

# ARE YOU READY?



#### **DUAL SIMPLEX ALGORITHM - SCENARIO**

Farmer Brown must determine how many acres of corn and wheat to plant this year. An acre of wheat yields 25 bushels of wheat and requires 10 hours of labour per week. An acre of corn yields 10 bushels of corn and requires 4 hours of labour per week. All wheat can be sold at R4 a bushel, and all corn can be sold at R3 a bushel. Seven acres of land and 40 hours per week of labour are available. Government regulations require that at least 30 bushels of corn be produced during the current year.

Let x1 = number of acres of wheat planted, and x2 = number of acres of corn planted. Using these decision variables, formulate an LP whose solution will tell Farmer Brown how to maximise the total revenue from wheat and corn.

Primary Linear Programming Model							
	Whe		Corn				
Max z =	100x1	+	30x2				
s.t.							
1			x2	≥	3Government		
2	x1	+	x2	≤	7Acres		
3	10x1	+	4x2	≤	40Labour		
	x1, x2	≥	(	)			

We will use Dual Simplex to solve this LP



First we need to put our LP into Canonical form:

Canonical Form: Dual Simplex Algorithm									
(z)	-	100x1	-	30x2	=		0		
-	x2	+	e1	=	-		3		
	x1	+	x2	+	s2	=	7		
	10x1	+	4x2	+	s3	=	40		

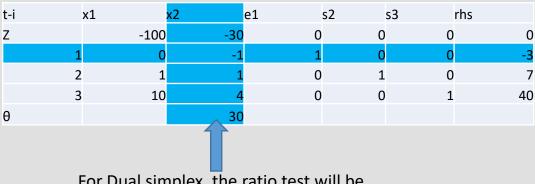
- For >= constraints, we subtract our excess variable and make the sign =. We
  then multiply the entire constraint by -1 to make sure the excess variable
  enters as a Basic Variable.
- For <= constraints, we just add the slack variable and make the sign =.
- For = constraints, we need to duplicate the constraint to make two constraints, but one will be <= and the other will be >= (this will still force it to be equals). We then repeat the above steps for the two constraints.



• First we need to put our LP into Canonical form:

Canonical Form: Dual Simplex Algorithm								
(z)	-	100x1	-	30x2	=		0	
-	x2	+	e1	=	-		3	
	x1	+	x2	+	s2	=	7	
	10x1	+	4x2	+	s3	=	40	

Now we can create our initial table:



For Dual simplex, the ratio test will be =ABS(z-value/pivot row value) ->=ABS(-30/-1)=30

Start with choosing a pivot row.
Only negative values in the RHS constraints can become the pivot row





t-i	x1	x2	e1	s2	s3	rhs			
Z		-100	-30	0	0	0	0		
	1	0	-1	1	0	0	-3		
	2	1	1	0	1	0	7		
	3	10	4	0	0	1	40		
θ			30						
	For Dual simplex, the ratio test will be =ABS(z-value/pivot row value) ->=ABS(-30/-1)=30								

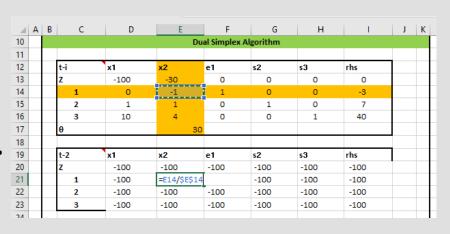
Start with choosing a pivot row. Only negative values in the RHS constraints can become the pivot row. Phase 1 will only be complete once there are no more negatives in the RHS column constraints. If there is more than one negative value in the RHS column of the constraints, you will choose the smallest value (or largest absolute of the negative values to get the biggest improvement).



• To get the new table, we will do the following: Drag the formula through the entire table.

4	Α	ВС	D	E	F	G	Н	1	J	K	L
10		Dual Simplex Algorithm									
11											
12		t-i	x1	x2	e1	<b>s2</b>	<b>s</b> 3	rhs			
13		Z	-100	I -30	Ιo	0	0	0		Ш	
14		1	0	-1	1	0	0	-3		Ш	
15		2	1	1	0	1	0	7		Ш	
16		3	10	4	0	0	1	40			
17		θ		3(	0						
18											
19		t-2	x1	x2	e1	<b>s</b> 2	<b>s</b> 3	rhs			
20		Z	=D13-(\$E1	3*D\$21)							
21	_	1	<u> </u>	Ī						Ш	
22		2									
23		3									
24	L										

Drag this formula through the entire row.





• In the new table, we do not have any negative values in the RHS column which means we are done with Dual Simplex and will move on to Primal Simplex.

t-2	x1	x2	e1	s2	s3	rhs	Rat	io test
Z		-100	0	-30	0	0	90	
	1	0	1	-1	0	0	3 #1	DIV/0!
	2	1	0	1	1	0	4	4
	3	10	0	4	0	1	28	2,8

We will again do our calculations to get our new table:

t-3*	x1	x2	e1	s2	s3	rhs	
Z		0	0	10	0	10	370
	1	0	1	-1	0	0	3
	2	0	0	3/5	1	- 1/10	1 1/5
	3	1	0	2/5	0	1/10	2 4/5

There are no more negative values in the z row, thus we have an optimal table.

### **EXERCISES**



- 1. Korean Auto manufactures luxury cars and trucks. The company believes that its most likely customers are high-income women and men. To reach these groups, Korean Auto has embarked on an ambitious TV advertising campaign and has decided to purchase 1-minute commercial spots on two types of programs: comedy shows and football games. Each comedy commercial is seen by 7 million high-income women and 2 million high-income women and 2 million high-income women and 12 million high-income men. A 1-minute comedy ad costs \\ \square 50 000, and a 1-minute football ad costs \square 100 000. Korean Auto would like the commercials to be seen by at least 28 million high-income women and 24 million high-income men.
  - Formulate a mathematical model of Korean Auto that can be used to meet its advertising requirements at minimum cost.
  - Solve the formulated Linear Programming Model of Korean Auto using the Dual Simplex Algorithm.

#### **EXERCISES**



2. A freshman's diet requires that she eat food that comes from the four "basic food groups" (chocolate cake, ice cream, soda, and cheesecake). At present, the following four foods are available for consumption: brownies, chocolate ice cream, cola, and pineapple cheesecake. Each brownie costs 50¢, each scoop of chocolate ice cream costs 20¢, each bottle of cola costs 30¢, and each piece of pineapple cheesecake costs 80¢. Each day, the student must ingest at least 500 calories, 6g of chocolate, 10g of sugar, and 8g of fat. The nutritional content per unit of each food is shown in the table.

Type of food	Calories	Chocolate (g)	Sugar (g)	Fat (g)
Brownie	400	3	2	2
Chocolate ice cream (1 scoop)	200	2	2	4
Cola (1 bottle)	150	0	4	1
Pineapple cheesecake (1 piece)	500	0	4	5

- Formulate a linear programming model that can be used to satisfy her daily nutritional requirements at minimum cost.
- Solve the formulated Linear Programming Model of the freshman's Diet using the Dual Simplex Algorithm.





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