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Mathematical sensitivity analysis – Adding constraints

Using the Limpopo furniture company, we will add constraints:

Primary Linear Programming Model								
	Desk		Table		Chair			
Max z	60x1	+	30x2	+	20x3			
s.t.								
	18x1	+	6x2	+	x3	≤	48	Lumber
	24x1	+	2x2	+	1.5x3	≤	20	Finishing
	32x1	+	1.5x2	+	0.5x3	≤	8	Carpentry
	x1, x2, x3	≥	0					

We will use the optimal table we got from using the Primal simplex algorithm:

T*	x1	x2	x3	s1	s2	s3	RHS
z	0	5	0	0	10	10	280
1	0	-2	0	1	2	-8	24
2	0	-2	1	0	2	-4	8
3	1	1 1/4	0	0	- 1/2	1 1/2	2

Mathematical sensitivity analysis – Adding constraints

Steps:

1. First write the constraint in canonical form and add it into the optimal table.
2. Second, make sure none of the decision variables that were basic, becomes non-basic. If so, first sort out the conflict.
3. Lastly, make sure the slack/excess added is a basic variable.

Example 1

Let's add the a constraint to restrict tables to a maximum of 5 -> $x_2 \leq 5$

T*	x1	x2	x3	s1	s2	s3	S4	RHS
z	0	5	0	0	10	10	0	280
1	0	-2	0	1	2	-8	0	24
2	0	-2	1	0	2	-4	0	8
3	1	1 1/4	0	0	- 1/2	1 1/2	0	2
4	0	1	0	0	0	0	1	5

In this case, x2 was non-basic so there is no conflict and S4 is basic.

Mathematical sensitivity analysis – Adding constraints

Example 2

Now let's add the constraint that will restrict chairs to a maximum of 5 →$x_3 \leq 5$

T*	x1	x2	x3	s1	s2	s3	S4	RHS
z	0	5	0	0	10	10	0	280
1	0	-2	0	1	2	-8	0	24
2	0	-2	1	0	2	-4	0	8
3	1	1 1/4	0	0	- 1/2	1 1/2	0	2
4	0	0	1	0	0	0	1	5

To solve this conflict, we will do the following:

b2	0	-2	1	0	2	-4	0	8
b4	0	0	1	0	0	0	1	5
b2 - b4	0	-2	0	0	2	-4	-1	3

T*	x1	x2	x3	s1	s2	s3	S4	RHS
***4 x -1**	0	2	0	0	-2	4	1	-3

Mathematical sensitivity analysis – Adding constraints

Add the new constraint into the optimal table:

T*	x1	x2	x3	s1	s2	s3	S4	RHS
z	0	5	0	0	10	10	0	280
1	0	-2	0	1	2	-8	0	24
2	0	-2	1	0	2	-4	0	8
3	1	1 1/4	0	0	- 1/2	1 1/2	0	2
4	0	2	0	0	-2	4	1	-3
Ratio test					5			

t-*	x1	x2	x3	s1	s2	s3	s4	RHS
z	0	15	0	0	0	30	5	265
1	0	0	0	1	0	-4	1	21
2	0	0	1	0	0	0	1	5
3	1	3/4	0	0	0	1/2	- 1/4	2 3/4
4	0	-1	0	0	1	-2	- 1/2	1 1/2

Let's compare the new optimal table with the optimal table before the constraint was added.

T*	x1	x2	x3	s1	s2	s3	RHS
z	0	5	0	0	10	10	280
1	0	-2	0	1	2	-8	24
2	0	-2	1	0	2	-4	8
3	1	1 1/4	0	0	- 1/2	1 1/2	2

Mathematical sensitivity analysis – Adding constraints

Example 3

Let’s add the a constraint to restrict desks to a minimum of 1 -> $x_1 \geq 1$

T*	x1	x2	x3	s1	s2	s3	e4	RHS
z	0	5	0	0	10	10	0	280
1	0	-2	0	1	2	-8	0	24
2	0	-2	1	0	2	-4	0	8
3	1	1 1/4	0	0	- 1/2	1 1/2	0	2
4	1	0	0	0	0	0	-1	1

To solve this conflict, we will do the following:

T*	x1	x2	x3	s1	s2	s3	e4	RHS
3	1	1 1/4	0	0	- 1/2	1 1/2	0	2
4	1	0	0	0	0	0	-1	1
***4**	0	1 1/4	0	0	- 1/2	1 1/2	1	1

Row 3 – Row 4

E4 is still basic, we can now add our new constraint to the optimal table:

Mathematical sensitivity analysis – Adding constraints

T*	x1	x2	x3	s1	s2	s3	e4	RHS
z	0	5	0	0	10	10	0	280
1	0	-2	0	1	2	-8	0	24
2	0	-2	1	0	2	-4	0	8
3	1	1 1/4	0	0	- 1/2	1 1/2	0	2
4	0	1 1/4	0	0	- 1/2	1 1/2	1	1

Exercises

1. Add a constraint that will restrict tables to manufacture a minimum of 1.
2. Korean Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Korean Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programmes: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2 million high-income women and 12 million high-income men. A 1-minute comedy ad costs ₩50 000, and a 1-minute football ad costs ₩100 000. Korean Auto would like the commercials to be seen by at least 28 million high-income women and 24 million high-income men. The formulated Linear Programming Model for the above mentioned problem is given below:

Primary Linear Programming Model						
		Comedy Add		Football Add		
Min z	50x1	+	100x2			
s.t.						
	17x1	+	2x2	≥	28HIW	
	22x1	+	12x2	≥	24HIM	
	x1, x2	≥	0			

Exercises

Having solved the above formulated problem using a Primal Simplex Algorithm, the Optimal Tableau is given below:

t-3*	x1	x2	e1	e2	rhs
Z	0	0	-5	-7 1/2	320
1	1	0	- 3/20	1/40	3 3/5
2	0	1	1/40	- 7/80	1 2/5

- Add the constraint that will allow a maximum of 3 comedy adds.
- Add the constraint that will allow a minimum of 4 comedy adds.
- Add the constraint that will allow a minimum of 2 football adds.

END

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