1 Introduction

Both the Newton-Raphson method and the Secant method were implemented and employed to find an extremum of a continuous multivariate function. If started on a point close enough to the extremum, both (very similar) methods were found to converge on the maximum which was known analytically in the case under consideration.

2 Algorithm Description

While the algorithm as such is quite simple, using it on the given function $F(x, y) = e^{-(x-x_0)^2-(y-y_0)^2}$ requires some setup.
Analogous to the univariate case, to find the (global) maximum of $F(x, y)$, we must find the root of $\vec{\nabla}F(x, y)$, or in other words, the solution of $\vec{0} = \vec{f}(x, y) \coloneqq \vec{\nabla}F(x, y)$ (see equation 1). The Jacobi matrix of $f(x, y)$ required by the Newton-Raphson method then is as described by equation 2.

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \vec{f}(x, y) \coloneqq \begin{pmatrix} \partial F(x, y)/\partial x \\ \partial F(x, y)/\partial y \end{pmatrix} = \begin{pmatrix} -2(x-x_0) \cdot e^{-(x-x_0)^2-(y-y_0)^2} \\ -2(y-y_0) \cdot e^{-(x-x_0)^2-(y-y_0)^2} \end{pmatrix} \tag{1}$$

$$\begin{aligned} \vec{\vec{J}}_f(x, y) &= \begin{pmatrix} \partial f_x(x, y)/\partial x & \partial f_x(x, y)/\partial y \\ \partial f_y(x, y)/\partial x & \partial f_y(x, y)/\partial y \end{pmatrix} = \begin{pmatrix} \partial^2 F(x, y)/\partial x^2 & \partial^2 F(x, y)/\partial x \partial y \\ \partial^2 F(x, y)/\partial y \partial x & \partial^2 F(x, y)/\partial x^2 \end{pmatrix} = \\ &= e^{-(x-x_0)^2-(y-y_0)^2} \cdot \begin{pmatrix} 4(x-x_0)-2 & 4(x-x_0)(y-y_0) \\ 4(x-x_0)(y-y_0) & 4(y-y_0)^2-2 \end{pmatrix} \end{aligned} \tag{2}$$

For brevity, we refer to the task description for the interpolating Jacobian of the Secant method.

With the only difference between the Newton-Raphson method and the Secant method being the way the Jacobian matrix is calculated, the algorithm then works as follows:

- Choose starting point $\vec{x}_0$ close enough to suspected maximum
- While $f(x_{n+1}) > \epsilon$
  - Do $\vec{x}_{n+1} = \vec{x}_n - (\vec{\vec{J}}_f(\vec{x}_n))^{-1} \cdot f(\vec{x}_n)$
- Output $\vec{x}_{n+1}$

3 Results

The program was implemented as described above and submitted with this report. For the given problem, the Jacobian matrices of the methods are almost equal, and both were found to converge for starting values x, y in the range $x \in [x_0 - \xi, x_0 + \xi], y \in [y_0 - \upsilon, y_0 + \upsilon]; \quad \xi = \upsilon = 0.35$ (see figures 1 and 2).

4 Discussion

Both methods were found to converge in an equal amount of steps. As expected, the starting coordinates have to be chosen close to the actual maximum for the methods to converge.

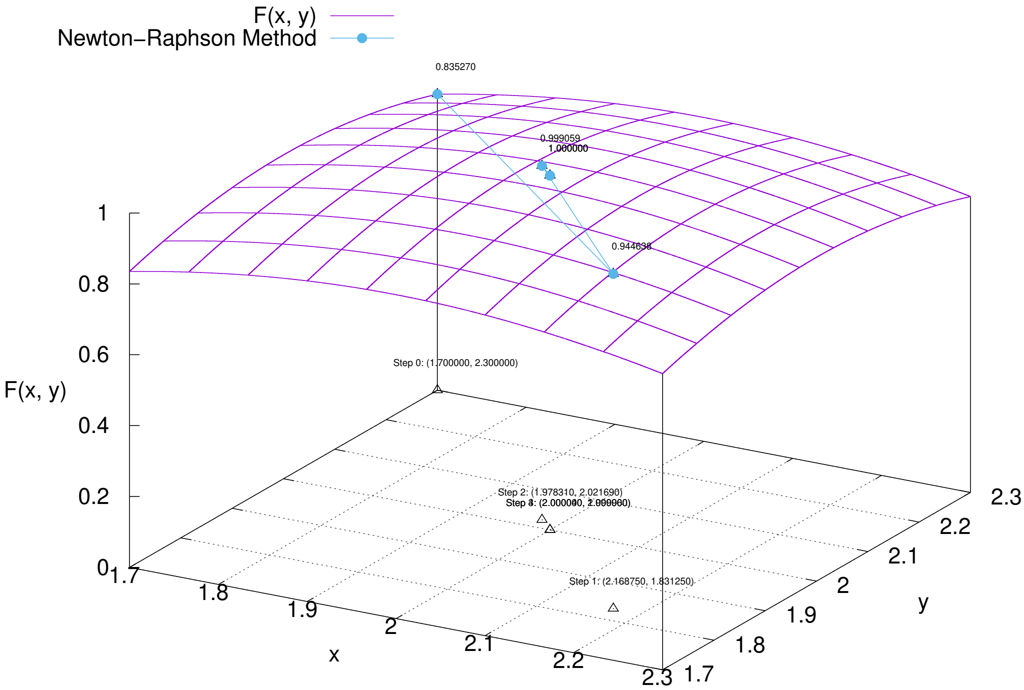

Figure 1: Convergence steps of Newton-Raphson method to find maximum of $F(x, y) = e^{-(x-x_0)^2-(y-y_0)^2}; x_0 = y_0 = 2$.

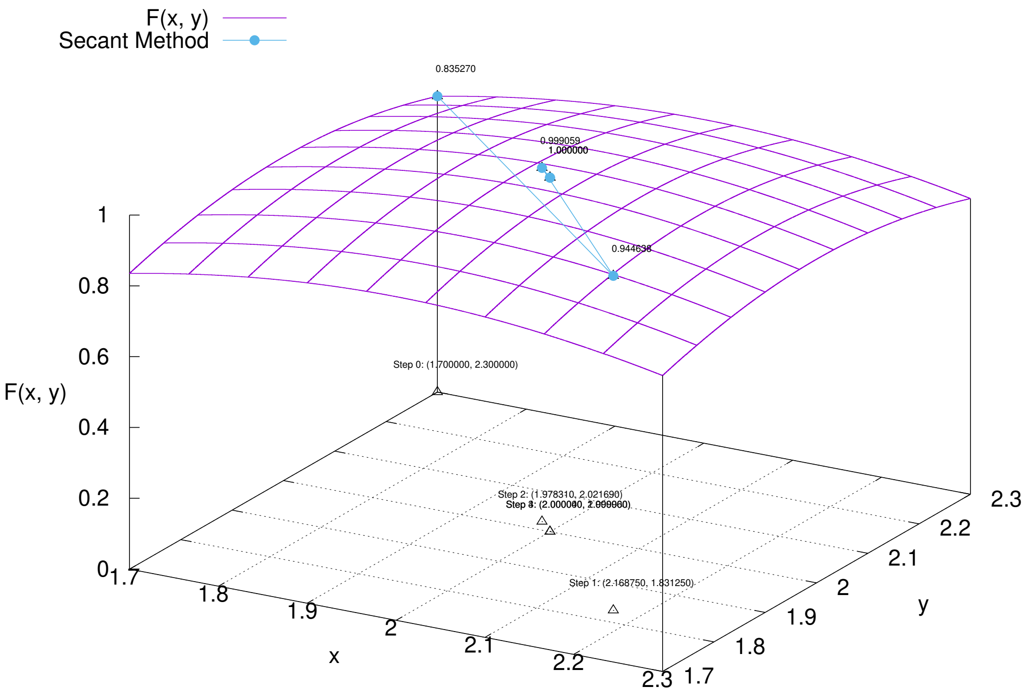

Figure 2: Convergence steps of Secant method to find maximum of $F(x, y) = e^{-(x-x_0)^2-(y-y_0)^2}; x_0 = y_0 = 2$.