

Stochastic Resonance in a LIF Neuron: Effects of Noise on Spike-Signal Synchronization

Introduction

Neural systems operate in intrinsically noisy environments, where stochastic fluctuations influence membrane dynamics and spike generation. While noise is often considered detrimental to information transmission, in nonlinear threshold systems it can play a functional role. In particular, **Stochastic Resonance (SR)** refers to the enhancement of weak signal detection at intermediate noise levels.

Neurons are natural examples of threshold systems, as action potentials are generated only when the membrane potential exceeds a critical value. In this context, noise **can improve the temporal alignment of spikes** with respect to a weak periodic input, enabling temporal coding quantified through phase-locking measures.

In this work, we investigate SR using the **Leaky Integrate-and-Fire (LIF)** model, first at the single-neuron level and then in populations of independent neurons, to assess how collective activity stabilizes noise assisted signal representations.

Objectives

In particular, the following hypotheses are tested:

- **H1 - Optimal noise level:** Spike-signal synchronization is maximal at an intermediate noise intensity.
- **H2 - Dependence on signal amplitude:** Increasing the signal amplitude shifts the optimal noise level toward lower values.
- **H3 - Dependence on operating point:** Bringing the neuron closer to threshold through an increased bias current reduces the noise level required for optimal synchronization.
- **H4 - High-noise degradation:** Excessive noise degrades temporal synchronization despite increased firing activity.
- **H5 - Population-level robustness:** Pooling the activity of multiple independent neurons produces more stable and robust temporal representations of the input signal.

Model and Methods

Neural dynamics are modeled using a Leaky Integrate-and-Fire (LIF) neuron, which describes a neuron as a threshold system integrating incoming current and generating spikes when the membrane potential exceeds a fixed threshold V_{th} . The subthreshold dynamics are governed by:

$$\tau_m \frac{dV}{dt} = -(V - E_L) + \frac{I(t)}{g_L}$$

where E_L is the leak reversal potential, g_L the leak conductance, and the $\tau_m = C_m/g_L$ membrane time constant. After a spike, the membrane potential is reset to V_{reset} , and the neuron enters an absolute refractory period.

The neuron is driven by a weak sinusoidal input:

$$I(t) = I_{bias} + A \sin(2\pi ft)$$

with parameters chosen such that the membrane potential remains strictly subthreshold in the absence of noise.

The distance from threshold is quantified by the **safety margin** $\Delta V = V_{th} - V_{reset}$.

Stochasticity is introduced by adding **Gaussian white noise** to the input current:

$$I_{tot}(t) = I_{sin}(t) + I_{noise}(t)$$

where $I_{noise}(t)$ is a zero-mean Gaussian process with intensity σ . Noise is implemented using an **Euler-Maruyama scheme**, ensuring consistency across time discretizations. Independent noise realizations are used across trials and neurons.

Spike-signal synchronization is quantified using the **Vector Strength**. Each spike time is t_k mapped to the phase of the sinusoidal input:

$$\phi_k = (2\pi f t_k) \bmod(2\pi)$$

and synchronization is measured as:

$$R = \left| \frac{1}{N} \sum_{k=1}^N e^{i\phi_k} \right|$$

with $R \in [0, 1]$. Vector Strength is computed only for trials with **at least 10 spikes** to ensure statistical reliability. This measure allows identification of the noise level that maximizes spike-signal synchronization. At the population level, the model is extended to **N independent and identical LIF neurons**, each receiving the same subthreshold input and independent noise realizations:

$$\tau_m \frac{dV_i}{dt} = -(V_i - V_L) + \frac{1}{g_L} [I_{sin}(t) + \sigma \xi_i(t)], \quad i = 1, \dots, N$$

Population activity is characterized by pooling spikes across neurons, yielding the population firing rate:

$$r_{pop}(t) = \frac{1}{N} \sum_{i=1}^N \sum_k \delta(t - t_k^{(i)})$$

This population readout is used to assess the robustness of spike-signal synchronization under stochastic resonance.

Results

Single Neuron Stochastic Resonance

Single Neuron stochastic resonance (SR) was investigated by analyzing the response of a LIF neuron to a subthreshold **10 Hz sinusoidal input** under increasing noise intensity σ . In the absence of noise, the membrane potential remains strictly subthreshold, resulting in a silent output despite the presence of a structured input signal.

As noise is introduced, spike-signal synchronization emerges and is quantified by the Vector Strength R . As shown in *Figure 1*, synchronization depends non-monotonically on the noise intensity. At low noise levels, spiking is rare and phase locking is poorly defined. At intermediate noise levels, noise-assisted threshold crossings become temporally structured with respect to the input phase, leading to **strong phase locking and maximal Vector Strength at an optimal noise intensity** ($\sigma_{opt} \approx 12.5$, $R \approx 0.97$).

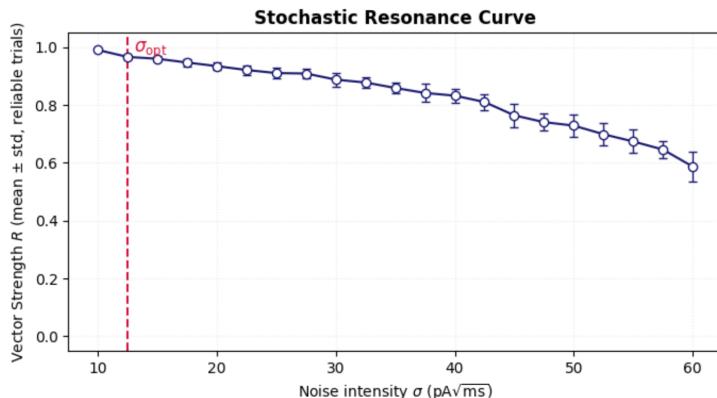


Figure 1: Stochastic Resonance Curve

At higher noise intensities ($\sigma = 60$) firing activity increases, however spike timing becomes progressively dominated by stochastic fluctuations, resulting in a **degradation of phase locking** ($R \approx 0.63$) despite increased firing. This behavior indicates that noise does not simply enhance firing but improves temporal coding only within a limited range.

Overall, these results confirm **H1**, demonstrating the existence of an optimal noise level that maximizes spike-signal synchronization and **H4**, showing that excessive noise degrades temporal coding by dominating spike timing.

Dependence on input parameters

Parametric analyses show that the optimal noise level σ_{opt} is not an intrinsic neuronal property but depends on the **distance between the deterministic input and the firing threshold**.

As shown in *Figure 2*, increasing the **sinusoidal amplitude** systematically shifts σ_{opt} toward lower values. Stronger inputs reduce the effective distance from threshold, shifting σ_{opt} toward lower values. Conversely, smaller amplitudes keep the neuron farther from the threshold, shifting the resonance peak toward higher noise levels, in agreement with **Hypothesis H2**.

A comparable trend is observed for the **DC bias current** I_{bias} . Similarly, increasing I_{bias} shifts σ_{opt} toward lower values by moving the operating point closer to threshold (*Figure 3*). Lower bias value, instead, require stronger noise to achieve stochastic resonance, confirming **Hypothesis H3**.

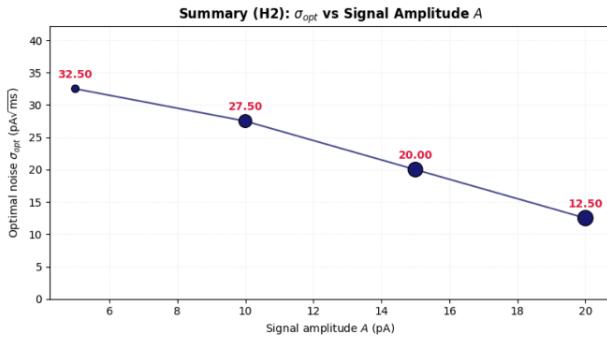


Figure 2: Dependence of the optimal noise level on signal amplitude

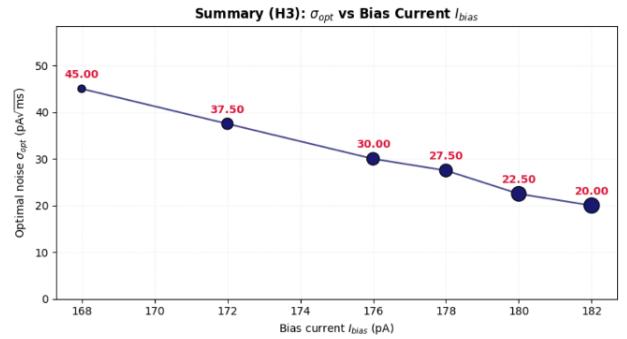


Figure 3: Dependence of the optimal noise level on bias current

Overall, amplitude and bias current jointly determine the neuronal operating point and control the **noise regime** in which stochastic resonance emerges.

Population level effects

The model was extended to a population of $N = 50$ independent LIF neurons receiving the same subthreshold sinusoidal input and independent noise realizations, in order to assess population-level signal encoding.

At the optimal noise level ($\sigma \approx 12.5$), individual neurons fire sparsely; however, population averaging reveals a clear temporal structure. As shown in *Figure 4*, the **pooled raster plot** and the corresponding **Peri-Stimulus Time Histogram** (PSTH) exhibit a robust oscillation at the input frequency (10 Hz). Phase pooling further confirms **strong collective synchronization**, with a high pooled Vector Strength ($R_{pooled} \approx 0.97$) and a narrow phase concentration around $\phi \approx \pi/2$.

At high noise levels ($\sigma \approx 60$), firing activity increases markedly, but temporal organization deteriorates, reflecting a transition to a noise-dominated regime. As illustrated in *Figure 4*, despite the increase in firing activity, spike timing becomes less structured and collective phase locking is reduced. Nevertheless, population

coding partially preserves signal power through averaging, highlighting a **dissociation between spike-level phase locking and population-level signal readout**.

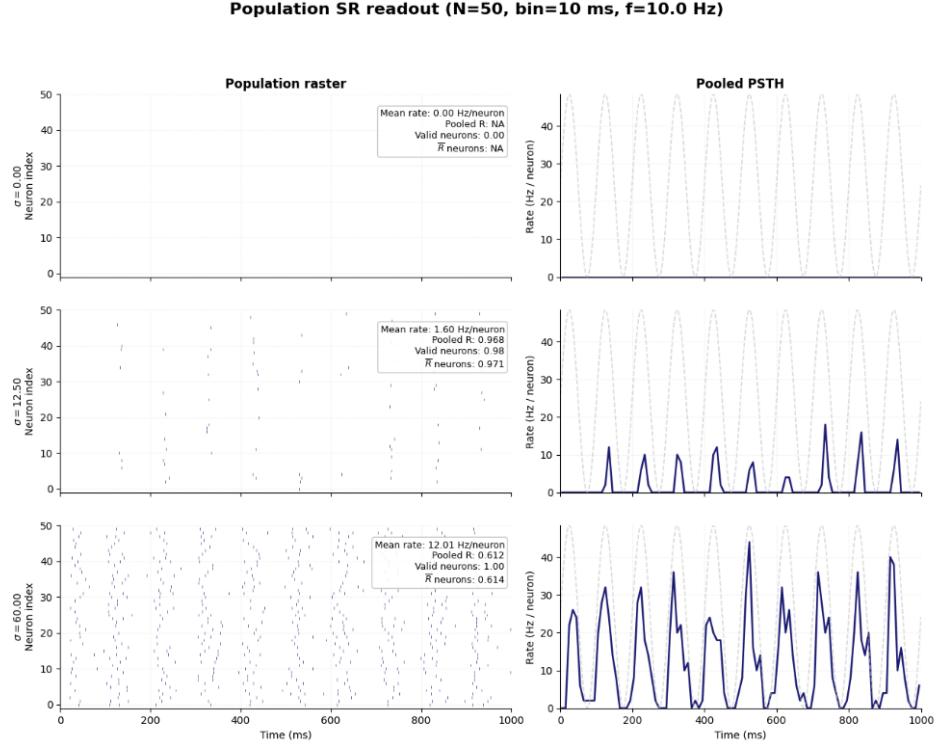


Figure 4: Population raster plot and pooled PSTH across noise regimes

Overall, these results support **Hypothesis H5**, showing that population averaging stabilizes temporal signal readout beyond single-neuron precision, while also reinforcing **H4**, as excessive noise degrades collective phase locking.

Conclusion and Limitations

This study investigates Stochastic Resonance in a Leaky Integrate-and-Fire neuron driven by a subthreshold sinusoidal input, at both single-neuron and population levels. The results show that noise can play a functional role in temporal signal encoding and that population averaging enhances the robustness of the readout. The present analysis is based on a simplified modeling framework. Neurons are assumed to be identical, uncoupled and driven by independent noise sources, neglecting synaptic interactions, neuronal heterogeneity and correlated fluctuations that are known to shape neural dynamics in biological circuits. Moreover, the minimal LIF model employed here cannot capture nonlinear membrane dynamics or history-dependent effects. Future work could address these limitations by incorporating neuronal diversity, synaptic coupling and shared noise sources, as well as by adopting more biophysically detailed neuron models. Extending the analysis to these conditions would provide deeper insight into the robustness and functional relevance of Stochastic Resonance in realistic neural networks.