1 Task 1: Heat2D on MPI

Initial remark: An attempt at solving the full task including subtasks c) and d) was made but time was not sufficient to eradicate bugs in subsections c) and d) for multgrid. Hence, the code is submitted as-is with partially functioning multigrid kernels which are deactivated by setting the gridCount to 1.

1.1 Implementation

The original code was refactored and extended to include the following:

- storing all grids in double* pointers instead of the original cumbersome double** pointers.
- extending gridLevel to hold MPI-specific information
- extending each level's grids to include halos for MPI communication
- introducing a new world struct for each MPI rank to hold grid-specific information
- using MPI_Cart_create and associated MPI calls to dynamically create a cartesian grid

1.2 Results

The CPU code was run as-is for a baseline reference. The MPI code was run for a single rank and then for 1, 2 and 4 full nodes. All outputs are given in appendix A.

1.3 Discussion

All runs exhibit the same L2Norm as required in the task specification. The strong scaling speed-up and efficiency are observed as follows.

1.3.1 Jacobi smoothing

- 24 threads: Speed-up xxx, efficiency xxx%
- 48 threads: Speed-up xxx, efficiency xxx%
- 96 threads: Speed-up xxx, efficiency xxx%

1.3.2 Residual calculation

- 24 threads: Speed-up xxx, efficiency xxx%
- 48 threads: Speed-up xxx, efficiency xxx%
- 96 threads: Speed-up xxx, efficiency xxx%

1.3.3 L2Norm calculation

- 24 threads: Speed-up xxx, efficiency xxx%
- 48 threads: Speed-up xxx, efficiency xxx%
- 96 threads: Speed-up xxx, efficiency xxx%

1.3.4 Overall

- 24 threads: Speed-up xxx, efficiency xxx%
- 48 threads: Speed-up xxx, efficiency xxx%
- 96 threads: Speed-up xxx, efficiency xxx%

Finally, looking at the overall efficiency it becomes evident that commmunication costs increase and efficiency decreases as we add ranks on multiple nodes as is to be expected. Optimizing communication could help with this to some extent, but ultimately the algorithm as-is suffers from having to perform an MPI_Allreduce for each rank to decide when to stop iterations.

2 Task 2: Communication-Tolerant Programming

2.1 Part a)

2.1.1 Implementation

Using OpenMP, the first hybrid model can be implemented by adding the line #pragma omp parallel for collapse(3) before calculating the Jacobi stencil. This can be done because the loops are perfectly nested and no data race is created. Further, as discussed on Piazza, the grid parameter was changed to 768 to allow for integer divisibility by the required number of ranks.

2.1.2 Results

Strangely, the L2 Norms achieved were somewhat inconsistent even though the directive of adapting the grid size to 768 given on Piazza was adhered to. This applies also to the given pure MPI code which was run otherwise unmodified. The pure MPI code was run as-is for a baseline reference on 1, 2 and 4 full nodes. The hybrid MPI/OpenMP code was run for a partial load on 1, 2 and 4 nodes and then for 1, 2 and 4 full nodes. All outputs are given in appendix B.

2.1.3 Discussion

Looking at the pure implementation, network communication becomes evident as expected when we increase from 2 to 4 nodes. For the hybrid implementation, it was attempted to unveil communication cost by running a partial load distributed across several nodes. The increasing significant MPI_Waital1 times when the isolated MPI ranks are distributed across several nodes reflect those relevant communication cost. Running the hybrid implentation on full nodes in an attempt to reduce the requirement for communication initially shows no improvement on 2 full nodes probably as the reduced communication between ranks is compensated by increasing message size as the ranks' grid fractions are larger in this case. However, when running the hybrid model on 4 full nodes, the MPI_Waital1 reduce by a factor of almost two as the benefits of less communication per se coincides with reduced message size, hinting that this benefit would likely increase when adding further nodes.

A Task 1 Euler outputs

Listing 1: Task 2: Collected Euler outputs.

```
MPI: 1 node(s) / 1 thread(s)

MPI: 1 node(s) / 1 thread(s)

MPI: 1 node(s) / 24 thread(s)

MPI: 2 node(s) / 48 thread(s)
```

```
MPI num_procs: 48
MPI dims_x:
MPI dims_y: 6
                         Grid0
          Time (s)
                                        Total
                   478.754 \\ 138.568
                                    478.754 \\ 138.568
Smoothing
Residual
Restriction
                   0.000
                                 0.000
                   0.000
Prolongation
                                 0.000
L2Norm
                   341.930
                                   341.930
Total
                   959 252
                                   959.252
Running Time : 20
MPI L2Norm: 70812.9164
                         20.232\,\mathrm{s}
MPI:
                  4 node(s) / 96 thread(s)
```

B Task 2 Euler outputs

Listing 2: Task 2: Collected Euler outputs.

```
1 \text{ node(s)} / 24 * 1 = 24 \text{ thread(s): FULL}
 Pure MPI:
 Execution Times:
        Compute:
                                                  1.0350s
        MPI_Irecv:
                                                  0.0003 \, s
        MPI_Isend:
                                                   0.0021 \, \mathrm{s}
        Packing:
                                                  0.0000s
                                                  0.0000s
        MPI_Waitall: 0.4012s
  Total Time:
                                                  1.4423 \, \mathrm{s}
 L2 Norm:
                                                  0.9811283499\\
 Hybrid MPI/OpenMP: 1 node(s) / 4 * 6 = 24 thread(s): FULL
[eu-c7-104-11:09836] SETTING BINDING TO CORE
[eu-c7-104-11:09836] MCW rank 0 bound to socket 0[core 0[hwt 0-1]], socket 0[core 1[hwt 0-1]], socket 0 0[core 2[hwt 0-1]], socket 0[core 3[hwt 0-1]], socket 0[core 4[hwt 0-1]], socket 0[core 5[hwt 0-1]]], socket 0[core 7[hwt 0-1]]], socket 0[core 7[hwt 0-1]]], socket 0[core 7[hwt 0-1]]], socket 0[core 10[hwt 0-1]]], socket 0[core 11[hwt 0-1]]], socket 1[core 12[hwt 0-1]]], socket 1[core 13[hwt 0-1]]], socket 1[core 14[hwt 0-1]]], socket 1[core 15[hwt 0-1]]], socket 1[core 16[hwt 0-1]]], socket 1[core 17[hwt 0-1]]], socket 1[core 18[hwt 0-1]]], socket 1[core 19[hwt 0-1]]], socket 1[core 20[hwt 0-1]]], socket 1[core 21[hwt 0-1]], socket 1[core 22[hwt 0-1]]], socket 1[core 22[hwt 0-1]]], socket 1[core 23[hwt 0-1]]]]
   [\mathrm{\,eu-c7\,-}104\,-}11:09836] SETTING BINDING TO CORE
 Execution Times:
        MPI_Irecv:
                                                  0.0003s
        MPI_Isend:
                                                  0.0041\,\mathrm{s}
        Packing:
Unpacking:
                                                  0.0000s
        MPI_Waitall: 2.2758s
 Total Time:
                                                  9.1238s
                                                  0.9887095465
 Hybrid MPI/OpenMP: 1 node(s) / 2 * 12 = 24 thread(s): FULL
MPI_Irecv:
                                                  0.0003s
        MPI_Isend:
                                                  0.0043\,\mathrm{s}
       Packing:
Unpacking:
                                                  0.0000s
        MPI_Waitall: 2.4361s
 Total Time:
                                                  10.1844s
 L2 Norm:
                                                  0.9887095464
                                                              2 \text{ node(s)} / 48 * 1 = 48 \text{ thread(s): FULL}
 Pure MPI:
 Execution Times:
```

```
0.5175 s
       Compute:
                                          0.0006s
       MPI_Irecv:
       MPI_Isend:
                                          0.0038s
       Packing:
                                          0.0000s
       Unpacking:
                                          0.0000s
       MPI_Waitall: 0.3561s
                                         0.9141s
0.9811283499
 Total Time:
 L2 Norm:
 Hybrid\ MPI/OpenMP\colon\ 2\ node(s)\ /\ 4\ *\ 6\ =\ 24\ thread(s)\colon\ PARTIAL
Execution Times:
      Compute:
MPI_Irecv:
                                         6.3606s
       MPI_Isend:
                                          0.0040 \, s
       Packing:
       Unpacking:
                                         0.0000s
       MPI_Waitall: 12.6248s
                                         19.1037s
 L2 Norm:
                                         0.9887095465
 Hybrid\ MPI/OpenMP\colon\ 2\ node(s)\ /\ 4\ *\ 12\ =\ 48\ thread(s)\colon FULL
3.8945s
       Compute:
       MPI_Irecv
       MPI Isend:
                                          0.0034 s
                                          0.0000 \, \mathrm{s}
       Packing:
       Unpacking:
                                          0.0000s
       MPI_Waitall: 22.2460s
 Total Time:
                                          26 3027 s
                                         0.9887095465
 L2 Norm:
 Pure MPI:
                                                   4 \text{ node(s)} / 96 * 1 = 96 \text{ thread(s): FULL}
 Execution Times:
                                         0.1653s
       Compute:
       MPI_Irecv:
MPI_Isend:
                                         0.0028 s
0.0018 s
       Packing:
                                          0.0000s
       Unpacking:
                                          0.0000s
      MPI_Waitall: 5.6531s
  Total Time:
                                          6.0211s
                                         0.9811237635
 L2 Norm:
 Hybrid MPI/OpenMP: 4 node(s) / 4 * 6 = 24 thread(s): PARTIAL
  [eu-c7-118-08:24464] SETTING BINDING TO CORE [eu-c7-101-13:17817] MCW rank 1 bound to socket 0[core 0[hwt 0-1]], socket 0[core 1[hwt 0-1]], socket
 \begin{bmatrix} \text{eu-c7} - 101 - 13:17817 \end{bmatrix} \text{ MCW } \text{rank 1 bound to socket 0[core 0[hwt 0-1]], socket 0[core 1[hwt 0-1]], socket } \\ & \rightarrow 0[\text{core 2[hwt 0-1]], socket 0[core 3[hwt 0-1]], socket 0[core 4[hwt 0-1]], socket 0[core 5[hwt 0-1]], socket 0[core 5[hwt 0-1]]], socket 0[core 0[hwt 0-1]], socket 0[core 1[hwt 0-1]], socket 0[core 0[hwt 0-1]], socket 0[core 1[hwt 0-1]], socket 0[core 1[hwt 0-1]], socket 0[core 2[hwt 0-1]], socket 0[core 3[hwt 0-1]], socket 0[core 4[hwt 0-1]], socket 0[core 5[hwt 0-1]], socket 0[core 0[hwt 0-1]], sock
 Execution Times
```

```
Compute:
              5.7847s
              0.0015\,\mathrm{s}
  MPI_Irecv:
  MPI_Isend:
              0.0061s
  Packing:
              0.0000s
  Unpacking:
              0.0000s
  MPI-Waitall: 12.4602s
Total Time:
              18.2548s
              0.9887095465
L2 Norm:
\label{eq:hybrid_MPI_OpenMP: 4 node(s) / 8 * 12 = 96 thread(s): FULL}
  Compute:
  MPI_Irecv:
             0.0017s
  MPI_Isend:
  Packing:
Unpacking:
              0.0000s
  MPI_Waitall: 10.4580s
Total Time:
              12.4772s
              0.9887095465
```