

COMS20011 – Data-Driven Computer Science

Problem Sheet MM05

1 – Calculate the result of the convolution $A*B$ in each of the examples below by hand.

(i) $A = (1 \ 2 \ 1)$ $B = (2 \ 2 \ 3 \ 3 \ 2)$

(ii) $A = (1 \ 1 \ 3 \ 1 \ 1)$ $B = (3 \ 3 \ -2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 3)$

(iii) $A = \begin{pmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 5 & 5 & 5 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 10 & 10 & 10 & 0 \\ 0 & 5 & 10 & 5 & 0 \\ 0 & 5 & 5 & 5 & 0 \end{pmatrix}$

Now verify your result using Matlab (using `conv` and use `help conv` to determine what convention Matlab uses when convolving at the border points) or using Python.

Answer:

H is the result by hand using the convention seen in the lecture. **M** is the result using Matlab and the convention used by `conv` (which does not normalise and leaves it to the user).

(i) *Without normalisation*

$$H = (9 \ 11 \ 11)$$

$$M = (2 \ 6 \ 9 \ 11 \ 11 \ 7 \ 2)$$

With normalisation

$$H = \frac{1}{4}(9 \ 11 \ 11)$$

$$M = (2 \ 6 \ 9 \ 11 \ 11 \ 7 \ 2)$$

$$M = M * 1/4$$

(ii) Normalisation factor is $\frac{1}{7}$

$$H = (-1 \ -1 \ 0 \ 7 \ 13) \quad M = (3 \ 6 \ 10 \ 9 \ -1 \ -1 \ 0 \ 7 \ 13 \ 14 \ 11 \ 5 \ 2)$$

(iii) Normalisation factor is $\frac{1}{8}$

$$H = \begin{pmatrix} -35 & 0 & 35 \\ -40 & 0 & 40 \\ -35 & 0 & 35 \end{pmatrix}$$

$$M = \begin{pmatrix} 0 & -5 & -5 & 0 & 5 & 5 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -30 & -40 & 0 & 40 & 30 & 0 \\ 0 & -25 & -35 & 0 & 35 & 25 & 0 \\ 0 & -15 & -20 & 0 & 20 & 15 & 0 \\ 0 & -5 & -5 & 0 & 5 & 5 & 0 \end{pmatrix}$$

2 – Imagine you have received a huge shipment of three varieties of fruits consisting of *Oranges*, *Satsumas*, and *Red Pears*. The fruit is unfortunately mixed up, but you have access to a vision system you can program to distinguish and separate the fruit as they pass in front of a camera on a conveyor belt one at a time. The camera is positioned to give a top-view of the fruit.

- (a) State no fewer than three features you would use in your design to distinguish between the different types. Very briefly explain why your features will pick the correct type each time considering that some measurements may be somewhat affected by noisy data from the image acquisition process.

Answer:

Shape of fruit, colour of fruit, and size of fruit.

Shape is probably enough to locate pears (roundish for both oranges and satsumas, not round for pear) but due to noise, maybe colour could be used as an additional cue. Size should be enough to locate satsumas, but best to be combined with colour for more certainty.

All three features should be combined to select oranges to deal with the possibly noisy measurements (i.e. larger than satsumas, rounder than pears, and more orangeness than pears).

- (b) Consider you had actually been asked to consider using up to 20 features for this task. Discuss what would you do to find out which features are significant (or which ones are redundant)?

Answer:

Apply some form of step-wise feature selection or feature elimination (e.g. to a training set of the data) and keep only those features whose eigenvalues correspond to say 90% to 95% of the variance in the data. These should hopefully be substantially fewer than 20 dimensions.

3 – Given a feature set X_i , $i = 1, \dots, 20$, we wish to find a subset Y of 9 features, that gives the best $P(\text{correct classification})$. Determine how many times the $P(\text{correct classification})$ function will have to be evaluated to find the best subset Y .

Answer:

This uses the Binomial Coefficient formula: this is a very standard mathematical formula which may NOT be given in an exam. Please learn it if you don't know it already.

$$20! / (9! * 11!) = 167960$$

4 – A 3x3 spatial filter with all elements set to **-1**, except the central element set to **16**, has a...

- (a) normalisation factor of $1/8$
- (b) normalisation factor of $1/16$
- (c) normalisation factor of $1/24$**
- (d) normalisation factor of $1/12$

Sum the absolute value of each element, i.e. $\text{abs}(-1)*8 + \text{abs}(16) = 24$

5 – The eigenvalues of a 6D dataset are: [17, 11, 8, 2, 0.65, 0.35]. What *variance* in the dataset do the first 3 eigenvalues represent?

- (a) 93.2%
- (b) 91.3%
- (c) 96.3%
- (d) 92.3%**

$$(17+11+8) / (17+11+8+2+0.65+0.35) = 0.92307$$

6 – Apply convolution filter M to matrix K at location (4,3) using 8-connectivity and 4-connectivity.

$$M = \begin{pmatrix} 1 & 1 & -1 \\ 4 & 8 & -4 \\ 1 & 1 & -1 \end{pmatrix} \quad K = \begin{pmatrix} 10 & 8 & 0 & 0 & 1 \\ 8 & 9 & 2 & 1 & 0 \\ 8 & 8 & 1 & 1 & 2 \\ 6 & 7 & \mathbf{1} & 1 & 3 \\ 4 & 8 & 2 & 0 & 3 \end{pmatrix}$$

Location (4,3) is highlighted above which has value $K(4,3) = 1$.

Applying M with 8-connectivity, the result is = -28

Applying M with 4-connectivity (i.e. $M = \begin{pmatrix} 0 & 1 & 0 \\ 4 & 8 & -4 \\ 0 & 1 & 0 \end{pmatrix}$), the result is = -13