1. $L=\{a^n b^{n-1} \mid n \geq 2\}$ is not regular

Proof

All words $w \in L$ have the form $\overbrace{a\ldots aa}^{n\ a's}\overbrace{b\ldots b}^{n-1\ b's}$

Assume L is regular. By the pumping lemma, there exists x, y, z such that $xy^nz \in L$ for any $n \geq 0$ $(y \neq \Lambda)$

There are 3 possibilities for y

1. $y = a\ldots a$ (at least 1).
2. $y = b\ldots b$ (at least 1)
3. $y = a\ldots ab\ldots b$ (the number of a's and b's is not necessarily the same)

Case 1 n a's n-1 b's

$\underbrace{a}_{x}\underbrace{\ldots aa}_{y}\underbrace{b\ldots b}_{z}$

$xyyyz \in L$ by the pumping lemma → This word has more a's than the one more a than b's that is required to be part of L.

Therefore $xyyyz \notin L$ for this case.

n a's n-1 b's

$\underbrace{a\ldots aa}_{x}\underbrace{b\ldots}_{y}\underbrace{b}_{z}$

$xyyyz \in L$ by the pumping lemma → This word has more b's than a's.

Therefore $xyyyz \notin L$ for this case.

n a's n-1 b's

$\underbrace{a}_{x}\underbrace{\ldots ab\ldots}_{y}\underbrace{b}_{z}$

$\underbrace{a\ldots a}_{x}\underbrace{a\ldots ab\ldots b}_{y}\underbrace{a\ldots ab\ldots b}_{y}\underbrace{a\ldots ab\ldots b}_{y}\underbrace{b\ldots b}_{z}$

$xyyyz \in L$ by the pumping lemma

$xyyyz$ contains ba and thus $xyyyz \notin L$

Because there is no x, y, z such that for any $n \geq 0$ $xy^nz \in L$, L is not regular.

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2. Let L be the language containing any string s followed by a number of b's equal to length(s).

Assume L is regular. There is a FA with N states that accepts L.
Let s be any string.

$s^n b^n$ is accepted by this FA

There are x, y, z such that $xyz = s^n b^n$

$xy^n z \in L$ for all $n > 0$

$length(x) + length(y) \leq N$

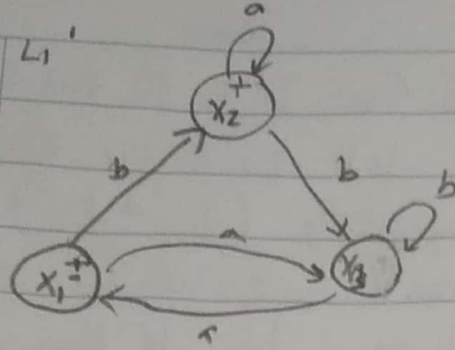
$xyz = \underbrace{s^n}_{x} \overbrace{\underbrace{b}_{y} \underbrace{\cdots b}_{z}}^{n\ b's}$. Thus y is equal to only the first b after the letters in s^n. Thus, $xyyz \in L$ ← by the second version of the pumping lemma.

But xyyz has an odd number of letters therefore it is not possible it split xyyz into two halves of equal length (i.e. length(s) can not be equal to the number of b's for xyyz)

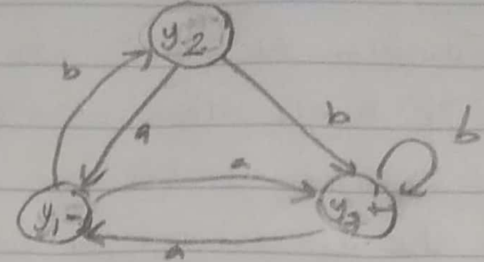
So $xyyz \notin L$ a contradiction

Therefore L is not regular

3. L_1'



L_2



$(L_1' + L_2)'$

		a	b
$\pm Z_1$	x_1, y_1	Z_2	Z_3
$+Z_2$	x_3, y_3	Z_1	Z_2
$+Z_3$	x_2, y_2	Z_4	Z_2
$+Z_4$	x_2, y_1	Z_5	Z_6
$+Z_5$	x_2, y_3	Z_4	Z_2
Z_6	x_3, y_2	Z_1	Z_2

$x_2, y_2 \xrightarrow{b} x_3, y_3$

$x_2, y_1 \xrightarrow{a} x_2, y_3$

$x_2, y_1 \xrightarrow{b} x_3, y_2$

$x_2, y_3 \xrightarrow{a} x_2, y_1$

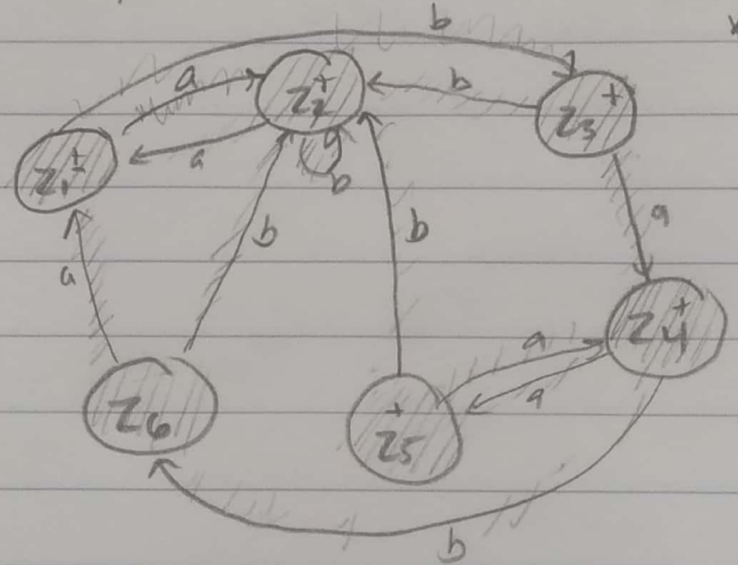
$x_2\,y_3 \xrightarrow{b} x_3, y_3$

$x_3, y_2 \xrightarrow{a} x_1, y_1$

$x_3, y_2 \xrightarrow{b} x_3, y_3$

blue = ///

arrow erased = ~~→~~



The language is not empty

As the union language is not empty the two FA are not equivalent

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4. 1. Transform r_1 and r_2 into deterministic FA. (Let L_1 be the FA for r_1 and let L_2 be the FA for r_2)

Note: If $r_1 \subseteq r_2$ then $r_1 \cap r_2 = r_1$

2. Construct a FA for $L_1 \cap L_2$ or equivalently $(L_1' + L_2')'$

3. Use the algorithm from problem two (chp 11, slide 7) to determine if the FA from step 2 if equal to L_1. If they are equal $r_1 \subseteq r_2$. If they are not then $r_1 \not\subseteq r_2$.

5. a) $(a+b)^* abb (a+b)^* + b(a+b)^*$

Any sequence of a's or/and b's followed by 'abb' followed by any sequence of a's or/and b's <u>or</u> 'b' followed by any sequence of a's or/and b's.

b) $S \rightarrow XabbX \rightarrow bXabbX \rightarrow b\Lambda abbX \rightarrow babb\Lambda \rightarrow babb$

$S \rightarrow bX \rightarrow bXb \rightarrow bXbb \rightarrow bXabb \rightarrow b\Lambda abb \rightarrow babb$