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Branch & Bound Simplex Algorithm

Pure Integer Programming models

Formulating a linear programming model

The Oakfield Corporation manufactures tables and chairs. A table requires 1 hour of labour and 9 square board metres of wood, and a chair requires 1 hour of labour and 5 square board metres of wood. Currently, 6 hours of labour and 45 square board metres of wood are available. Each table contributes R8 to profit, and each chair contributes R5 to profit.

1. Formulate an Integer Programming Model to maximise Oakfield's profit.
2. Solve the formulated Integer Programming Model using the Branch & Bound Simplex Algorithm.

Formulating a linear programming model

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- First, we need to declare our decision variables.
  $x_i = The\ number\ of\ i\ manufactured. i = 1 = Tables, 2 = Chairs$
- Next, we will create the objective function. Each table contributes R8 to profit, and each chair contributes R5 to profit.
  $$\max z = 8x_1 + 5x_2$$
- Next, we will create the constraints. We will start with the labour constraint: A table requires 1 hour of labour, a chair requires 1 hour of labour. Currently, 6 hours of labour are available.
  $$s.t. x_1 + x_2 \leq 6$$

Formulating a linear programming model

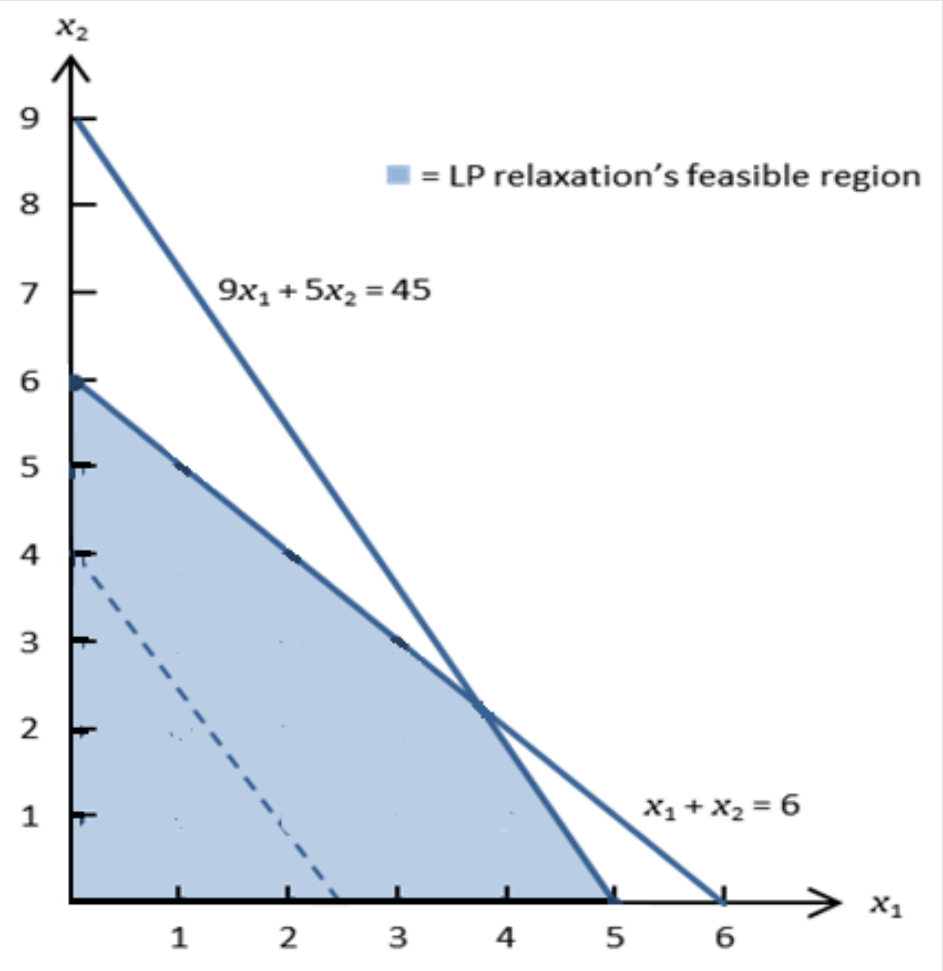
- For the next constraint we will look at the carpentry hours constraint: a table requires 9 square board metres of wood, a chair requires 5 square board metres of wood. Currently, 45 square board metres of wood are available.

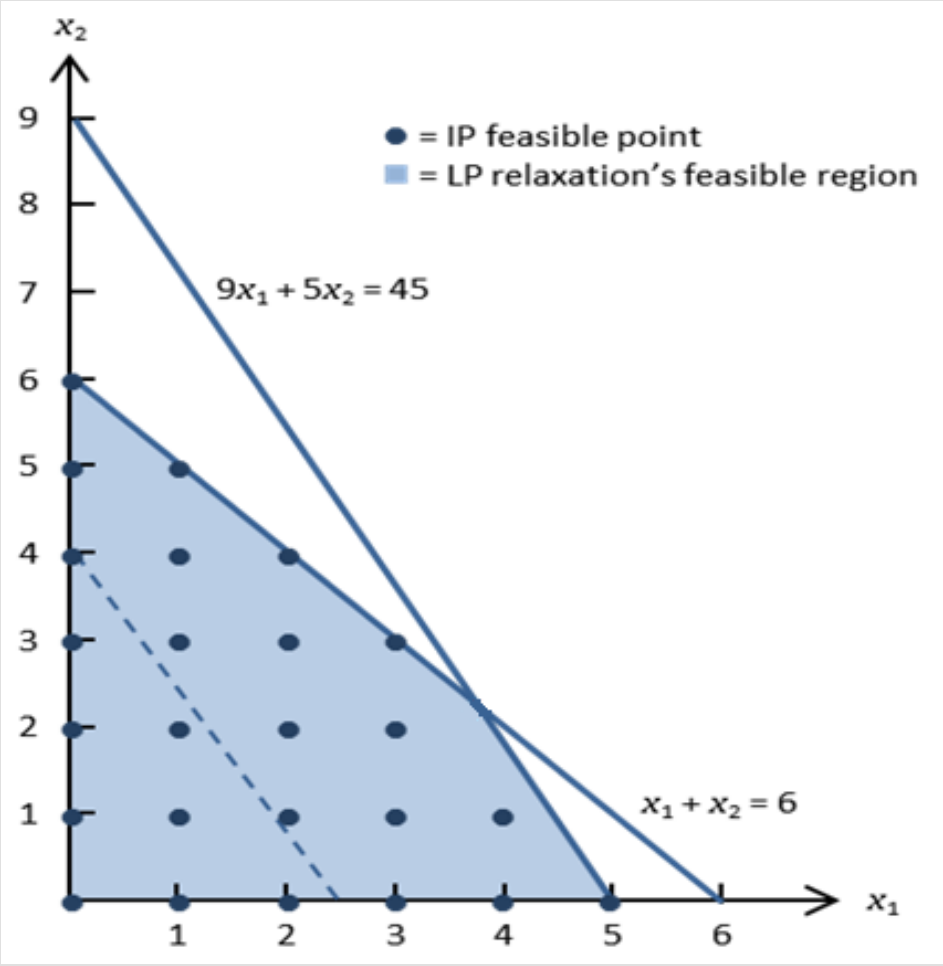
$$s.t. 9x_1 + 5x_2 \leq 45$$

- Lastly, we need our sign restrictions:

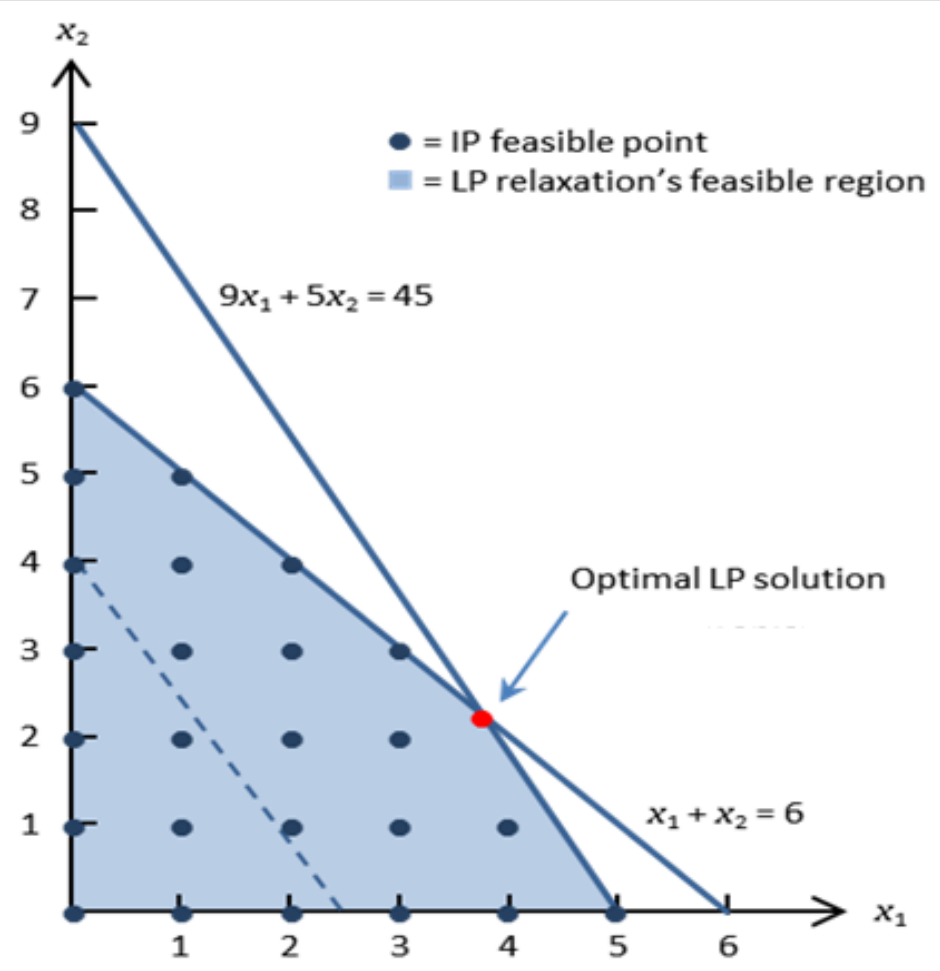
$$x_1, x_2 \geq 0$$
$$x_1, x_2 \; integers$$

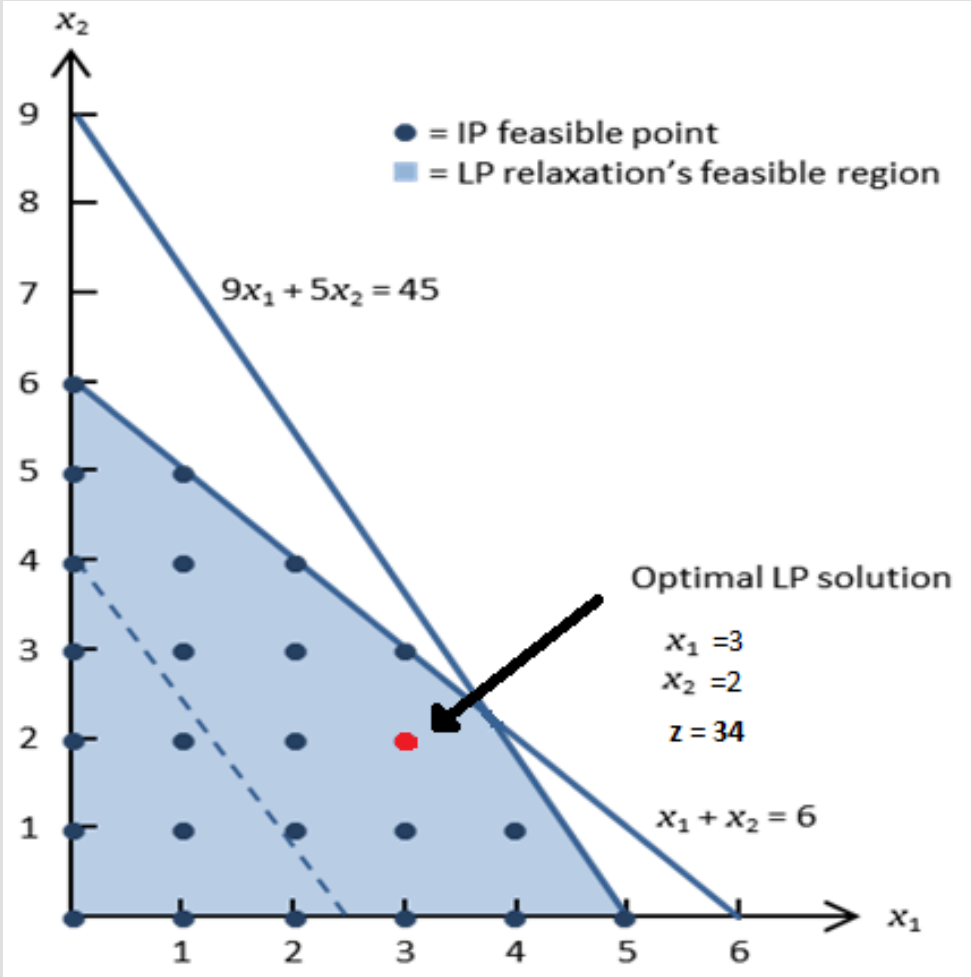
Graphical Solution





Graphical Solution (Continue)





Solver

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	A	B	C	D	E	F	G	H
1		Solver:						
2			x1	x2	ref.	sign	rhs	
3		var.						
4		obj.	8	5	=sumproduct($C$3:$D$3;C4:D4)			
5		s.t.	1	1	1 R x 2 C	<=	6	
6			9	5		<=	45	
7								

Solver (Continue)

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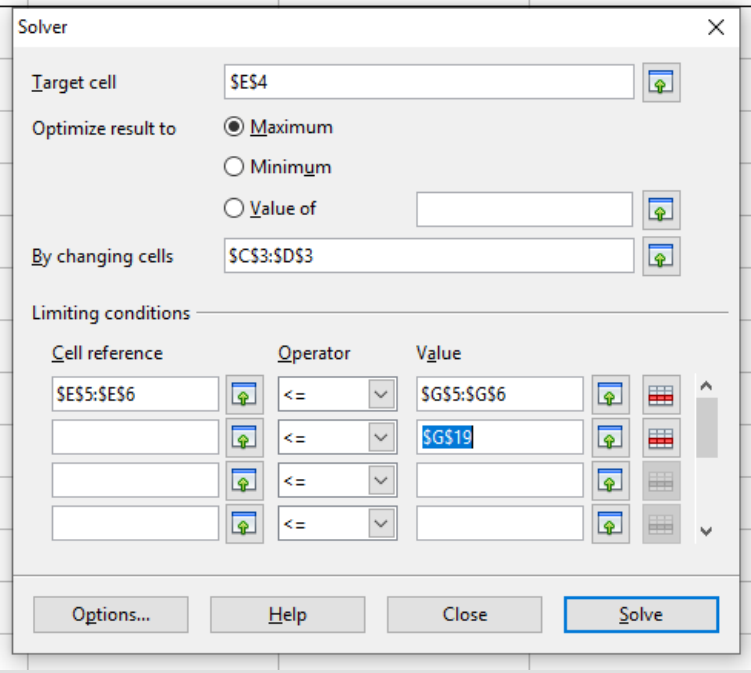
	A	B	C	D	E	F	G
1		Solver:					
2			x1	x2	ref.	sign	rhs
3		var.					
4		obj.	8	5	0		
5		s.t.	1	1	0	<=	6
6			9	5	0	<=	45
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							
19							



Solver (Continue)

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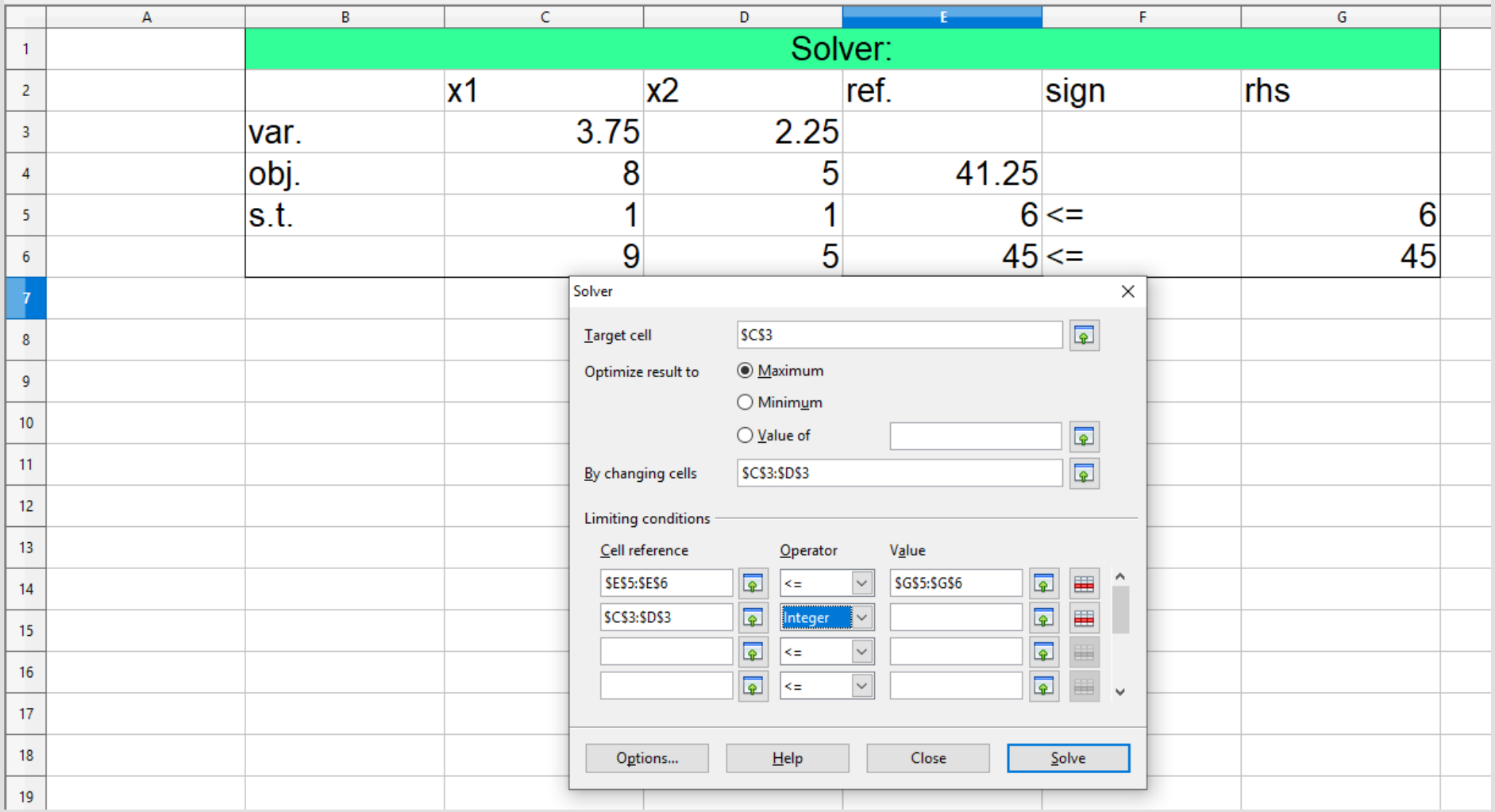
Solver:					
	x1	x2	ref.	sign	rhs
var.	3.75	2.25			
obj.	8	5	41.25		
s.t.	1	1	6	<=	6
	9	5	45	<=	45

Solver (Continue)



	A	B	C	D	E	F	G
1		Solver:					
2			x1	x2	ref.	sign	rhs
3		var.	3.75	2.25			
4		obj.	8	5	41.25		
5		s.t.	1	1	6	<=	6
6			9	5	45	<=	45

Solver (Continue)

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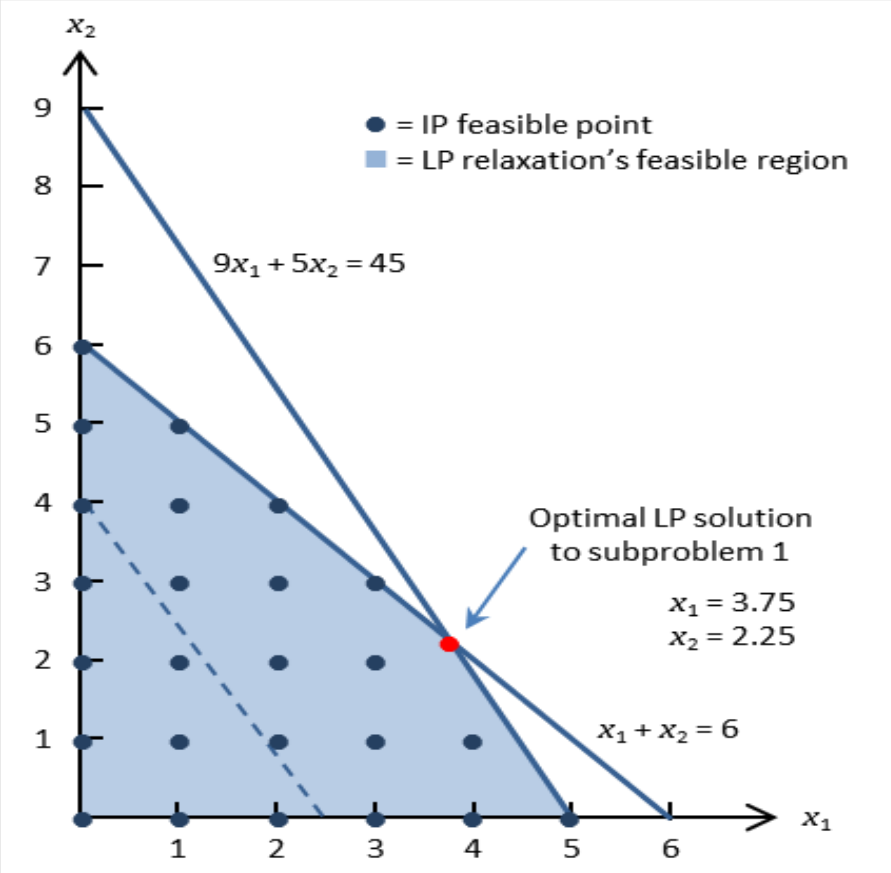
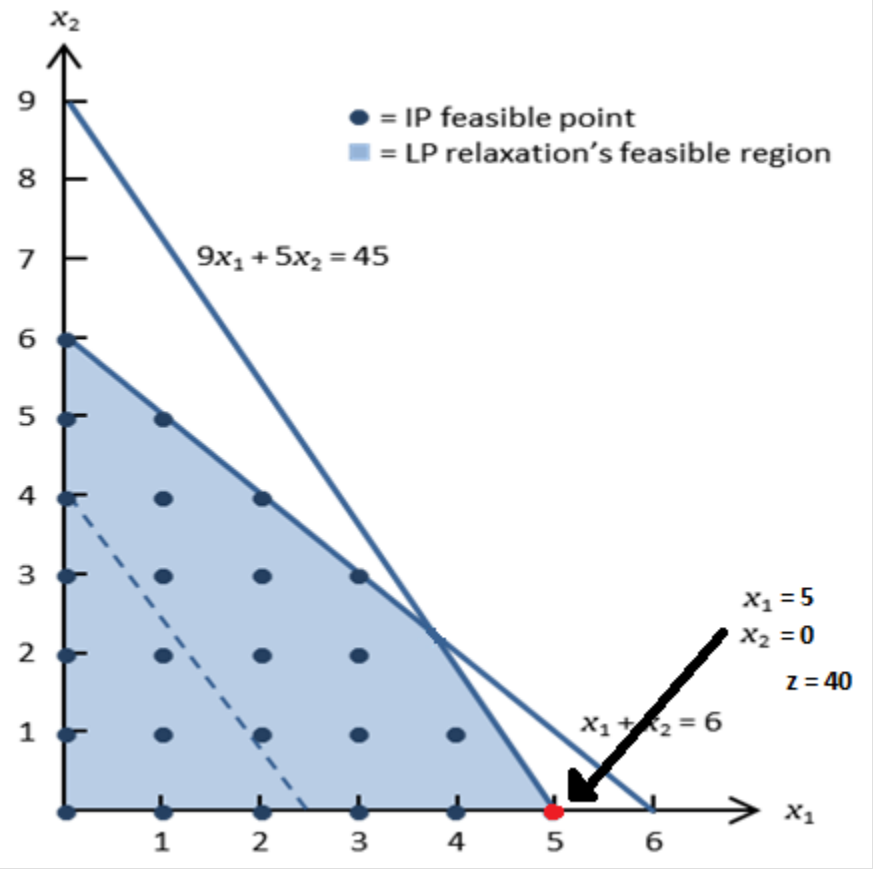
Solver:					
	x1	x2	ref.	sign	rhs
var.	5	0			
obj.	8	5	40		
s.t.	1	1	5	<=	6
	9	5	45	<=	45

Graphical Solution (Final)





Relaxed Integer Programming Model

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Integer Programming Model: Relaxed					
Max z =	$8x_1$	+	$5x_2$		
s.t	x_1	+	x_2	≤	6
	$9x_1$	+	$5x_2$	≤	45
	x_1, x_2	≥	0		

Canonical Form

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Canonical Form: Branch & Bound Algorithm								
	(z)	-	$8x_1$	-	$5x_2$	=	0	
		x_1	+	x_2	+	s_1	=	6
		$9x_1$	+	$5x_2$	+	s_2	=	45

Primal Simplex Solution

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T-i	x1	x2	s1	s2	rhs	θ
z	-8	-5	0	0	0	
1	1	1	1	0	6	6
2	9	5	0	1	45	5

Initial

T-2	x1	x2	s1	s2	rhs	θ
z	0	- 5/9	0	8/9	40	
1	0	4/9	1	- 1/9	1	2 1/4
2	1	5/9	0	1/9	5	9

T-3*	x1	x2	s1	s2	rhs
z	0	0	1 1/4	3/4	41 1/4
1	0	1	2 1/4	- 1/4	2 1/4
2	1	0	-1 1/4	1/4	3 3/4

1/4
3/4

choose the one closest to 0.5

If both are the same distance, choose the lower subscript.

Sub-Problem 1: $x_1 \leq 3$
Sub-Problem 2: $x_1 \geq 4$

Branch & Bound Simplex Algorithm Sub-Problem 1

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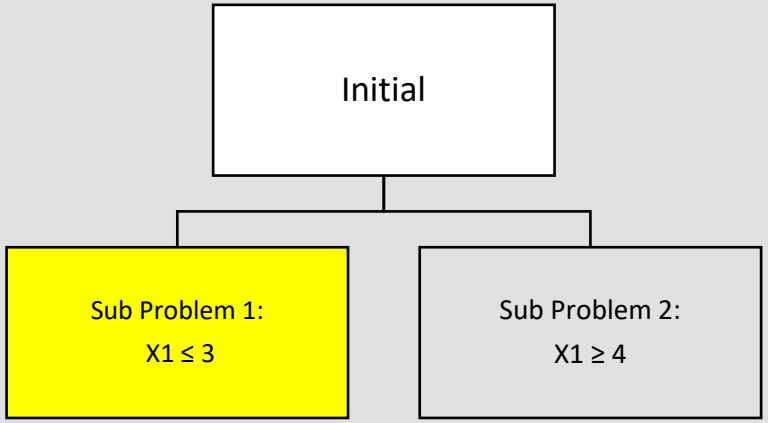


T-3*	x1	x2	s1	s2	rhs
z	0	0	1 1/4	3/4	41 1/4
1	0	1	2 1/4	- 1/4	2 1/4
2	1	0	-1 1/4	1/4	3 3/4

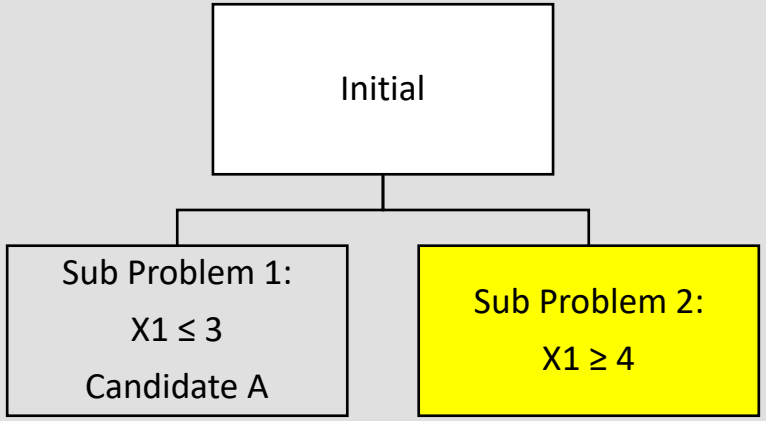
Candidate A

T-3	x1	x2	s1	s2	s3	rhs
2	1	0	-1 1/4	1/4	0	3 3/4
3	1	0	0	0	1	3
2-3	0	0	-1 1/4	1/4	-1	3/4
3 x -1	0	0	1 1/4	- 1/4	1	- 3/4

T-3	x1	x2	s1	s2	s3	rhs
z	0	0	1 1/4	3/4	0	41 1/4
1	0	1	2 1/4	- 1/4	0	2 1/4
2	1	0	-1 1/4	1/4	0	3 3/4
3	0	0	1 1/4	- 1/4	1	- 3/4
θ				3		

T-4	x1	x2	s1	s2	s3	rhs
z	0	0	5	0	3	39
1	0	1	1	0	-1	3
2	1	0	0	0	1	3
3	0	0	-5	1	-4	3

Branch & Bound Simplex Algorithm Sub-Problem 2



T-3*	x1	x2	s1	s2	rhs
z	0	0	1 1/4	3/4	41 1/4
1	0	1	2 1/4	- 1/4	2 1/4
2	1	0	-1 1/4	1/4	3 3/4

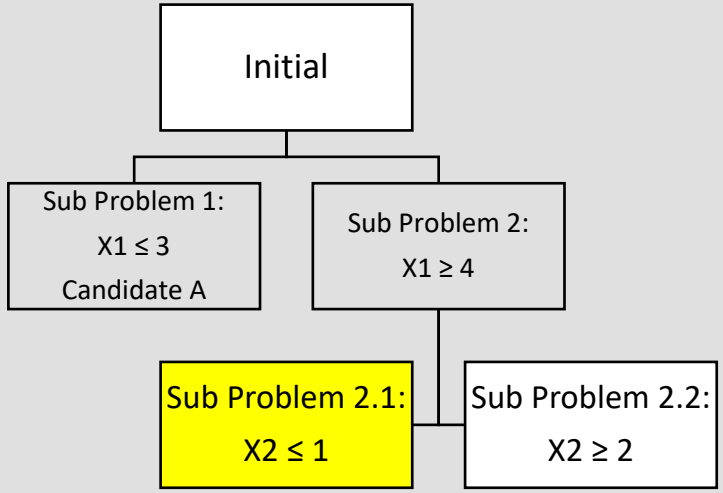
Sub-Problem 2.1: x2 ≤ 1
Sub-Problem 2.2: x2 ≥ 2

T-3	x1	x2	s1	s2	e3	rhs
2	1	0	-1 1/4	1/4	0	3 3/4
3	1	0	0	0	-1	4
2-3	0	0	-1 1/4	1/4	1	- 1/4

T-3	x1	x2	s1	s2	e3	rhs
z	0	0	1 1/4	3/4	0	41 1/4
1	0	1	2 1/4	- 1/4	0	2 1/4
2	1	0	-1 1/4	1/4	0	3 3/4
3	0	0	-1 1/4	1/4	1	- 1/4
θ			1			

T-4	x1	x2	s1	s2	e3	rhs
z	0	0	0	1	1	41
1	0	1	0	1/5	1 4/5	1 4/5
2	1	0	0	0	-1	4
3	0	0	1	- 1/5	- 4/5	1/5

Branch & Bound Simplex Algorithm Sub-Problem 2.1



T-4	x1	x2	s1	s2	e3	rhs
z	0	0	0	1	1	41
1	0	1	0	1/5	1 4/5	1 4/5
2	1	0	0	0	-1	4
3	0	0	1	- 1/5	- 4/5	1/5

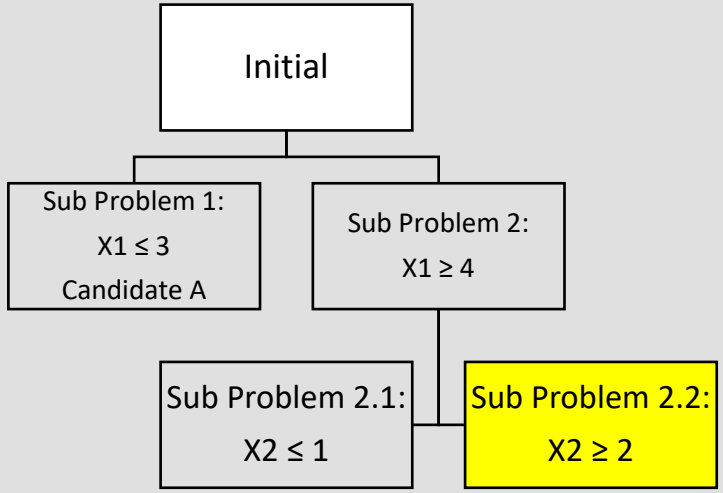
Sub-Problem 2.1.1: x1 ≤ 4
Sub-Problem 2.1.2: x1 ≥ 5

T-4	x1	x2	s1	s2	e3	s4	rhs
1	0	1	0	1/5	1 4/5	0	1 4/5
4	0	1	0	0	0	1	1
1-4	0	0	0	1/5	1 4/5	-1	4/5
4 x -1	0	0	0	- 1/5	-1 4/5	1	- 4/5

T-4	x1	x2	s1	s2	e3	s4	rhs
z	0	0	0	1	1	0	41
1	0	1	0	1/5	1 4/5	0	1 4/5
2	1	0	0	0	-1	0	4
3	0	0	1	- 1/5	- 4/5	0	1/5
4	0	0	0	- 1/5	-1 4/5	1	- 4/5
θ				5	5/9		

T-5	x1	x2	s1	s2	e3	s4	rhs
z	0	0	0	8/9	0	5/9	40 5/9
1	0	1	0	0	0	1	1
2	1	0	0	1/9	0	- 5/9	4 4/9
3	0	0	1	- 1/9	0	- 4/9	5/9
4	0	0	0	1/9	1	- 5/9	4/9

Branch & Bound Simplex Algorithm Sub-Problem 2.2



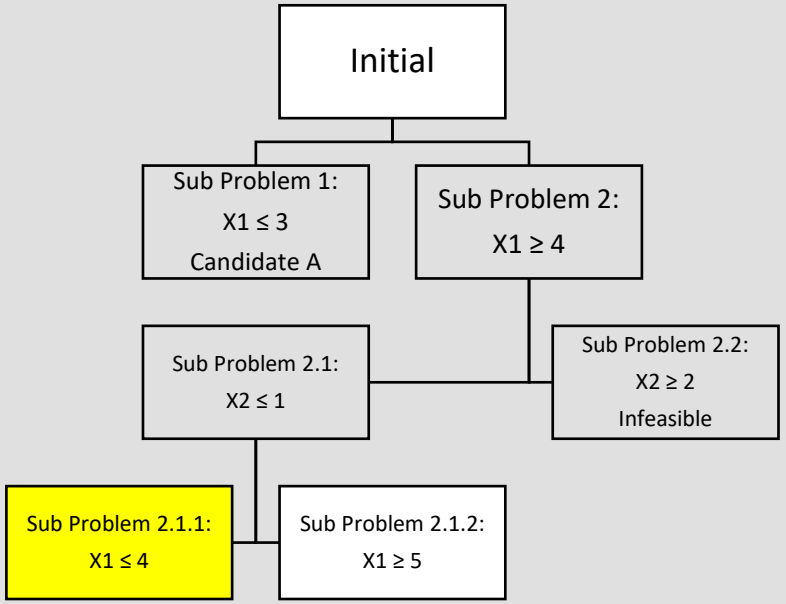
T-4	x1	x2	s1	s2	e3	e4	rhs
z	0	0	0	1	1	0	41
1	0	1	0	1/5	1 4/5	0	1 4/5
2	1	0	0	0	-1	0	4
3	0	0	1	- 1/5	- 4/5	0	1/5

T-4	x1	x2	s1	s2	e3	e4	rhs
1	0	1	0	1/5	1 4/5	0	1 4/5
4	0	1	0	0	0	-1	2
1-4	0	0	0	1/5	1 4/5	1	- 1/5

T-4	x1	x2	s1	s2	e3	e4	rhs
z	0	0	0	1	1	0	41
1	0	1	0	1/5	1 4/5	0	1 4/5
2	1	0	0	0	-1	0	4
3	0	0	1	- 1/5	- 4/5	0	1/5
4	0	0	0	1/5	1 4/5	1	- 1/5

Infeasible

Branch & Bound Simplex Algorithm Sub-Problem 2.1.1



T-5	x1	x2	s1	s2	e3	s4	rhs
z	0	0	0	8/9	0	5/9	40 5/9
1	0	1	0	0	0	1	1
2	1	0	0	1/9	0	- 5/9	4 4/9
3	0	0	1	- 1/9	0	- 4/9	5/9
4	0	0	0	1/9	1	- 5/9	4/9

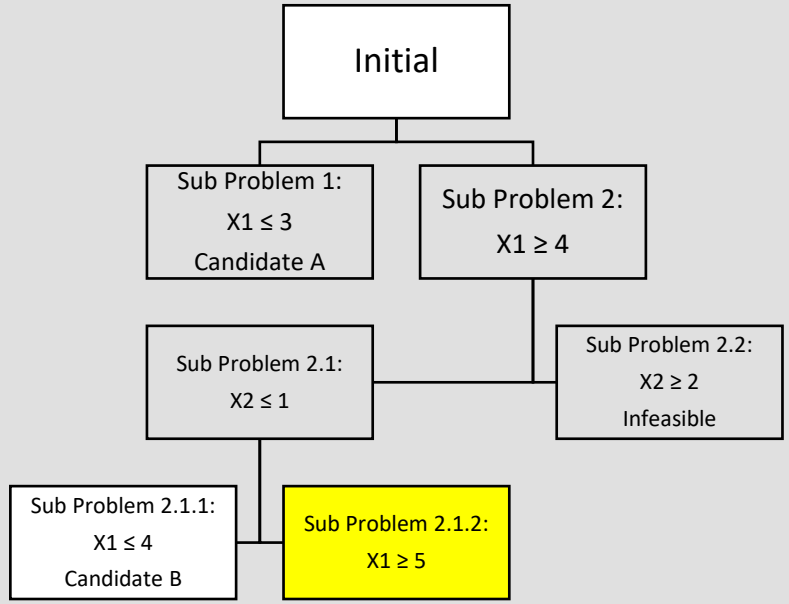
T-5	x1	x2	s1	s2	e3	s4	s5	rhs
2	1	0	0	1/9	0	- 5/9	0	4 4/9
5	1	0	0	0	0	0	1	4
2-5	0	0	0	1/9	0	- 5/9	-1	4/9
5 x -1	0	0	0	- 1/9	0	5/9	1	- 4/9

T-5	x1	x2	s1	s2	e3	s4	s5	rhs
z	0	0	0	8/9	0	5/9	0	40 5/9
1	0	1	0	0	0	1	0	1
2	1	0	0	1/9	0	- 5/9	0	4 4/9
3	0	0	1	- 1/9	0	- 4/9	0	5/9
4	0	0	0	1/9	1	- 5/9	0	4/9
5	0	0	0	- 1/9	0	5/9	1	- 4/9
θ				8				

T-6	x1	x2	s1	s2	e3	s4	s5	rhs
z	0	0	0	0	0	5	8	37
1	0	1	0	0	0	1	0	1
2	1	0	0	0	0	0	1	4
3	0	0	1	0	0	-1	-1	1
4	0	0	0	0	1	0	1	0
5	0	0	0	1	0	-5	-9	4

Cadindate B

Branch & Bound Simplex Algorithm Sub-Problem 2.1.2



T-5	x1	x2	s1	s2	e3	s4	rhs
z	0	0	0	8/9	0	5/9	40 5/9
1	0	1	0	0	0	1	1
2	1	0	0	1/9	0	- 5/9	4 4/9
3	0	0	1	- 1/9	0	- 4/9	5/9
4	0	0	0	1/9	1	- 5/9	4/9

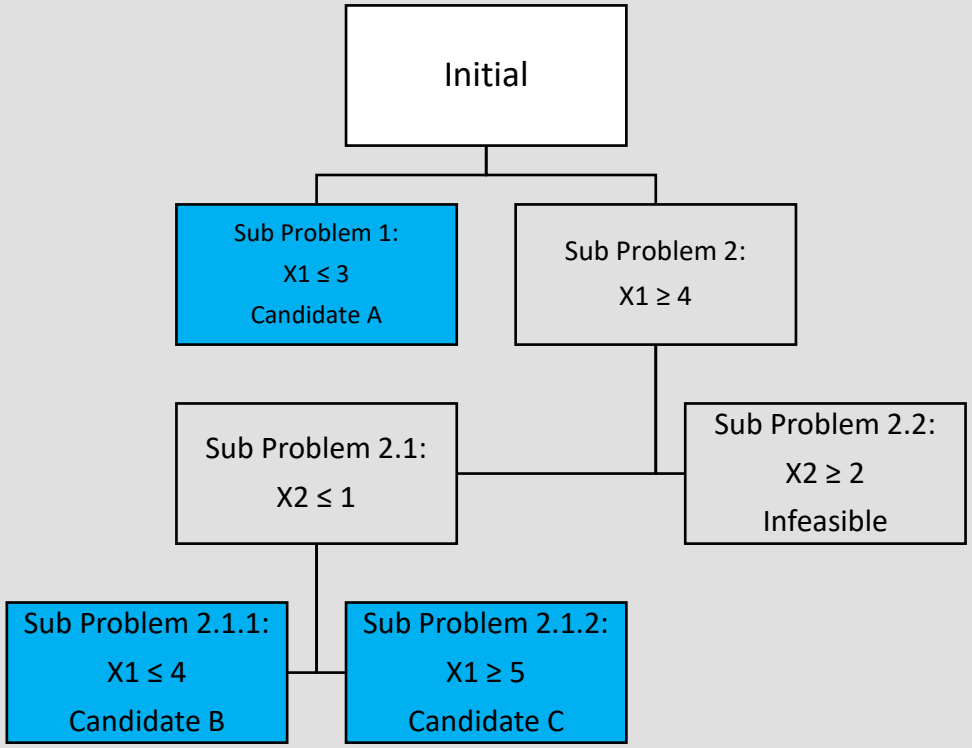
T-5	x1	x2	s1	s2	e3	s4	e5	rhs
2	1	0	0	1/9	0	- 5/9	0	4 4/9
5	1	0	0	0	0	0	-1	5
2-5	0	0	0	1/9	0	- 5/9	1	- 5/9

T-5	x1	x2	s1	s2	e3	s4	e5	rhs
z	0	0	0	8/9	0	5/9	0	40 5/9
1	0	1	0	0	0	1	0	1
2	1	0	0	1/9	0	- 5/9	0	4 4/9
3	0	0	1	- 1/9	0	- 4/9	0	5/9
4	0	0	0	1/9	1	- 5/9	0	4/9
5	0	0	0	1/9	0	- 5/9	1	- 5/9
θ						1		

T-6	x1	x2	s1	s2	e3	s4	e5	rhs
z	0	0	0	1	0	0	1	40
1	0	1	0	1/5	0	0	1 4/5	0
2	1	0	0	0	0	0	-1	5
3	0	0	1	- 1/5	0	0	- 4/5	1
4	0	0	0	0	1	0	-1	1
5	0	0	0	- 1/5	0	1	-1 4/5	1

Candidate C

Branch & Bound Simplex Algorithm Candidate Selection



Candidate A

T-4	x1	x2	s1	s2	s3	rhs
z	0	0	5	0	3	39
1	0	1	1	0	-1	3
2	1	0	0	0	1	3
3	0	0	-5	1	-4	3

Cadindate B

T-6	x1	x2	s1	s2	e3	s4	s5	rhs
z	0	0	0	0	0	5	8	37
1	0	1	0	0	0	1	0	1
2	1	0	0	0	0	0	1	4
3	0	0	1	0	0	-1	-1	1
4	0	0	0	0	1	0	1	0
5	0	0	0	1	0	-5	-9	4

Candidate C

T-6	x1	x2	s1	s2	e3	s4	e5	rhs
z	0	0	0	1	0	0	1	40
1	0	1	0	1/5	0	0	1 4/5	0
2	1	0	0	0	0	0	-1	5
3	0	0	1	- 1/5	0	0	- 4/5	1
4	0	0	0	0	1	0	-1	1
5	0	0	0	- 1/5	0	1	-1 4/5	1

Best

Exercises:

Found in the document called ‘Branch & Bound Simplex Algorithm Exercises’

END

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