Botlhale Village

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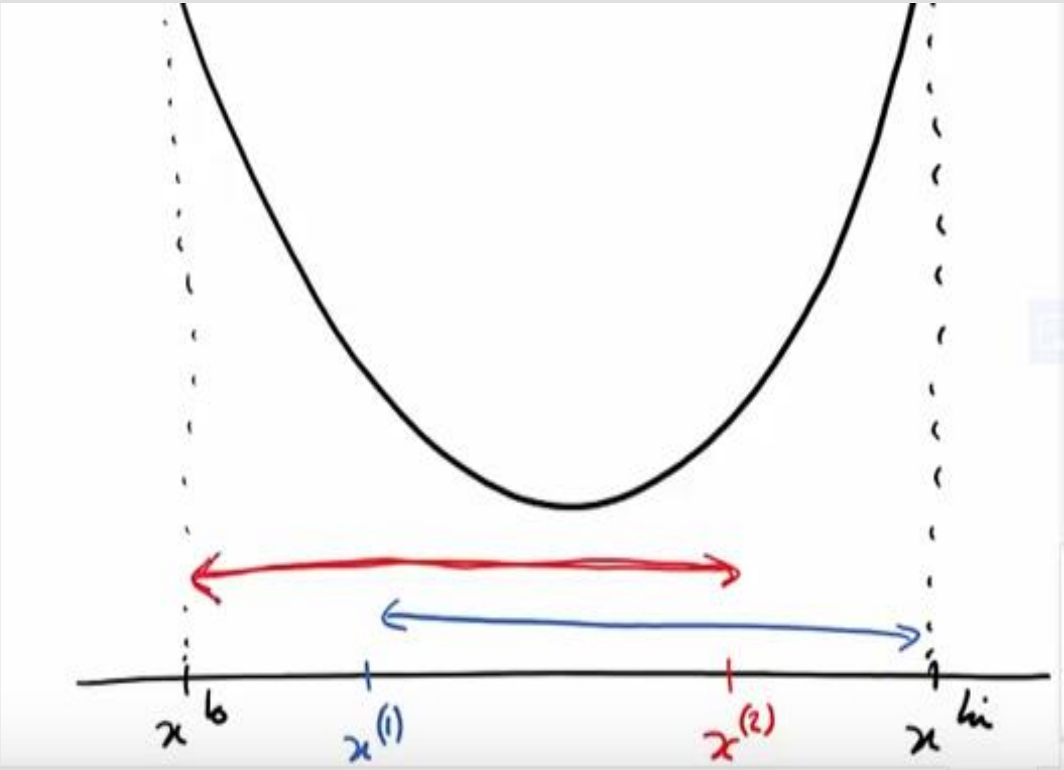
ARE YOU READY?

GOLDEN SECTION SEARCH

- This method divides the search more efficiently closing in on the optima in fewer iterations than the equal interval method.
- We will use the Golden Ratio = 1,618… The **Golden ratio** is a special number found by dividing a line into two parts so that the longer part divided by the smaller part is also equal to the whole length divided by the longer part. It is often symbolized using phi, after the 21st letter of the Greek alphabet.
- We will use the Golden section ratio conjugate which is =0,618…
- The Golden Section Search method is used to find the maximum or minimum of a unimodal function (A unimodal function contains only one minimum or maximum on the interval [a, b].)
- With this method, it uses an interval for the function and try to get a smaller and smaller interval until it reaches the minimum or maximum point.
- We need to make sure the interval we choose is unimodal (there must be at least one minimum or maximum).

GOLDEN SECTION SEARCH

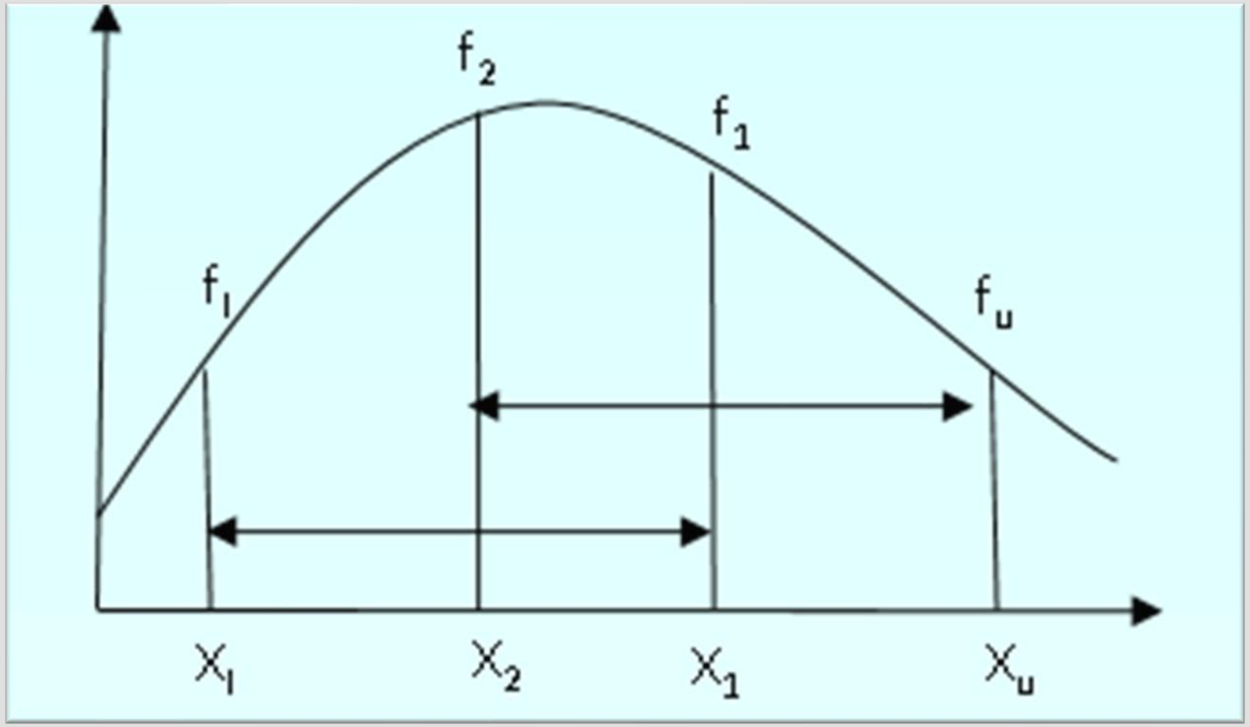
- Let's focus on a minimum function first:
- The distance between x-low and x2 is the same distance between x-high and x1. We are dealing with a ratio and we can use the golden ratio to determine which ratio will be our next search region.



DETERMINING THE NEW SEARCH REGION

Now showing with a maximum function:

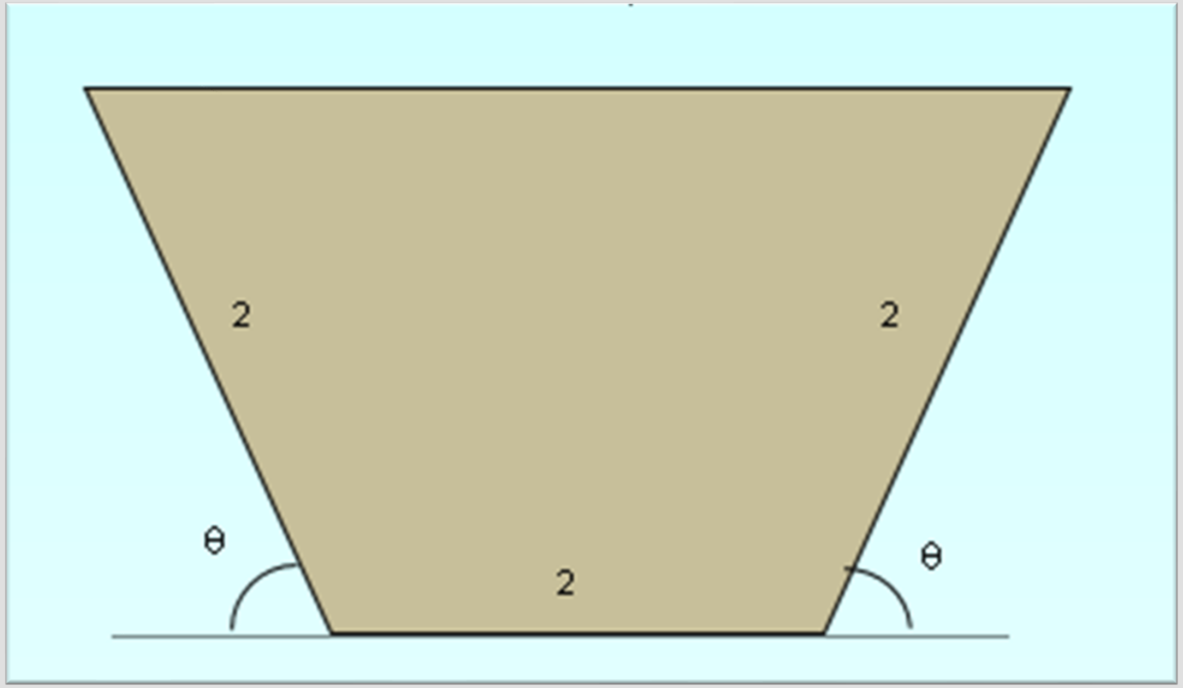
- If $f(x_2) > f(x_1)$ then the new interval is $[x_{lower},\ x_2,\ x_1]$
- If $f(x_2) < f(x_1)$ then the new interval is $[x_2,\ x_1,\ x_{upper}]$
- All that is left to do is to determine the location of the second intermediate point.



EXAMPLE

- The cross-sectional area of a gutter with equal base and edge length of 2 is given by:
  - $A = 4sin\theta(1 + cos\theta)$

- Find the angle θ which **maximizes** the cross-sectional area of the gutter. Using an initial interval of $\left[0, \frac{\pi}{2}\right]$ use an initial $\varepsilon = 0.05$. Show the calculations for the first two iterations.



EXAMPLE

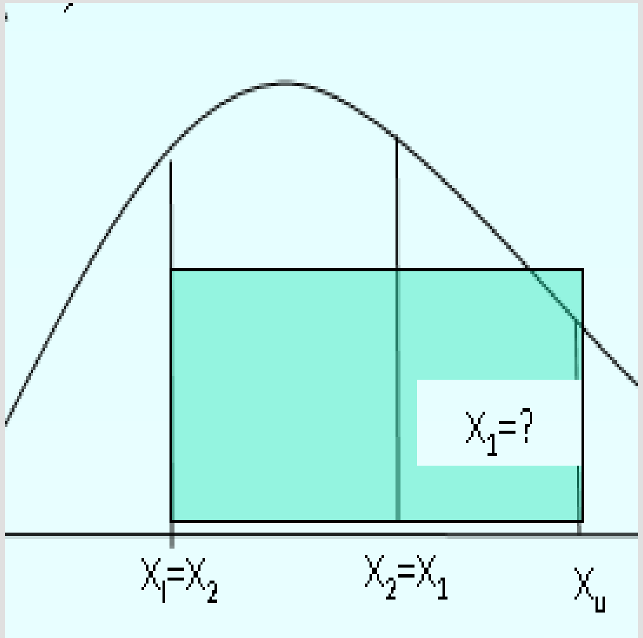
- The function to be maximized is $f(x) = 4sinx(1 + cosx)$
- **Iteration 1:** Given the values for the boundaries of $x_{lower} = 0$ and $x_{upper} = \frac{\pi}{2}$; we can calculate the initial intermediate points as follows:
  - $d = \frac{\sqrt{5}-1}{2} \times (x_{upper} - x_{lower}) = \frac{\sqrt{5}-1}{2} \times \left(\frac{\pi}{2}\right) = 0{,}97080552$
  - $X_1 = X_{lower} +$ d$=$ $0{,}97080552$
  - $=> f(0{,}97080552)$=5,16542807
  - $X_2 = X_{upper} -$ d$=$ $0{,}59999081$
  - $=> f(0{,}59999081)$=4,12260439

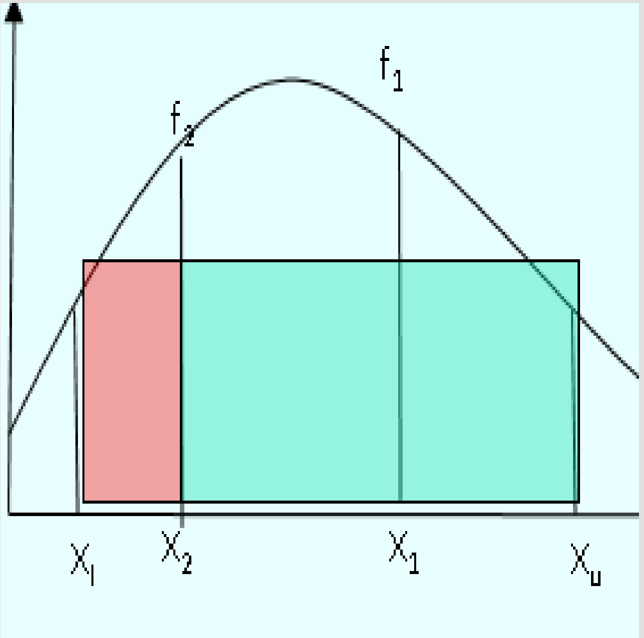
EXAMPLE

- To check the stopping criteria; the difference between x_{upper} andx_{lower} must be calculated and we can stop once this value is < 0,05:
  - $x_{upper} - x_{lower} = 1{,}57079633 - 0$

$= 1{,}57079633$

- We can see that $f(x_1)$ is the largest of the two and we need to maximize the function. Showing us that x_1 is our center point and $x_2 = x_{lower}$. Eliminating the previous lower bound.
- Our upper bound remains the same.





EXAMPLE

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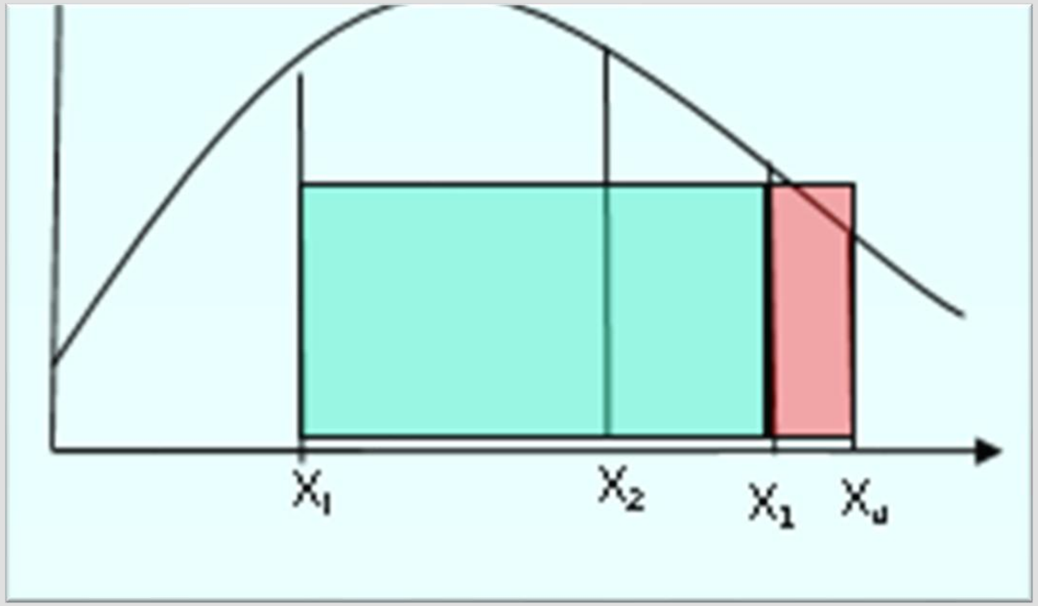
- **Iteration 2:** Now our lower boundary changed.
  - $\boldsymbol{x_{lower} = 0,59999081, x_{upper} = 1,57079633}$
- we can calculate the intermediate points as follows:
  - $d = \frac{\sqrt{5}-1}{2} \times \left(x_{upper} - x_{lower}\right) = \frac{\sqrt{5}-1}{2} \times (0{,}97080552)$ =0,59999081
  - $X_1 = X_{lower} +$ d= $1{,}19998161 \;=>\; f(1{,}19998161)$=5,07911028
  - $X_2 = X_{upper} -$ d= $0{,}97080552 \;=>\; f(0{,}97080552$)=5,16542807

EXAMPLE

- To check the stopping criteria; the difference between x_{upper} and x_{lower} must be calculated and we can stop once this value is < 0,05:
  - $x_{upper} - x_{lower} = \mathbf{1,57079633 - 0,59999081} =$ 0,97080552 (not <0,05 so we need to iterate again)
- We can see that $f(x_2)$ is the largest of the two and we need to maximize the function. Showing us that x_2 is our center point and $x_1 = x_{upper}$. Eliminating the previous upper bound.
- Our lower bound remains the same.



EXAMPLE

- The question asked that we show the calculations for the first two iterations only. Now we can complete our table without showing calculations:

xlower	xupper	d	x1	x2	f(x1)	f(x2)	ε
0	1,57079633	0,97080552	0,97080552	0,59999081	5,16542807	4,12260439	1,57079633
0,59999081	1,57079633	0,59999081	1,19998161	0,97080552	5,07911028	5,16542807	0,97080552
0,59999081	1,19998161	0,37081471	0,97080552	0,8291669	5,16542807	4,94181759	0,59999081
0,8291669	1,19998161	0,2291761	1,058343	0,97080552	5,19550836	5,16542807	0,37081471
0,97080552	1,19998161	0,14163862	1,11244414	1,058343	5,17433279	5,19550836	0,2291761
0,97080552	1,11244414	0,08753748	1,058343	1,02490666	5,19550836	5,19355978	0,14163862
1,02490666	1,11244414	0,05410114	1,07900779	1,058343	5,19092799	5,19550836	0,08753748
1,02490666	1,07900779	0,03343634	1,058343	1,04557145	5,19550836	5,19613868	0,05410114
1,02490666	1,058343	0,0206648	1,04557145	1,0376782	5,19613868	5,19568071	0,03343634

If you need to minimize the objective function, you need to choose the smallest between $f(x_1)$ and $f(x_2)$

EXAMPLE

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- Now that we reached the stopping criteria, we can conclude what the optimal solution is by using the following formula:
  - $\frac{x_{upper}+x_{lower}}{2} = \frac{1,058343+1,02490666}{2} = 1,04162483$
  - $f(1,04162483) = 5,19599088$

- Thus we can conclude that the optimal solution to the problem happens at approximately 60 degrees which is 1,0472 radians ($1,04162483 \times \frac{180}{\pi}$) and gives a maximum cross-sectional area of 5,19599088.

Exercises

1. Using the Golden Section Search Algorithm, minimize the following function: $f(x) = x^3 - 6x$ starting at the interval [0; 3]. Show the calculations for the first iteration.
2. Using the Golden Section Search Algorithm, maximize the following function: $f(x) = -x^2 - 1$ starting at the interval $-1 \leq x \leq 0.75$. Show the calculations for the first iteration. The final interval of uncertainty having a length of 0.25
3. Using the Golden Section Search Algorithm, maximize the following function: $f(x) = x - e^x$ starting at the interval $-1 \leq x \leq 3$. Show the calculations for the first iteration. The final interval of uncertainty having a length of 0.6

END

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