

Lecture 2: Language and NumPy basics

FIE463: Numerical methods in Macroeconomics and Finance using Python

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See GitHub repository for notebooks and data:

<https://github.com/richardfoltyn/FIE463-V25>

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2 Language and NumPy basics

In this unit, we start exploring the Python language, covering the following topics:

1. Basic syntax
2. Built-in data types
3. NumPy arrays

2.1 Basic syntax

- Everything after a # character (until the end of the line) is a comment and will be ignored.
- Variables are created using the assignment operator =.
- Variable names are case sensitive.
- Whitespace characters matter (unlike in most languages)!

- Python uses indentation (usually 4 spaces) to group statements, for example loop bodies, functions, etc.
- You don't need to add a character to terminate a line, unlike in some languages.
- You can use the `print()` function to inspect almost any object.

```
[1]: # First example

# create a variable named 'text' that stores the string 'Hello, world!'
text = 'Hello, world!'

# print contents of 'text'
print(text)
```

Hello, world!

In Jupyter notebooks and interactive command-line environments, we can also display a value by simply writing the variable name.

```
[2]: text
```

```
[2]: 'Hello, world!'
```

Alternatively, we don't even need to create a variable but can instead directly evaluate expressions and print the result:

```
[3]: 2*3
```

```
[3]: 6
```

This does not print anything in *proper* Python script files (ending in `.py`) that are run through the interpreter, though.

Calling `print()` is also useful if we want to display multiple expressions from a single notebook cell, as otherwise only the last value is shown:

```
[4]: text = 'Hello world!'
var = 1
text          # does NOT print contents of text
var           # prints only value of var
```

```
[4]: 1
```

```
[5]: print(text)    # print text explicitly
var          # var is shown automatically
```

Hello world!

```
[5]: 1
```

2.2 Built-in data types

Python is a dynamically-typed language:

- Unlike in C or Fortran, you don't need to declare a variable or its type.
- You can inspect a variable's type using the built-in `type()` function, but you rarely need to do this.

We now look at the most useful built-in data types:

Basic types

- integers (`int`)

- floating-point numbers (float)
- boolean (bool)
- strings (str)

Containers (or collections)

- tuples (tuple)
- lists (list)
- dictionaries (dict)

2.2.1 Integers and floats

Integers and floats (floating-point numbers) are the two main built-in data types to store numerical data (we ignore complex numbers in this course). Floating-point is the standard way to represent real numbers on computers since these cannot store real numbers with arbitrary precision.

```
[6]: # Integer variables
i = 1
type(i)
```

```
[6]: int
```

```
[7]: # Floating-point variables
x = 1.0
type(x)
```

```
[7]: float
```

```
[8]: # A name can reference any data type:
# Previously, x was a float, now it's an integer!
x = 1
type(x)
```

```
[8]: int
```

It is good programming practice to specify floating-point literals using a decimal point. It makes a difference in a few cases (especially when using NumPy arrays, or Python extensions such as Numba or Cython):

```
[9]: x = 1.0          # instead of x = 1
```

A boolean (bool) is a special integer type that can only store two values, True and False. We create booleans by assigning one of these values to a variable:

```
[10]: x = True
x = False
type(x)
```

```
[10]: bool
```

Boolean values are most frequently used for conditional execution, i.e., a block of code is run only when some variable is True. We study conditional execution in the next unit.

2.2.2 Strings

The string data type stores sequences of characters:

```
[11]: # Strings need to be surrounded by single (') or double (") quotes!
institution = 'Norwegian School of Economics'
institution = "Norwegian School of Economics"
```

Strings support various operations some of which we explore in the exercises at the end of this section. For example, we can use the addition operation `+` to concatenate strings:

```
[12]: # Define two strings
str1 = 'Python'
str2 = 'course'

# Concatenate strings using +
str1 + ' ' + str2
```

```
[12]: 'Python course'
```

An extremely useful variant of strings are the so-called *f-strings*. These allow us to dynamically insert a variable value into a string, a feature we'll use throughout this course.

```
[13]: # Approximate value of pi
pi = 3.1415

# Use f-strings to embed the value of the variable version inside the string
s = f'Pi is approximately equal to {pi}'
s
```

```
[13]: 'Pi is approximately equal to 3.1415'
```

2.2.3 Tuples

Tuples represent an *ordered, immutable collection* of items which can have different data types. They are created whenever several items are separated by commas:

```
[14]: # A tuple containing a string, an integer and a float
items = 'foo', 1, 1.0
items
```

```
[14]: ('foo', 1, 1.0)
```

The parenthesis are optional, but improve readability:

```
[15]: items = ('foo', 1, 1.0)      # equivalent way to create a tuple
items
```

```
[15]: ('foo', 1, 1.0)
```

We use brackets `[]` to access an element in a tuple (or any other container object). Elements in tuples need to be accessed by their position or *index*.

```
[16]: first = items[0]            # variable first now contains 'foo'
first
```

```
[16]: 'foo'
```

Python indices are 0-based, so 0 references the *first* element, 1 the second element, etc.

```
[17]: second = items[1]           # second element
second
```

```
[17]: 1
```

Tuples and any other Python collections support automatic unpacking if we want to extract multiple (or all) values at once:

```
[18]: first, second, third = items

# Print first element
first
```

```
[18]: 'foo'
```

If we are not interested in extracting all items, we can collect any remaining items using a `*` as follows:

```
[19]: first, *rest = items

# Rest contains a list of all remaining items
rest
```

```
[19]: [1, 1.0]
```

Tuples are *immutable*, which means that the contents of a tuple cannot be changed. (Technically, the *references* to elements stored in the tuple cannot be changed.)

```
[20]: # This raises an error!
items = 'foo', 1, 1.0
items[0] = 123
```

```
TypeError: 'tuple' object does not support item assignment
```

2.2.4 Lists

Lists are like tuples, except that they can be modified (i.e., they are *mutable*). We create lists using brackets:

```
[21]: # Create list which contains a string, an integer and a float
lst = ['foo', 1, 1.0]
lst
```

```
[21]: ['foo', 1, 1.0]
```

Accessing list items works the same way as with tuples

```
[22]: lst[0] # print first item
```

```
[22]: 'foo'
```

Lists items can be modified:

```
[23]: lst[0] = 'bar' # first element is now 'bar'
lst
```

```
[23]: ['bar', 1, 1.0]
```

Lists are full-fledged objects that support various operations, for example

```
[24]: lst.insert(0, 'abc') # insert element at position 0
lst.append(2.0) # append element at the end
del lst[3] # delete the 4th element
lst
```

```
[24]: ['abc', 'bar', 1, 2.0]
```

The built-in function `len()` returns the number of elements in a list (and any other container object)

```
[25]: len(lst)
```

```
[25]: 4
```

2.2.5 Dictionaries

Dictionaries are container objects that map keys to values.

- Both keys and values can be (almost any) Python objects, even though we often use strings as keys.
- Dictionaries are created using curly braces: `{key1: value1, key2: value2, ...}`, or by using the `dict()` constructor `dict(key1=value1, key2=value2, ...)`.

For example, to create a dictionary with three items we write

```
[26]: dct = {  
      'institution': 'NHH',  
      'course': 'Python course',  
      'year': 2025  
      }  
      dct
```

```
[26]: {'institution': 'NHH', 'course': 'Python course', 'year': 2025}
```

The alternative way to create dictionaries using the `dict()` constructor is less powerful and supports only keys that are strings. For most cases, this is sufficient:

```
[27]: # Alternative way to define the same dictionary  
      dct = dict(institution='NHH', course='Python course', year=2025)  
      dct
```

```
[27]: {'institution': 'NHH', 'course': 'Python course', 'year': 2025}
```

Specific values are accessed using the syntax `dct[key]`:

```
[28]: dct['institution']
```

```
[28]: 'NHH'
```

We can use the same syntax to either modify an existing key or add a new key-value pair:

```
[29]: dct['course'] = 'Introduction to Python'      # modify value of existing key  
      dct['city'] = 'Bergen'                       # add new key-value pair  
      dct
```

```
[29]: {'institution': 'NHH',  
      'course': 'Introduction to Python',  
      'year': 2025,  
      'city': 'Bergen'}
```

Moreover, we can use the methods `keys()` and `values()` to get the collection of a dictionary's keys and values:

```
[30]: dct.keys()
```

```
[30]: dict_keys(['institution', 'course', 'year', 'city'])
```

```
[31]: dct.values()
```

```
[31]: dict_values(['NHH', 'Introduction to Python', 2025, 'Bergen'])
```

When we try to retrieve a key that is not in the dictionary, this will produce an error:

```
[32]: dct['country']
```

```
KeyError: 'country'
```

One way to get around this is to use the `get()` method which accepts a default value that will be returned whenever a key is not present:

```
[33]: dct.get('country', 'Norway')    # return 'Norway' if 'country' is  
                                         # not a valid key
```

```
[33]: 'Norway'
```

2.3 NumPy arrays

NumPy is a library that allows us to efficiently store and access (mainly) numerical data and apply numerical operations similar to those available in Matlab.

- NumPy is not part of the core Python project.
- Python itself has an array type, but there is really no reason to use it. Use NumPy!
- NumPy types and functions are not built-in, we must first import them to make them visible. We do this using the `import` statement.

The convention is to make NumPy functionality available using the `np` namespace:

```
[34]: # Access functionality from NumPy using the 'np' short-hand  
import numpy as np
```

2.3.1 Creating arrays

Creating arrays from other Python objects

Arrays can be created from other objects such as lists and tuples by calling `np.array()`

```
[35]: # Create array from list [1,2,3]  
arr = np.array([1, 2, 3])  
arr
```

```
[35]: array([1, 2, 3])
```

```
[36]: # Create array from tuple  
arr = np.array((1.0, 2.0, 3.0))  
arr
```

```
[36]: array([1., 2., 3.])
```

```
[37]: # Create two-dimensional array from nested list  
arr = np.array([[1, 2, 3], [4, 5, 6]])  
arr
```

```
[37]: array([[1, 2, 3],
           [4, 5, 6]])
```

Array creation routines

Additionally, NumPy offers a multitude of functions to create new arrays from scratch.

```
[38]: # Create a 1-dimensional array with 10 elements, initialize values to 0.
arr = np.zeros(10)
arr
```

```
[38]: array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0.])
```

```
[39]: arr1 = np.ones(5)      # vector of five ones
arr1
```

```
[39]: array([1., 1., 1., 1., 1.])
```

We can also create sequences of integers using the `np.arange()` function:

```
[40]: arr2 = np.arange(5)    # vector [0,1,2,3,4]
arr2
```

```
[40]: array([0, 1, 2, 3, 4])
```

`np.arange()` accepts initial values and increments as optional arguments. The end value is *not* included.

```
[41]: start = 2
end = 10
step = 2
arr3 = np.arange(start, end, step)
arr3
```

```
[41]: array([2, 4, 6, 8])
```

As in Matlab, there is a `np.linspace()` function that creates a vector of uniformly-spaced real values.

```
[42]: # Create 11 elements, equally spaced on the interval [0.0, 1.0]
arr5 = np.linspace(0.0, 1.0, 11)
arr5
```

```
[42]: array([0. , 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1. ])
```

We create arrays of higher dimension by specifying the desired shape. Shapes are specified as tuple arguments; for example, the shape of an $m \times n$ matrix is (m, n) .

```
[43]: mat = np.ones((2,2))    # Create 2x2 matrix of ones
mat
```

```
[43]: array([[1., 1.],
           [1., 1.]])
```

2.3.2 Reshaping arrays

The `reshape()` method of an array object can be used to reshape it to some other (conformable) shape.

```
[44]: # Create vector of 4 elements and reshape it to a 2x2 matrix
mat = np.arange(4).reshape((2,2))
mat
```



```
[44]: array([[0, 1],
           [2, 3]])
```

```
[45]: # reshape back to vector of 4 elements
vec = mat.reshape(4)
vec
```

```
[45]: array([0, 1, 2, 3])
```

We use -1 to let NumPy automatically compute the size of *one* remaining dimension.

```
[46]: # with 2 dimensions, second dimension must have size 2
mat = np.arange(4).reshape((2, -1))
mat
```

```
[46]: array([[0, 1],
           [2, 3]])
```

If we want to convert an arbitrary array to a vector, we can alternatively use the `flatten()` method.

```
[47]: mat.flatten()
```

```
[47]: array([0, 1, 2, 3])
```

Important: the reshaped array must have the same number of elements!

```
[48]: mat = np.arange(6).reshape((2,-1))
mat.reshape((2,2))      # Cannot reshape 6 into 4 elements!
```

```
ValueError: cannot reshape array of size 6 into shape (2,2)
```

2.3.3 Indexing

Single element indexing

To retrieve a single element, we specify the element's index on each axis ("axis" is the NumPy terminology for an array dimension).

- Remember that just like Python in general, NumPy arrays use 0-based indices.
- Unlike lists or tuples, NumPy arrays support multi-dimensional indexing.

```
[49]: import numpy as np

mat = np.arange(6).reshape((3,2))
mat
```

```
[49]: array([[0, 1],
           [2, 3],
           [4, 5]])
```

```
[50]: mat[0,1]      # returns element in row 1, column 2
```

```
[50]: 1
```

It is important to pass multi-dimensional indices as a tuple within brackets, i.e., `[0,1]` in the above example. We could alternatively write `mat[0][1]`, which would give the same result:

```
[51]: mat[0][1]      # don't do this!
```

```
[51]: 1
```

This is substantially less efficient, though, as it first creates a sub-dimensional array `mat[0]`, and then applies the second index to this array.

Index slices

There are numerous ways to retrieve a subset of elements from an array. The most common way is to specify a triplet of values `start:stop:step` called `slice` for some axis.

Indexing with slices can get quite intricate. Some basic rules:

- all tokens in `start:stop:step` are optional, with the obvious default values. We could therefore write `::` to include all indices, which is the same as `:`
- The end value is *not* included. Writing `vec[0:n]` does not include element with index n !
- Any of the elements of `start:stop:step` can be negative.
 - If `start` or `stop` are negative, elements are counted from the end of the array: `vec[:-1]` retrieves the whole vector except for the last element.
 - If `step` is negative, the order of elements is reversed.

```
[52]: vec = np.arange(5)

# These are equivalent ways to return the WHOLE vector
vec[0:5:1]      # all three tokens present
vec[:, :]       # omit all tokens
vec[:, :]       # omit all tokens
vec[:5]         # end value only
vec[-5:]        # start value only, using negative index
```

```
[52]: array([0, 1, 2, 3, 4])
```

You can reverse the order like this:

```
[53]: vec[::-1]
```

```
[53]: array([4, 3, 2, 1, 0])
```

With multi-dimensional arrays, the above rules apply for each dimension.

```
[54]: # Create a 2x3 matrix
mat = np.arange(6).reshape((2,3))
mat
```

```
[54]: array([[0, 1, 2],
            [3, 4, 5]])
```

```
[55]: # Retrieve only the first and third columns:
mat[:,0:2,0:3:2]
```

```
[55]: array([[0, 2],
            [3, 5]])
```

We can omit indices for higher-order dimensions if all elements should be included.

```
[56]: mat[1]      # includes all columns of row 2; same as mat[1,:]
```

```
[56]: array([3, 4, 5])
```

We cannot omit the indices for *leading* axes, though! If an entire leading axis is to be included, we specify this using `:`

```
[57]: mat[:, 1] # includes all rows of column 2
```

```
[57]: array([1, 4])
```

Indexing lists and tuples

The basic indexing rules we have covered so far also apply to the built-in tuple and list types. However, list and tuple do not support advanced indexing available for NumPy arrays which we study in later units.

```
[58]: # Apply start:stop:step indexing to tuple
      tpl = (1,2,3)
      tpl[:3:2]
```

```
[58]: (1, 3)
```

2.3.4 Numerical data types (advanced)

We can explicitly specify the numerical data type when creating NumPy arrays.

So far we haven't done so, and then NumPy does the following:

- Functions such as `zeros()` and `ones()` default to using `np.float64`, a 64-bit floating-point data type (this is also called *double precision*)
- Other functions such as `arange()` and `array()` inspect the input data and return a corresponding array.
- Most array creation routines accept a `dtype` argument which allows you to explicitly set the data type.

Examples:

```
[59]: import numpy as np

      # Floating-point arguments return array of type np.float64
      arr = np.arange(1.0, 5.0, 1.0)
      arr.dtype
```

```
[59]: dtype('float64')
```

```
[60]: # Integer arguments return array of type np.int64
      arr = np.arange(1,5,1)
      arr.dtype
```

```
[60]: dtype('int64')
```

Often we don't care about the data type too much, but keep in mind that

- Floating-point has limited precision, even for integers if these are larger than (approximately) 10^{16}
- Integer values cannot represent fractional numbers and (often) have a more limited range.

This might lead to surprising consequences:

```
[61]: # Create integer array
      arr = np.ones(5, dtype=np.int64)
      # Store floating-point in second element
      arr[1] = 1.234
      arr
```

```
[61]: array([1, 1, 1, 1, 1])
```

The array is unchanged because it's impossible to represent 1.234 as an integer value!

The take-away is to explicitly write floating-point literal values and specify a floating-point dtype argument when we want data to be interpreted as floating-point values. For example, always write 1.0 instead of 1, unless you *really* want an integer!

2.4 Optional exercises

Exercise 1: string operations

Experiment with operators applied to strings and integers:

1. Define two string variables using the values 'Hello' and 'World', and concatenate them using +. Modify your solution to add a space.
2. Define a string variable 'NHH' and multiply it by 2 using *. What happens?
3. Define a string variable 'Hello' and use the += assignment operator to append another string 'World'.

The += operator is one of several operators in Python that combine assignment with another operation, such as addition. In this particular case, these statements are equivalent:

```
a += b
a = a + b
```

Exercise 2: string formatting with f-strings

We frequently want to create strings that incorporate integer and floating-point data, possibly formatted in a particular way.

Python offers quite powerful formatting capabilities which can become so complex that they are called the *Format Specification Mini-Language* (see the [docs](#)). In this exercise, we explore a small but useful subset of formatting instructions.

A format specification is a string that contains one or several {}, for example:

```
[62]: version = 3.13
      f'The current version of Python is {version}'
```

```
[62]: 'The current version of Python is 3.13'
```

What if we want to format the float 3.13 in a particular way? We can augment the {} to achieve that goal. For example, if the data to be formatted is of type integer, we can specify

- {}:w:d where w denotes the total field width and d indicates that the data type is an integer.

To print an integer into a field that is 3 characters wide, we would thus write {}:3d.

For floats we have additional options:

- {}:w.df specifies that a float should be formatted using a field width w and d decimal digits.

To print a float into a field of 10 characters using 5 decimal digits, we would thus specify {}:10.5f.

- {}:w.de is similar, but instead uses scientific notation with exponents.

This is particularly useful for very large or very small numbers.

- {}:w.dg, where g stands for *general* format, is a superset of f and e formatting. Either fixed or exponential notation is used depending on a number's magnitude.

In all these cases the field width w is optional and can be omitted. Python then uses as many characters as are required.

Now what we have introduced the formatting language, you are asked to perform the following exercises:

1. Modify the above f-string so that only the first decimal digit of the Python version is printed.
2. Modify the above f-string, but truncate the Python version to *not* include any decimal digits. Does this work with the integer formatting specification '{:d}'?
3. Print π using a precision of 10 decimal digits. *Hint:* the value of π is available as
4. Print e^{10} , computed as `exp(10.0)`, using exponential notation and three decimal digits. *Hint:* To use the exponential function, you need to import it using

```
from math import pi
```

```
from math import exp
```

Exercise 3: operations on tuples and lists

Create two lists `a` and `b` with the values 1, 2, 3 and 'a', 'b', 'c', respectively. Perform the following tasks and examine their results:

1. Concatenate the two lists using `+`.
2. Multiply the list `a` by the integer 2.
3. Append the elements ['x', 'y', 'z'] to `b` using the `+=` operator. Alternatively, do this using the list method `extend()`. Is the list `b` modified in place?
4. Append the integer 10 to `b` using the `+=` operator.
5. Duplicate the list `a` using the `*` operator. Is the list `a` modified in place?

Repeat steps 1-5 using *tuples* instead of lists.

Finally, create a list and a tuple and try to add them using `+`. Does this work?

2.5 Solutions

Solution for exercise 1

```
[63]: # 1. string concatenation using addition
      str1 = 'Hello'
      str2 = 'World'

      # Concatenate two strings using +
      str1 + str2
```

```
[63]: 'HelloWorld'
```

Note that this does not insert a space inbetween, so we have to do this manually:

```
[64]: str1 + ' ' + str2
```

```
[64]: 'Hello World'
```

```
[65]: # 2. string multiplication by integers
      str1 = 'NH'
      # Repeat string using multiplication!
      str1 * 2
```

```
[65]: 'NHHNHH'
```

```
[66]: # 3. Append using +=  
str1 = 'Hello'  
str1 += ' World'      # Append ' World' to value in str1, assign result to str1  
str1
```

```
[66]: 'Hello World'
```

Solution for exercise 2

```
[67]: # 1. Print Python version with only one decimal digit  
version = 3.13  
f'The current version of Python is {version:.1f}'
```

```
[67]: 'The current version of Python is 3.1'
```

```
[68]: # 2. Truncate all decimal digits  
# To do this, we use floating-point formatting with 0 decimal digits.  
f'The current version of Python is {version:.0f}'
```

```
[68]: 'The current version of Python is 3'
```

Note that this does not work with the integer formatting specification because that one does not accept any float-valued variables:

```
[69]: f'The current version of Python is {version:d}'
```

```
ValueError: Unknown format code 'd' for object of type 'float'
```

```
[70]: # 3. Print pi using 10 decimal digits  
from math import pi  
f'The first 10 digits of pi: {pi:.10f}'
```

```
[70]: 'The first 10 digits of pi: 3.1415926536'
```

```
[71]: # 4. Print exp(10.0) using three decimal digits and exponential notation  
from math import exp  
f'exp(10.0) = {exp(10.0):.3e}'
```

```
[71]: 'exp(10.0) = 2.203e+04'
```

Solution for exercise 3

List operators

```
[72]: # Create lists  
a = [1, 2, 3]  
b = ['a', 'b', 'c']
```

```
[73]: # 1. Adding two lists concatenates the second list to the first  
# and returns a new list object  
a + b
```

```
[73]: [1, 2, 3, 'a', 'b', 'c']
```

```
[74]: # 2. multiplication of list and integer duplicates the list!
      # (as opposed to multiplying each element by 2)
      a * 2
```

```
[74]: [1, 2, 3, 1, 2, 3]
```

```
[75]: # 3. Extending a list in place using +=
      # This does not return a new list but instead operates directly on b.
      b += ['x', 'y', 'z']
      b
```

```
[75]: ['a', 'b', 'c', 'x', 'y', 'z']
```

This is the same as using the `extend()` list method:

```
[76]: # Recreate original list b
      b = ['a', 'b', 'c']
      b.extend(['x', 'y', 'z'])
      b
```

```
[76]: ['a', 'b', 'c', 'x', 'y', 'z']
```

```
[77]: # 4. Append the integer 10. Note that we cannot directly append
      # the integer as such, this produces an error:
      b += 10
```

```
TypeError: 'int' object is not iterable
```

Instead, we have to embed the integer in a list if we want to use `+=`, or alternatively, we can use the `append()` method.

```
[78]: # Append single integer, wrap it in a list first
      b += [10]

      # Alternatively, use append()
      # b.append(10)
```

```
[79]: # 5. Replicating list in place using *=
      a *= 2
      a
```

```
[79]: [1, 2, 3, 1, 2, 3]
```

Tuple operators

```
[80]: # Create tuples
      a = 1, 2, 3
      b = 'a', 'b', 'c'
```

```
[81]: # 1. Adding two tuples concatenates the second tuple to the first
      # and returns a new tuple object
      a + b
```

```
[81]: (1, 2, 3, 'a', 'b', 'c')
```

```
[82]: # 2. Multiplication of tuple and integer replicates the tuple!
      a * 2
```

```
[82]: (1, 2, 3, 1, 2, 3)
```

```
[83]: # 3. Extending tuple in place
b += ('x', 'y', 'z')
b
```

```
[83]: ('a', 'b', 'c', 'x', 'y', 'z')
```

It might be surprising that this works since a tuple is an immutable collection. However, what happens is that the original tuple is discarded and the reference `a` now points to a newly created tuple.

When appending a single item to a tuple, we need to embed it in a tuple just as we did for the list earlier.

```
[84]: # Append integer 10 to tuple
b += (5, )
```

Similarly, if we replicate a tuple with `*` “in place” this actually returns a new tuple:

```
[85]: # 5. Replicate tuple in place using *=
a *= 2
a
```

```
[85]: (1, 2, 3, 1, 2, 3)
```

Tuple and list operators

We cannot mix tuples and lists as operands!

```
[86]: # Create list
a = [1, 2, 3]

# Create tuple
b = 'a', 'b', 'c'

# Cannot concatenate list and tuple!
a + b
```

```
TypeError: can only concatenate list (not "tuple") to list
```