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# ARE YOU READY?

# SCENARIO



Four jobs must be processed on a single machine. The time required to process each job and the date the job is due are shown in the table.

| Job | Days required to complete Job | Due date |
|---|---|---|
| 1 | 6 | End of day 8 |
| 2 | 4 | End of day 4 |
| 3 | 5 | End of day 12 |
| 4 | 8 | End of day 16 |

# SCENARIO



The delay of a job is the number of days after the due date that a job is completed (if a job is completed on time or early, the job's delay is zero).

In what order should the jobs be processed to minimise the total delay of the four jobs?

# Formulating the IP

- Decision variables:

$x_{ij} = if\ job\ i\ is\ completed\ in\ position\ j\ (1)\ or\ not\ (0)\ where\ i=j=1, 2, 3, 4$

- Sign restrictions:

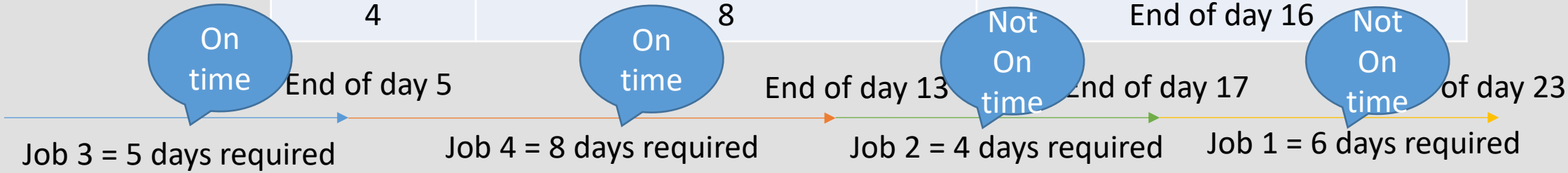
$$xij = 0\ or\ 1$$

# EXAMPLE - > NOT THE SOLUTION



Four jobs must be processed on a single machine. The time required to process each job and the date the job is due are shown in the table.

| Job | Days required to complete Job | Due date |
|---|---|---|
| 1 | 6 | End of day 8 |
| 2 | 4 | End of day 4 |
| 3 | 5 | End of day 12 |
| 4 | 8 | End of day 16 |



# Branch and Bound Machine Scheduling Algorithm

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Position 4

| Problem 1 | Problem 2 | Problem 3 | Problem 4 |
|---|---|---|---|
| x14 | x24 | x34 | x44 |
| Time required= 6+4+5+8= 23 days | Time required= 6+4+5+8= 23 days | Time required= 6+4+5+8= 23 days | Time required= 6+4+5+8= 23 days |
| Overdue= 23-8= 15 days | Overdue= 23-4= 19 days | Overdue= 23-12= 11 days | Overdue= 23-16= 7 days * |
| | | | |

# Branch and Bound Machine Scheduling Algorithm



Position 4 -> x44

Position 3

| Problem 4.1 | Problem 4.2 | Problem 4.3 |
|---|---|---|
| x44 & x13 | x44 & x23 | x44 & x33 |
| Time required= 6+4+5= 15 days | Time required= 6+4+5= 15 days | Time required= 6+4+5= 15 days |
| Overdue= 15-8= 7 days | Overdue= 15-4= 11 days | Overdue= 15-12= 3 days |
| Total overdue = 7+7=14 days | Total overdue = 7+11=18 days | Total overdue = 7+3=10 days * |
| | | |

# Branch and Bound Machine Scheduling Algorithm



Position 4 -> x44
Position 3 -> x33
Position 2

| Problem 4.3.1 |
| --- |
| x44 & x33 & x12 |
| Time required= 6+4= 10 days |
| Overdue= 10-8= 2 days |
| Total overdue = 10+2=12 days * |
| |

| Problem 4.3.2 |
| --- |
| x44 & x33 & x22 |
| Time required= 6+4= 10 days |
| Overdue= 10-4= 6 days |
| Total overdue = 10+6=16 days |
| |

# Branch and Bound Machine Scheduling Algorithm



Position 4 -> x44
Position 3 -> x33
Position 2 -> x12
Position 1

| Problem 4.3.1.1 |
| --- |
| x44 & x33 & x12 & x21 |
| Time required= 4 days |
| Overdue= 4-4= 0 days |
| Total overdue = 12+0= 12 days * |
| Candidate A |

As soon as you have a candidate (all jobs were placed), you need to back track level by level until the last position that were checked first. If you find a total overdue amount <= the total overdue amount of the candidate, you will branch from that sub problem to see if you will get something better or an alternative solution. If the sub-problem was branched, you ignore it. If the total overdue amount is > the total overdue amount of the candidate, it will not possibly improve so eliminate that sub-problem with the **current** candidate.

# Branch and Bound Machine Scheduling Algorithm



Position 4 -> x44
Position 3 -> x33
Position 2

| Problem 4.3.1 |
| --- |
| x44 & x33 & x12 |
| Time required= 6+4= 10 days |
| Overdue= 10-8= 2 days |
| Total overdue = 10+2=12 days * |
| |

| Problem 4.3.2 |
| --- |
| x44 & x33 & x22 |
| Time required= 6+4= 10 days |
| Overdue= 10-4= 6 days |
| Total overdue = 10+6=16 days |
| Eliminated by Candidate A |

# Branch and Bound Machine Scheduling Algorithm

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Position 4 -> x44

Position 3

| Problem 4.1 | Problem 4.2 | Problem 4.3 |
|---|---|---|
| x44 & x13 | x44 & x23 | x44 & x33 |
| Time required= 6+4+5= 15 days | Time required= 6+4+5= 15 days | Time required= 6+4+5= 15 days |
| Overdue= 15-8= 7 days | Overdue= 15-4= 11 days | Overdue= 15-12= 3 days |
| Total overdue = 7+7=14 days | Total overdue = 7+11=18 days | Total overdue = 7+3=10 days * |
| Eliminated by Candidate A | Eliminated by Candidate A | |

# Branch and Bound Machine Scheduling Algorithm

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Position 4

| Problem 1 | Problem 2 | Problem 3 | Problem 4 |
|---|---|---|---|
| x14 | x24 | x34 | x44 |
| Time required= 6+4+5+8= 23 days | Time required= 6+4+5+8= 23 days | Time required= 6+4+5+8= 23 days | Time required= 6+4+5+8= 23 days |
| Overdue= 23-8= 15 days | Overdue= 23-4= 19 days | Overdue= 23-12= 11 days* | Overdue= 23-16= 7 days * |
| Eliminated by Candidate A | Eliminated by Candidate A | | |

# Branch and Bound Machine Scheduling Algorithm



Position 4 -> x34

Position 3

| Problem 3.1 | Problem 3.2 | Problem 3.3 |
|---|---|---|
| x34 & x13 | x34 & x23 | x34 & x43 |
| Time required= 6+4+8= 18 days | Time required= 6+4+8= 18 days | Time required= 6+4+8= 18 days |
| Overdue= 18-8= 10 days | Overdue= 18-4= 14 days | Overdue= 18-16= 2 days |
| Total overdue= 11+10= 21 days | Total overdue= 11+14= 25 days | Total overdue= 11+2= 13 days |
| Eliminated by Candidate A | Eliminated by Candidate A | Eliminated by Candidate A |

# Branch and Bound Machine Scheduling Algorithm



Position 4 -> x44
Position 3 -> x33
Position 2 -> x12
Position 1 -> x21

| **Problem 4.3.1.1** |
| --- |
| x44 & x33 & x12 & x21 |
| Time required= 4 days |
| Overdue= 4-4= 0 days |
| Total overdue = 12+0= 12 days * |
| Candidate A |

# Exercises

Four jobs must be processed on a single machine. The time required performing each job and the due date of each job are shown in the table.

| Job | Time to perform job (Minutes) | Due date of job |
|---|---|---|
| 1 | 7 | End of minute 14 |
| 2 | 5 | End of minute 13 |
| 3 | 9 | End of minute 18 |
| 4 | 11 | End of minute 15 |

Solve the formulated Integer Programming Model using the Branch & Bound Machine Scheduling Algorithm.

# END

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