Hash Table

Shusen Wang

Hash Function

A hash function $h(k)$ is a function that maps the key k to a fixed range.

- For example, $h(k) = k \bmod 11$ maps integer keys to the range $\{0, 1, 2, \cdots, 10\}$.
- $h(20) = 20 \bmod 11 = 9$.

Hash Function

A hash function $h(k)$ is a function that maps the key k to a fixed range.

- Hash function can map strings to integers.
- For example, $h(\text{“}ABC\text{”}) = (65 + 66 + 67) \bmod 53 = 39$.

Hash Function

A hash function $h(k)$ is a function that maps the key k to a fixed range.

- Hash function can map strings to integers.
- For example, $h(\text{“}ABC\text{”}) = (65 + 66 + 67) \bmod 53 = 39$.

```
int h(char key[], int length) {
      sum = 0;
      for (i=0; i<length; i++)
            sum += key[i];
      return sum % 53;
      // divisor should be a prime
}
```

```
ASCII('A') = 65
ASCII('B') = 66
      ...
ASCII('Z') = 90
```

Hash Table

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

- This is called a key-value pair.

Hash Table

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

- Keys must be unique.

Hash Table

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

- Values may not be unique.

Hash Table

Indices

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

Hash Table

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

- The hash table has n rows.
- Hash function h maps keys to indices in $\{0, 1, 2, \cdots, n-1\}$.
- E.g., $h(\text{“James”}) = 5$.

Hash Table

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

Insertion

- Given a key-value pair (k, v).
- Compute $i = h(k)$.
- Insert (k, v) to the i-th row.

Hash Table

	Key (name)	Value (phone)
0		
1	Willian	5013892745
2	John	2012636901
3		
4	Cindy	9732502971
5	James	5512636588
6		

Search

- Given a key k.
- Compute $i = h(k)$.
- Go to the i-th row to find the key and value.

Hash Collision

Hash Collision

- If $k_1 = k_2$, then $h(k_1) = h(k_2)$.
- However, $h(k_1) = h(k_2)$ does not guarantee $k_1 = k_2$.

Hash Collision

- If $k_1 = k_2$, then $h(k_1) = h(k_2)$.
- However, $h(k_1) = h(k_2)$ does not guarantee $k_1 = k_2$.
- Example:
  - $h(k) = k \bmod 11$.
  - $k_1 = 25$ and $k_2 = 3$.
  - $h(k_1) = h(k_2) = 3$ ➔ Hash collision!

Hash Collision

- If $k_1 = k_2$, then $h(k_1) = h(k_2)$.
- However, $h(k_1) = h(k_2)$ does not guarantee $k_1 = k_2$.
- Example:
  - $h(k) = k \bmod 11$.
  - $k_1 = 25$ and $k_2 = 3$.
  - $h(k_1) = h(k_2) = 3$ ➔ Hash collision!

Hash collision: $k_1 \neq k_2$, but $h(k_1) = h(k_2)$.

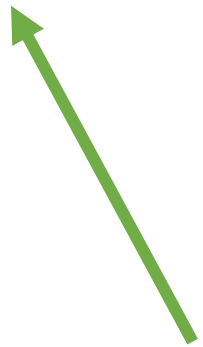
Hash Collision

	Key	Value
0		
1		
2		
3		
4		
5		
6		
7		
8		
9		

- Keys are integers; values are strings.
- $h(k) = (k^2 + 3) \bmod 10.$
- We are inserting key-values pairs to the table.

Hash Collision

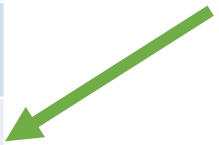
	Key	Value
0		
1		
2	3	James
3		
4		
5		
6		
7		
8		
9		



- To insert `(3,"James")`.
- $h(3) = (3^2 + 3) \bmod 10 = 2.$
- Insert `(3,"James")` to the 2nd row.

Hash Collision

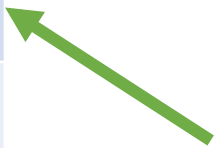
	Key	Value
0		
1		
2	3	James
3		
4		
5		
6		
7	32	Mary
8		
9		



- To insert `(32,"Mary")`.
- $h(32) = (32^2 + 3) \bmod 10 = 7.$
- Insert `(32,"Mary")` to the 7th row.

Hash Collision

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9		



- To insert `(1,"Michael")`.
- $h(1) = (1^2 + 3) \bmod 10 = 4.$
- Insert `(1,"Michael")` to the 4th row.

Hash Collision

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9		

- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(2,"Larry")` to the 7th row.

Collision!

Solutions

- **Separate chaining:** One row can store multiple key-value pairs (using a linked list).
- **Linear probing:** Use the next vacant row.
- **Quadratic probing:** Jump to some vacant row.

Separate Chaining

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9		

- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(2,"Larry")` to the 7th row.

Collision!

Insertion

	Key	Value	
0			
1			
2	3	James	
3			
4	1	Michael	
5			
6			
7	32	Mary	⇝ (2,Larry)
8			
9			

- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(2,"Larry")` to the 7th row.

Insertion

	Key	Value	
0			
1			
2	3	James	
3			
4	1	Michael	
5			
6			
7	32	Mary	⇝ (2,Larry)
8			
9			

- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7.$
- Insert `(8,"Bill")` to the 7th row.

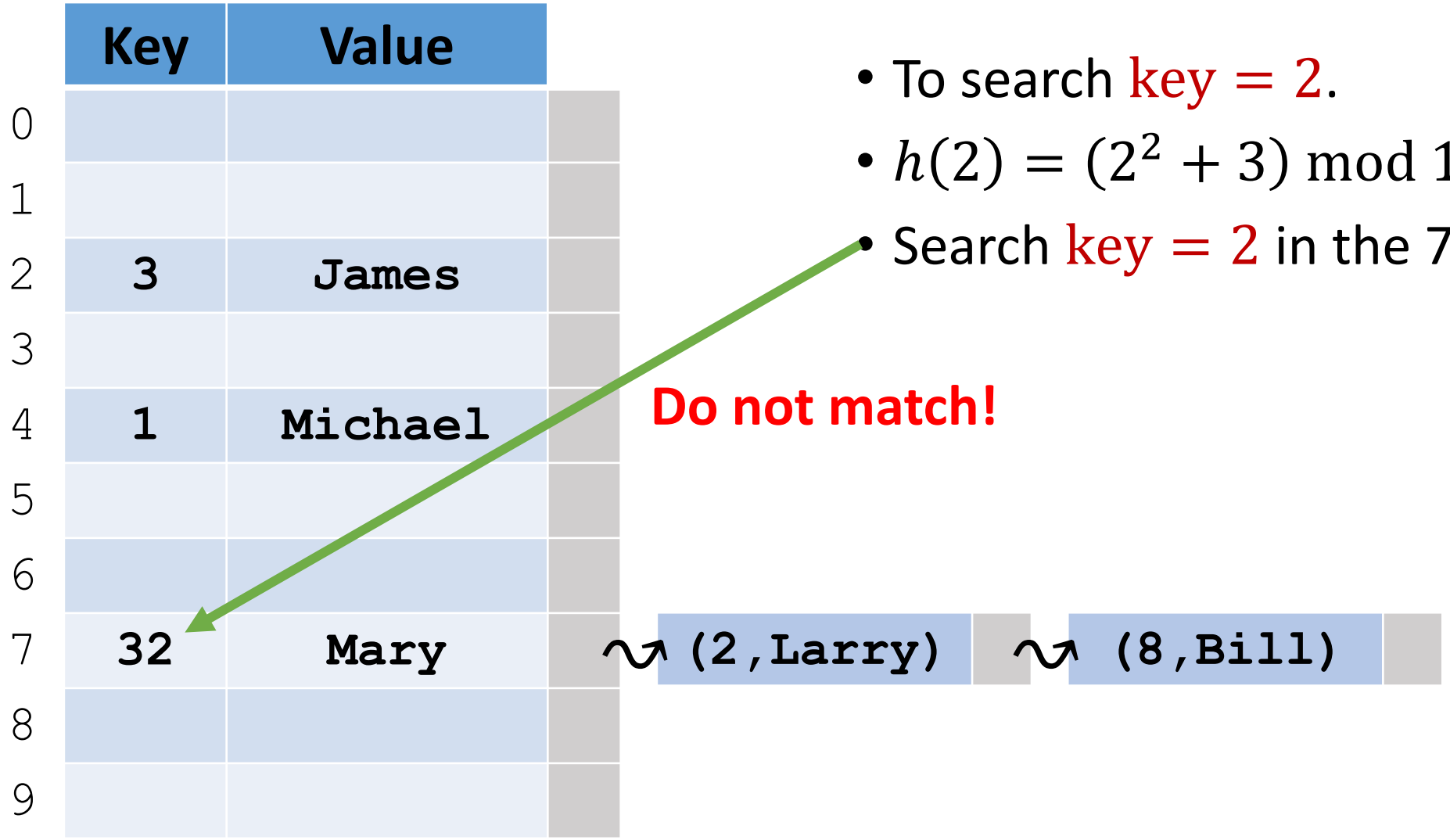
Collision!

Insertion

	Key	Value	
0			
1			
2	3	James	
3			
4	1	Michael	
5			
6			
7	32	Mary	⇝ (2,Larry) ⇝ (8,Bill)
8			
9			

- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7.$
- Insert `(8,"Bill")` to the 7th row.

Search



- To search $\text{key} = 2$.
- $h(2) = (2^2 + 3) \bmod 10 = 7$.
- Search $\text{key} = 2$ in the 7th row.

Do not match!

Search

	Key	Value	
0			
1			
2	3	James	
3			
4	1	Michael	
5			
6			
7	32	Mary	⇝ (2,Larry) ⇝ (8,Bill)
8			
9			

- To search $\text{key} = 2$.
- $h(2) = (2^2 + 3) \bmod 10 = 7$.
- Search $\text{key} = 2$ in the 7th row.
- Go to the linked list to search $\text{key} = 2$.

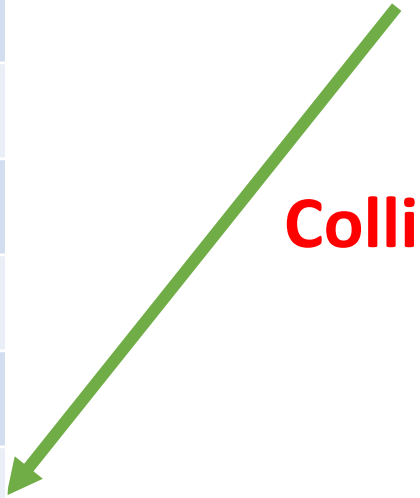
Found!

Linear Probing

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9	6	Cindy

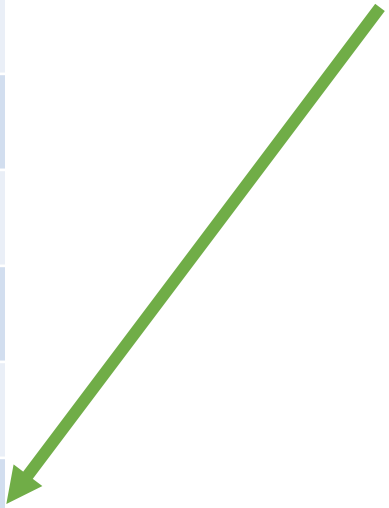
- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(2,"Larry")` to the 7th row.



Collision!

Insertion

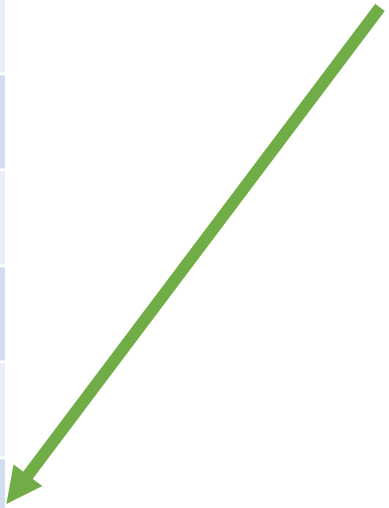
	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9	6	Cindy



- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(2,"Larry")` to the 7th row. (Fail.)
- Try the 8th row (which is empty.)

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy

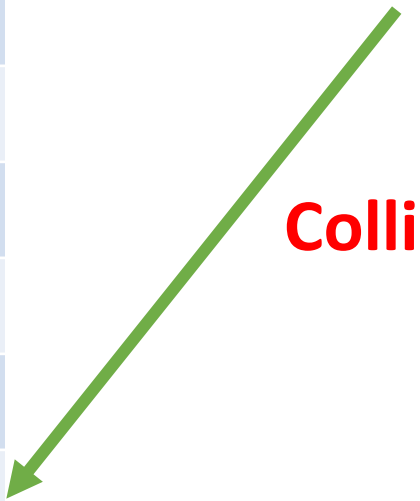


- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7$.
- Insert `(2,"Larry")` to the 7th row. (Fail.)
- Try the 8th row (which is empty.)
- Insert `(2,"Larry")` to the 8th row.

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy

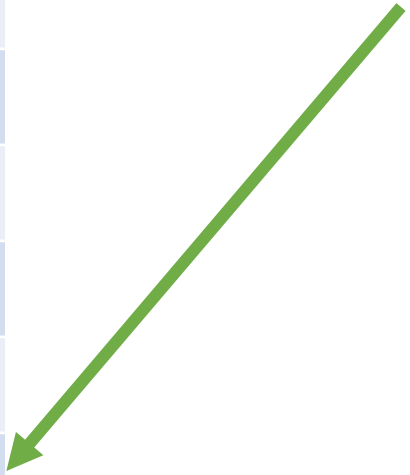
- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7.$
- Insert `(8,"Bill")` to the 7th row.



Collision!

Insertion

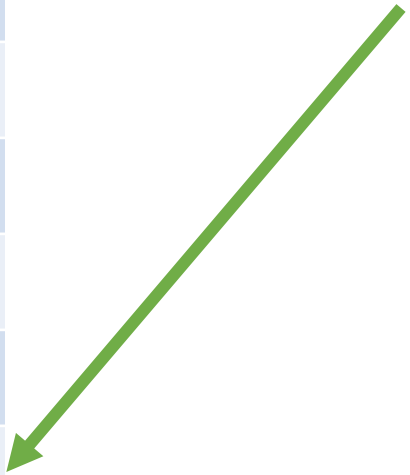
	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Try the 8th row.

Insertion

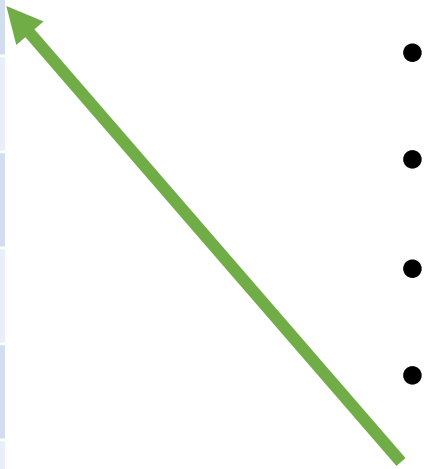
	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Try the 8th row. (Fail.)
- Try the 9th row.

Insertion

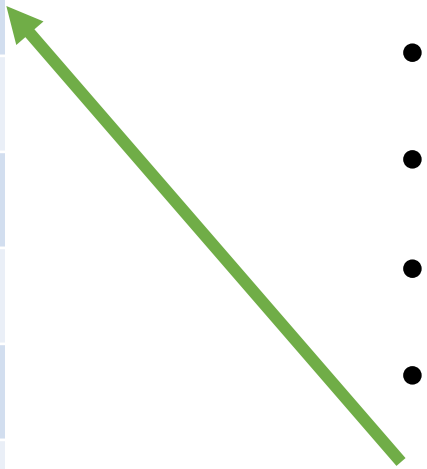
	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Try the 8th row. (Fail.)
- Try the 9th row. (Fail.)
- Try the 0th row (which is empty.)

Insertion

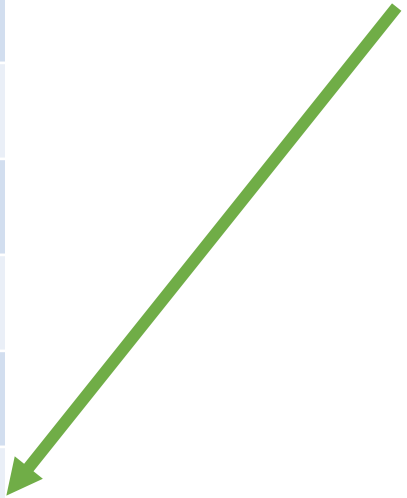
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To insert `(8,"Bill")`.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Try the 8th row. (Fail.)
- Try the 9th row. (Fail.)
- Try the 0th row (which is empty.)
- Insert `(8,"Bill")` to the 0th row.

Search

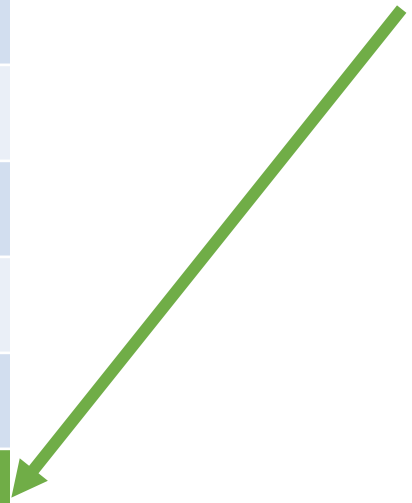
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row.

Search

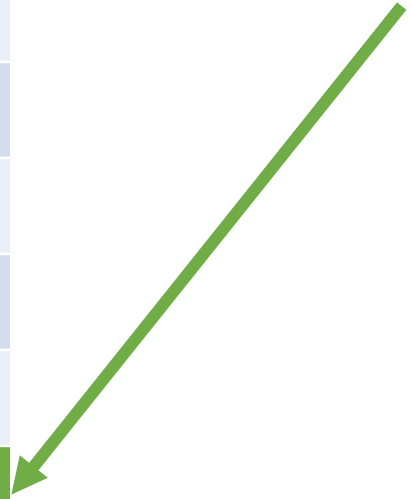
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)

Search

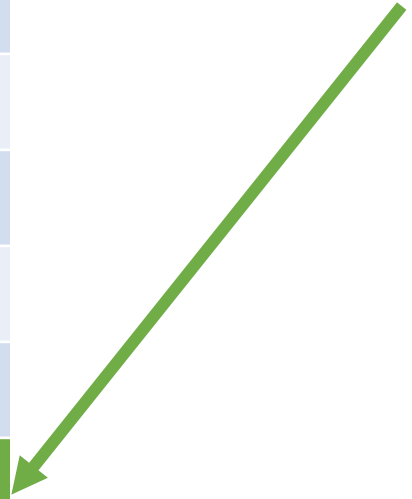
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Look for $\text{key} = 8$ in the 8th row. (Not found.)

Search

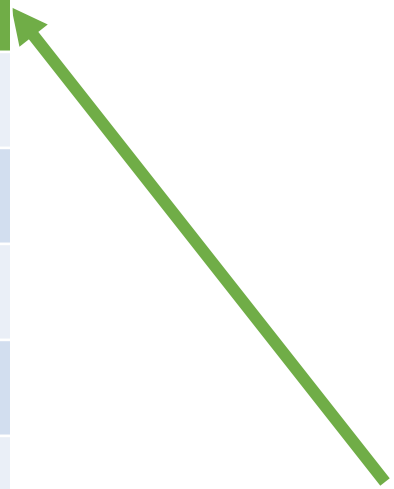
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Look for $\text{key} = 8$ in the 8th row. (Not found.)
- Look for $\text{key} = 8$ in the 9th row. (Not found.)

Search

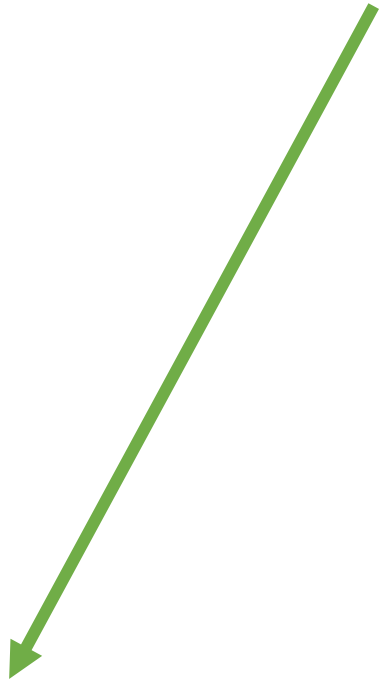
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Look for $\text{key} = 8$ in the 8th row. (Not found.)
- Look for $\text{key} = 8$ in the 9th row. (Not found.)
- Look for $\text{key} = 8$ in the 0th row. (Found!)

Search

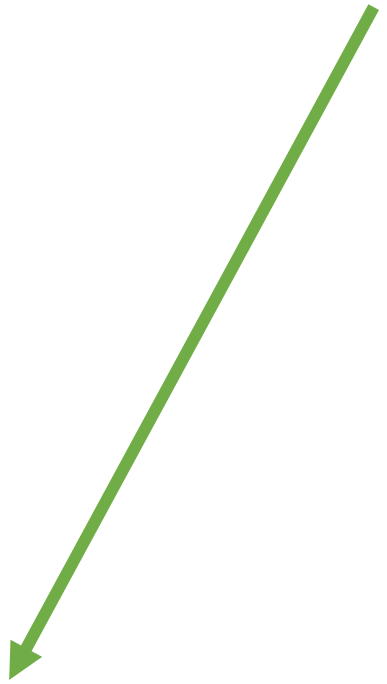
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 4$.
- $h(4) = (4^2 + 3) \bmod 10 = 9$.
- Look for $\text{key} = 4$ in the 9th row.

Search

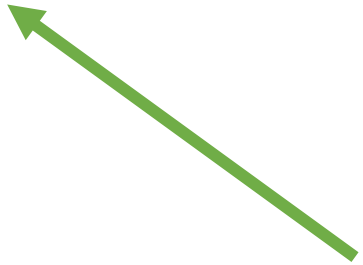
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 4$.
- $h(4) = (4^2 + 3) \bmod 10 = 9$.
- Look for $\text{key} = 4$ in the 9th row. (Not found.)

Search

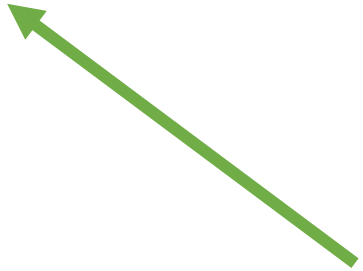
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 4$.
- $h(4) = (4^2 + 3) \bmod 10 = 9$.
- Look for $\text{key} = 4$ in the 9th row. (Not found.)
- Look for $\text{key} = 4$ in the 0th row. (Not found.)

Search

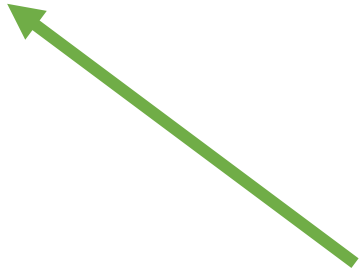
	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 4$.
- $h(4) = (4^2 + 3) \bmod 10 = 9$.
- Look for $\text{key} = 4$ in the 9th row. (Not found.)
- Look for $\text{key} = 4$ in the 0th row. (Not found.)
- Look for $\text{key} = 4$ in the 1st row. (Not found.)

Search

	Key	Value
0	8	Bill
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 4$.
- $h(4) = (4^2 + 3) \bmod 10 = 9$.
- Look for $\text{key} = 4$ in the 9th row. (Not found.)
- Look for $\text{key} = 4$ in the 0th row. (Not found.)
- Look for $\text{key} = 4$ in the 1st row. (Not found.)

$\text{key} = 4$ is not in the table!

Quadratic Probing

Quadratic Probing

- If $h(k)$ causes a collision, then look for an empty row.
- For $i = 1, 2, 3, \cdots$, try the rows

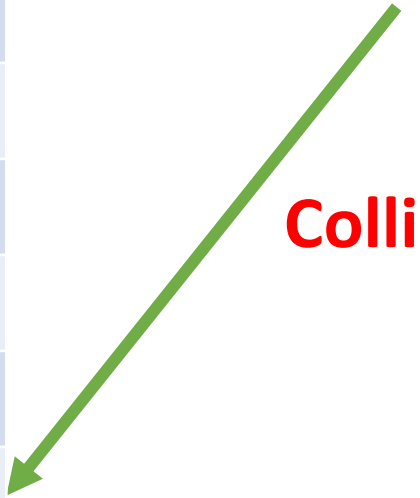
$$(h(k) + i^2) \bmod \text{TableSize}.$$

- Place the key-value pair in the first vacant cell.

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9	6	Cindy

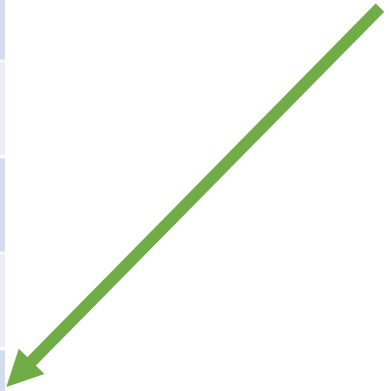
- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(2,"Larry")` to the 7th row.



Collision!

Insertion

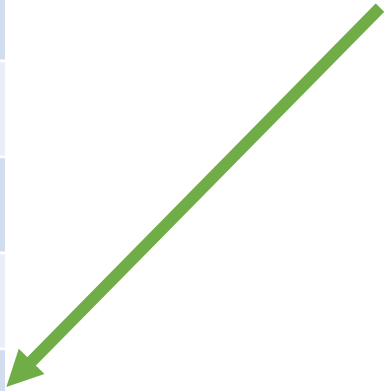
	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8		
9	6	Cindy



- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7$.
- Insert `(2,"Larry")` to the 7th row. (Fail.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Try the 8th row.

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy

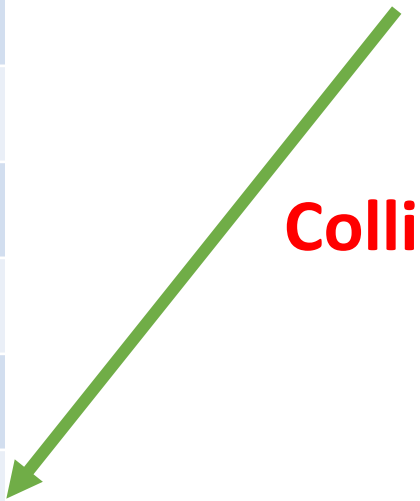


- To insert `(2,"Larry")`.
- $h(2) = (2^2 + 3) \bmod 10 = 7$.
- Insert `(2,"Larry")` to the 7th row. (Fail.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Try the 8th row.

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy

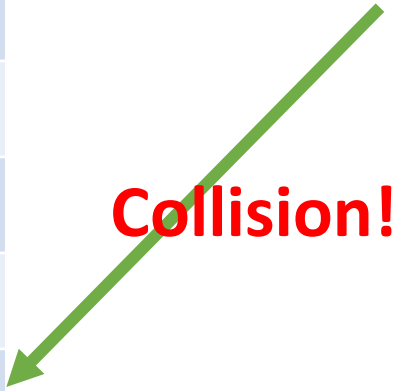
- To insert `(8,"Bill")`.
- $h(8) = (2^2 + 3) \bmod 10 = 7.$
- Insert `(8,"Bill")` to the 7th row.



Collision!

Insertion

	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy

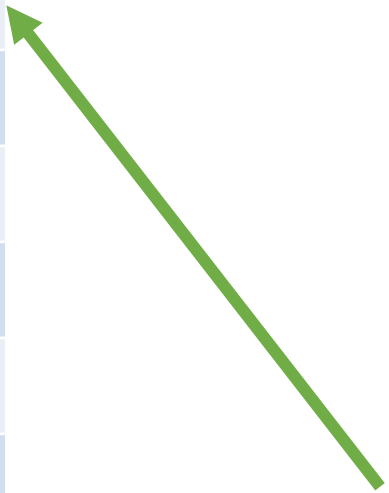


Collision!

- To insert `(8,"Bill")`.
- $h(8) = (2^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Try the 8th row.

Insertion

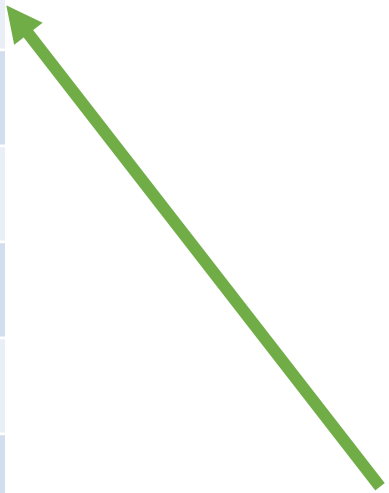
	Key	Value
0		
1		
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To insert `(8,"Bill")`.
- $h(8) = (2^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Try the 8th row.
- Quadratic probing: $(7 + 2^2) \bmod 10 = 1$.
- Try the 1st row.

Insertion

	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To insert `(8,"Bill")`.
- $h(8) = (2^2 + 3) \bmod 10 = 7$.
- Insert `(8,"Bill")` to the 7th row. (Fail.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Try the 8th row.
- Quadratic probing: $(7 + 2^2) \bmod 10 = 1$.
- Try the 1st row.

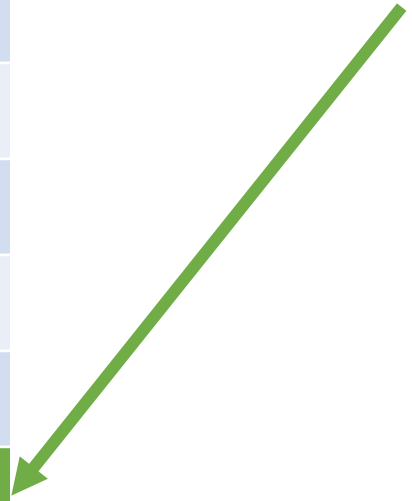
Search

	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy

- To search $\text{key} = 8$.

Search

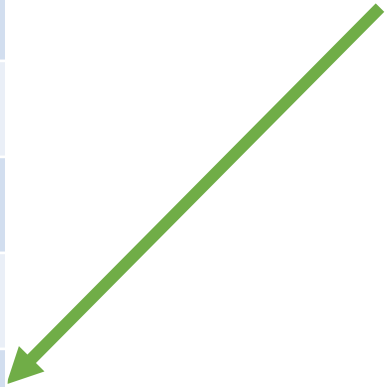
	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)

Search

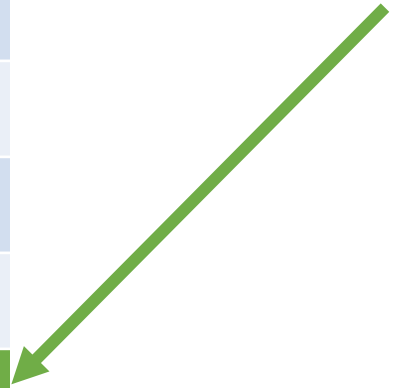
	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Look for $\text{key} = 8$ in the 8th row.

Search

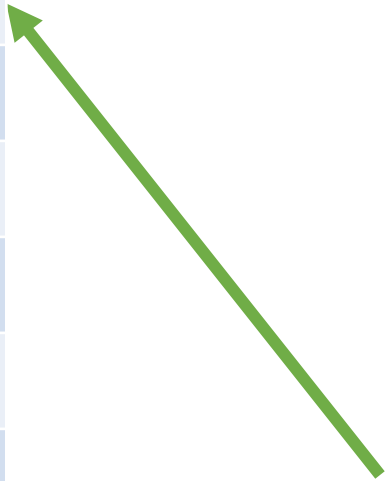
	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Look for $\text{key} = 8$ in the 8th row. (Not found.)

Search

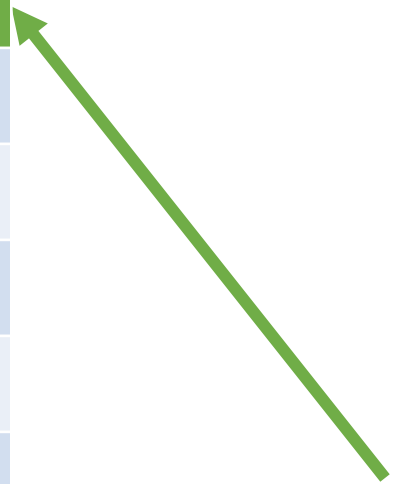
	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Look for $\text{key} = 8$ in the 8th row. (Not found.)
- Quadratic probing: $(7 + 2^2) \bmod 10 = 1$.
- Look for $\text{key} = 8$ in the 1st row.

Search

	Key	Value
0		
1	8	Bill
2	3	James
3		
4	1	Michael
5		
6		
7	32	Mary
8	2	Larry
9	6	Cindy



- To search $\text{key} = 8$.
- $h(8) = (8^2 + 3) \bmod 10 = 7$.
- Look for $\text{key} = 8$ in the 7th row. (Not found.)
- Quadratic probing: $(7 + 1^2) \bmod 10 = 8$.
- Look for $\text{key} = 8$ in the 8th row. (Not found.)
- Quadratic probing: $(7 + 2^2) \bmod 10 = 1$.
- Look for $\text{key} = 8$ in the 1st row.

Questions

Insertion

	Key	Value
0	8	Bill
1	3	James
2		
3	1	Michael
4	32	Mary
5		
6	2	Larry
7		
8	6	Cindy
9		
10		

- Hash function:

$$h(k) = (k^2 + 2) \bmod 11.$$

- **Q1:** Where to insert $(12, \text{Chelsea})$ if linear probing is used?
- **Q2:** Where to insert $(12, \text{Chelsea})$ if quadratic probing is used?

Thank You!