1 Introduction

To measure the fractal dimension of the percolating cluster in a square site lattice, both the Sandbox Algorithm and the Box Counting algorithm as described in [1] were implemented. The largerst cluster found by the Hoshen-Kopelman algorithm [2] was used as percolating cluster.

2 Algorithm Description

The Hoshen-Kopelman algorithm implementation was taken over from the previously submitted exercise 03.
Both the Sandbox and the Box Counting algorithm were implemented as described in the course lecture notes [1].
For the box counting method, the choice was made to double the cell size at each step until reaching the lattice size L, i.e. $\epsilon \in \{1, 2, 4, 8, \ldots, L\}$.

3 Results

The program was implemented as described above and submitted with this report. A square site lattice of side length $L = 1024$ and occupation probabilities $p \in \{0.58, p_c = 0.592746, 0.61\}$ were used. As in the previous exercise, `C++`'s Mersenne Twister `mt19937` was used to initialize the lattices.

3.1 Experiment 1: Sandbox Algorithm

The box masses $M(R)$ where M equals the numbers of sites occcupied by the largest (i.e. percolating) cluster at occupation probability p in a box of increasing side length R on a site lattice with side length $L = 1024$, were recorded for $R \in \{3, 4, \cdots, L\}$. The results were plotted as $\log(M(R))$ versus $\log(R)$ as shown in figure 1.

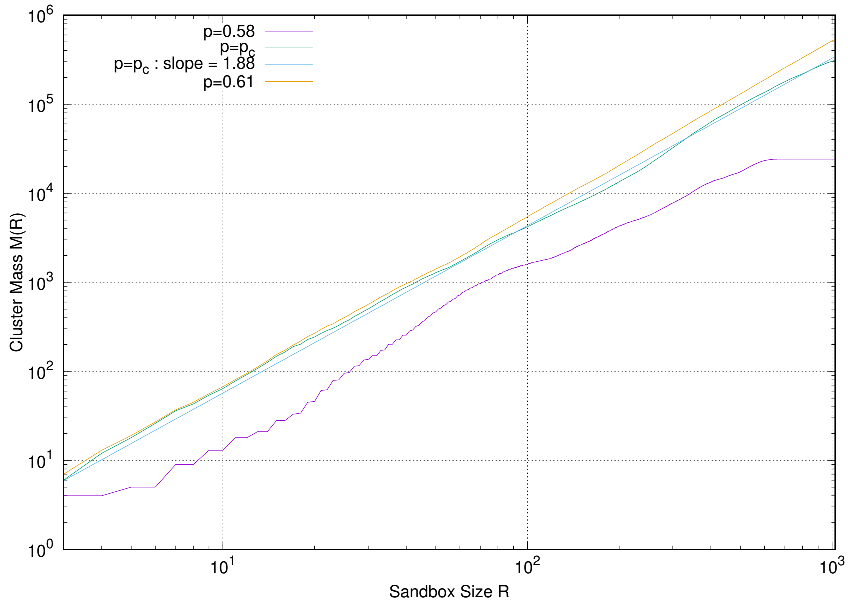

Figure 1: Sandbox Algorithm Results for $p \in \{0.58, p_c, 0.61\}$.

3.2 Experiment 2: Box Counting Algorithm

The number of non-empty grid cells $N(\epsilon)$ where ϵ is the grid constant were recorded for $L = 1024$ and $\epsilon \in \{1, 2, 4, 8, \ldots, L\}$ as described above. The results were plotted as $\log(N(\epsilon))$ versus $\log(\frac{1}{\epsilon})$ as shown in figure 2.

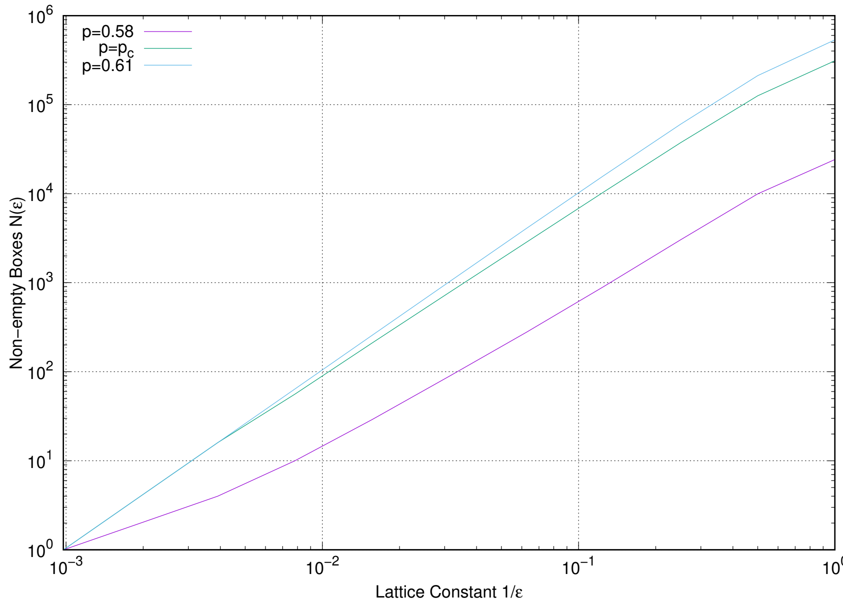

Figure 2: Box Counting Algorithm Results for $p \in \{0.58, p_c, 0.61\}$.

4 Discussion

The slope of 1.88 of the line for $p = p_c$ in figure 1 obtained from the Sandbox algorithm is well in line with the theoretical value of $d_f = \frac{91}{48} \approx 1.89$ for a 2D lattice and thus as expected. For some reason reason, `gnuplot` couldn't be brought to fit a slope to the line for $p = p_c$ in figure 2 obtained from the Box Counting algorithm. However, a visual inspection reveals that the slope is just a tad under 2 and thus in line with both the sandbox algorithm results and theoretical expectations.

References

[1] Herrmann, H. J., Singer, H. M., Mueller L., Buchmann, M.-A.,
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[2] Hoshen, J., Kopelman, R.,
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