Botlhale Village
Working together for ICT innovation and growth in Africa

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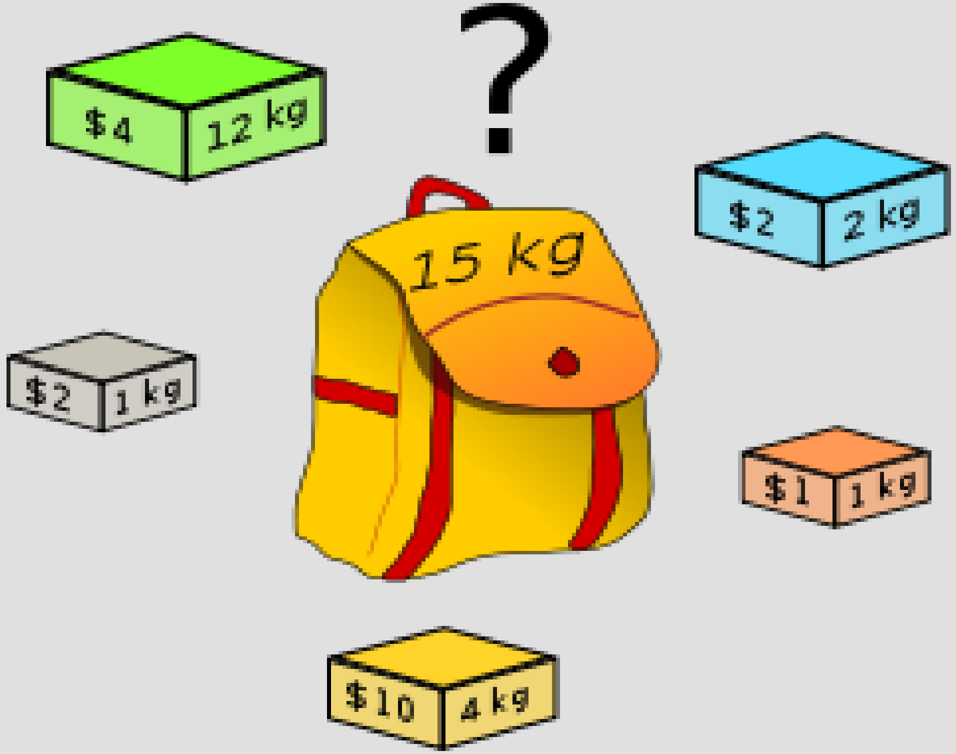
ARE YOU READY?

SCENARIO

Which boxes should be chosen to maximise the amount of money while keeping the overall weight under or equal to 15 kg?



Formulating the IP

- Decision variables:

$$x_i = If\ item\ i\ is\ taken(1) or\ not\ (0)\ where$$
$$i = items\ 1 - 5$$

- Objective function:

$$max\ z = 4x_1 + 2x_2 + 2x_3 + x_4 + 10x_5$$

- Constraint:

$$12x_1 + 2x_2 + x_3 + x_4 + 4x_5 \leq 15$$

- Sign restrictions:

$$x_i\ (i = items\ 1 - 5) = 0\ or\ 1$$

Solver

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Solver									
	x1	x2	x3	x4	x5	Ref.	Sign	b	
Var,	0	1	1	1	1				
Obj.	4	2	2	1	10	15			
s.t. 1	12	2	1	1	4	8	≤	15	

Branch & Bound Algorithm- Knapsack method

Ratio Test		
Item	z_i/c_i	Rank
x_1	$\frac{4}{12} = \frac{1}{3} = 0{,}333$	5
x_2	$\frac{2}{2} = 1$	3
x_3	$\frac{2}{1} = 2$	2
x_4	$\frac{1}{1} = 1$	4
x_5	$\frac{10}{4} = 2{,}5$	1

Integer Programming Model												
max z =	4x1	+	2x2	+	2x3	+	x4	+	10x5			
s.t	12x1	+	2x2	+	x3	+	x4	+	4x5	≤	15	
	xi = 0 or 1											

Sub-Problem	
$x_5 = 1$	15-4=11
$x_3 = 1$	11-1=10
$x_2 = 1$	10-2=8
$x_4 = 1$	8-1=7
x_1 =7/12	7-12

Sub-P 1: x1 = 0

Sub-P 2: x1 = 1

Branch & Bound Algorithm- Knapsack method

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Ratio Test	
Item	Rank
x_1	5
x_2	3
x_3	2
x_4	4
x_5	1

Integer Programming Model												
max z =	4x1	+	2x2	+	2x3	+	x4	+	10x5			
s.t	12x1	+	2x2	+	x3	+	x4	+	4x5	≤	15	
	xi = 0 or 1											

Sub-P 1: x1 = 0

Sub-Problem 1	
$* x_1 = 0$	15-0=15
$x_5 = 1$	15-4=11
$x_3 = 1$	11-1=10
$x_2 = 1$	10-2=8
$x_4 = 1$	8-1=7

z = $10 + 2 + 2 + 1 = 15$

Candidate A

Best Candidate

Sub-P 2: x1 = 1

Sub-Problem 2	
$* x_1 = 1$	15-12=3
$x_5 = 3/4$	3-4
$x_3 = 0$	
$x_2 = 0$	
$x_4 = 0$	

Sub-P 2.1: x5 = 0

Sub-P 2.2: x5 = 1

Branch & Bound Algorithm- Knapsack method

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Ratio Test	
Item	Rank
x_1	5
x_2	3
x_3	2
x_4	4
x_5	1

Integer Programming Model												
max z =	4x1	+	2x2	+	2x3	+	x4	+	10x5			
s.t	12x1	+	2x2	+	x3	+	x4	+	4x5	≤	15	
	xi = 0 or 1											

Sub-P 2.1: x5 = 0

Sub-Problem 1	
$* x_1 = 1$	15-12=3
$* x_5 = 0$	3-0=3
$x_3 = 1$	3-1=2
$x_2 = 1$	2-2=0
$x_4 = 0$	Stays 0

z $= 4 + 2 + 2 = 8$

Candidate B

Sub-P 2.2: x5 = 1

Sub-Problem 2	
$* x_1 = 1$	15-12=3
$* x_5 = 1$	3-4
$x_3 = 0$	
$x_2 = 0$	
$x_4 = 0$	

Infeasible

Branch & Bound Algorithm- Knapsack method

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Sub-Problem	
$x_5 = 1$	15-4=11
$x_3 = 1$	11-1=10
$x_2 = 1$	10-2=8
$x_4 = 1$	8-1=7
x_1 =7/12	7-12

Sub-P 1: x1 = 0

z = $10 + 2 + 2 + 1 = 15$

Candidate A

Best Candidate

Sub-P 2: x1 = 1

Sub-P 2.1: x5 = 0

z =$4 + 2 + 2 = 8$

Candidate B

Sub-P 2.2: x5 = 1

Infeasible

Exercises

Stocks & Bonds is considering four investments. Investment 1 will yield a net present value (NPV) of R16 000; investment 2, an NPV of R22 000; investment 3, an NPV of R12 000; and investment 4, an NPV of R8 000. Each investment requires a certain cash outflow at the present time: investment 1, R5 000; investment 2, R7 000; investment 3, R4 000; and investment 4, R3 000. Currently, R14 000 is available for investment.

Integer Programming Model										
max z =	$16x_1$	+	$22x_2$	+	$12x_3$	+	$8x_4$			
s.t	$5x_1$	+	$7x_2$	+	$4x_3$	+	$3x_4$	≤	14	*Funds Restriction*
	$x_i = 0$ or 1									

Solve using Branch & Bound Algorithm – Knapsack method

More exercises can be found in the document 'Knapsack Problems (Exercises)'

END

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