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ARE YOU READY?

Location – The lockbox problem

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OverCharge receives credit card payments from four regions of the country (West, Central, East, and South). The average daily value of payments mailed by customers from each region is as follows: the West, R70 000; Central, R50 000; the East, R60 000; the South, R40 000. OverCharge must decide where customers should mail their payments. Because OverCharge can earn 20% annual interest by investing these revenues, it would like to receive payments as quickly as possible. OverCharge is considering setting up operations to process payments (often referred to as lockboxes) in four different cities: Cape Town, Johannesburg, Durban, and Port Elizabeth. The average number of days (from time payment is sent) until a check clears and OverCharge can deposit the money depends on the city to which the payment is mailed, as shown in the table.

Location – The lockbox problem

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Average number of days from mailing of payment until payment clears				
	To			
From	City 1 (Cape Town)	City 2 (Johannesburg)	City 3 (Durban)	City 4 (Port Elizabeth)
Region 1 West	2	6	8	8
Region 2 Central	6	2	5	5
Region 3 East	8	5	2	5
Region 4 South	8	5	5	2

For example, if a check is mailed from the West to Port Elizabeth, it would take an average of 8 days before OverCharge could earn interest on the check.

Location – The lockbox problem

The annual cost of running a lockbox in any city is R50 000.

Formulate an IP that OverCharge can use to minimise the sum of costs due to lost interest and lockbox operations. Assume that each region must send all its money to a single city and that there is no limit on the amount of money that each lockbox can handle.

Location – formulating the IP

- Decision variables:

$$x_{ij} = If\ region\ i\ sends\ payments\ to\ city\ j\ (1) or\ not\ (0)\ where$$
$$i = 1 = West, 2 = Central, 3 = East, 4 = South\ and$$
$$j = 1 = CT, 2 = Jhb, 3 = Durban, 4 = PE$$
$$y_j = If\ a\ lockbox\ is\ operated\ in\ city\ j\ (1) or\ not (0)$$

- Objective function:

$$\min z = 28x_{11} + 84x_{12} + 112x_{13} + 112x_{14} +$$
$$60x_{21} + 20x_{22} + 50x_{23} + 50x_{24} +$$
$$96x_{31} + 60x_{32} + 24x_{33} + 60x_{34} +$$
$$64x_{41} + 40x_{42} + 40x_{43} + 16x_{44} +$$
$$50y_1 + 50y_2 + 50y_3 + 50y_4$$

Calculation of Annual lost of Interest				
Assignment		Decision variable	Annual lost of Interest (R)	
From Region	To City			
West	Cape Town	x_{11}	$0{,}20 \times 70000 \times 2$	$= 28000$
West	Johannesburg	x_{12}	$0{,}20 \times 70000 \times 6$	$= 84000$
West	Durban	x_{13}	$0{,}20 \times 70000 \times 8$	$= 112000$
West	Port Elizabeth	x_{14}	$0{,}20 \times 70000 \times 8$	$= 112000$
Central	Cape Town	x_{21}	$0{,}20 \times 50000 \times 6$	$= 60000$
Central	Johannesburg	x_{22}	$0{,}20 \times 50000 \times 2$	$= 20000$
Central	Durban	x_{23}	$0{,}20 \times 50000 \times 5$	$= 50000$
Central	Port Elizabeth	x_{24}	$0{,}20 \times 50000 \times 5$	$= 50000$
East	Cape Town	x_{31}	$0{,}20 \times 60000 \times 8$	$= 96000$
East	Johannesburg	x_{32}	$0{,}20 \times 60000 \times 5$	$= 60000$
East	Durban	x_{33}	$0{,}20 \times 60000 \times 2$	$= 24000$
East	Port Elizabeth	x_{34}	$0{,}20 \times 60000 \times 5$	$= 60000$
South	Cape Town	x_{41}	$0{,}20 \times 40000 \times 8$	$= 64000$
South	Johannesburg	x_{42}	$0{,}20 \times 40000 \times 5$	$= 40000$
South	Durban	x_{43}	$0{,}20 \times 40000 \times 5$	$= 40000$
South	Port Elizabeth	x_{44}	$0{,}20 \times 40000 \times 2$	$= 16000$

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Location – formulating the IP

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- The first constraints are to make sure each region sends payments to a single city:

$$s.t. x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

- Sign restrictions:

$$x_{ij} = 0 \; or \; 1$$
$$y_j = 0 \; or \; 1$$

Location – Solver

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		x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	x41	x42	x43	x44	y1	y2	y3	y4	ref	Sign	b
sign		Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin			
var		1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0			
obj		28	84	112	112	60	20	50	50	96	60	24	60	64	40	40	16	50	50	50	50	88		
s.t. 1		1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
2		0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	=	1
3		0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	=	1
4		0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	1	=	1

	x11	x12	x13	x14	x21	x22	x23	x24	x31	x32	x33	x34	x41	x42	x43	x44	y1	y2	y3	y4	ref	Sign	b
sign	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin	Bin			
var	1	0	0	0	0	1	0																
obj	28	84	112	112	60	20	50														88		
s.t. 1	1	1	1	1	0	0	0														1	=	1
2	0	0	0	0	1	1	1														1	=	1
3	0	0	0	0	0	0	0														1	=	1
4	0	0	0	0	0	0	0														1	=	1

Solver Parameters

Set Objective: $V$6

To: Max Min Value Of: 0

By Changing Variable Cells:

$B$5:$U$5

Subject to the Constraints:

$B$5:$U$5 = binary
$V$7:$V$10 = $X$7:$X$10

Add
Change
Delete
Reset All
Load/Save

Make Unconstrained Variables Non-Negative

Select a Solving Method: Simplex LP

Options

Solving Method

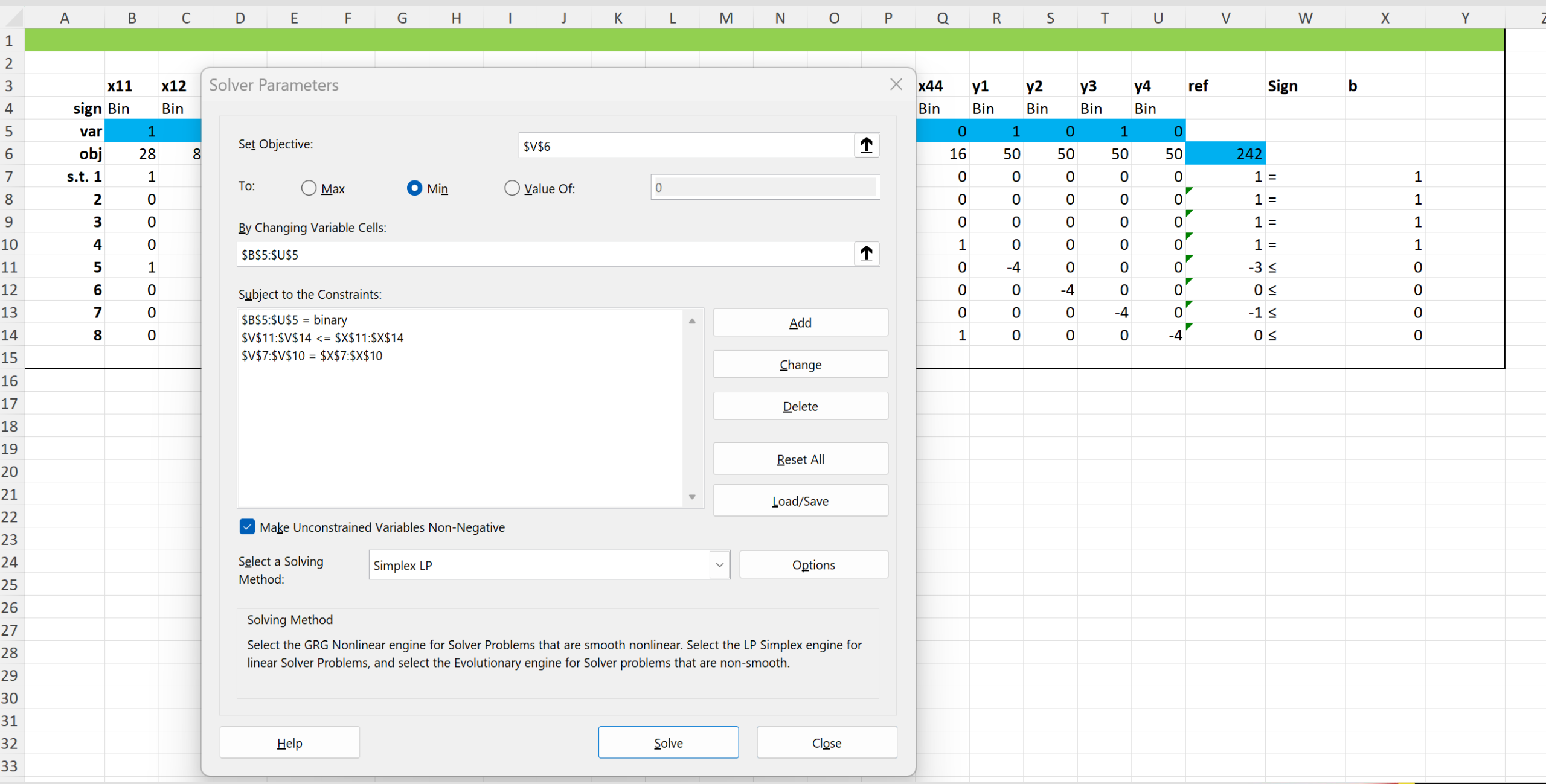
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

Help Solve Close

Location – formulating the IP

- Now we need to add constraints to make sure if payments are sent to a city, that the city has a lockbox:

$$x_{11} + x_{21} + x_{31} + x_{41} - 4y_1 \leq 0$$
$$x_{12} + x_{22} + x_{32} + x_{42} - 4y_2 \leq 0$$
$$x_{13} + x_{23} + x_{33} + x_{43} - 4y_3 \leq 0$$
$$x_{14} + x_{24} + x_{34} + x_{44} - 4y_4 \leq 0$$



	x11	x12	x44	y1	y2	y3	y4	ref	Sign	b
sign	Bin	Bin	Bin	Bin	Bin	Bin	Bin			
var	1		0	1	0	1	0			
obj	28	8	16	50	50	50	50	242		
s.t. 1	1		0	0	0	0	0	1	=	1
2	0		0	0	0	0	0	1	=	1
3	0		0	0	0	0	0	1	=	1
4	0		1	0	0	0	0	1	=	1
5	1		0	-4	0	0	0	-3	≤	0
6	0		0	0	-4	0	0	0	≤	0
7	0		0	0	0	-4	0	-1	≤	0
8	0		1	0	0	0	-4	0	≤	0

Location – If-Then

- If customers in region 1 send their payments to city 1, then no other customers may send their payments to city 1.
- If $x_{11} > 0, then\ x_{21} + x_{31} + x_{41} \leq 0$

$$x_{21} + x_{31} + x_{41} - 3d_1 \leq 0$$

$$x_{11} + 3d_1 \leq 3$$

- Remember to declare the dummy variable and specify it is 0 or 1

$$d_1 = dummy\ variable\ 0\ or\ 1$$

x_{21}	+	x_{31}	+	x_{41}	−	$3d_1$	≤	0			
x_{11}	+	$3d_1$	≤	3							
If $x_{11} = 1$ *and* $x_{21}, x_{31}, x_{41} = 0$											
(0)	+	(0)	+	(0)	−	3(0)	≤	0	T	TRUE	Keeps $d_1 = 0$
(1)	+	3(0)	≤	3					T		
If $x_{11} = 1$ *and* $x_{21}, x_{31}, x_{41} = 1$											
(1)	+	(1)	+	(1)	−	3(0)	≤	0	F	FALSE	Changes $d_1 = 1$
(1)	+	3(0)	≤	3					T		
(1)	+	(1)	+	(1)	−	3(1)	≤	0	T	FALSE	These values will not be allowed
(1)	+	3(1)	≤	3					F		
If $x_{11} = 0$ *and* $x_{21}, x_{31}, x_{41} = 1$											
(1)	+	(1)	+	(1)	−	3(0)	≤	0	F	FALSE	Changes $d_1 = 1$
(0)	+	3(0)	≤	3					T		
(1)	+	(1)	+	(1)	−	3(1)	≤	0	T	TRUE	These values will be allowed
(0)	+	3(1)	≤	3					T		

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Location – If-Then

Exercises

Location Problems (Exercises).pdf

END

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