Cutting Plane Algorithm (Exercises)

Pure Integer Programming Problem 1

$$\max z = 5x_1 + 2x_2$$

$$\text{s.t.} \quad 3x_1 + x_2 \leq 12$$

$$x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

$$x_1, x_2 \text{ integer}$$

1. Solve the formulated Integer Programming Model using the Cutting Plane Algorithm.

Pure Integer programming Problem 2

$$\min z = 50x_1 + 100x_2$$

$$\text{s.t.} \quad 7x_1 + 2x_2 \geq 28 \qquad \text{(HIW constraint)}$$

$$2x_1 + 12x_2 \geq 24 \qquad \text{(HIM constraint)}$$

$$x_1, x_2 \geq 0$$

$$x_1, x_2 \text{ integer}$$

1. Solve the formulated Integer Programming Model using the Cutting Plane Algorithm.

Pure Integer Programming Problem 3

$$\max z = 4x_1 + 3x_2$$

$$\text{s.t.} \quad 4x_1 + 9x_2 \leq 26$$

$$8x_1 + 5x_2 \leq 17$$

$$x_1, x_2 \geq 0$$

$$x_1, x_2 \text{ integer}$$

1. Solve the formulated Integer Programming Model using the Cutting Plane Algorithm.

Mixed Integer Programming Problem 1

$$\max z = \quad 3x_1 + x_2$$

$$\text{s.t.} \quad 5x_1 + 2x_2 \leq 10$$

$$4x_1 + x_2 \leq 7$$

$$x_1, x_2 \geq 0$$

$$x_2 \text{ integer}$$

1. Solve the formulated Integer Programming Model using the Cutting Plane Algorithm.

Mixed Integer Programming Problem 2

$$\min z = \quad 3x_1 + x_2$$

$$\text{s.t.} \quad x_1 + 5x_2 \geq 8$$

$$x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

$$x_1 \text{ integer}$$

1. Solve the formulated Integer Programming Model using the Cutting Plane Algorithm.

Farmer Brown (Acres)

Farmer Brown must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labour per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labour per week. All wheat can be sold at R4 a bushel, and all corn can be sold at R3 a bushel. Seven acres of land and 40 hours per week of labour are available. Government regulations require that at least 30 bushels of corn be produced during the current year.

1. Let x1 = number of acres of wheat planted, and x2 = number of acres of corn planted. Using these decision variables, formulate an Integer Programming Model whose solution will tell Farmer Brown how to maximise the total revenue from wheat and corn. Assume that fractional acres of land cannot be used.
2. Solve the formulated Integer Programming Model of Farmer Brown using the Cutting Plane Algorithm.

Korean Auto

Korean Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Korean Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programmes: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income men. Each football commercial is seen by 2

million high-income women and 12 million high-income men. A 1-minute comedy ad costs ¥50 000, and a 1-minute football ad costs ¥100 000. Korean Auto would like the commercials to be seen by at least 28 million high-income women and 24 million high-income men.

1. Formulate an Integer Programming Model of Korean Auto that can be used to meet its advertising requirements at minimum cost. Assume that fractional adds cannot be purchased.
2. Solve the formulated Integer Programming Model of Korean Auto using the Cutting Plane Algorithm.

Post Office Employees

A post office requires different numbers of full-time employees on different days of the week. The number of full-time employees required to work on each day is given in the table. Union rules state that each full-time employee must work five consecutive days and then receive two days off. For example, an employee who works Monday to Friday must be off on Saturday and Sunday. The post office wants to meet its daily requirements using only full-time employees.

Requirements for Post Office	
Day	**Number of Full-time Employees Required**
1 = Monday	17
2 = Tuesday	13
3 = Wednesday	15
4 = Thursday	19
5 = Friday	14
6 = Saturday	16
7 = Sunday	11

1. Formulate an Integer Programming Model that the post office can use to minimise the number of full-time employees who must be hired. Assume that a fractional person cannot be scheduled.
2. Solve the formulated Integer Programming Model of the Post Office Employees using the Cutting Plane Algorithm.