Central Limit Theorem for $\hat{\pi}_i$

Beate Franke

Stochastic Processes Group, University College London, UK

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CLT in Olhede and Wolfe (2012) [1]

Assumption

simple graph, $A_{ij} \sim \text{Bern}(p_{ij})$ with $p_{ij} = \pi_i \pi_j$

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$\Rightarrow$

$$\frac{\hat{\pi}_i - \pi_i}{\sqrt{\pi_i/||\pi||_1}} \xrightarrow{d} N(0,1)$$

Generalization of CLT

Additional Assumptions

$$\mathbb{E}(d_i|\pi) \text{ is growing to } \infty, \qquad \frac{||\pi||_2^2}{||\pi||_1} \overset{n\to\infty}{\to} 0.$$

Generalization of CLT

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Then, $$\frac{\hat{\pi}_i - \pi_i}{\sqrt{\pi_i/||\pi||_1}} \xrightarrow{d} N(0,1).$$

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Then, $$\frac{\hat{\pi}_i - \pi_i}{\sqrt{\pi_i/||\pi||_1 - \pi_i^2(||\pi||_2^2/||\pi||_1^2)}} \xrightarrow{d} N(0,1).$$

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$\mathrm{Var}(d_i|\pi)$ is growing to ∞.

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Power law case ✓

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Erdös R*é*nyi graph ✓

References

S. C. Olhede and P. J. Wolfe.
What is a Degree Distribution?
arXiv:1211.6537v1, 2012.