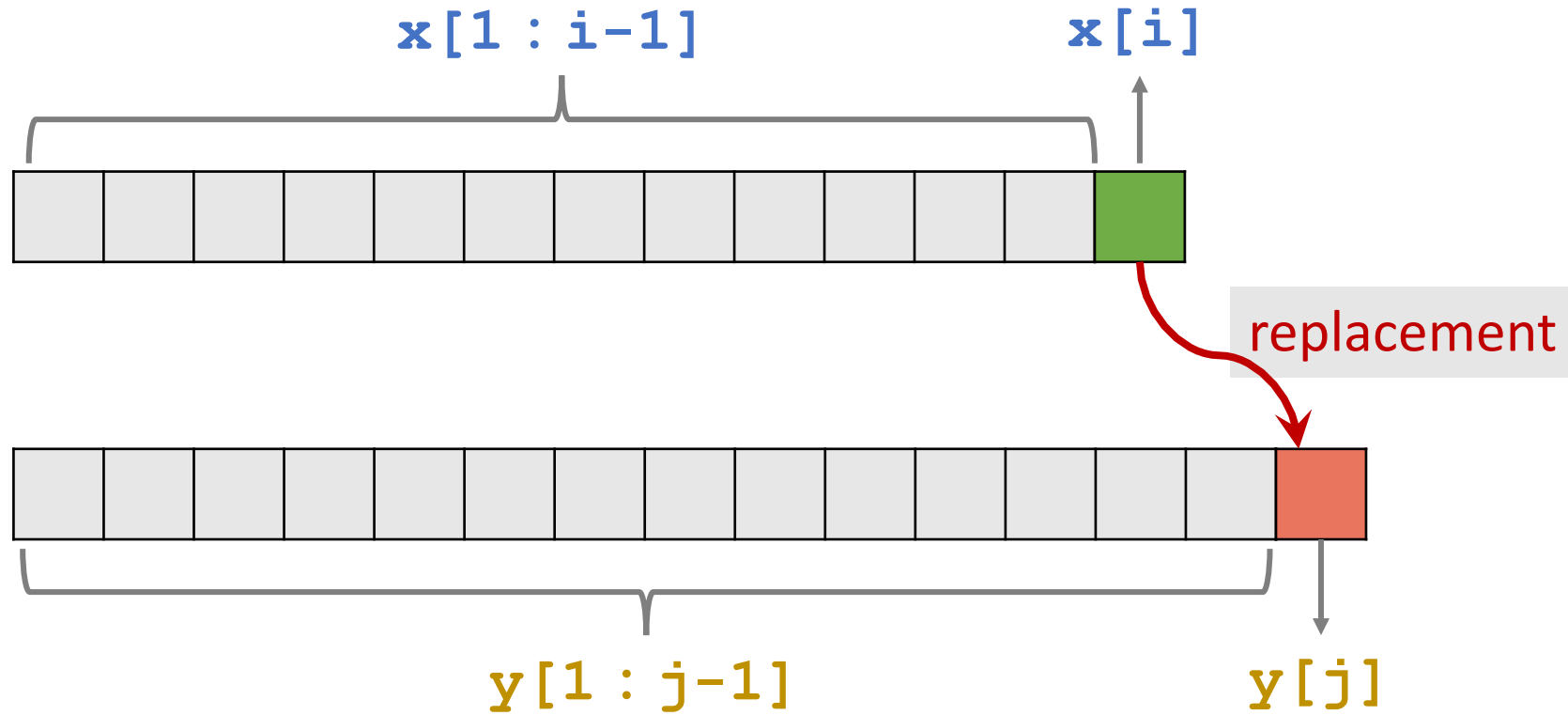


If $x[i] = y[j]$, then $D[i][j] = D[i-1][j-1]$.

Case 2: $x[i] \neq y[j]$

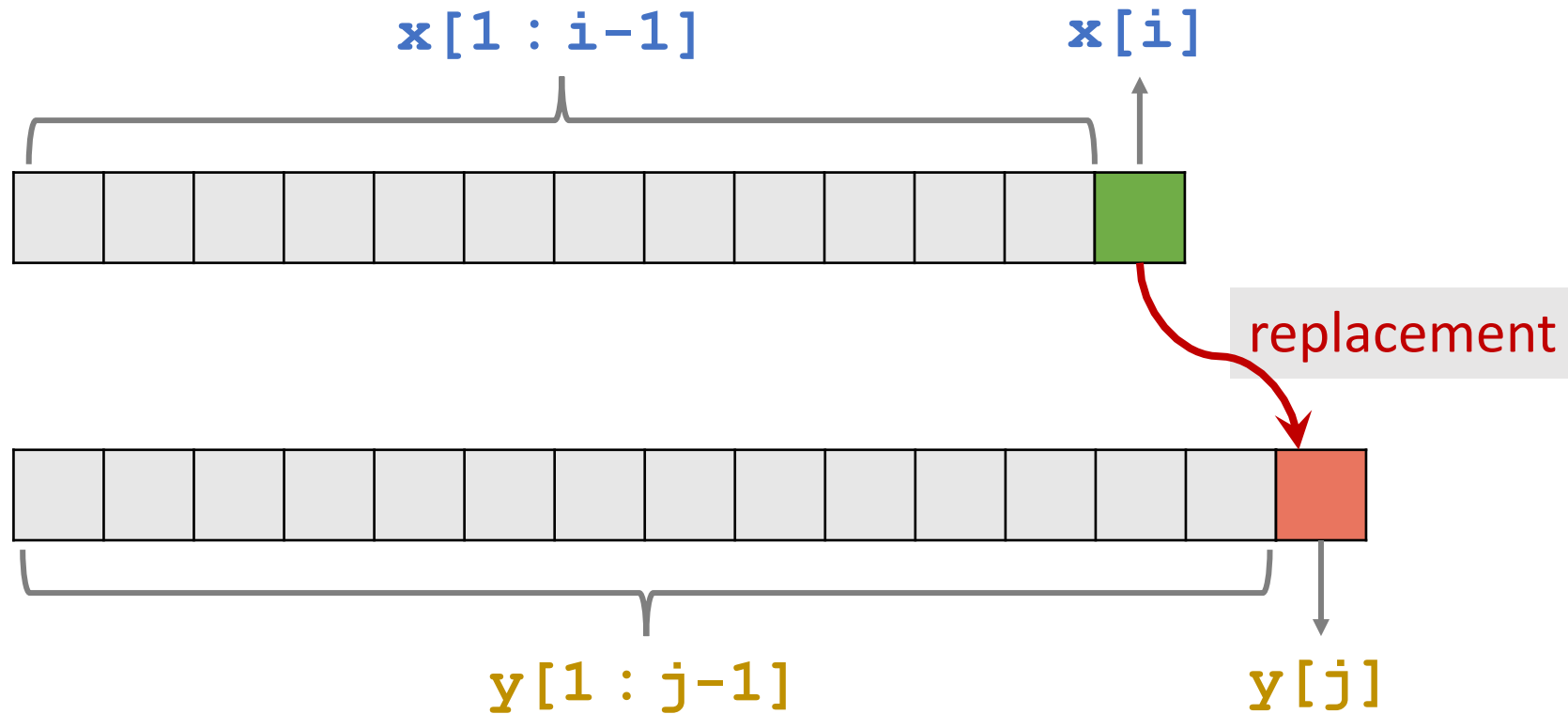


Case 2(A): Replacement



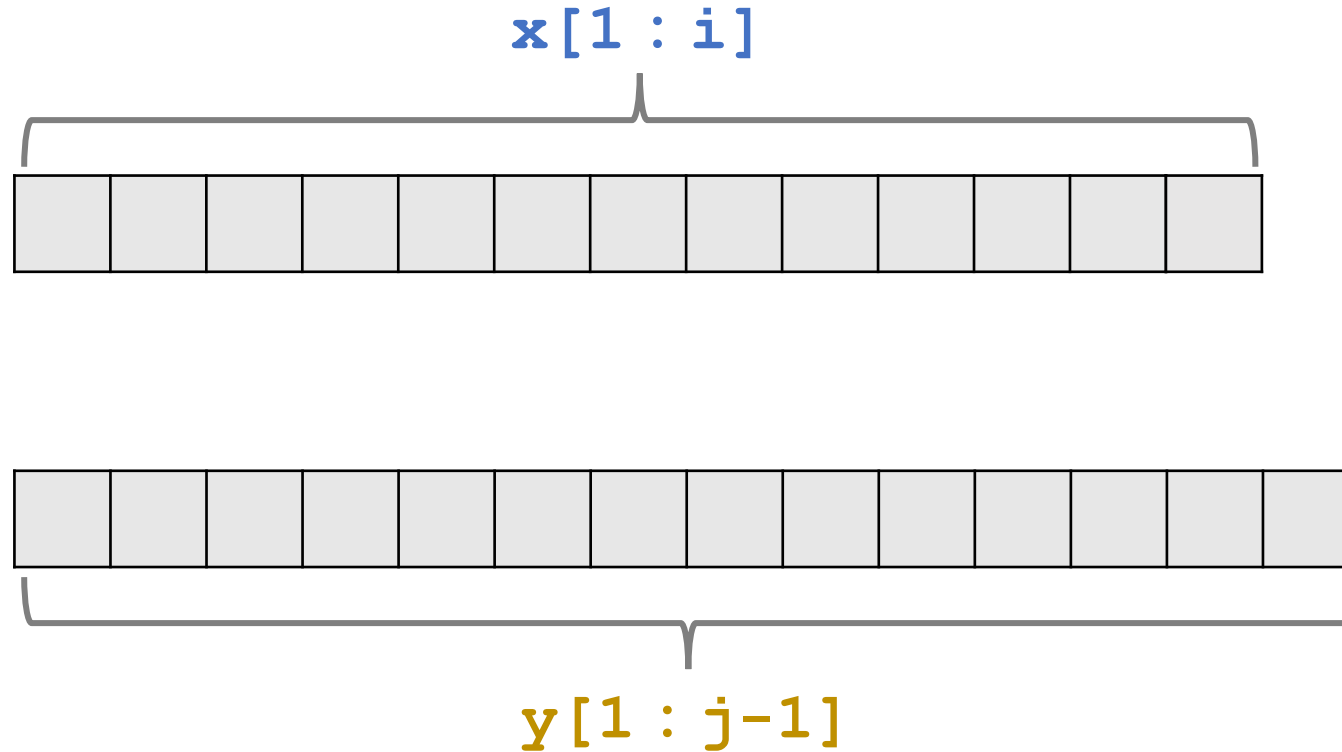
Case 2(A): $y[j]$ is obtained by replacing $x[i]$.

Case 2(A): Replacement



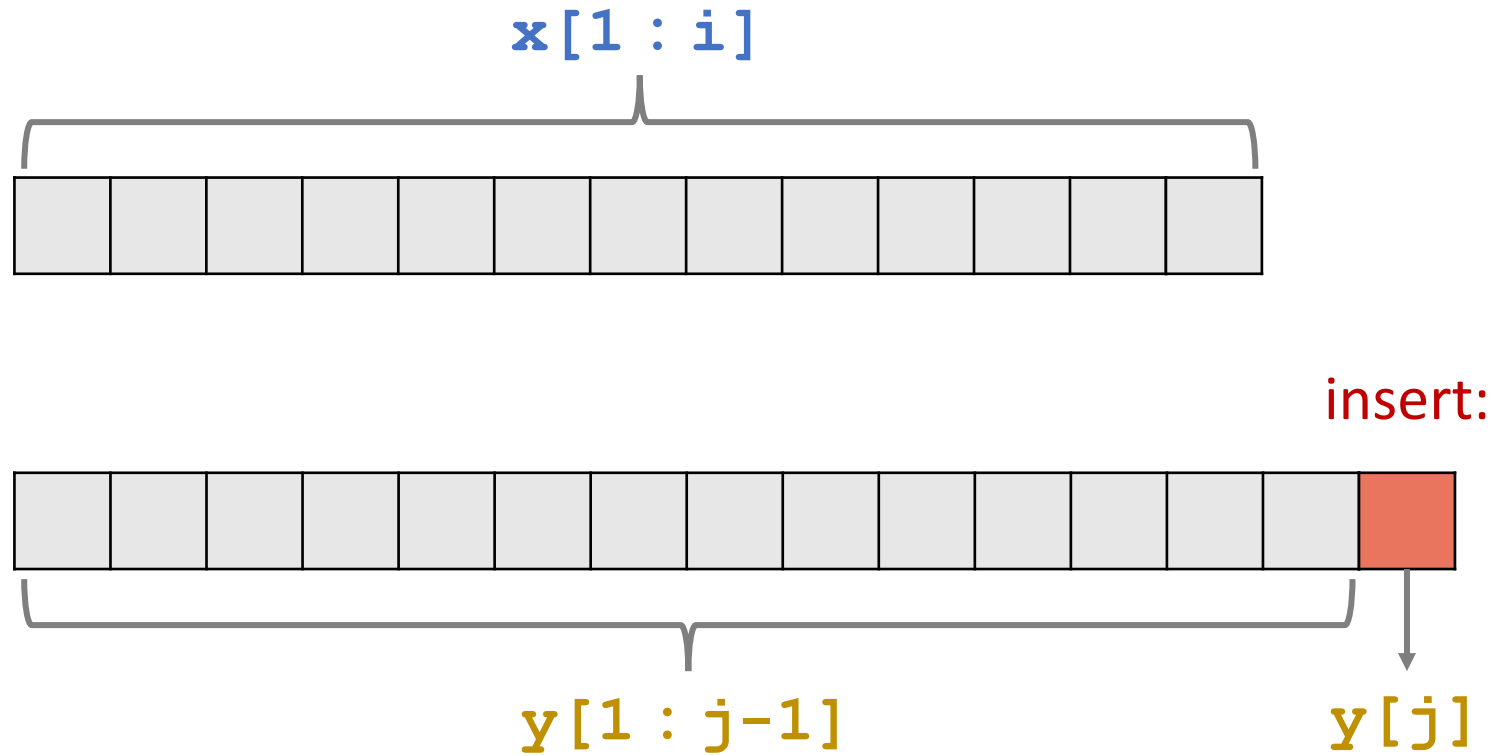
In the case of replacement, $D[i][j] = D[i-1][j-1] + 1$.

Case 2(B): Insertion



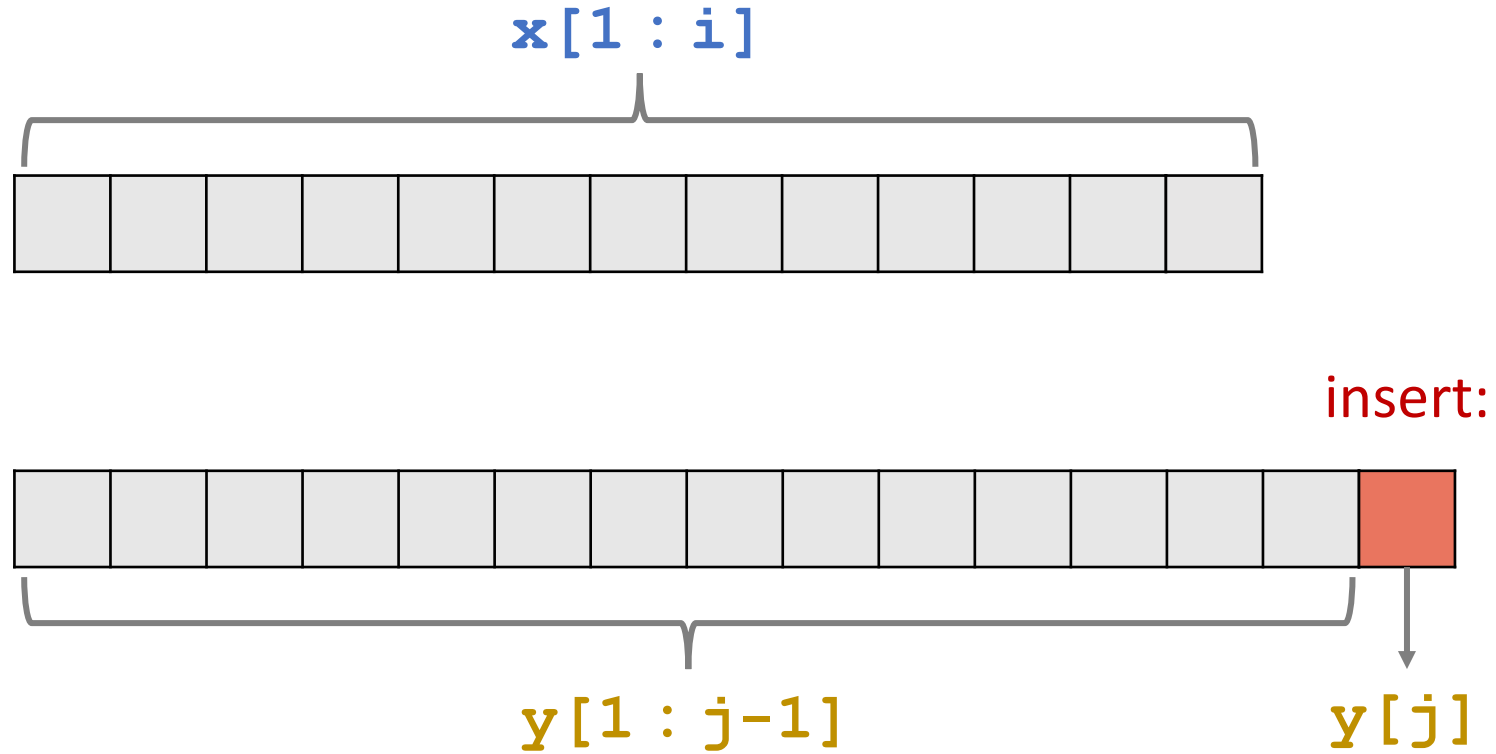
Case 2(B): $y[j]$ is obtained by insertion.

Case 2(B): Insertion



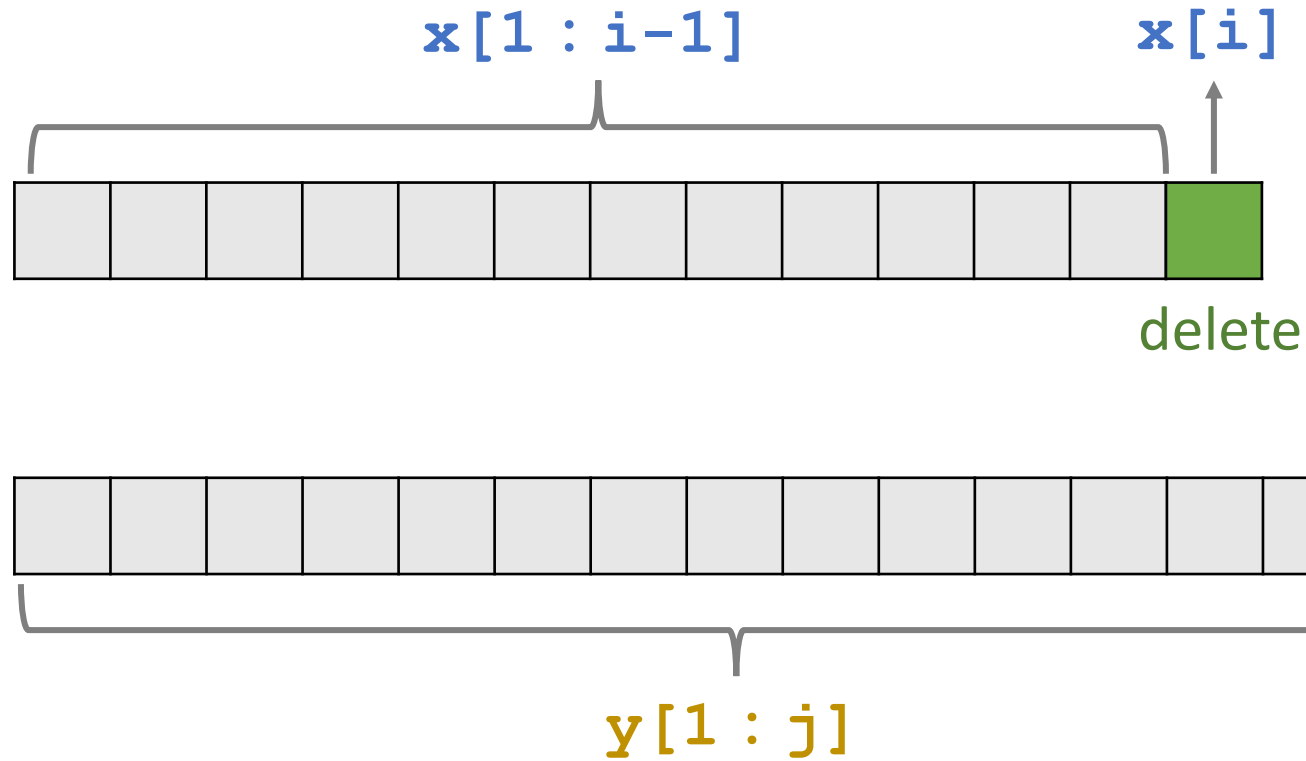
Case 2(B): $y[j]$ is obtained by insertion.

Case 2(B): Insertion



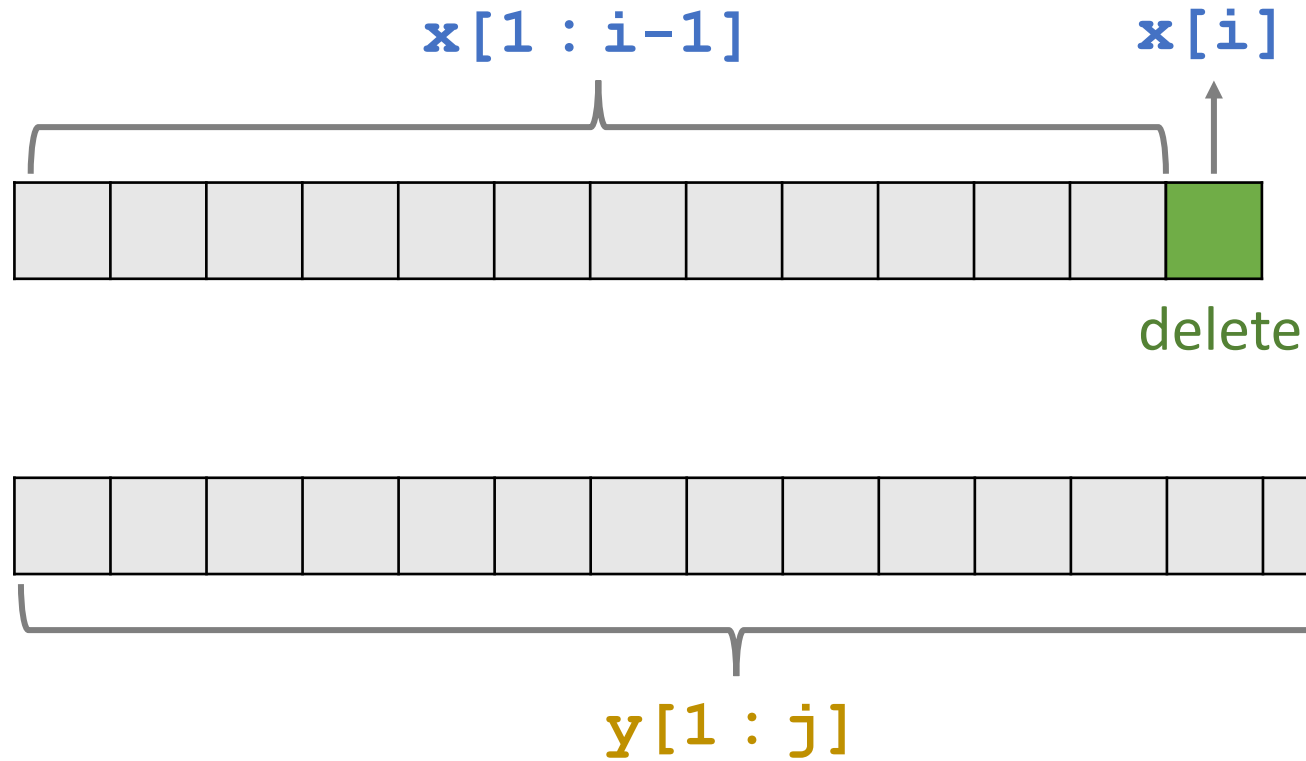
In the case of insertion, $D[i][j] = D[i][j-1] + 1$.

Case 2(C): Deletion



Case 2(C): $y[1:j]$ is obtained by deleting $x[i]$.

Case 2(C): Deletion



In the case of deletion, $D[i][j] = D[i-1][j] + 1$.

Case 2: $x[i] \neq y[j]$

In the case of replacement, $D[i][j] = D[i-1][j-1] + 1$.

In the case of insertion, $D[i][j] = D[i][j-1] + 1$.

In the case of deletion, $D[i][j] = D[i-1][j] + 1$.

Case 2: $x[i] \neq y[j]$

In the case of replacement, $D[i][j] = D[i-1][j-1] + 1.$

In the case of insertion, $D[i][j] = D[i][j-1] + 1.$

In the case of deletion, $D[i][j] = D[i-1][j] + 1.$

Case 2: $x[i] \neq y[j]$

In the case of replacement, $D[i][j] = D[i-1][j-1] + 1$.

In the case of insertion, $D[i][j] = D[i][j-1] + 1$.

In the case of deletion, $D[i][j] = D[i-1][j] + 1$.

- In sum, if $x[i] \neq y[j]$, then

$$D[i][j] = 1 + \min \begin{cases} D[i-1][j-1] \\ D[i][j-1] \\ D[i-1][j] \end{cases}.$$

Optimal Substructure

- **Case 1:** If $x[i] = y[j]$, then

$$D[i][j] = D[i - 1][j - 1].$$

- **Case 2:** If $x[i] \neq y[j]$, then

$$D[i][j] = 1 + \min \begin{cases} D[i - 1][j - 1] \\ D[i][j - 1] \\ D[i - 1][j] \end{cases}.$$

Example

R E L A T I V E

ACTIVE

[illegible]

Example

RELATIVE								
A								
C								
T						5		
I								
V								
E								

Case 1: $x[i] = y[j]$

	R E L A T I V E								
A C T I V E									
					<i>d</i>				

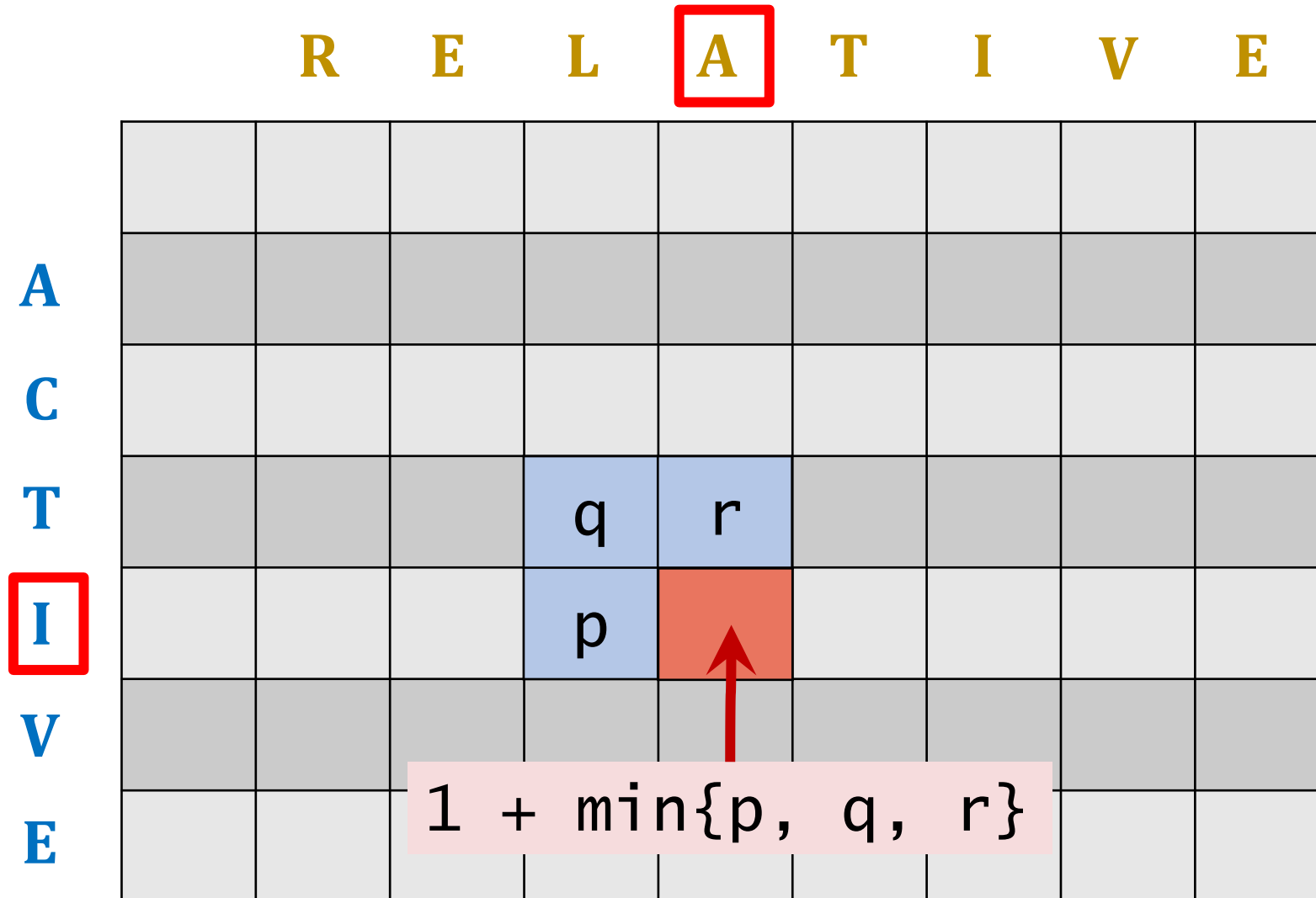
Case 1: $x[i] = y[j]$

	R E L A T I V E							
A C T I V E								
					<i>d</i>			
						<i>d</i>		

Case 2: $x[i] \neq y[j]$

	RELATIVE							
ACTIVE				A				
				q	r			
				p				

Case 2: $x[i] \neq y[j]$



Dynamic Programming

Inputs: two strings

R E L A T I V E

ACTIVE

[illegible]

Base Case

NULL RELATIVE

NULL

A

C

T

I

V

E

Base Case

NULL RELATIVE

NULL	0								
A									
C									
T									
I									
V									
E									

Base Case

NULL RELATIVE

NULL	0	1	2	3	4	5	6	7	8
A									
C									
T									
I									
V									
E									

Base Case

	NULL	R	E	L	A	T	I	V	E
NULL	0	1	2	3	4	5	6	7	8
A									
C									
T									
I									
V									
E									

Base Case

NULL		R	E	L	A	T	I	V	E
NULL A C T I V E	0	1	2	3	4	5	6	7	8
	1								
	2								
	3								
	4								
	5								
	6								

x[1] ≠ y[1]

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	?							
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] \neq y[1]$

R E L A T I V E

A

C

T

I

V

E

0	1	2	3	4	5	6	7	8
1	?							
2	$\min\{1, 0, 1\} + 1$							
3								
4								
5								
6								

x[1] ≠ y[2]

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	?						
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] \neq y[2]$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	?						
C	2	$\min\{1, 1, 2\} + 1$							
T	3								
I	4								
V	5								
E	6								

$x[1] \neq y[3]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3					
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] = y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	?				
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] = y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	?				
C	2								
T	3								
I	4								
V	5								
E	6								

$L[1][4] = L[0][3]$

x[1] ≠ y[5]

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4			
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] \neq y[6]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5		
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] \neq y[7]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	
C	2								
T	3								
I	4								
V	5								
E	6								

$x[1] \neq y[8]$

R E L A T I V **E**

A

C

T

I

V

E

0	1	2	3	4	5	6	7	8
1	1	2	3	3	4	5	6	7
2								
3								
4								
5								
6								



$x[2] \neq y[1]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2							
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[2]$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2						
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[3]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3					
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4				
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[5]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4			
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[6]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5		
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[7]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	
T	3								
I	4								
V	5								
E	6								

$x[2] \neq y[8]$

R E L A T I V **E**

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6 → 7	
T	3								
I	4								
V	5								
E	6								

$x[3] \neq y[1]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3							
I	4								
V	5								
E	6								

$x[3] \neq y[2]$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3						
I	4								
V	5								
E	6								

$x[3] \neq y[3]$

		R	E	L	A	T	I	V	E
A	0	1	2	3	4	5	6	7	8
C	1	1	2	3	3	4	5	6	7
T	2	2	2	3	4	4	5	6	7
I	3	3	3	3					
V	4								
E	5								
	6								

$x[3] \neq y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4				
I	4								
V	5								
E	6								

$x[3] = y[5]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	?			
I	4								
V	5								
E	6								

$x[3] = y[5]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	?			
I	4								
V	5								
E	6								

$L[3][5] = L[2][4]$

$x[3] \neq y[6]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5		
I	4								
V	5								
E	6								

$x[3] \neq y[7]$

R E L A T I **V** E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5 → 6		
I	4								
V	5								
E	6								

$x[3] \neq y[8]$

R E L A T I V **E**

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6 → 7	
I	4								
V	5								
E	6								

$x[4] \neq y[1]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4							
V	5								
E	6								

$x[4] \neq y[2]$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4						
V	5								
E	6								

$x[4] \neq y[3]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4					
V	5								
E	6								

$x[4] \neq y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4				
V	5								
E	6								

$x[4] \neq y[5]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5			
V	5								
E	6								

$x[4] = y[6]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	?		
V	5								
E	6								

$x[4] = y[6]$

		R	E	L	A	T	I	V	E
A	0	1	2	3	4	5	6	7	8
C	1	1	2	3	3	4	5	6	7
T	2	2	2	3	4	4	5	6	7
I	3	3	3	3	4	4	5	6	7
V	4	4	4	4	4	5	?		
E	5								
	6								

$L[4][6] = L[3][5]$

$x[4] \neq y[7]$

R E L A T I **V** E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4 → 5		
V	5								
E	6								

$x[4] \neq y[8]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5								
E	6								

$x[5] \neq y[1]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5							
E	6								

$x[5] \neq y[2]$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5						
E	6								

$x[5] \neq y[3]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5					
E	6								

$x[5] \neq y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5				
E	6								

$x[5] \neq y[5]$

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5			
E	6								

$x[5] \neq y[6]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5		
E	6								

$x[5] = y[7]$

R E L A T I **V** E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	?	
E	6								

$x[5] = y[7]$

R E L A T I **V** E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	?	
E	6						$L[5][7] = L[4][6]$		

$x[5] \neq y[8]$

R E L A T I V **E**

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4 → 5	
E	6								

$x[6] \neq y[1]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6							

$x[6] = y[2]$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	?						

$$x[6] = y[2]$$

R **E** L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	?		$L[6][2] = L[5][1]$				

$x[6] \neq y[3]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6					

$x[6] \neq y[4]$

R E L **A** T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6				

$x[6] \neq y[5]$

R E L A **T** I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6			

$x[6] \neq y[6]$

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6		

$x[6] \neq y[7]$

R E L A T I **V** E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	

$x[6] = y[8]$

R E L A T I V **E**

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	?

$x[6] = y[8]$

R E L A T I V **E**

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	4	5	
E	6	6	5	6	6	6	5		?

$L[6][8] = L[5][7]$

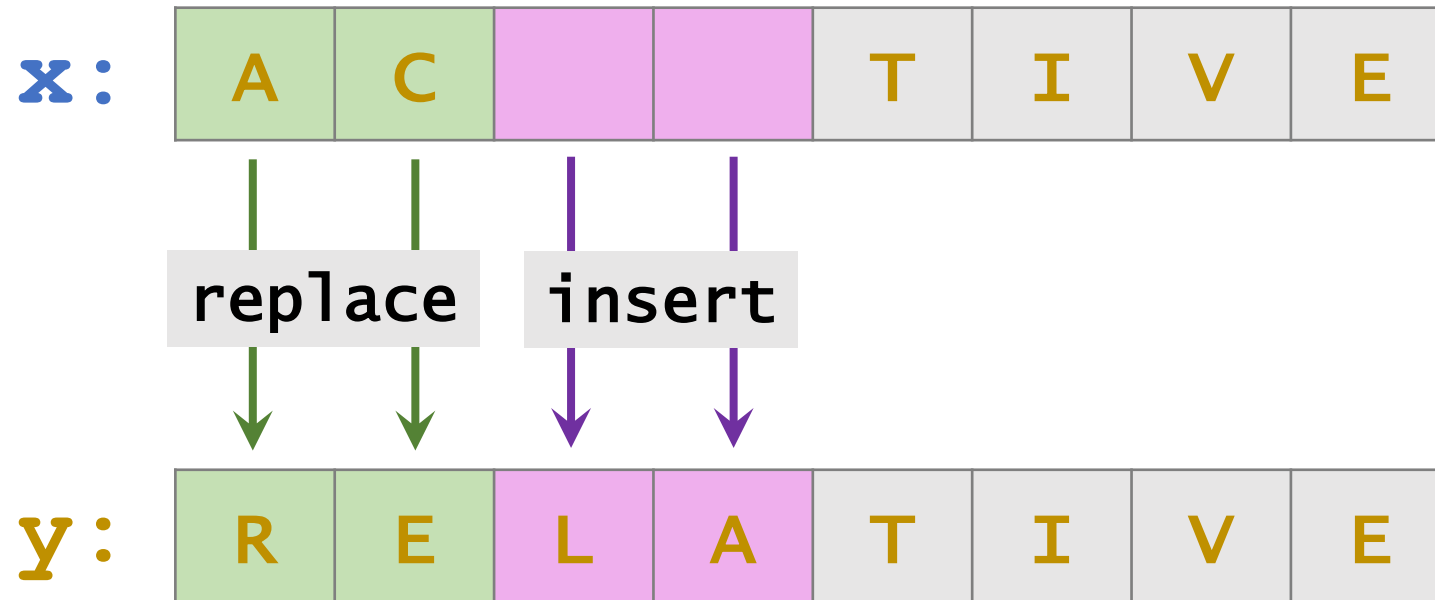
End of Procedure

R E L A T I V E

**A
C
T
I
V
E**

0	1	2	3	4	5	6	7	8
1	1	2	3	3	4	5	6	7
2	2	2	3	4	4	5	6	7
3	3	3	3	4	4	5	6	7
4	4	4	4	4	5	4	5	6
5	5	5	5	5	5	5	4	5
6	6	6	5	6	6	6	5	4

Edit Distance = 4



Summary

Optimal Substructure

Case 1: $x[i] = y[j]$.

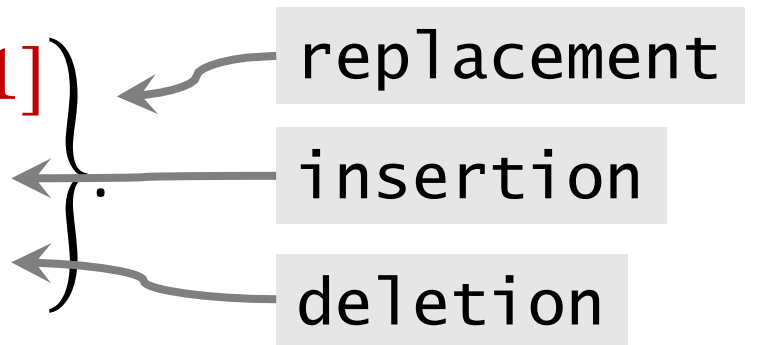
- Recursion: $D[i][j] = D[i - 1][j - 1]$.

Optimal Substructure

Case 1: $x[i] = y[j]$.

- Recursion: $D[i][j] = D[i - 1][j - 1]$.

Case 2: $x[i] \neq y[j]$.

- Recursion: $D[i][j] = 1 + \min \begin{cases} D[i - 1][j - 1] \\ D[i][j - 1] \\ D[i - 1][j] \end{cases}$
- 
- The diagram shows three arrows pointing from labels to the min expression. The label 'replacement' points to $D[i - 1][j - 1]$. The label 'insertion' points to $D[i][j - 1]$. The label 'deletion' points to $D[i - 1][j]$.

Time Complexity

- Inputs:
 - String x . (The length is m .)
 - String y . (The length is n .)
- Size of the table is $(m + 1) \times (n + 1)$.
- $O(1)$ time for filling each entry of the table.
- Thus, the overall time complexity is $O(mn)$.

Thank You!

Find the edits from the table


What are the edits?

R E L A T I V E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4


What are the edits?

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4




What are the edits?

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4



What are the edits?

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4



What are the edits?

		R	E	L	A	T	I	V	E
	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

What are the edits?

insert

E

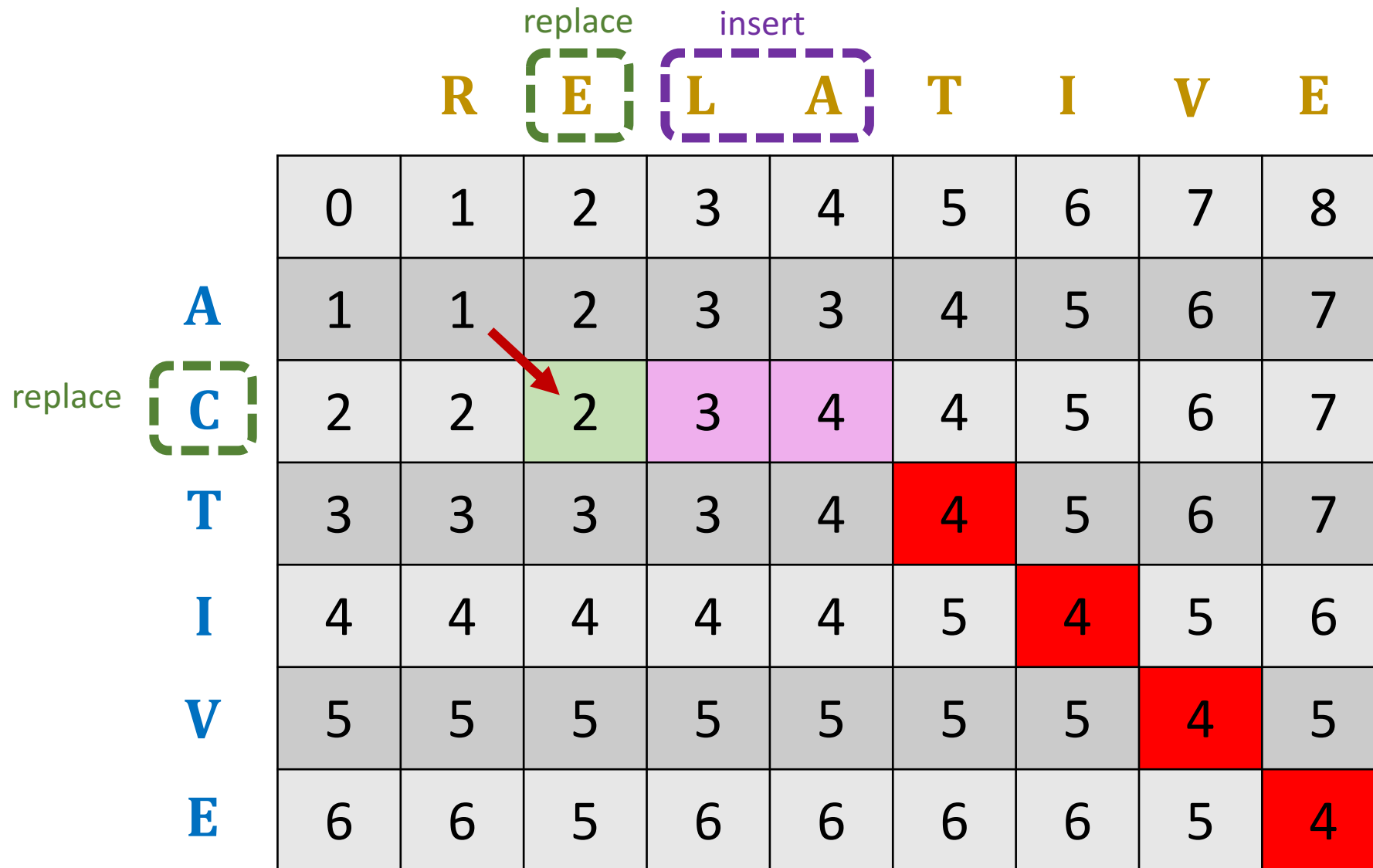
A

I

E

	0	1	2	3	4	5	6	7	8
A	1	1	2	3	3	4	5	6	7
C	2	2	2	3	4	4	5	6	7
T	3	3	3	3	4	4	5	6	7
I	4	4	4	4	4	5	4	5	6
V	5	5	5	5	5	5	5	4	5
E	6	6	5	6	6	6	6	5	4

What are the edits?



What are the edits?

