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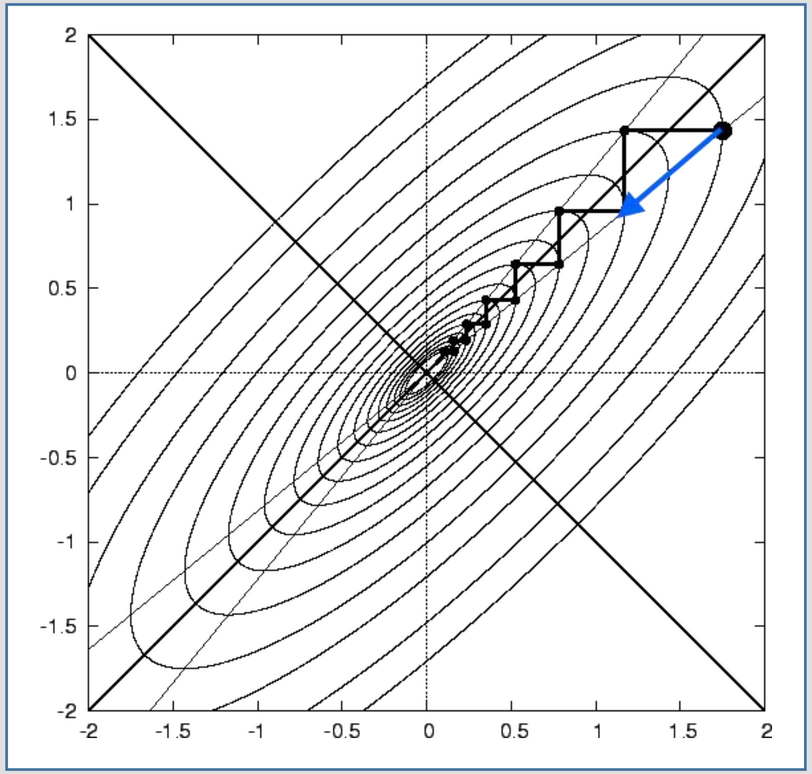
STEEPEST ASCENT/DESCENT ALGORITHM

- Also known as the multi-dimensional gradient method



MULTI-VARIABLE SEARCH

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- We will break down a multi-variable problem into a single-variable problem in which we can use the Golden Section Search Algorithm.
- The method of steepest ascent is a method whereby the experimenter proceeds sequentially along the path of steepest ascent, that is, along the path of maximum increase in the predicted response.
- The method of steepest descent is a method whereby the experimenter proceeds sequentially along the path of steepest descent, that is, along the path of maximum decrease in the predicted response.

STEEPEST ASCENT/DESCENT ALGORITHM

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1. Step 1: Start from an initial guessed point and look for a local optimal solution along the gradient.
2. Step 2: The gradient at the initial solution is calculated to find a direction to travel. The gradient of the function can be denoted as $\nabla f(x)$.
3. Step 3: Find the step size "h" along the calculated direction (can also use the golden section algorithm or Analytical Method).
4. Step 4: Substitute "h" back into formula and find the new coordinate. Repeat steps 2, 3 and 4 until you find an approximate solution. You might need to substitute the new coordinate into your objective function to see if you are moving in the correct direction at first (this depends on how many local optimal points there are and where you initially started). If you want to find a maximum, you will want to objective function value to increase and if you want to find a minimum , you will want to objective function value to decrease.

Note: When the step size changes direction, you can assume you were at a local minimum/maximum previously (or more accurately close to one). The same applies when the gradient at a specific point is 0 (then you are on a local minimum/maximum).

DETERMINE THE MINIMUM FUNCTION F(X,Y)=$x^2 + y^2 + 2x + 4$

- Use the point $\vec{x}_0 = \begin{Bmatrix} x_0 \\ y_0 \end{Bmatrix} = \{2, 1\}$ as the initial estimate (this is our initial guess) of the optimal solution.
- Check the min function f(2, 1)=13
- **Iteration 1:** To calculate the gradient; the partial derivatives must be evaluated as follows:
  - $\frac{\partial f}{\partial x} = 2x + 2$
  - $\frac{\partial f}{\partial y} = 2y$
  - **Now we have the gradient of the function, but we need to evaluate the gradient at the initial point.**
  - Substitute the initial point: $\frac{\partial f}{\partial x} = 2(2) + 2 = 6$ and $\frac{\partial f}{\partial y} = 2(1) = 2$

DETERMINE THE MINIMUM FUNCTION F(X,Y)=$x^2 + y^2 + 2x + 4$

- Now we have a vector for our gradient $\nabla f = 6i + 2j$
- This gradient will tell us in which direction we should move and we will use the following formula to determine our new point:
  - $\vec{x}_{(i+1)} = \vec{x}_{(i)} + h\nabla\vec{f}$
    - h = the step size and we don't know the size.
  - $\vec{x}_{(i+1)} = \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} + h \begin{Bmatrix} 6 \\ 2 \end{Bmatrix} = \begin{Bmatrix} 2 + 6h \\ 1 + 2h \end{Bmatrix}$ **(we use the gradient we just obtained.**
- Now we will substitute our values into our function:
  - $f\left(\vec{x}_{(i+1)}\right) = (2 + 6h)^2 + (1 + 2h)^2 + 2(2 + 6h) + 4 = g(h)$
  - $g(h) = 40h^2 + 40h + 13$

DETERMINE THE MINIMUM FUNCTION F(X,Y)=$x^2 + y^2 + 2x + 4$

- With this new function $g(h)$, we can use the Golden Section Search to find the optimal solution, because we only have one variable (h).
- OR because this is a simple function, we can take the first derivative of $g(h)$ and determine h. To get g_{min} , we set $\frac{dg}{dh} = 0 = 80h + 40 => h = -0.5$
- Now we saw that the step size to minimise the function *f* is $h = -0.5$
- We can no substitute h into $\begin{Bmatrix} 2 + 6h \\ 1 + 2h \end{Bmatrix} = \begin{Bmatrix} 2 + 6(-0{,}5) \\ 1 + 2(-0{,}5) \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix}$ **which will give us our new point.**

DETERMINE THE MINIMUM FUNCTION F(X,Y)=$\boldsymbol{x^2 + y^2 + 2x + 4}$

- Let's check if this new point is a better solution:
  - $f(2,1) = 13$ and $f(-1,0) = 3$;This shows that we have a better solution as we are looking for a minimum.
- **Iteration 2:** Now our initial point is $(-1,0)$ and we need to calculate the gradient at this point.
  - $\boldsymbol{\frac{\partial f}{\partial x} = 2x + 2}$
  - $\boldsymbol{\frac{\partial f}{\partial y} = 2y}$
  - **We still have the gradient of the function, but we need to evaluate the gradient at the initial point.**
  - Substitute the initial point: $\boldsymbol{\frac{\partial f}{\partial x} = 2(-1) + 2 = 0}$ and $\boldsymbol{\frac{\partial f}{\partial y} = 2(0) = 0}$

DETERMINE THE MINIMUM FUNCTION F(X,Y)=$x^2 + y^2 + 2x + 4$

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- Now we have a vector for our gradient $\nabla f = 0i + 0j$
- This shows us that the gradient is equals to 0 and thus indicated that we have our optimal solution.
- The minimum of our function is at point $(-1, 0)$ and $f_{min}(-1, 0) = 3$

Exercises

1. Using the Hessian matrix, prove that the following function has a local maximum: $f(x, y) = xy + y - x^2 - y^2$.
2. Using the steepest ascent/descent algorithm, find the local maximum starting at the point [1; 1] for the previous question. Stop after the 6th iteration.
3. Using the Hessian matrix, prove that the following function has a local maximum: $f(x_1, x_2) = -(x_1 - 3)^2 - (x_2 - 2)^2$.
4. Using the steepest ascent/descent algorithm, find the local maximum starting at the point [1; 1] for the previous question.

END

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