Q1) Find the greatest common divisor of 252 and 198 using the Euclidean algorithm.

Solution: Successive uses of the division algorithm give:

$$\begin{aligned} 252 &= 1 \cdot 198 + 54 \\ 198 &= 3 \cdot 54 + 36 \\ 54 &= 1 \cdot 36 + 18 \\ 36 &= 2 \cdot 18. \end{aligned}$$

so

$$\gcd(252, 198) = 18$$

Q2) Find the greatest common divisor of 414 and 662 using the Euclidean algorithm.

Solution: Successive uses of the division algorithm give:

$$\begin{aligned} 662 &= 414 \cdot 1 + 248 \\ 414 &= 248 \cdot 1 + 166 \\ 248 &= 166 \cdot 1 + 82 \\ 166 &= 82 \cdot 2 + 2 \\ 82 &= 2 \cdot 41. \end{aligned}$$

Hence, $\gcd(414, 662) = 2$, because 2 is the last nonzero remainder.

Q3) Find the expantion of the followings:

a) What is the decimal expansion of the integer that has $(1\,0101\,1111)_2$ as its binary expansion?

Solution: $(1\,0101\,1111)_2 = 1 \cdot 2^8 + 0 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 1 \cdot 2^4$
$+ 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = 351.$

b) What is the decimal expansion of the number with octal expansion $(7016)_8$?

Solution: Using the definition of a base b expansion with $b = 8$

$$(7016)_8 = 7 \cdot 8^3 + 0 \cdot 8^2 + 1 \cdot 8 + 6 = 3598.$$

c) What is the decimal expansion of the number with hexadecimal expansion $(2AE0B)_{16}$?

Solution: Using the definition of a base b expansion with $b = 16$

$$(2AE0B)_{16} = 2 \cdot 16^4 + 10 \cdot 16^3 + 14 \cdot 16^2 + 0 \cdot 16 + 11 = 175627.$$

Q4 Find The Expantion of the followings :

a) Find the octal expansion of $(12345)_{10}$.

Solution: First, divide 12345 by 8 to obtain

$$12345 = 8 \cdot 1543 + 1.$$

Successively dividing quotients by 8 gives

$$\begin{aligned} 1543 &= 8 \cdot 192 + 7, \\ 192 &= 8 \cdot 24 + 0, \\ 24 &= 8 \cdot 3 + 0, \\ 3 &= 8 \cdot 0 + 3. \end{aligned}$$

Hence, $(12345)_{10} = (30071)_8.$

b) Find the binary expansion of $(241)_{10}$.

Solution: First divide 241 by 2 to obtain

$$241 = 2 \cdot 120 + 1.$$

Successively dividing quotients by 2 gives

$$\begin{aligned}
120 &= 2 \cdot 60 + 0,\\
60 &= 2 \cdot 30 + 0,\\
30 &= 2 \cdot 15 + 0,\\
15 &= 2 \cdot 7 + 1,\\
7 &= 2 \cdot 3 + 1,\\
3 &= 2 \cdot 1 + 1,\\
1 &= 2 \cdot 0 + 1.
\end{aligned}$$

Hence, $(241)_{10} = (1111\ 0001)_2$.