1. a) i) baa

state	input (tape)	stack
start	baaΔ...	Δ...
READ$_1$	$\emptyset$aaΔ...	Δ...
PUSH b	$\emptyset$aaΔ...	bΔ...
READ 2	$\emptyset\emptyset$aΔ...	bΔ...
READ3	$\emptyset\emptyset\emptyset\Delta$...	bΔ...
POP	$\emptyset\emptyset\emptyset\Delta$...	Δ...
READ4	$\emptyset\emptyset\emptyset\emptyset\Delta$...	Δ...
POP	$\emptyset\emptyset\emptyset\emptyset\Delta$...	Δ...
Accept	$\emptyset\emptyset\emptyset\emptyset\Delta$...	Δ...

ii) baba

state	input	stack
start	baba	Δ
Read 1	aba	Δ
Push b	aba	b
Read 2	ba	b
Read 3	ba	b
reject	ba	b

b) i) bbaa

Note: input = tape

state	input	stack
start	bbaaΔ...	Δ...
Read 1	Øbaa Δ...	Δ...
Push b	ØbaaΔ...	bΔ...
Read 2	ØØaaΔ...	bΔ...
Push b	ØØaaΔ...	bbΔ...
Read 2	ØØØaΔ...	bbΔ...
Read 3	ØØØØΔ...	bbΔ...
Pop	ØØØØΔ...	bΔ...
Read 4	ØØØØΔΔ...	bΔ...
Pop	ØØØØΔΔ...	Δ...
Reject	ØØØØΔΔ...	Δ...

ends in reject

bbaaaa

state	input	stack
start	bbaaaa	Δ...
Read 1	Øbaaaa	Δ...
Push b	Øbaaaa	bΔ...
Read 2	ØØaaaa	bΔ...
Push b	ØØaaaa	bbΔ...
Read 2	ØØØaaa	bbΔ...
Read 3	ØØØØaa	bbΔ...
Pop	ØØØØaa	bΔ...
Read 4	ØØØØØa	bΔ...
Read 3	ØØØØØØΔ...	bΔ...
Pop	ØØØØØØΔ...	Δ...
Read 4	ØØØØØØΔ...	Δ...
Pop	ØØØØØØΔΔ...	Δ...
accept	ØØØØØØΔΔ...	Δ...

ends in accept

(ends in accept)

c) Words that have one or more b's followed by sequence of a's where the number of a's is twice the number of b's.

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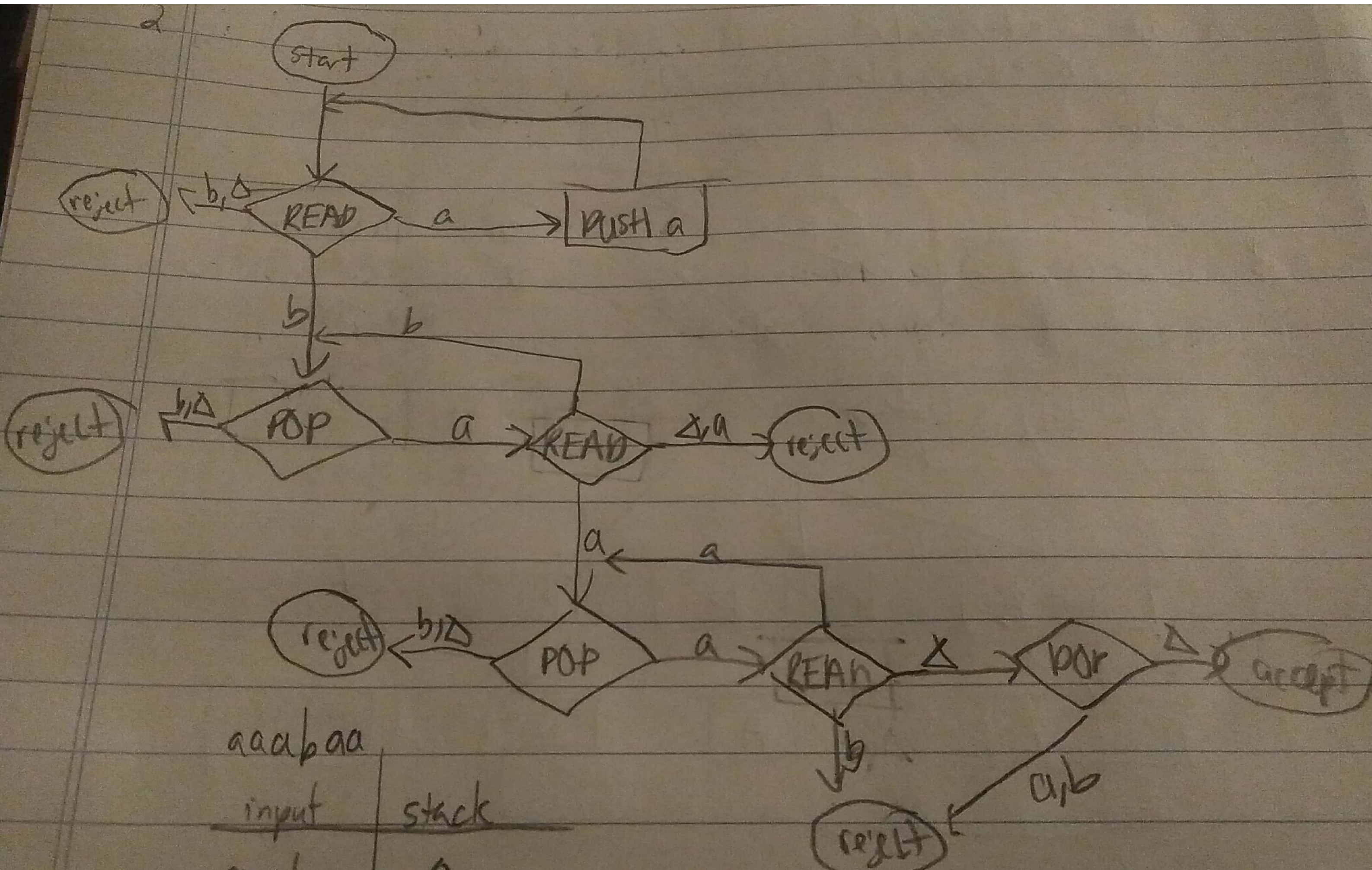
d) $S \rightarrow bXaa \mid \Delta$

$X \rightarrow baa \mid bXaa$

e) No, the language is not regular

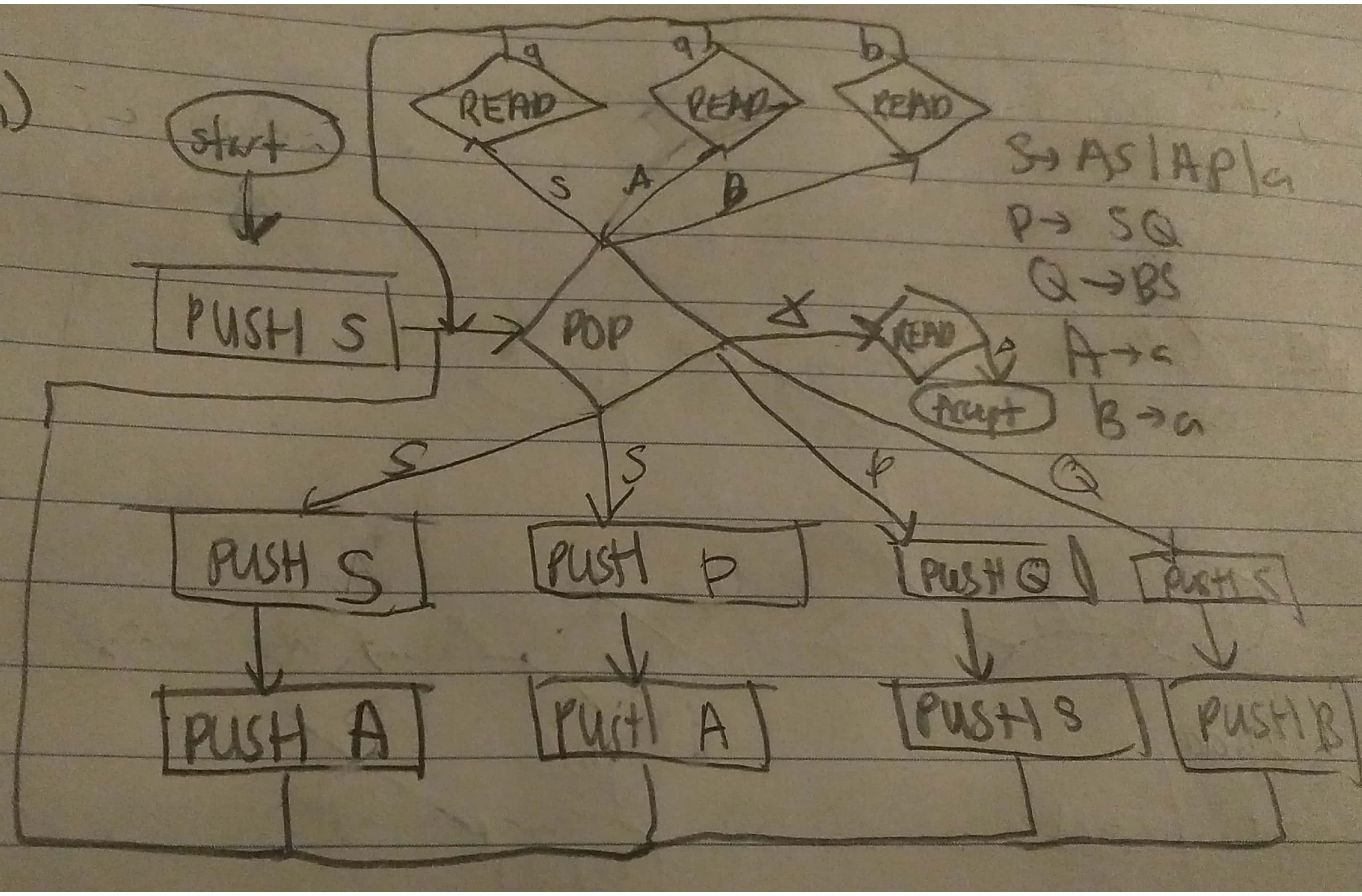
2



aaabaa

input	stack
aaaba[illegible]	Δ

3. a)



$S \to AS \mid AP \mid a$

$P \to SQ$

$Q \to BS$

$A \to a$

$B \to a$

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b) state	input	stack
Start	$aa\Delta$...	Δ...
Push S	$aa\Delta$...	$S\Delta$...
Pop	$aa\Delta$...	Δ...
Push S	$aa\Delta$...	$S\Delta$...
Push A	$aa\Delta$...	$AS\Delta$...
Pop	$aa\Delta$...	$S\Delta$...
Read	$\emptyset a\Delta$...	$S\Delta$...
Pop	$\emptyset a\Delta$...	Δ...
Read	$\emptyset\emptyset\Delta$...	Δ...
Pop	$\emptyset\emptyset\Delta$...	Δ...
Read	$\emptyset\emptyset\Delta$.	Δ...
Accept	$\emptyset\emptyset\Delta$.	Δ...

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4 Assume L is context free. There is a grammar G in CNF which generates L with p productions of the form Nonterm → NonT NonT

Let $N = 2^p$

$w = a^N b^{3N} a^N \in L$ and length $(w) > 2^p$
$= 2^p \cdot 3$

By the pumping lemma there exists u, v, x, y, z s.t. $w = uvxyz$
$x \neq \Lambda$ et $v \neq \Lambda$ ou $y \neq \Lambda$
$\forall n \geq 1 \quad uv^n xy^n z \in L$

Note: there is exactly one factor ab and one factor ba in w

Case 1: v or y contains ab or ba
$uv^2 xy^2 z$ contains ab twice or ba twice
$\therefore \notin L$

Case 2: v and y contain only a's or b's but not both ($v \neq \Lambda$ or $y \neq \Lambda$)
Consider $uv^2 xy^2 z$
In all cases, there is a max of 2 sequences of a's and/or b's that are increased, always leading to a word not in L.

There is no way to decompose w into u, v, x, y, z st $w = uvxyz$ and $\forall n, n \geq 1 \quad uv^n xy^n z \in L$

Contradiction, thus L is not context free

5. a) If L_2 is context free there is a grammar in CNF which generates L_2. The grammar is below

$S \to aXbYa$

$X \to aX \mid a$

$Y \to bYa \mid ba$

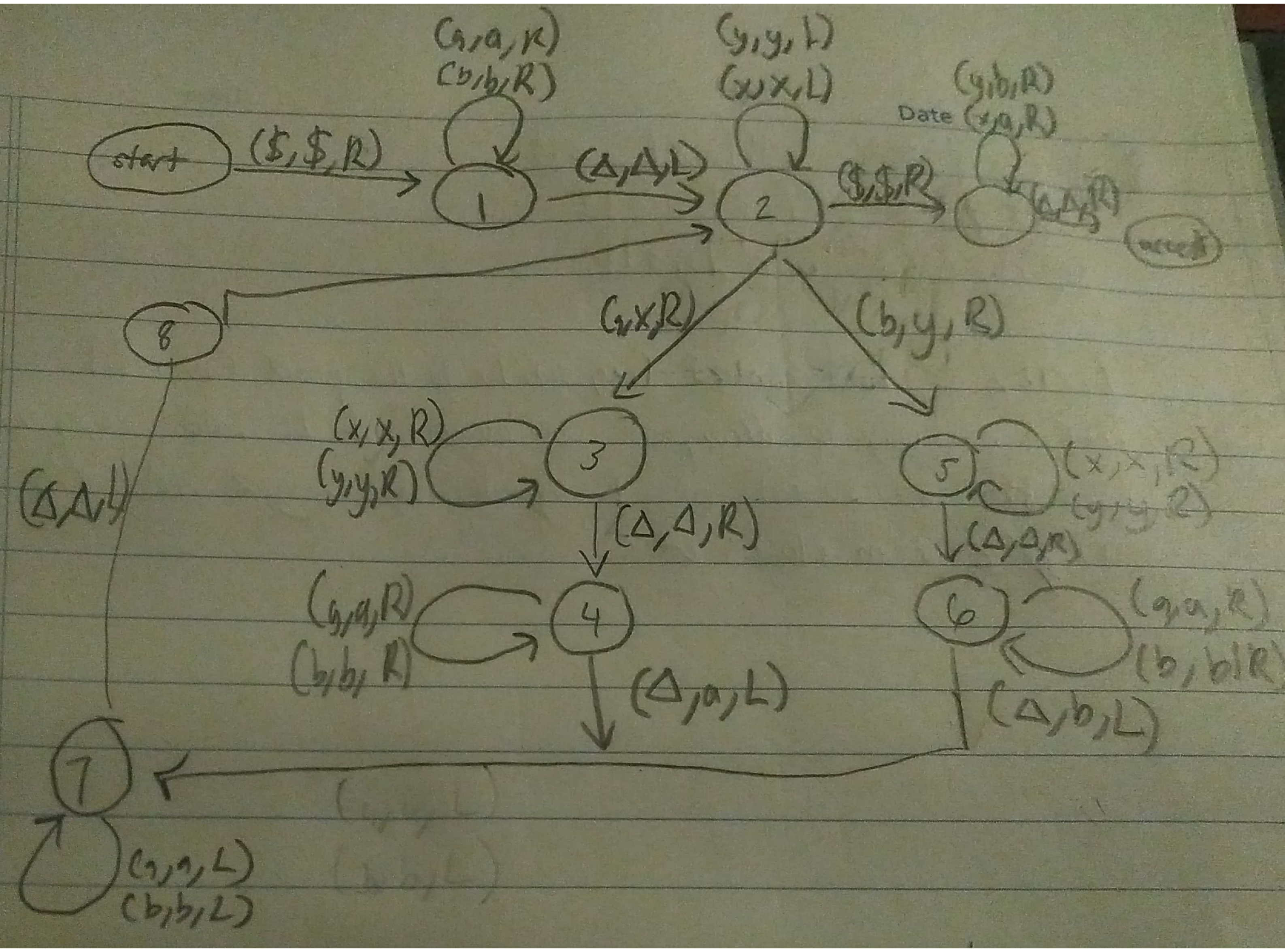
Date

5. b) $L_1 = \{a^{n+m} b^n a^m \mid n \geq 1, m \geq 1\}$

$L_2 = \{a^i b^k a^k \mid i \geq 1, k \geq 1\}$

$L_1 \cap L_2 = \{a^{2n} b^n a^n \mid n \geq 1\}$

$L_1 \cap L_2$ is not context-free, similar to the proof from 4.
There is no way to decompose an element of L into u, v, x, y, z such that an element of L equals u, v, x, y, z and $\forall n, n \geq 1$ $uv^n xy^n z$ is an element of L.


(a,a,R)
(b,b,R)
(y,y,L)
(x,x,L)
(y,b,R)
Date (x,a,R)
start
($,$,R)
1
(Δ,Δ,L)
2
($,$,R)
(Δ,Δ,R)
accept
8
(x,x,R)
(b,y,R)
(x,x,R)
(y,y,R)
3
5
(x,x,R)
(y,y,R)
(Δ,Δ,R)
(Δ,Δ,R)
(Δ,Δ,L)
(a,a,R)
(b,b,R)
4
6
(a,a,R)
(b,b,R)
(Δ,a,L)
(Δ,b,L)
7
(a,a,L)
(b,b,L)