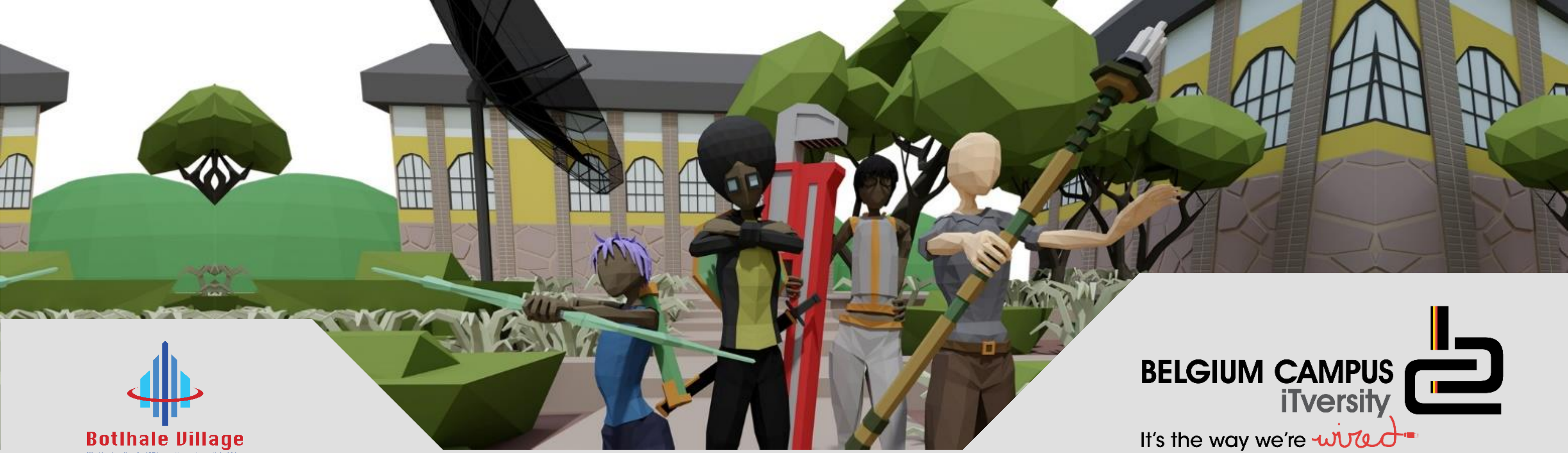

Botlhale Village

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ARE YOU READY?

Formulating a linear programming model

The Limpopo Furniture Company manufactures desks, tables, and chairs. The manufacture of each type of furniture requires lumber and two types of skilled labour: finishing and carpentry. The amount of each resource needed to make each type of furniture is as follows: 8 meters of lumber, 4 finishing hours and 2 carpentry hours to make a desk. 6 meters of lumber, 2 finishing hours and 1.5 carpentry hours to make a table. 1 meter of lumber, 1.5 finishing hours and 0.5 carpentry hours to make a chair.

Currently, 48 board meters of lumber, 20 finishing hours, and 8 carpentry hours are available. A desk sells for R60, a table for R30, and a chair for R20. Because the available resources have already been purchased, Limpopo wants to maximize total revenue.

Formulating a linear programming model

- First we need to declare our decision variables.

  $x_i = The\ number\ of\ i\ manufactured. i = 1 = Desks, 2 = Tables, 3 = Chairs$

- Next we will create the objective function. A desk sells for R60, a table for R30, and a chair for R20.

$$\max z = 60x_1 + 30x_2 + 20x_3$$

- Next we will create the constraints. We will start with the lumber constraint: 8 meters of lumber to make a desk, 6 meters of lumber to make a table and 1 meter of lumber to make a chair. Currently, 48 board meters of lumber is available.

$$s.t. 8x_1 + 6x_2 + 1x_3 \leq 48$$

Formulating a linear programming model

- For the next constraint we will look at the finishing hours constraint: 4 finishing hours to make a desk, 2 finishing hours to make a table and 1.5 finishing hours to make a chair. Currently, 20 finishing hours are available.

$$s.t.\ 4x_1 + 2x_2 + 1.5x_3 \leq 20$$

- For the next constraint we will look at the carpentry hours constraint: 2 carpentry hours to make a desk, 1.5 carpentry hours to make a table and 0.5 carpentry hours to make a chair. Currently, 8 carpentry hours are available.

$$s.t.\ 2x_1 + 1.5x_2 + 0.5x_3 \leq 8$$

- Lastly we need our sign restrictions:

$$x_1, x_2, x_3 \geq 0$$

Primal Simplex

- The first step will be to put the formulated LP into canonical form:

Canonical Form: Primal Simplex Algorithm										
	(z)	-	60x1	-	30x2	-	20x2	=		0
		8x1	+	6x2	+	x2	+	s1	=	48
		4x1	+	2x2	+	1.5x2	+	s2	=	20
		2x1	+	1.5x2	+	0.5x2	+	s3	=	8

- Now we can create our initial table:

t-i	x1	x2	x3	s1	s2	s3	rhs	θ
z	-60	-30	-20	0	0	0	0	
1	8	6	1	1	0	0	48	6
2	4	2	1 1/2	0	1	0	20	5
3	2	1 1/2	1/2	0	0	1	8	4

=You will select the Right-hand side value/pivot column value
-> =48/8=6
You will drag the formula down.

In the initial table, we first check if it is a max or min problem to get our pivot column. Then we do a ratio test to get the pivot row.

Primal Simplex

- To get the new table, we will do the following:

Primal Simplex Algorithm

t-i	x1	x2	x3	s1	s2	s3	rhs	θ
z	-60	-30	-20	0	0	0	0	
1	8	6	1	1	0	0	48	6
2	4	2	1 1/2	0	1	0	20	5
3	2	1 1/2	1/2	0	0	1	8	4

t-2	x1	x2	x3	s1	s2	s3	rhs	θ
z	=D13-($D13*D$22)							
1								
2								
3								

- Drag the formula through the entire table.

Primal Simplex Algorithm

t-i	x1	x2	x3	s1	s2	s3	rhs	θ
z	-60	-30	-20	0	0	0	0	
1	8	6	1	1	0	0	48	6
2	4	2	1 1/2	0	1	0	20	5
3	2	1 1/2	1/2	0	0	1	8	4

t-2	x1	x2	x3	s1	s2	s3	rhs	θ
z	-60	-30	-20	0	0	0	0	
1	-60	-30	-20	0	0	0	0	
2	-60	-30	-20	0	0	0	0	
3	=D16/D16		-20	0	0	0	0	

- Drag this formula through the entire row.

Primal Simplex

- In the new table, we still have a negative value in our objective function z row and we need to make this value our pivot row and do a ratio test:

t-2	x1	x2	x3	s1	s2	s3	rhs	θ
z	0	15	-5	0	0	30	240	
1	0	0	-1	1	0	-4	16	-16
2	0	-1	1/2	0	1	-2	4	8
3	1	3/4	1/4	0	0	1/2	4	16

- We will again do our calculations to get our new table:

t-3*	x1	x2	x3	s1	s2	s3	rhs
z	0	5	0	0	10	10	280
1	0	-2	0	1	2	-8	24
2	0	-2	1	0	2	-4	8
3	1	1 1/4	0	0	- 1/2	1 1/2	2

There are no more negative values in the z row, thus we have an optimal table.

Exercises:

1. Santa's Workshop employs elves to manufacture two types of wooden toys: soldiers and trains. A soldier sells for R27 and uses R10 worth of raw materials. Each soldier that is manufactured increases Santa's variable labour and overhead costs by R14. A train sells for R21 and uses R9 worth of raw materials. Each train built increases Santa's variable labour and overhead costs by R10. The manufacture of wooden soldiers and trains requires two types of skilled labour: carpentry and finishing. A soldier requires 2 hours of finishing labour and 1 hour of carpentry labour. A train requires 1 hour of finishing and 1 hour of carpentry labour. Each week, Santa can obtain all the needed raw material from the forests of Lapland, but only 100 finishing hours and 80 carpentry hours from his elves. Demand for trains is unlimited, but at most 40 soldiers are bought each week. Santa wants to maximize weekly profit (revenues − costs).
   - Formulate a mathematical model of Santa's situation that can be used to maximise Santa's weekly profit.
   - Solve the formulated Linear Programming Model of Santa using the Primal Simplex Algorithm.

Exercises:

1. Accessories Inc. manufactures belts and shoes. A belt requires 2 square metres of leather and 1 hour of skilled labour. A pair of shoes requires 3 square metres of leather and 2 hours of skilled labour. As many as 25 square metres of leather and 15 hours of skilled labour can be purchased at a price of R5/m2 of leather and R10/hour of skilled labour. A belt sells for R23, and a pair of shoes sells for R40. Accessories Inc. wants to maximise profits (revenues − costs).
    1. Formulate a mathematical model of Accessories Inc. that can be used to maximise Accessories Inc. weekly profit.
    2. Solve the formulated Linear Programming Model of Accessories Inc. using the Primal Simplex Algorithm.

END

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