Botlhale Village
Working together for ICT innovation and growth in Africa

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ARE YOU READY?

SCENARIO

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- Gandalf the Grey lives in Gondor, Middle Earth. He is responsible for the communities in Gondor, Isengard, Rivendell, Lothlórien, Rohan and Mordor. Each Yule, he visits each of these communities. The distance between each community (called cities in the problem, in miles) is shown in the tables.

- What order of visiting the communities will minimise the total distance travelled?

SCENARIO

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Distance between cities		
Gondor	Isengard	264
	Rivendell	434
	Lothlorien	328
	Rohan	116
	Mordor	174
Isengard	Gondor	132
	Rivendell	290
	Lothlorien	201
	Rohan	79
	Mordor	119

Distance between cities		
Rivendell	Gondor	217
	Isengard	580
	Lothlorien	226
	Rohan	606
	Mordor	909
Lothlorien	Gondor	164
	Isengard	402
	Rivendell	113
	Rohan	196
	Mordor	294

Distance between cities		
Rohan	Gondor	58
	Isengard	158
	Rivendell	303
	Lothlorien	392
	Mordor	441
Mordor	Gondor	87
	Isengard	237
	Rivendell	455
	Lothlorien	588
	Rohan	662

SCENARIO

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Distance between cities		
1.Gondor	2.Isengard	264
	3.Rivendell	434
	4.Lothlorien	328
	5.Rohan	116
	6.Mordor	174
2.Isengard	1.Gondor	132
	3.Rivendell	290
	4.Lothlorien	201
	5.Rohan	79
	6.Mordor	119

Distance between cities		
3.Rivendell	1.Gondor	217
	2.Isengard	580
	4.Lothlorien	226
	5.Rohan	606
	6.Mordor	909
4.Lothlorien	1.Gondor	164
	2.Isengard	402
	3.Rivendell	113
	5.Rohan	196
	6.Mordor	294

Distance between cities		
5.Rohan	1.Gondor	58
	2.Isengard	158
	3.Rivendell	303
	4.Lothlorien	392
	6.Mordor	441
6.Mordor	1.Gondor	87
	2.Isengard	237
	3.Rivendell	455
	4.Lothlorien	588
	5.Rohan	662

Formulating the IP

- Decision variables:

$x_{ij} = if\ Gandalf\ leaves\ city\ i\ and\ travels\ next\ to\ city\ j\ (1)\ or\ not\ (0)\ where\ i{=}j{=}1{=}Gondor,\ 2{=}Isengard,\ 3{=}Rivendell,\ 4{=}Lothlorien,\ 5{=}Rohan,\ 6{=}Mordor$

$$U_i = dummy\ variable$$

- Objective function:

$$\begin{aligned} min\ z = {} & 264x_{12} + 434x_{13} + 328x_{14} + 116x_{15} + 174x_{16} + \\ & 132x_{21} + 290x_{23} + 201x_{24} + 79x_{25} + 119x_{26} + \\ & 217x_{31} + 580x_{32} + 226x_{34} + 606x_{35} + 909x_{36} + \\ & 164x_{41} + 402x_{42} + 113x_{43} + 196x_{45} + 294x_{46} + \\ & 58x_{51} + 158x_{52} + 303x_{53} + 392x_{54} + 441x_{56} + \\ & 87x_{61} + 237x_{62} + 455x_{63} + 588x_{64} + 662x_{65} \end{aligned}$$

- Arriving once in a city constraints:

$$x_{21} + x_{31} + x_{41} + x_{51} + x_{61} = 1$$
$$x_{12} + x_{32} + x_{42} + x_{52} + x_{62} = 1$$

Formulating the IP

$$x_{13} + x_{23} + x_{43} + x_{53} + x_{63} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{54} + x_{64} = 1$$
$$x_{15} + x_{25} + x_{35} + x_{45} + x_{65} = 1$$
$$x_{16} + x_{26} + x_{36} + x_{46} + x_{56} = 1$$

- Leaving a city once constraints:

$$x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1$$
$$x_{21} + x_{23} + x_{24} + x_{25} + x_{26} = 1$$
$$x_{31} + x_{32} + x_{34} + x_{35} + x_{36} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{45} + x_{46} = 1$$
$$x_{51} + x_{52} + x_{53} + x_{54} + x_{56} = 1$$
$$x_{61} + x_{62} + x_{63} + x_{64} + x_{65} = 1$$

Formulating the IP

- Sub-tour constraints:

$$
\begin{aligned}
U_2 - U_3 + 6x_{23} &\leq 5 \\
U_2 - U_4 + 6x_{24} &\leq 5 \\
U_2 - U_5 + 6x_{25} &\leq 5 \\
U_2 - U_6 + 6x_{26} &\leq 5 \\
U_3 - U_2 + 6x_{32} &\leq 5 \\
U_3 - U_4 + 6x_{34} &\leq 5 \\
U_3 - U_5 + 6x_{35} &\leq 5 \\
U_3 - U_6 + 6x_{36} &\leq 5 \\
U_4 - U_2 + 6x_{42} &\leq 5 \\
U_4 - U_3 + 6x_{43} &\leq 5 \\
U_4 - U_5 + 6x_{45} &\leq 5 \\
U_4 - U_6 + 6x_{46} &\leq 5
\end{aligned}
$$

Formulating the IP

$$U_5 - U_2 + 6x_{52} \leq 5$$
$$U_5 - U_3 + 6x_{53} \leq 5$$
$$U_5 - U_4 + 6x_{54} \leq 5$$
$$U_5 - U_6 + 6x_{56} \leq 5$$
$$U_6 - U_2 + 6x_{62} \leq 5$$
$$U_6 - U_3 + 6x_{63} \leq 5$$
$$U_6 - U_4 + 6x_{64} \leq 5$$
$$U_6 - U_5 + 6x_{65} \leq 5$$

- Sign restrictions:

$$x_{ij} = 0 \; or \; 1$$
$$U_i \geq 0 \; and \; integers$$

Using Solver in the next slides is just to help you understand how to find any sub tours, but you will not set up the IP with Solver so you need to include all sub tours in your IP.

SOLVING USING HEURISTIC – NEAREST NEIGHBOUR

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Distance between cities		
1.Gondor	2.Isengard	264
	3.Rivendell	434
	4.Lothlorien	328
	5.Rohan	116
	6.Mordor	174
2.Isengard	1.Gondor	132
	3.Rivendell	290
	4.Lothlorien	201
	5.Rohan	79
	6.Mordor	119

Distance between cities		
3.Rivendell	1.Gondor	217
	2.Isengard	580
	4.Lothlorien	226
	5.Rohan	606
	6.Mordor	909
4.Lothlorien	1.Gondor	164
	2.Isengard	402
	3.Rivendell	113
	5.Rohan	196
	6.Mordor	294

Distance between cities		
5.Rohan	1.Gondor	58
	2.Isengard	158
	3.Rivendell	303
	4.Lothlorien	392
	6.Mordor	441
6.Mordor	1.Gondor	87
	2.Isengard	237
	3.Rivendell	455
	4.Lothlorien	588
	5.Rohan	662

SOLVING USING HEURISTIC – NEAREST NEIGHBOUR (NNH)

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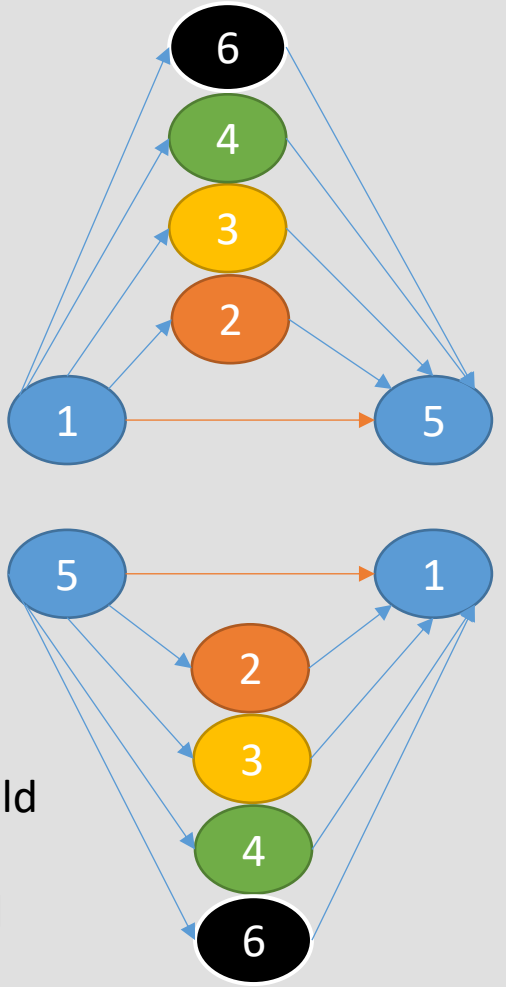
		Routes Available				
Initial Route	x_{1i}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
Route 1	x_{15}	x_{52}	x_{53}	x_{54}	x_{56}	
Route 2	$x_{15} \rightarrow x_{52}$	x_{23}	x_{24}	x_{26}		
Route 3	$x_{15} \rightarrow x_{52} \rightarrow x_{26}$	x_{63}	x_{64}			
Route 4	$x_{15} \rightarrow x_{52} \rightarrow x_{26} \rightarrow x_{63}$	x_{34}				
Route 5	$x_{15} \rightarrow x_{52} \rightarrow x_{26} \rightarrow x_{63} \rightarrow x_{34}$	none				
Return	$x_{15} \rightarrow x_{52} \rightarrow x_{26} \rightarrow x_{63} \rightarrow x_{34} \rightarrow x_{41}$					
Z=	$x_{15} \rightarrow x_{52} \rightarrow x_{26} \rightarrow x_{63} \rightarrow x_{34} \rightarrow x_{41}$	$116 + 158 + 119 + 455 + 226 + 164$				
Z=	1238					

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

Route		Detour				Detour Length
x_{15}	116	x_{12}	x_{25}	264	79	$= (264 + 79) - 116 = 227$
x_{15}	116	x_{13}	x_{35}	434	606	$= (434 + 606) - 116 = 924$
x_{15}	116	x_{14}	x_{45}	328	196	$= (328 + 196) - 116 = 408$
x_{15}	116	x_{16}	x_{65}	174	662	$= (174 + 662) - 116 = 720$
x_{51}	58	x_{52}	x_{21}	158	132	$= (158 + 132) - 58 = 232$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$
x_{51}	58	x_{54}	x_{41}	392	164	$= (392 + 164) - 58 = 498$
x_{51}	58	x_{56}	x_{61}	441	87	$= (441 + 87) - 58 = 470$

$$x_{12} \rightarrow x_{25} \rightarrow x_{51}$$

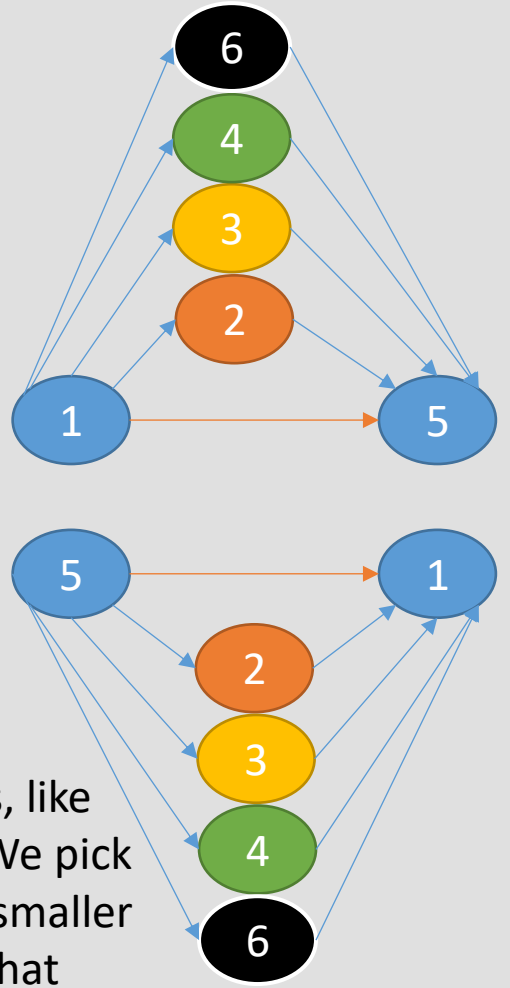


Due to the table being asymmetrical, x15 could be the shortest from the origin, but x21 could be the shortest back to the origin, you will need to compare the two, pick the shortest and complete the route. E.g. if x15 is 116 and x21 is 100 instead of 132 (it is just an example and the actual value was 132 so x51 was chosen), we would pick x21 and our initial start will be adding detours for x12 + x21 (completing the route back to or from the origin).

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

Route		Detour				Detour Length
x_{15}	116	x_{12}	x_{25}	264	79	$= (264 + 79) - 116 = 227$
x_{15}	116	x_{13}	x_{35}	434	606	$= (434 + 606) - 116 = 924$
x_{15}	116	x_{14}	x_{45}	328	196	$= (328 + 196) - 116 = 408$
x_{15}	116	x_{16}	x_{65}	174	662	$= (174 + 662) - 116 = 720$
x_{51}	58	x_{52}	x_{21}	158	132	$= (158 + 132) - 58 = 232$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$
x_{51}	58	x_{54}	x_{41}	392	164	$= (392 + 164) - 58 = 498$
x_{51}	58	x_{56}	x_{61}	441	87	$= (441 + 87) - 58 = 470$

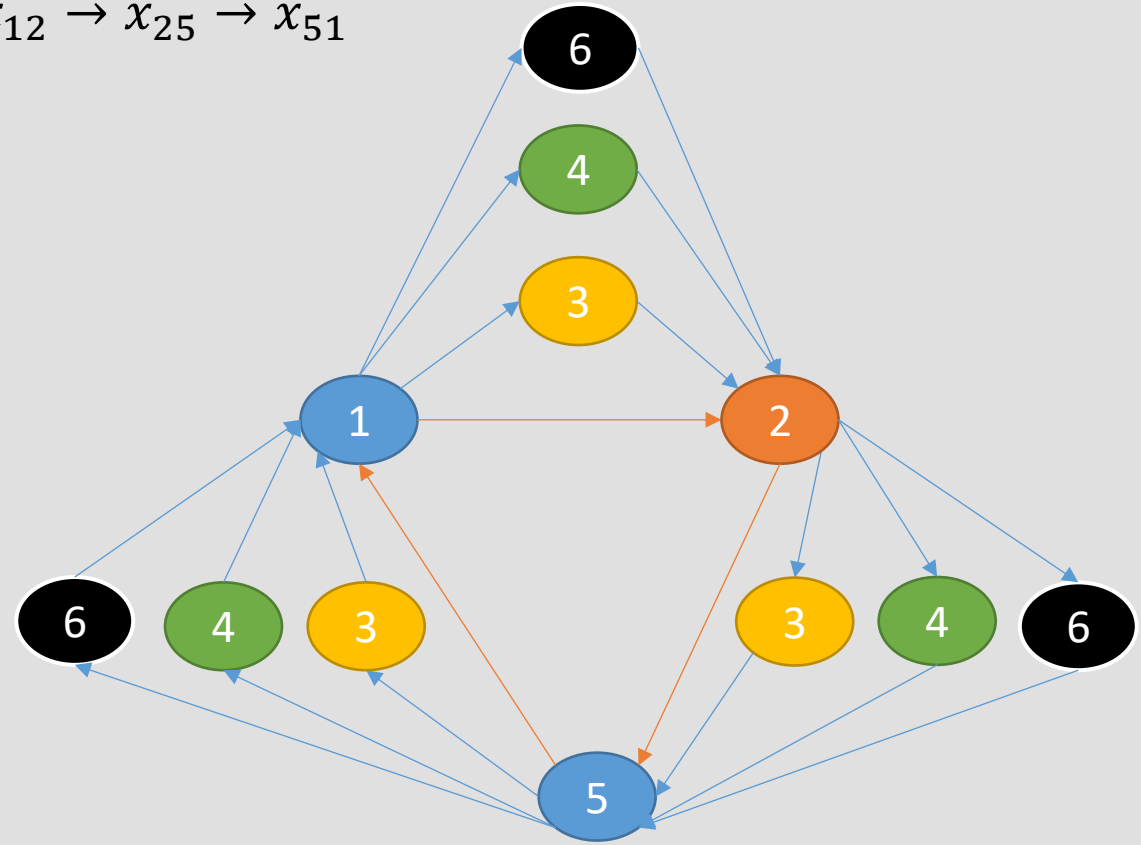
$$x_{12} \rightarrow x_{25} \rightarrow x_{51}$$



Due to the table being asymmetrical, x15 and x51 will not necessarily be the same amounts, like above (we need to check from the origin as well as to the origin if the to is maybe better). We pick the smallest and then complete the route like above. **If** x51 with a detour of x54 + x41 was smaller (e.g. 200 instead of 498 which was not the case and why it was NOT chosen), you will pick that one and complete the route with what was not replaced $x_{15} \rightarrow x_{54} \rightarrow x_{41}$

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

$x_{12} \rightarrow x_{25} \rightarrow x_{51}$



SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

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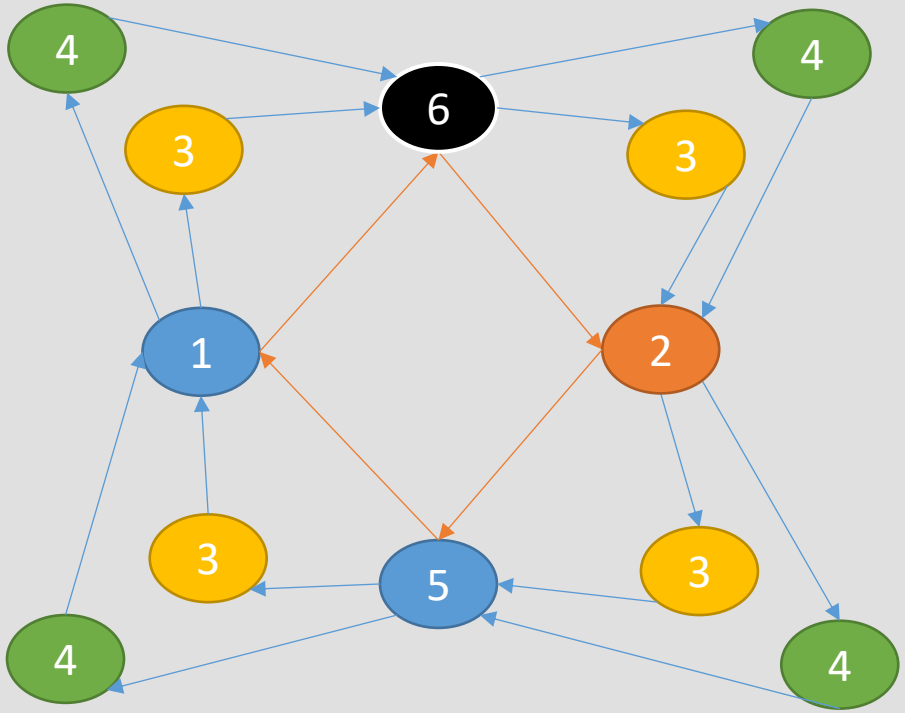
Route		Detour				Detour Length
x_{12}	264	x_{13}	x_{32}	434	580	$= (434 + 580) - 264 = 750$
x_{12}	264	x_{14}	x_{42}	328	402	$= (328 + 402) - 264 = 466$
x_{12}	264	x_{16}	x_{62}	174	237	$= (174 + 237) - 264 = 147$
x_{25}	79	x_{23}	x_{35}	290	606	$= (290 + 606) - 79 = 817$
x_{25}	79	x_{24}	x_{45}	201	196	$= (201 + 196) - 79 = 318$
x_{25}	79	x_{26}	x_{65}	119	662	$= (119 + 662) - 79 = 702$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$
x_{51}	58	x_{54}	x_{41}	392	164	$= (392 + 164) - 58 = 498$
x_{51}	58	x_{56}	x_{61}	441	87	$= (441 + 87) - 58 = 470$

$$x_{16} \rightarrow x_{62} \rightarrow x_{25} \rightarrow x_{51}$$

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

$$x_{16} \rightarrow x_{62} \rightarrow x_{25} \rightarrow x_{51}$$



SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

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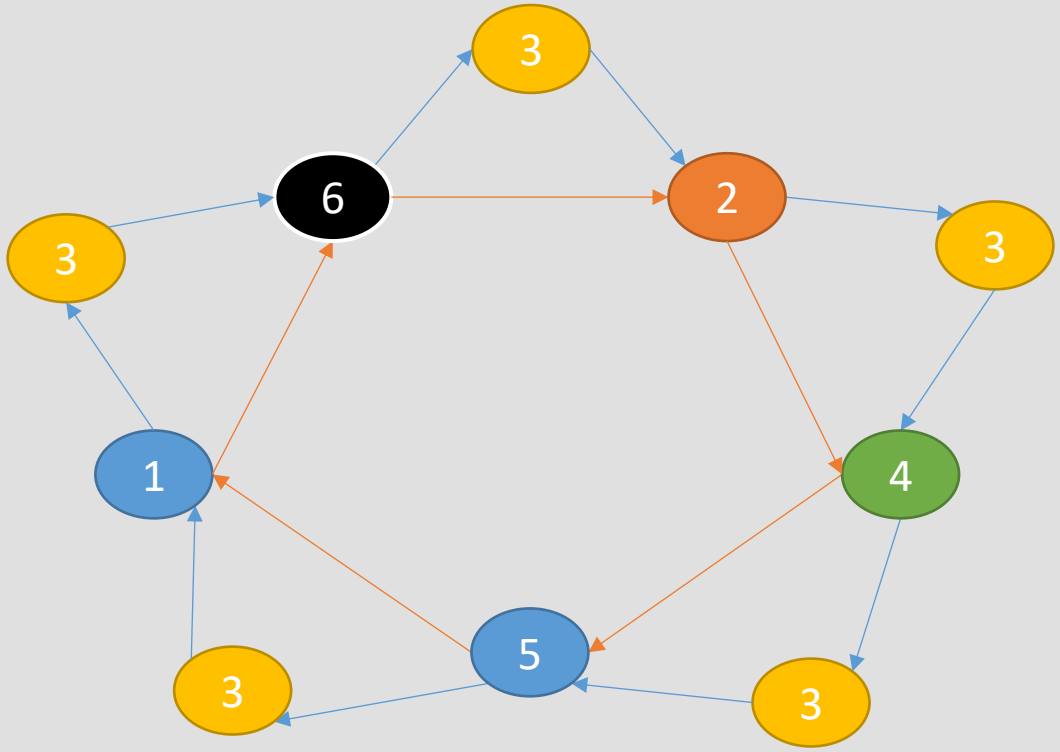
Route		Detour				Detour Length
x_{16}	174	x_{13}	x_{36}	434	909	$= (434 + 909) - 174 = 1169$
x_{16}	174	x_{14}	x_{46}	328	294	$= (328 + 294) - 174 = 448$
x_{62}	237	x_{63}	x_{32}	455	580	$= (455 + 580) - 237 = 798$
x_{62}	237	x_{64}	x_{42}	588	402	$= (588 + 402) - 237 = 753$
x_{25}	79	x_{23}	x_{35}	290	606	$= (290 + 606) - 79 = 817$
x_{25}	79	x_{24}	x_{45}	201	196	$= (201 + 196) - 79 = 318$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$
x_{51}	58	x_{54}	x_{41}	392	164	$= (392 + 164) - 58 = 498$

$$x_{16} \rightarrow x_{62} \rightarrow x_{24} \rightarrow x_{45} \rightarrow x_{51}$$

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

$$x_{16} \rightarrow x_{62} \rightarrow x_{24} \rightarrow x_{45} \rightarrow x_{51}$$



SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

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Route		Detour				Detour Length
x_{16}	174	x_{13}	x_{36}	434	909	$= (434 + 909) - 174 = 1169$
x_{62}	237	x_{63}	x_{32}	455	580	$= (455 + 580) - 237 = 798$
x_{24}	201	x_{23}	x_{34}	290	226	$= (290 + 226) - 201 = 315$
x_{45}	196	x_{43}	x_{35}	113	606	$= (113 + 606) - 196 = 523$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$

$$x_{16} \rightarrow x_{62} \rightarrow x_{23} \rightarrow x_{34} \rightarrow x_{45} \rightarrow x_{51}$$

$$z = 174 + 237 + 290 + 226 + 196 + 58$$
$$z = 1181$$

Exercises

2. Each day, African Petroleum manufactures four types of petrol: lead-free premium (LFP), lead-free regular (LFR), leaded premium (LP), and leaded regular (LR). Because of cleaning and resetting of machinery, the time required to produce a batch of petrol depends on the type of petrol last produced. For example, it takes longer to switch between a lead-free petrol and a leaded petrol than it does to switch between two lead-free types of petrol. The time (in minutes) required to manufacture each day's petrol requirements are shown in the Asymmetrical table. Assume that you need to start with the LFR petrol.

Last-produced petrol	Petrol to be produced next			
	LFR	LFP	LR	LP
LFR	-	50	120	140
LFP	60	-	140	110
FR	90	130	-	60
FP	130	120	80	-

Exercises

- Formulate an Integer Programming Model that will solve the given problem.
- Solve the formulated Integer Programming Model using Solver.
- Solve the formulated Integer Programming Model using the NNH and CIH.

Last-produced petrol	Petrol to be produced next			
	LFR	LFP	LR	LP
LFR	-	50	120	140
LFP	60	-	140	110
LR	90	130	-	60
LP	130	120	80	-

END

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