Az informatika számítástudományi alapjai

0. feladatsor

1.7. Describe each of the following infinite sets using the format $\{\underline{\hspace{2cm}} \mid n \in \mathcal{N}\}$, without using "..." in the expression on the left side of the vertical bar.

a. $\{0, -1, 2, -3, 4, -5, \ldots\}$

b. $\{\{0\}, \{1\}, \{2\}, \ldots\}$

c. $\{\{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 2, 3\}, \ldots\}$

d. $\{\{0\}, \{0, 1\}, \{0, 1, 2, 3\}, \{0, 1, 2, 3, 4, 5, 6, 7\}, \{0, 1, \ldots, 15\}, \{0, 1, 2, \ldots, 31\}, \ldots\}$

$\mathcal{N}$ a természetes számok halmaza.

1.8. In each case below, find an expression for the indicated set, involving A, B, C, and any of the operations $\cup$, $\cap$, $-$, and $'$.

a. $\{x \mid x \in A$ or $x \in B$ but not both$\}$

b. $\{x \mid x$ is an element of exactly one of the three sets A, B, and $C\}$

c. $\{x \mid x$ is an element of at most one of the three sets A, B, and $C\}$

d. $\{x \mid x$ is an element of exactly two of the three sets A, B, and $C\}$

A vesszőzés a komplementerképzést jelenti, azaz A' azoknak a dolgoknak a halmazát jelöli, amik nincsenek benne A-ban.

1.12. a. How many elements are there in the set

$$\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\{\emptyset, \{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}\}\}?$$

1.11. In each case below, say whether the given statement is true for the universe $(0, 1) = \{x \in \mathcal{R} \mid 0 < x < 1\}$, and say whether it is true for the universe $[0, 1] = \{x \in \mathcal{R} \mid 0 \leq x \leq 1\}$. For each of the four cases, you should therefore give two true-or-false answers.

a. $\forall x (\exists y (x > y))$

b. $\forall x (\exists y (x \geq y))$

c. $\exists y (\forall x (x > y))$

d. $\exists y (\forall x (x \geq y))$

1.23. In each case below, a relation on the set $\{1, 2, 3\}$ is given. Of the three properties, reflexivity, symmetry, and transitivity, determine which ones the relation has. Give reasons.

   a. $R = \{(1, 3), (3, 1), (2, 2)\}$

   b. $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$

   c. $R = \emptyset$

1.25. Each case below gives a relation on the set of all nonempty subsets of $\mathcal{N}$. In each case, say whether the relation is reflexive, whether it is symmetric, and whether it is transitive.

   a. R is defined by: ARB if and only if $A \subseteq B$.

   b. R is defined by: ARB if and only if $A \cap B \neq \emptyset$.

   c. R is defined by: ARB if and only if $1 \in A \cap B$.

$\mathcal{N}$ a természetes számok halmaza.