

Task Assignment & Hungarian Algorithm (Exercises)

Random Jobs

Five employees are available to perform four jobs. The time it takes each person to perform each job is given in the table. A dash indicates that the person cannot do that particular job.

Person	Time (hours)			
	Job 1	Job 2	Job 3	Job 4
1	22	18	30	18
2	18	—	27	22
3	26	20	28	28
4	16	22	—	14
5	21	—	25	28

1. Formulate an Integer Programming Model to determine the assignment of employees to jobs that minimises the total time required to perform the four jobs.
2. Solve the formulate Integer Programming Model using the Hungarian Algorithm.

Bus Routes

The Polokwane board of education is taking bids on the city's four school bus routes. Four companies have made the bids in the table.

Company	Bids			
	Route 1	Route 2	Route 3	Route 4
1	R4 000	R5 000	—	—
2	—	R4 000	—	R4 000
3	R3 000	—	R2 000	—
4	—	—	R4 000	R5 000

1. Suppose each bidder can be assigned only one route. Formulate an Integer Programming Model to assign the companies to routes to minimise Polokwane's cost of running the four bus routes.
2. Solve the formulate Integer Programming Model using the Hungarian Algorithm.
3. Suppose that each company can be assigned two routes. Formulate an Integer Programming Model to assign the companies to routes to minimise Polokwane's cost of running the four bus routes.
4. Solve the formulate Integer Programming Model using the Hungarian Algorithm.

Doc Councillman

Doc Councillman is putting together a relay team for the 400-meter relay. Each swimmer must swim 100 meters of breaststroke, back stroke, butterfly, or freestyle. Doc believes that each swimmer will attain the times given in the table. To minimise the team's time for the race, which swimmer should swim which stroke?

	Time (seconds)			
Swimmer	Free	Breast	Fly	Back
Gary Hall	54	54	51	53
Mark Spitz	51	57	52	52
Jim Montgomery	50	53	54	56
Chet Jastremski	56	54	55	53

1. Formulate an Integer Programming Model to determine the assignment of swimmer to strokes that minimises the total time required to perform the four strokes.
2. Solve the formulate Integer Programming Model using the Hungarian Algorithm.

Desert Island

Tom Cruise, Freddy Prince Jr., Harrison Ford, and Matt LeBlanc are marooned on a desert island with Jennifer Aniston, Courteney Cox, Gwyneth Paltrow, and Julia Roberts. The “compatibility measures”, in the below table, indicates how much happiness each couple would experience if they spent all their time together. The happiness earned by a couple is proportional to the fraction of time they spend together. For example, if Freddy Prince Jr. and Gwyneth Paltrow spend half their time together, they earn happiness of $\frac{1}{2}(9) = 4.5$.

	Jennifer Aniston	Courteney Cox	Gwyneth Paltrow	Julia Roberts
Tom Cruise	7	5	8	2
Freddy Prince Jr.	7	8	9	4
Harrison Ford	3	5	7	9
Matt LeBlanc	5	5	6	7

1. Let x_{ij} be the fraction of time that the i th man spends with the j th woman. The goal of the eight people is to maximize the total happiness of the people on the island. Formulate an Linear Programming Model whose optimal solution will yield the optimal values of the x_{ij} 's.
2. Explain why the optimal solution in the first part will have four $x_{ij} = 1$ and twelve $x_{ij} = 0$. The optimal solution requires that each person spend all his or her time with one person of the opposite sex, so this result is often referred to as a Marriage Theorem.
3. Determine the marriage partner for each person using the Hungarian Algorithm.
4. Do you think the Proportionality Assumption of Linear Programming is valid in this situation?