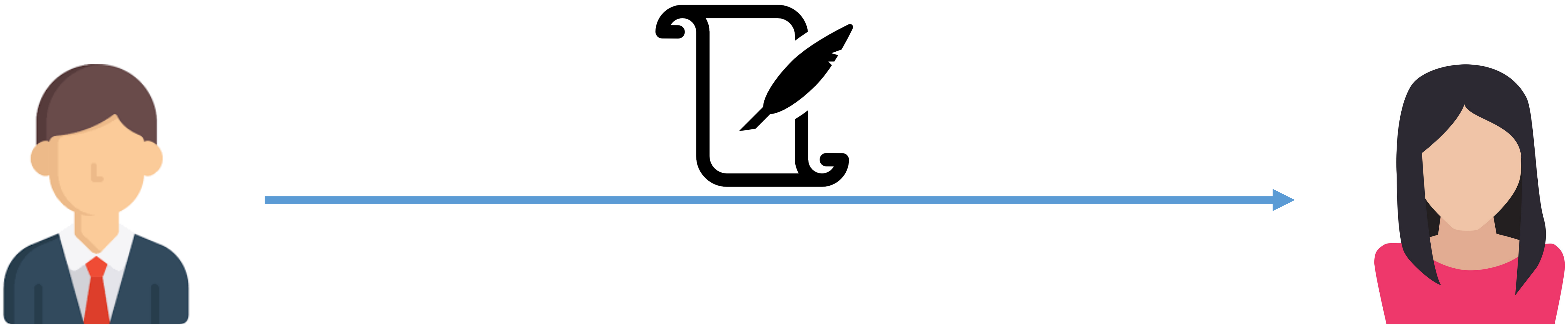
Public Key and RSA Algorithm

Shusen Wang

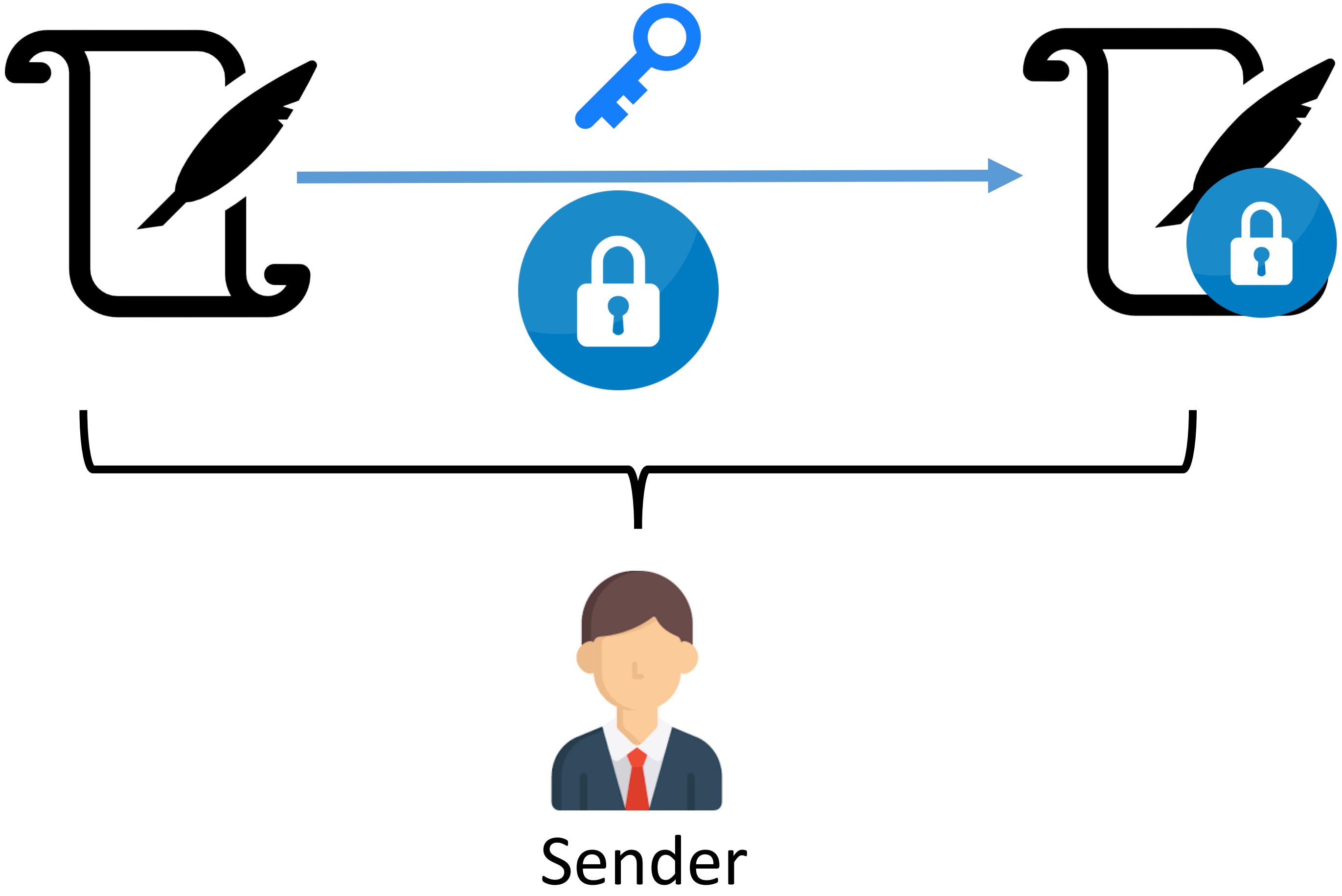
Sending Messages



Sender

Receiver

Symmetric Encryption



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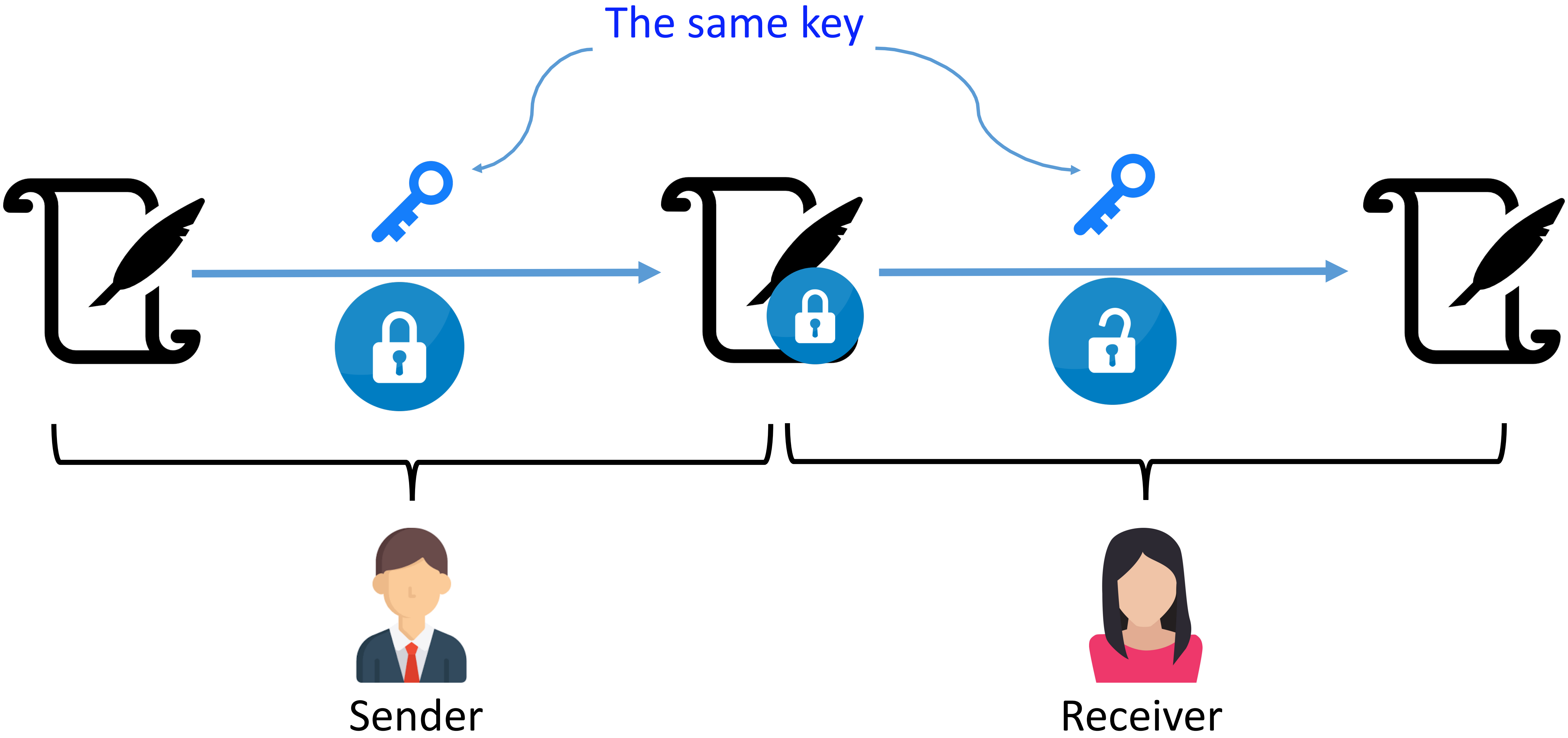
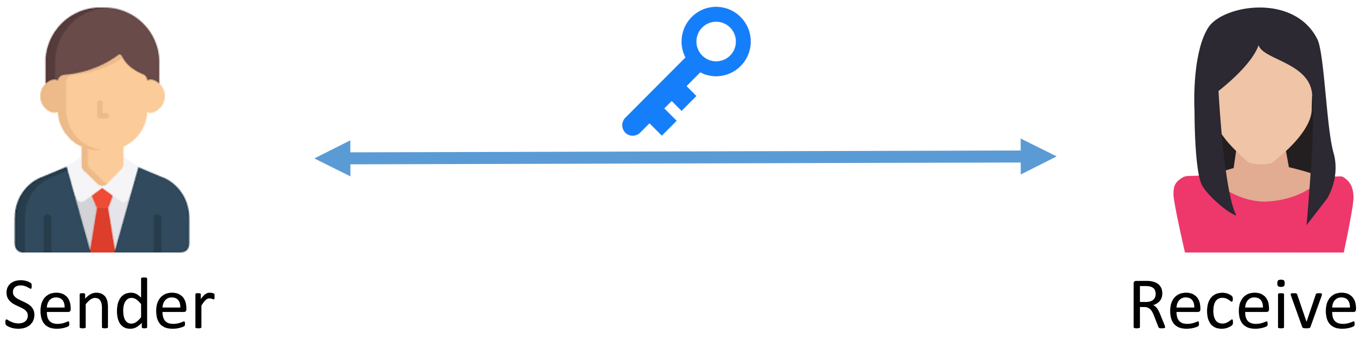


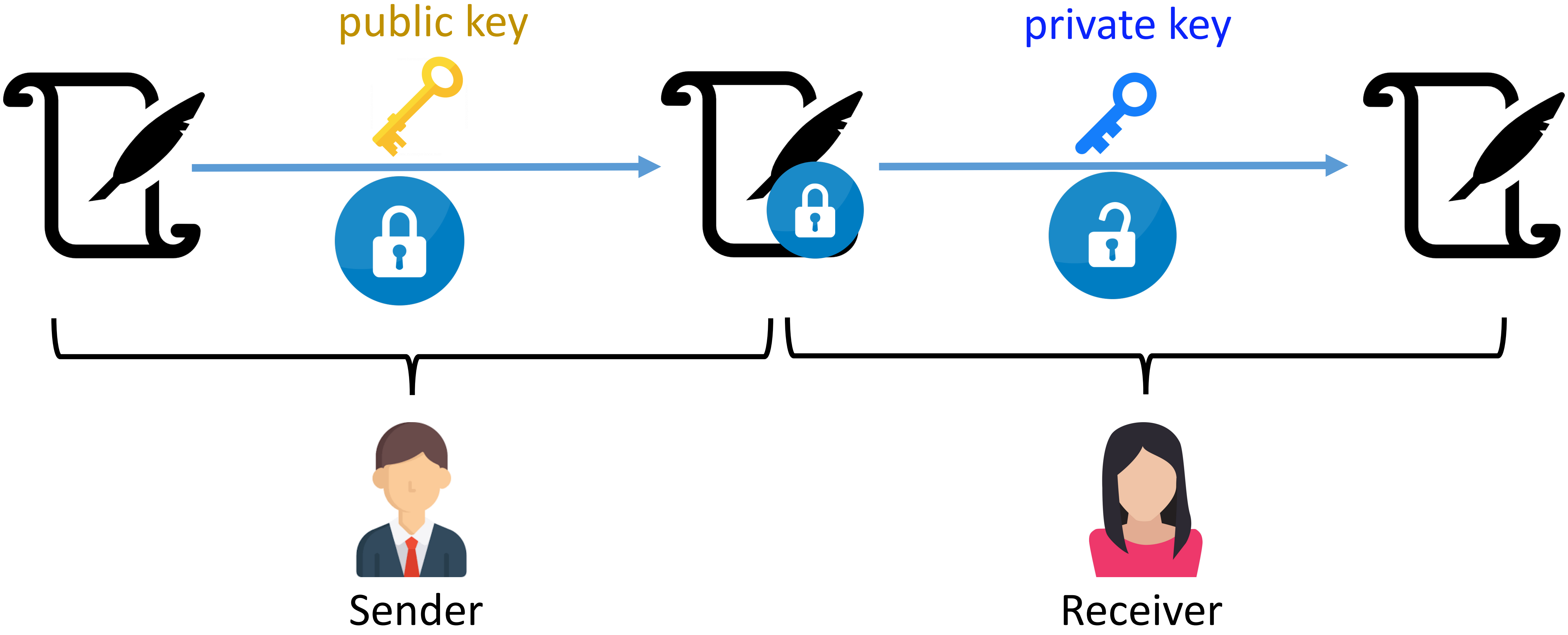
Symmetric Encryption

- The sender and receiver use the same secret key to encrypt and decrypt information.
- The sender and receiver have to agree upon the key.



Difficulty: How to exchange the secret key safely?

Asymmetric Encryption



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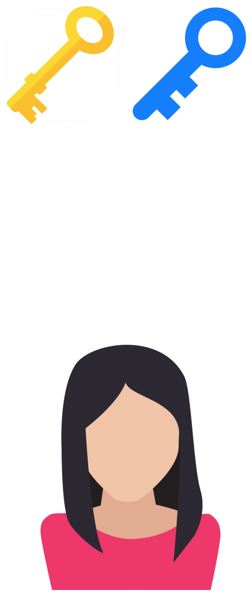
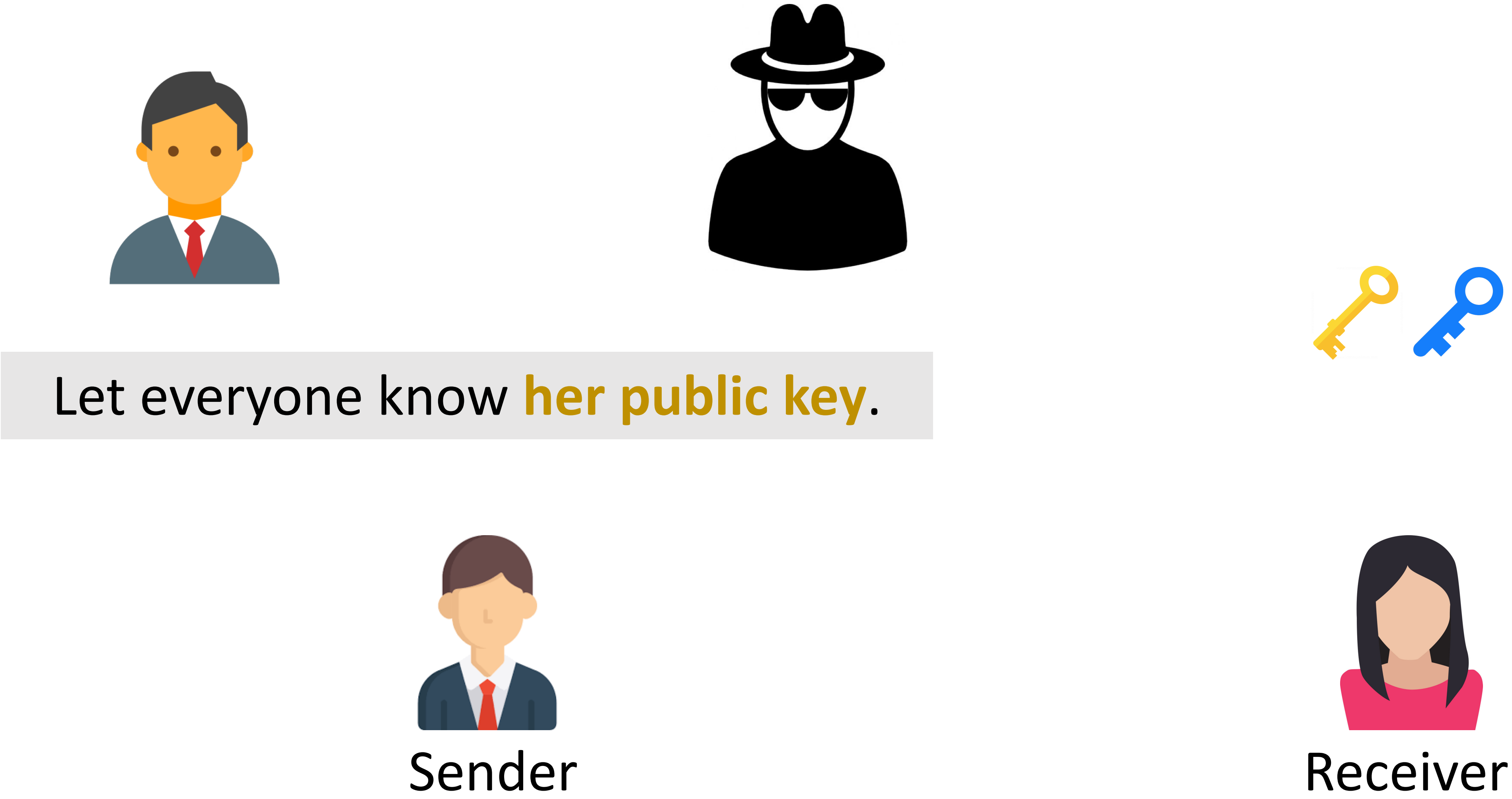


Receiver

Asymmetric Encryption

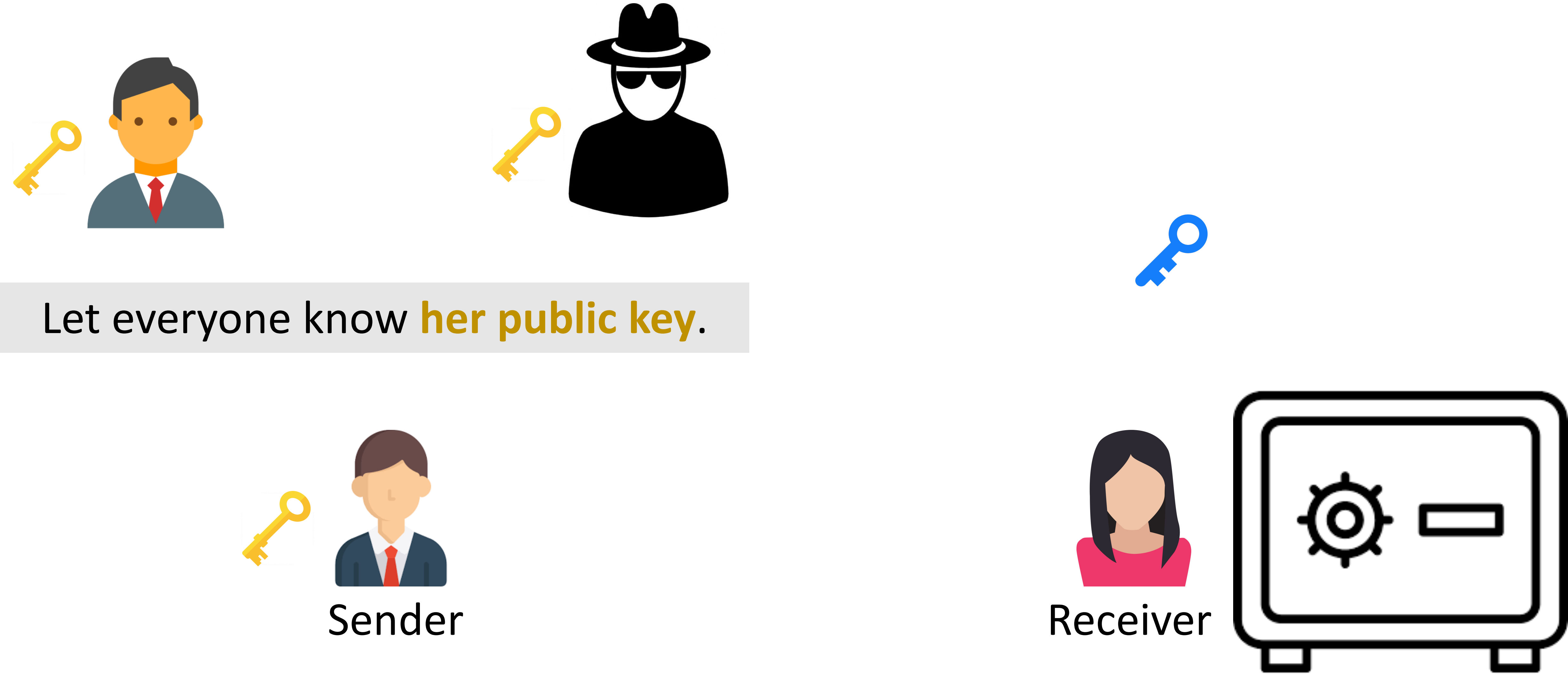


Let everyone know **her public key**.

Sender

Receiver

Asymmetric Encryption



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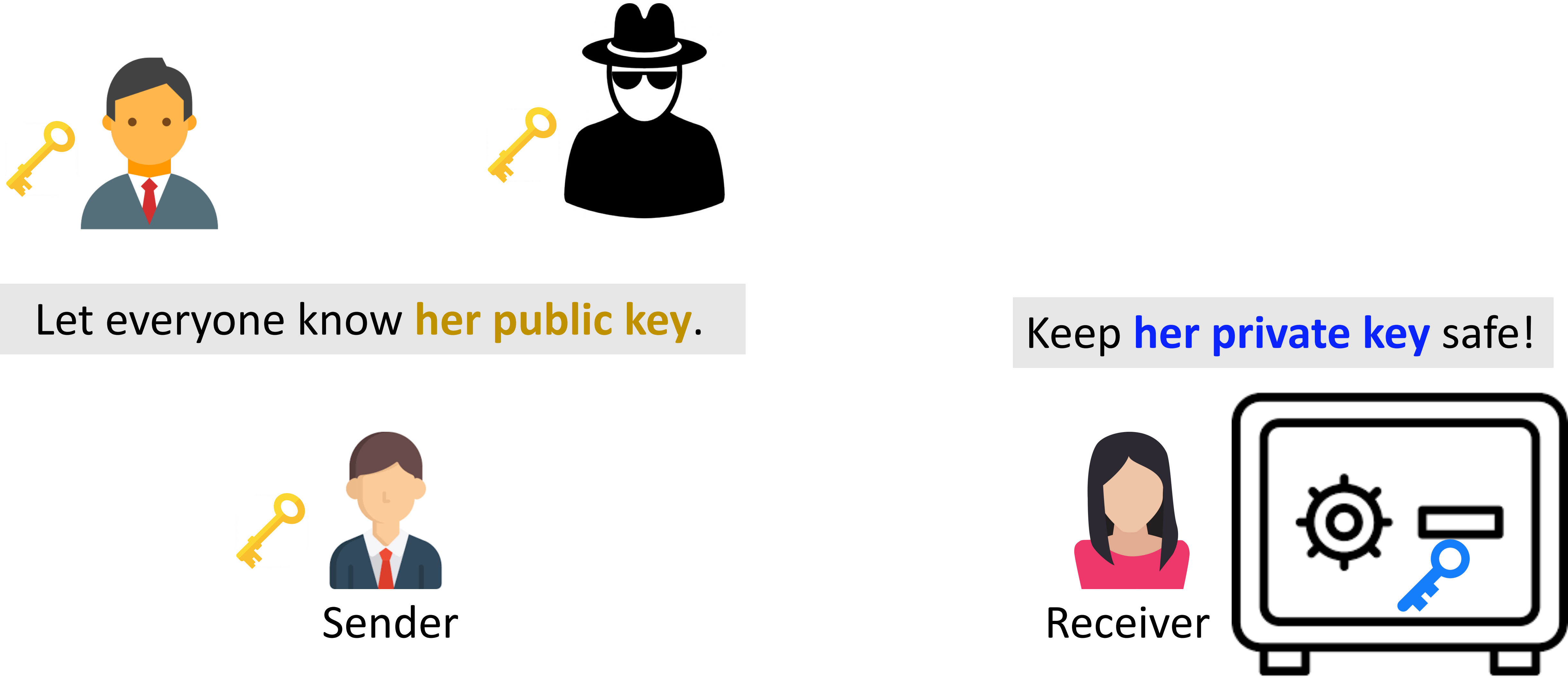
Asymmetric Encryption



Let everyone know **her public key**.

Keep **her private key** safe!

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Receiver

Properties of Asymmetric Encryption

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3. Given E, one cannot easily figure out D.

   - Everyone has the public key.

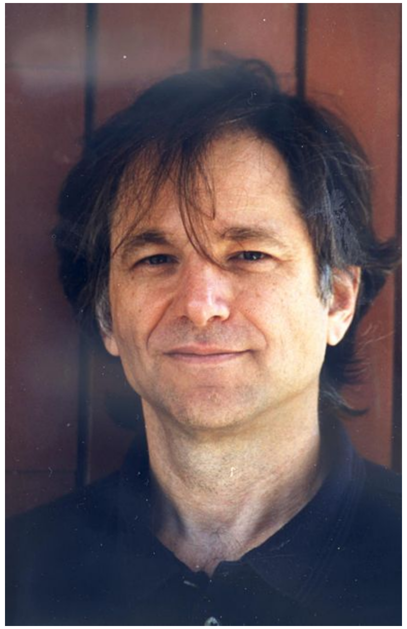
   - They cannot thereby infer the private key.

RSA Algorithm

- RSA (Rivest–Shamir–Adleman) is one of the first public-key cryptosystems and is widely used for secure data transmission.

Ron Rivest

Adi Shamir



Leonard Adleman

Turing Award 2002, for their ingenious contribution for making public-key cryptography useful in practice.

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Theorem: $D\big(E(M)\big) = M$ for certain e, d, n.

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3. $t = (p - 1)(q - 1)$.
4. Find a large integer d such that $gcd(d\ ,\ t) = 1,$ where gcd means greatest common divisor.
5. Find e such that $mod(d * e\ ,\ t) =\ 1.$

Thank You!