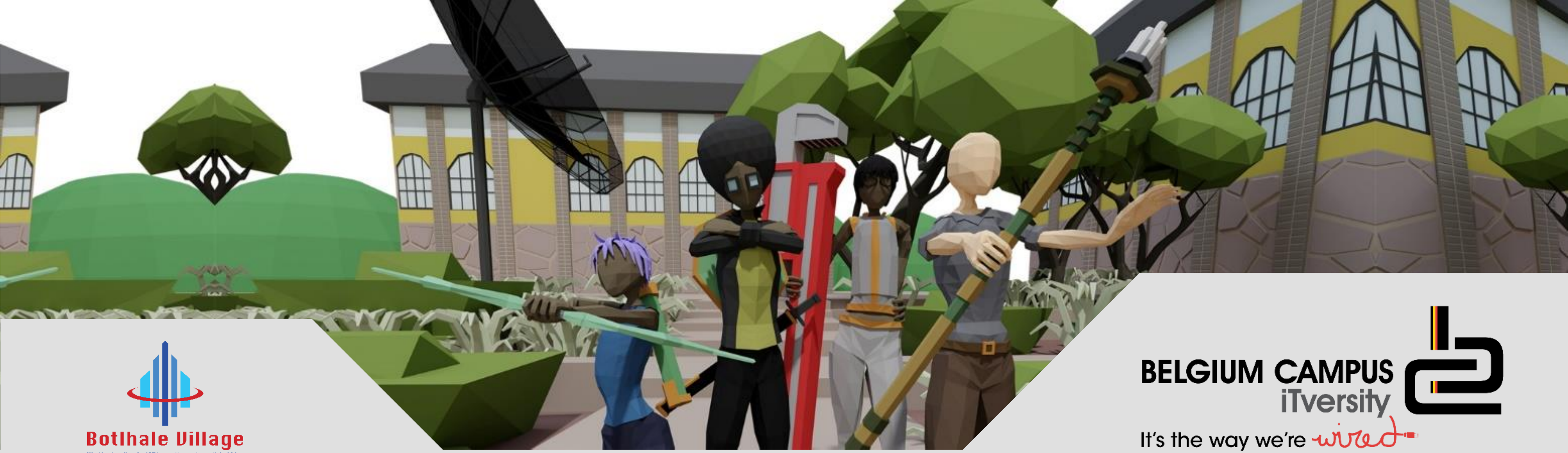

Botlhale Village

Working together for ICT innovation and growth in Africa

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ARE YOU READY?

Task Assignment IP

T-i	α	β	γ
A	1	2	4
B	5	3	4
C	5	4	8

Example 1

- Assign the agents A, B, C to the activities α, β, γ. The cost for each agent to perform each activity is given by the following cost matrix:
- First we need to declare the decision variables:
  $x_{ij} = Agent\ i\ assigned\ to\ activity\ j\ where\ i = 1 = A, 2 = B, 3 = C\ and\ j = 1 = \alpha, 2 = \beta, 3 = \gamma$
- Next we will create the objective function:
  $\min z = x_{11} + 2x_{12} + 4x_{13} + 5x_{21} + 3x_{22} + 4x_{23} + 5x_{31} + 4x_{32} + 8x_{33}$
- In the first few constraints we need to make sure that only one activity is assigned per agent:

$$
\begin{aligned}
s.t.\, x_{11} + x_{12} + x_{13} &= 1 \\
x_{21} + x_{22} + x_{23} &= 1 \\
x_{31} + x_{32} + x_{33} &= 1
\end{aligned}
$$

Task Assignment IP

- In the next few constraints we need to make sure that only one agent is assigned per activity:

$$
\begin{aligned}
s.t.\, x_{11} + x_{21} + x_{31} &= 1 \\
x_{12} + x_{22} + x_{32} &= 1 \\
x_{13} + x_{23} + x_{33} &= 1
\end{aligned}
$$

- Lastly, we need to add our sign restrictions:

$$x_{ij} = 0 \; or \; 1$$

Hungarian Algorithm

Steps:

- **Step 1**: Add dummy row/column if the matrix is not even.
- **Step 2**: Row reduction
- **Step 3**: Column reduction
- **Step 4**: Conditional check. If the amount of lines = matrix size, table is optimal, stop and continue to **Step 6**. If the amount of lines ≠ matrix size, table is not optimal, continue to **Step 5**.
- **Step 5a**: Choose the smallest value in the table that is not covered by any lines. We will call this value ***a***
- **Step 5b**: Subtract ***a*** from every uncovered element in the matrix.
- **Step 5c**: Add ***a*** to every element in the matrix that is covered twice (+where two lines cross).
- **Step 5d**: Leave the values as they are if they are covered by one line. Calculate the new matrix and go back to **Step 4**.
- **Step 6:** Select the appropriate 0s and apply the selection to the initial matrix, disregarding dummy rows or columns. Calculate the optimal task assignment z objective by calculating the sum of all selected values in the original matrix.

Hungarian Algorithm

- Now we will solve the Task assignment IP we created in the previous slide:

T-i	α	β	γ
A	1	2	4
B	5	3	4
C	5	4	8

- Row reduction:

T-i	α	β	γ
A	1	2	4
B	5	3	4
C	5	4	8

T-i	α	β	γ
A	0	1	3
B	2	0	1
C	1	0	4

- Column reduction:

T-i	α	β	γ
A	0	1	3
B	2	0	1
C	1	0	4

T-i	α	β	γ
A	0	1	2
B	2	0	0
C	1	0	3

Hungarian Algorithm

- Conditional check:

T-i	α	β	γ
A	0	1	2
B	2	0	0
C	1	0	3

- Choosing 0s:

T-2*	α	β	γ
A	0	1	2
B	2	0	0
C	1	0	3

- Substitute value from initial table:

T-2*	α	β	γ
A	1	2	4
B	5	3	4
C	5	4	8

- Total cost: $1 + 4 + 4 = 9$ or $\min z = x_{11} + 2x_{12} + 4x_{13} + 5x_{21} + 3x_{22} + 4x_{23} + 5x_{31} + 4x_{32} + 8x_{33}$
- $\min z = (1) + 2(0) + 4(0) + 5(0) + 3(0) + 4(1) + 5(0) + 4(1) + 8(0) = 9$

Task Assignment IP

Example 2

Assign the agents A, B, C, D, and E to the activities α, β, γ, and δ. The cost for each agent to perform each activity is given by the following cost matrix:

T-i	α	β	γ	δ
A	10	19	8	15
B	10	18	7	17
C	13	16	9	14
D	12	19	8	18
E	14	17	10	19

- First we need to declare the decision variables:

  $x_{ij} = Agent\ i\ assigned\ to\ activity\ j\ where\ i = 1 = A, 2 = B, 3 = C, 4 = D, 5 = E\ and\ j = 1 = \alpha, 2 = \beta, 3 = \gamma, 4 = \delta$

- Next we will create the objective function:

$$\min \quad \begin{aligned} z = 10x_{11} + 19x_{12} + 8x_{13} + 15x_{14} + \\ 10x_{21} + 18x_{22} + 7x_{23} + 17x_{24} + \\ 13x_{31} + 16x_{32} + 9x_{33} + 14x_{34} + \\ 12x_{41} + 19x_{42} + 8x_{43} + 18x_{44} + \\ 14x_{51} + 17x_{52} + 10x_{53} + 19x_{54} \end{aligned}$$

Task Assignment IP

- In the first few constraints we need to make sure that only one activity is assigned per agent:

$$
\begin{aligned}
s.t.\ x_{11} + x_{12} + x_{13} + x_{14} &\leq 1 \\
x_{21} + x_{22} + x_{23} + x_{24} &\leq 1 \\
x_{31} + x_{32} + x_{33} + x_{34} &\leq 1 \\
x_{41} + x_{42} + x_{43} + x_{44} &\leq 1 \\
x_{51} + x_{52} + x_{53} + x_{54} &\leq 1
\end{aligned}
$$

- In the next few constraints we need to make sure that only one agent is assigned per activity:

$$
\begin{aligned}
s.t.\ x_{11} + x_{21} + x_{31} + x_{41} + x_{51} &= 1 \\
x_{12} + x_{22} + x_{32} + x_{42} + x_{52} &= 1 \\
x_{13} + x_{23} + x_{33} + x_{43} + x_{53} &= 1 \\
x_{14} + x_{24} + x_{34} + x_{44} + x_{54} &= 1
\end{aligned}
$$

- Lastly, we need to add our sign restrictions:

$$x_{ij} = 0\ or\ 1$$

Hungarian Algorithm

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- Now we will solve the Task assignment IP we created in the previous slide:

T-i	α	β	γ	δ
A	10	19	8	15
B	10	18	7	17
C	13	16	9	14
D	12	19	8	18
E	14	17	10	19

- Add dummy column:

T-i	α	β	γ	δ	ε
A	10	19	8	15	19
B	10	18	7	17	19
C	13	16	9	14	19
D	12	19	8	18	19
E	14	17	10	19	19

- Row reduction:

T-2	α	β	γ	δ	ε
A	10	19	8	15	19
B	10	18	7	17	19
C	13	16	9	14	19
D	12	19	8	18	19
E	14	17	10	19	19

T-2	α	β	γ	δ	ε
A	2	11	0	7	11
B	3	11	0	10	12
C	4	7	0	5	10
D	4	11	0	10	11
E	4	7	0	9	9

Hungarian Algorithm

- Column reduction:

T-2	α	β	γ	δ	ε
A	2	11	0	7	11
B	3	11	0	10	12
C	4	7	0	5	10
D	4	11	0	10	11
E	4	7	0	9	9

T-2	α	β	γ	δ	ε
A	0	4	0	2	2
B	1	4	0	5	3
C	2	0	0	0	1
D	2	4	0	5	2
E	2	0	0	4	0

- Conditional check:

T-2	α	β	γ	δ	ε
A	0	4	0	2	2
B	1	4	0	5	3
C	2	0	0	0	1
D	2	4	0	5	2
E	2	0	0	4	0

- When the table is not optimal do the following:

T-3	α	β	γ	δ	ε
A	0	4	1	2	2
B	0	3	0	4	2
C	2	0	1	0	1
D	1	3	0	4	1
E	2	0	1	4	0

Hungarian Algorithm

- Conditional check:

T-3	α	β	γ	δ	ε
A	0	4	1	2	2
B	0	3	0	4	2
C	2	0	1	0	1
D	1	3	0	4	1
E	2	0	1	4	0

- When the table is not optimal do the following:

T-4	α	β	γ	δ	ε
A	0	3	1	1	1
B	0	2	0	3	1
C	3	0	2	0	1
D	1	2	0	3	0
E	3	0	2	4	0

- Conditional check:

T-4	α	β	γ	δ	ε
A	0	3	1	1	1
B	0	2	0	3	1
C	3	0	2	0	1
D	1	2	0	3	0
E	3	0	2	4	0

Hungarian Algorithm

- When the table is optimal do the following:

T-4	α	β	γ	δ	ε
A	0	3	1	1	1
B	~~0~~	~~2~~	0	3	1
C	~~3~~	~~0~~	~~2~~	0	1
D	1	2	0	3	0
E	3	0	2	4	0

T-5*	α	β	γ	δ	ε
A	10	19	8	15	19
B	10	18	7	17	19
C	13	16	9	14	19
D	12	19	8	18	19
E	14	17	10	19	19

- Total Cost= 10 + 7 + 14 + 17 = 48

Exercises

- Five employees are available to perform four jobs. The time it takes each person to perform each job is given in the table. A dash indicates that the person cannot do that particular job.

	Time (hours			
Person	Job 1	Job 2	Job 3	Job 4
1	22	18	30	18
2	18	—	27	22
3	26	20	28	28
4	16	22	—	14
5	21	—	25	28

  - Formulate an Integer Programming Model to determine the assignment of employees to jobs that minimizes the total time required to perform the four jobs.
  - Solve the formulate Integer Programming Model using the Hungarian Algorithm.

Exercises

- The Polokwane board of education is taking bids on the city's four school bus routes. Four companies have made the bids in the table:

	Bids			
Company	Route 1	Route 2	Route 3	Route 4
1	R4 000	R5 000	—	—
2	—	R4 000	—	R4 000
3	R3 000	—	R2 000	—
4	—	—	R4 000	R5 000

  - Suppose each bidder can be assigned only one route. Formulate an Integer Programming Model to assign the companies to routes to minimize Polokwane's cost of running the four bus routes.
  - Solve the formulate Integer Programming Model using the Hungarian Algorithm
  - Suppose that each company can be assigned two routes. Formulate an Integer Programming Model to assign the companies to routes to minimize Polokwane's cost of running the four bus routes.
  - Solve the formulate Integer Programming Model using the Hungarian Algorithm

Exercises

- Assign the agents A, B, C, D, and E to the activities α, β, γ, and δ. The cost for each agent to perform each activity is given by the following cost matrix:

	Task				
Agent	**α**	**β**	**γ**	**δ**	**ε**
A	20	38	16	30	28
B	20	36	14	34	24
C	26	32	18	28	26
D	24	38	16	38	20
E	28	34	20	38	18
F	30	34	28	38	36

  - Formulate an Integer Programming Model to determine the assignment of agents to activities that minimizes the total costs required to perform the different activities.
  - Solve the formulate Integer Programming Model using the Hungarian Algorithm.

END

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