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# SCENARIO

Four jobs must be processed on a single machine. The time required to process each job and the date the job is due are shown in the table.

Job	Days required to complete Job	Due date	Penalty/day
1	6	End of day 8	R8
2	4	End of day 4	R10
3	5	End of day 12	R15
4	8	End of day 16	R20

# SCENARIO

The delay of a job is the number of days after the due date that a job is completed (if a job is completed on time or early, the job's delay is zero).

In what order should the jobs be processed to minimise the total delay of the four jobs?

## Formulating the IP

- Decision variables:

$x_{ij}$  = if job  $i$  is completed in position  $j$  (1) or not (0) where  
 $i=j=1, 2, 3, 4$

- Sign restrictions:

$$x_{ij} = 0 \text{ or } 1$$

# Branch and Bound Machine Scheduling Algorithm

Position 4

Problem 1	Problem 2	Problem 3	Problem 4
x14	x24	x34	x44
Time required= $6+4+5+8=23$ days	Time required= $6+4+5+8=23$ days	Time required= $6+4+5+8=23$ days	Time required= $6+4+5+8=23$ days
Overdue= $23-8=15$ days	Overdue= $23-4=19$ days	Overdue= $23-12=11$ days	Overdue= $23-16=7$ days
Penalty= $15 \times 8 = R120$ *	Penalty= $19 \times 10 = R190$	Penalty= $11 \times 15 = R165$	Penalty= $7 \times 20 = R140$

# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x14

Position 3

Problem 1.1	Problem 1.2	Problem 1.3
X14 & x23	X14 & x33	X14 & x43
Time required= $4+5+8=17$ days	Time required= $4+5+8=17$ days	Time required= $4+5+8=17$ days
Overdue= $17-4=13$ days	Overdue= $17-12=5$ days	Overdue= $17-16=1$ days
Penalty= $13 \times 10= R130$	Penalty= $5 \times 15= R75$	Penalty= $1 \times 20= R20$
Total penalty= $120+130= R250$	Total penalty= $120+75= R195$	Total penalty= $120+20= R140$ *

# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x14

Position 3 -> x43

Position 2

## Problem 1.3.1

X14 & x43 & x22

Time required=  $4+5=9$  days

Overdue=  $9-4=5$  days

Penalty=  $5 \times 10 = R50$

Total penalty=  $140+50=$   
R190

## Problem 1.3.2

X14 & x43 & x32

Time required=  $4+5=9$  days

Overdue=  $9-12=0$  days

Penalty=  $0 \times 15 = R0$

Total penalty=  $140+0=$   
R140 \*

# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x14

Position 3 -> x43

Position 2 -> x32

Position 1 -> x21

## Problem 1.3.2.1

X14 & x43 & x32 & x21

Time required= 4= 4 days

Overdue= 4-4= 0 days

Penalty= 0x10= R0

Total penalty= 140+0=  
R140 \*

Candidate A

As soon as you have a candidate (all jobs were placed), you need to back track level by level until the last position that were checked first. If you find a penalty  $\leq$  the penalty of the candidate, you will branch from that sub problem to see if you will get something better or an alternative solution. If the sub-problem was branched, you ignore it. If the penalty is  $>$  the penalty of the candidate, it will not possibly improve so eliminate that sub-problem with the **current** candidate.



# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x14

Position 3 -> x43

Position 2

Problem 1.3.1	Problem 1.3.2
X14 & x43 & x22	X14 & x43 & x32
Time required= $4+5=9$ days	Time required= $4+5=9$ days
Overdue= $9-4=5$ days	Overdue= $9-12=0$ days
Penalty= $5 \times 10= R50$	Penalty= $0 \times 15= R0$
Total penalty= $140+50= R190$	Total penalty= $140+0= R140 *$
Eliminated by Candidate A	

# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x14  
Position 3

Problem 1.1	Problem 1.2	Problem 1.3
X14 & x23	X14 & x33	X14 & x43
Time required= $4+5+8= 17$ days	Time required= $4+5+8= 17$ days	Time required= $4+5+8= 17$ days
Overdue= $17-4= 13$ days	Overdue= $17-12= 5$ days	Overdue= $17-16= 1$ days
Penalty= $13 \times 10= R130$	Penalty= $5 \times 15= R75$	Penalty= $1 \times 20= R20$
Total penalty= $120+130= R250$	Total penalty= $120+75= R195$	Total penalty= $120+20= R140$ *
Eliminated by Candidate A	Eliminated by Candidate A	

# Branch and Bound Machine Scheduling Algorithm

Position 4

Problem 1	Problem 2	Problem 3	Problem 4
x14	x24	x34	x44
Time required= $6+4+5+8=23$ days	Time required= $6+4+5+8=23$ days	Time required= $6+4+5+8=23$ days	Time required= $6+4+5+8=23$ days
Overdue= $23-8=15$ days	Overdue= $23-4=19$ days	Overdue= $23-12=11$ days	Overdue= $23-16=7$ days
Penalty= $15 \times 8 = R120$ *	Penalty= $19 \times 10 = R190$	Penalty= $11 \times 15 = R165$	Penalty= $7 \times 16 = R140$ *
	Eliminated by Candidate A	Eliminated by Candidate A	

# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x44

Position 3

Problem 4.1	Problem 4.2	Problem 4.3
X44 & x13	X44 & x23	X44 & x33
Time required= $6+4+5=15$ days	Time required= $6+4+5=15$ days	Time required= $6+4+5=15$ days
Overdue= $15-8=7$ days	Overdue= $15-4=11$ days	Overdue= $15-12=3$ days
Penalty= $7 \times 8 = R56$	Penalty= $11 \times 10 = R110$	Penalty= $3 \times 20 = R60$
Total penalty= $140+56 = R196$	Total penalty= $140+110 = R250$	Total penalty= $140+60 = R200$
Eliminated by Candidate A	Eliminated by Candidate A	Eliminated by Candidate A

# Branch and Bound Machine Scheduling Algorithm

Position 4 -> x14

Position 3 -> x43

Position 2 -> x32

Position 1 -> x21

## Problem 1.3.2.1

X14 & x43 & x32 & x21

Time required= 4= 4 days

Overdue= 4-4= 0 days

Penalty= 0x10= R0

Total penalty= 140+0=  
R140 \*

Candidate A

## Exercises

Four jobs must be processed on a single machine. The time required performing each job and the due date of each job are shown in the table.


Job	Time to perform job (Days)	Due date of job	Penalty (R000)
1	4	Day 4	4
2	5	Day 2	5
3	2	Day 13	7
4	3	Day 8	2


Solve the formulated Integer Programming Model using the Branch & Bound Machine Scheduling Algorithm.

# END




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