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1. $S \subset T \Rightarrow S^* \subset T^*$, direct proof.

1) By definition, $S \subset T$ means every element of S is also in T but S and T are not equal.

By definition, S^* contains the smallest superset of S that contains the empty string Λ and is closed under the string concatenation operation.

By definition, T^* contains the smallest superset of T that contains the empty string Λ and is closed under the string concatenation operation.

~~By definition, as S^* is a superset of S, all elements in S are also in S^*.~~

~~By definition, as T^* is a superset of T, all element in T are also in T^*.~~

As every element in S is also in T, S^* can be build from T as S^* is build by concatinating words from S and all words from S are contained in T.

As S^* can be formed by concatenating some words from T (the words that are contained in both S and T) including the empty string. And, T^* can be formed by concatenating all words from T including the empty string therefore every element of S^* is also in T^* and $S^* \subset T^*$. To clarify S^* and T^* are not equal as there exists a word in T that is not in S and that word can be concatinated to a word from S^* to form a word that is not in S^*.

2. a) Λ plus all sequences of aaa's and bbb's.
b) $S = \{aa, ab, ba, bb\}$

3. 1. aaa, aab, aba, abb, baa, bab, bba and bbb are in 3MULT
2. If x and y are in 3MULT, x concatinated with y is in 3MULT

b) 1. Λ, a and b are in NOTBB

2. If x is in NOTBB, and if x ends with a, xa and xb are in NOTBB. If x is in NOTBB and if x ends with b, xa is in NOTBB. If x is Λ, a or b are in NOTBB.

Note xa: means x concatinated with a in the order x then a.
xb: means x concatinated with b in the order x then b.

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4. a) $(a+b)^*(aa+bb+ba)+\Lambda$

b) $b^*(a(aa)^*b^*)^*$

5. a) The pairs $(ba)^*b$ and $b(ab)^*$ are equivalent as they both define only all words that have at least 1 b, have an odd number of letters, start and end with b, and don't have any double letters (i.e. don't have any double a's or double b's).

b) The pairs $(aa^*bb^*)^*$ and $\Lambda + a(a+b)^*b$ are equivalent as they both define only Λ plus words that ~~start~~ with ~~an~~ a, end with a b, and have and combination of Λ, a's, and b's in between.

6. a) babb, abb

b) a, aa

7. a) Assumption: $\Sigma = \{a, b, c\}$

All words that start and end with the same letter.

b) All words that end with / are b or baaa.