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ARE YOU READY?

SCENARIO

Gandalf the Grey lives in Gondor, Middle Earth. He is responsible for the communities in Gondor, Isengard, Rivendell, Lothlórien, and Rohan. Each Yule, he visits each of these communities. The distance between each community (called cities in the problem, in miles) is shown in the table.

	1. Gondor	2. Isengard	3. Rivendell	4. Lothlorien	5. Rohan
1. Gondor	0	132	217	164	58
2. Isengard	132	0	290	201	79
3. Rivendell	217	290	0	113	303
4. Lothlorien	164	201	113	0	196
5. Rohan	58	79	303	196	0

SCENARIO

What order of visiting the communities will minimise the total distance travelled?

Formulating the IP

- Decision variables:

$x_{ij} = if\ Gandalf\ leaves\ city\ i\ and\ travels\ next\ to\ city\ j\ (1)\ or\ not\ (0)\ where$ *i=j=1=Gondor, 2=Isengard, 3=Rivendell, 4=Lothlorien, 5=Rohan*

- Objective function:

$$\min z = 132x_{12} + 217x_{13} + 164x_{14} + 58x_{15} + 132x_{21} + 290x_{23} + 201x_{24} + 79x_{25} + 217x_{31} + 290x_{32} + 113x_{34} + 303x_{35} + 164x_{41} + 201x_{42} + 113x_{43} + 196x_{45} + 58x_{51} + 79x_{52} + 303x_{53} + 196x_{54}$$

- Arriving once in a city constraints:

$$x_{21} + x_{31} + x_{41} + x_{51} = 1$$
$$x_{12} + x_{32} + x_{42} + x_{52} = 1$$
$$x_{13} + x_{23} + x_{43} + x_{53} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{54} = 1$$
$$x_{15} + x_{25} + x_{35} + x_{45} = 1$$

Formulating the IP

- Leaving a city once constraints:

$$x_{12} + x_{13} + x_{14} + x_{15} = 1$$
$$x_{21} + x_{23} + x_{24} + x_{25} = 1$$
$$x_{31} + x_{32} + x_{34} + x_{35} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{45} = 1$$
$$x_{51} + x_{52} + x_{53} + x_{54} = 1$$

- Sign restrictions:

$$x_{ij} = 0 \; or \; 1$$

Using Solver in the next slides is just to help you understand how to find any sub tours, but you will not set up the IP with Solver so you need to include all sub tours in your IP.

Solver without sub-tour constraints

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$x_{34} = 1$ $x_{43} = 1$

	x11	x12	x13	x14	x15	x21	x22	x23	x24	x25	x31	x32	x33	x34	x35	x41	x42	x43	x44	x45	x51	x52	x53	x54	x55	ref.	sign	cons.
Var	0	1	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0			
min z	2000	132	217	164	58	132	2000	290	201	79	217	290	2000	113	303	164	201	113	2000	196	58	79	303	196	2000	495		
s.t. 1	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	=	1
2	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	1	=	1
3	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	=	1
4	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	1	=	1
5	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	1	=	1
6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
7	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	=	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	=	1
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	=	1

$x_{12} = 1$ $x_{25} = 1$ $x_{51} = 1$

Add constraint to eliminate sub-tour $\boldsymbol{x_{34}}$->$\boldsymbol{x_{43}}$

- Decision variables:

$$U_i = dummy\ variable$$

- First sub-tour constraints:

$$U_3 - U_4 + 5x_{34} \leq 4$$
$$U_4 - U_3 + 5x_{43} \leq 4$$

- Sign restrictions:

$$U_i \geq 0\ and\ integers$$

Solver without all sub-tour constraints

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$x_{25} = 1$ $x_{52} = 1$

	x11	x12	x13	x14	x15	x21	x22	x23	x24	x25	x31	x32	x33	x34	x35	x41	x42	x43	x44	x45	x51	x52	x53	x54	x55	U3	U4	ref.	sign	cons.
Var	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	4	0			
min z	2000	132	217	164	58	132	2000	290	201	79	217	290	2000	113	303	164	201	113	2000	196	58	79	303	196	2000	0	0	652		
s.t. 1	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	=	1
2	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	=	1
3	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	=	1
4	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	1	=	1
5	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	=	1
6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
7	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	1	=	1
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	1	=	1
11	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	1	-1	4	<=	4
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	-1	1	1	<=	4

$x_{14} = 1$ $x_{43} = 1$ $x_{31} = 1$

Add constraint to eliminate sub-tour $\boldsymbol{x_{25}}$->$\boldsymbol{x_{52}}$

- Next sub-tour constraints:

$$U_2 - U_5 + 5x_{25} \leq 4$$
$$U_5 - U_2 + 5x_{52} \leq 4$$

Add constraint to eliminate sub-tour x_{25}->x_{52}

$x_{15} = 1$ $x_{52} = 1$ $x_{24} = 1$ $x_{43} = 1$ $x_{31} = 1$

	x11	x12	x13	x14	x15	x21	x22	x23	x24	x25	x31	x32	x33	x34	x35	x41	x42	x43	x44	x45	x51	x52	x53	x54	x55	U3	U4	U2	U5	ref.	sign	cons.
Var	0	0	0	0	1	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0			
min z	2000	132	217	164	58	132	2000	290	201	79	217	290	2000	113	303	164	201	113	2000	196	58	79	303	196	2000	0	0	0	0	668		
s.t. 1	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	=	1
2	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	1	=	1
3	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	1	=	1
4	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	=	1
5	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	=	1
6	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
7	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
8	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	=	1
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	1	=	1
11	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	0	1	<=	4
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	-1	1	0	0	4	<=	4
13	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	1	<=	4
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	0	0	0	0	-1	1	4	<=	4

Final IP

x_{ij} = *if Gandalf leaves city i and travels next to city j (1) or not (0) where i=j=1=Gondor, 2=Isengard, 3=Rivendell, 4=Lothlorien, 5=Rohan*

$$U_i = dummy\ variable$$

$$\begin{aligned} \min z = 132x_{12} + 217x_{13} + 164x_{14} + 58x_{15} + \\ 132x_{21} + 290x_{23} + 201x_{24} + 79x_{25} + \\ 217x_{31} + 290x_{32} + 113x_{34} + 303x_{35} + \\ 164x_{41} + 201x_{42} + 113x_{43} + 196x_{45} + \\ 58x_{51} + 79x_{52} + 303x_{53} + 196x_{54} \end{aligned}$$

Arriving once in a city constraints:

$$x_{21} + x_{31} + x_{41} + x_{51} = 1$$
$$x_{12} + x_{32} + x_{42} + x_{52} = 1$$
$$x_{13} + x_{23} + x_{43} + x_{53} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{54} = 1$$
$$x_{15} + x_{25} + x_{35} + x_{45} = 1$$

Formulating the IP

- Leaving a city once constraints:

$$x_{12} + x_{13} + x_{14} + x_{15} = 1$$
$$x_{21} + x_{23} + x_{24} + x_{25} = 1$$
$$x_{31} + x_{32} + x_{34} + x_{35} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{45} = 1$$
$$x_{51} + x_{52} + x_{53} + x_{54} = 1$$

- Sub-tour constraints:

$$U_2 - U_3 + 5x_{23} \leq 4$$
$$U_2 - U_4 + 5x_{24} \leq 4$$
$$U_2 - U_5 + 5x_{25} \leq 4$$
$$U_3 - U_2 + 5x_{32} \leq 4$$
$$U_3 - U_4 + 5x_{34} \leq 4$$

Formulating the IP

$$U_3 - U_5 + 5x_{35} \leq 4$$
$$U_4 - U_2 + 5x_{42} \leq 4$$
$$U_4 - U_3 + 5x_{43} \leq 4$$
$$U_4 - U_5 + 5x_{45} \leq 4$$
$$U_5 - U_2 + 5x_{52} \leq 4$$
$$U_5 - U_3 + 5x_{53} \leq 4$$
$$U_5 - U_4 + 5x_{54} \leq 4$$

- Sign restrictions:

$$x_{ij} = 0 \; or \; 1$$
$$U_i \geq 0 \; and \; integers$$

Testing subtour constraints

Test:

$$x_{15} = 1 \rightarrow x_{52} = 1 \rightarrow x_{24} = 1 \rightarrow x_{43} = 1 \rightarrow x_{31} = 1$$

$$U_5 = 0 \rightarrow U_2 = 1 \rightarrow U_4 = 2 \rightarrow U_3 = 3 \rightarrow U_1 = 0$$

U_5	$-$	U_2	$+$	$5x_{52}$	$\leq$	4	
U_2	$-$	U_5	$+$	$5x_{25}$	$\leq$	4	
(0)	$-$	(1)	$+$	5(1)	$\leq$	4	TRUE
(1)	$-$	(0)	$+$	5(0)	$\leq$	4	TRUE
(0)	$-$	(1)	$+$	5(1)	$\leq$	4	TRUE
(1)	$-$	(0)	$+$	5(1)	$\leq$	4	FALSE

SOLVING USING HEURISTIC – NEAREST NEIGHBOUR

	1. Gondor	2. Isengard	3. Rivendell	4. Lothlorien	5. Rohan
1. Gondor	0	132	217	164	58
2. Isengard	132	0	290	201	79
3. Rivendell	217	290	0	113	303
4. Lothlorien	164	201	113	0	196
5. Rohan	58	79	303	196	0

SOLVING USING HEURISTIC – NEAREST NEIGHBOUR (NNH)

		Routes Available			
Initial Route	x_{1i}	x_{12}	x_{13}	x_{14}	x_{15}
Route 1	x_{15}	x_{52}	x_{53}	x_{54}	
Route 2	$x_{15} \rightarrow x_{52}$	x_{23}	x_{24}		
Route 3	$x_{15} \rightarrow x_{52} \rightarrow x_{24}$	x_{43}			
Route 4	$x_{15} \rightarrow x_{52} \rightarrow x_{24} \rightarrow x_{43}$	none			
Return	$x_{15} \rightarrow x_{52} \rightarrow x_{24} \rightarrow x_{43} \rightarrow x_{31}$				
Z=	$x_{15} \rightarrow x_{52} \rightarrow x_{24} \rightarrow x_{43} \rightarrow x_{31}$	$58 + 79 + 201 + 113 + 271$			
Z=	668				

SOLVING USING HEURISTIC – CHEAPEST INSERTION

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	1. Gondor	2. Isengard	3. Rivendell	4. Lothlorien	5. Rohan
1. Gondor	0	132	217	164	58
2. Isengard	132	0	290	201	79
3. Rivendell	217	290	0	113	303
4. Lothlorien	164	201	113	0	196
5. Rohan	58	79	303	196	0

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

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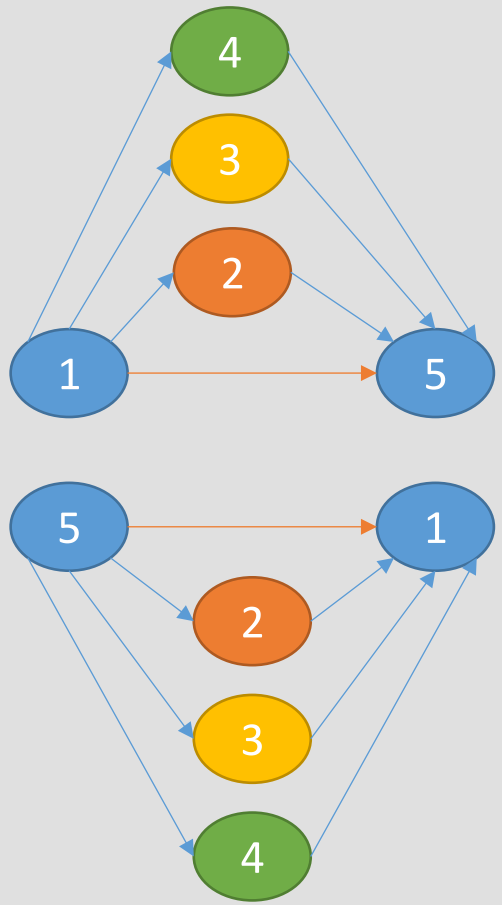
Route		Detour				Detour Length
x_{15}	58	x_{12}	x_{25}	132	79	$= (132 + 79) - 58 = 153$
x_{15}	58	x_{13}	x_{35}	271	303	$= (217 + 303) - 58 = 462$
x_{15}	58	x_{14}	x_{45}	164	196	$= (164 + 196) - 58 = 302$
x_{51}	58	x_{52}	x_{21}	79	132	$= (79 + 132) - 58 = 153$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$
x_{51}	58	x_{54}	x_{41}	196	164	$= (196 + 164) - 58 = 302$



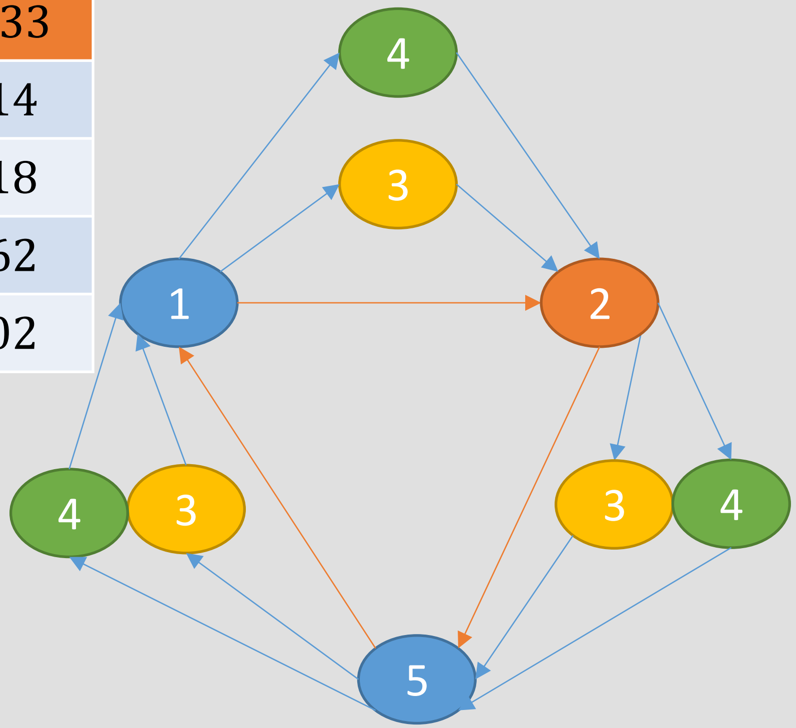
$$x_{12} \rightarrow x_{25} \rightarrow x_{51}$$

Due to the table being symmetrical, x15 and x51 will be the same amounts (we need to check from the origin as well as to the origin if the to is maybe better). To keep the same standard, we start with the origin route (pick the top one).

SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

Route		Detour				Detour Length
x_{12}	132	x_{13}	x_{32}	217	290	$= (217 + 290) - 132 = 375$
x_{12}	132	x_{14}	x_{42}	164	201	$= (164 + 201) - 132 = 233$
x_{25}	79	x_{23}	x_{35}	290	303	$= (290 + 303) - 79 = 514$
x_{25}	79	x_{24}	x_{45}	201	196	$= (201 + 196) - 79 = 318$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$
x_{51}	58	x_{54}	x_{41}	196	164	$= (196 + 164) - 58 = 302$

$$x_{14} \to x_{42} \to x_{25} \to x_{51}$$



SOLVING USING HEURISTIC – CHEAPEST INSERTION (CIH)

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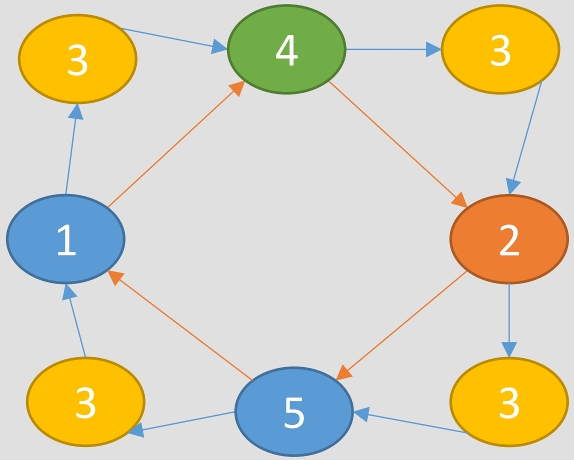
Route		Detour				Detour Length
x_{14}	164	x_{13}	x_{34}	217	113	$= (217 + 113) - 164 = 166$
x_{42}	201	x_{43}	x_{32}	113	290	$= (113 + 290) - 201 = 202$
x_{25}	79	x_{23}	x_{35}	290	303	$= (290 + 303) - 79 = 514$
x_{51}	58	x_{53}	x_{31}	303	217	$= (303 + 217) - 58 = 462$



$$x_{13} \rightarrow x_{34} \rightarrow x_{42} \rightarrow x_{25} \rightarrow x_{51}$$

$$z = 217 + 113 + \ 201 + \ 79 + \ 58$$

$$z = 668$$

Exercises

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1. Each day, African Petroleum manufactures four types of petrol: lead-free premium (LFP), lead-free regular (LFR), leaded premium (LP), and leaded regular (LR). Because of cleaning and resetting of machinery, the time required to produce a batch of petrol depends on the type of petrol last produced. For example, it takes longer to switch between a lead-free petrol and a leaded petrol than it does to switch between two lead-free types of petrol. The time (in minutes) required to manufacture each day's petrol requirements are shown in the symmetrical table. Assume that you need to start with the LFR petrol.

Last-produced petrol	Petrol to be produced next			
	LFR	LFP	LR	LP
LFR	-	50	120	140
LFP		-	140	110
LR			-	60
LP				-

Exercises

- Formulate an Integer Programming Model that will solve the given problem.
- Solve the formulated Integer Programming Model using Solver.
- Solve the formulated Integer Programming Model using the NNH and CIH.

END

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