香港電腦奧林匹克競賽
Hong Kong Olympiad in Informatics

Constructive Algorithms & Special Tasks (I)

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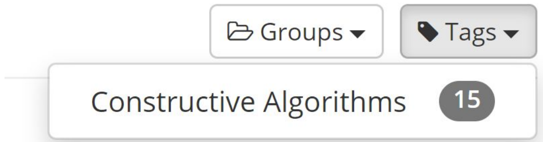
Outline

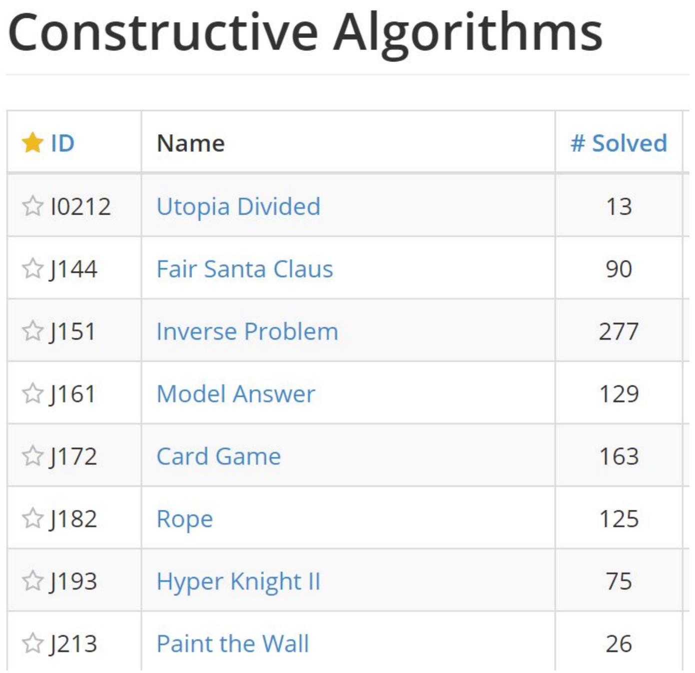
1. Constructive Algorithms
2. Interactive Tasks

3. Minicomp on these two types of tasks

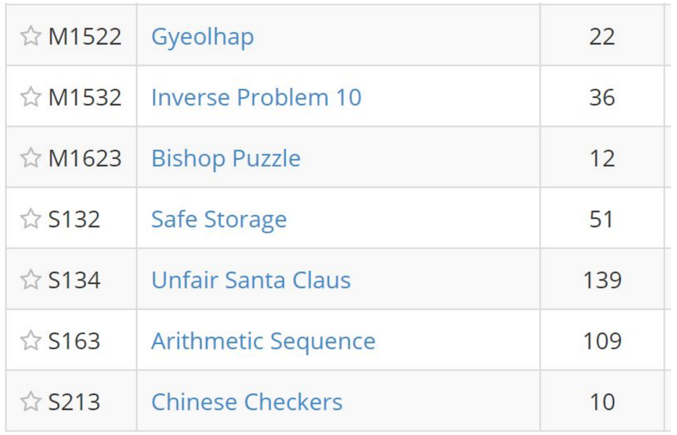
Constructive Algorithm

What is it?





Constructive Algorithms



ID	Name	# Solved
I0212	Utopia Divided	13
J144	Fair Santa Claus	90
J151	Inverse Problem	277
J161	Model Answer	129
J172	Card Game	163
J182	Rope	125
J193	Hyper Knight II	75
J213	Paint the Wall	26
M1522	Gyeolhap	22
M1532	Inverse Problem 10	36
M1623	Bishop Puzzle	12
S132	Safe Storage	51
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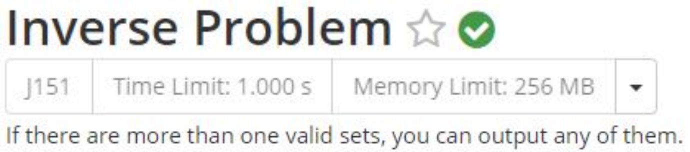
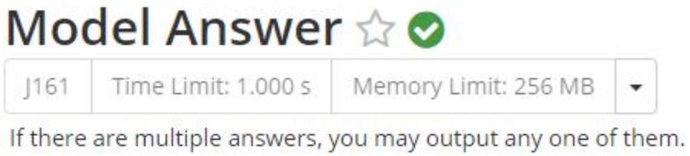
S213	Chinese Checkers	10

Constructive Problems?


Inverse Problem ☆ ✓

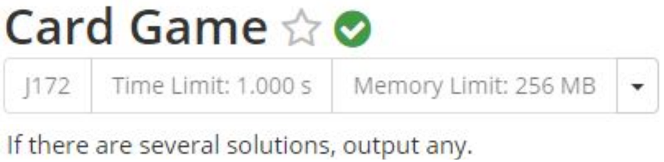
J151 | Time Limit: 1.000 s | Memory Limit: 256 MB

If there are more than one valid sets, you can output any of them.


Model Answer ☆ ✓

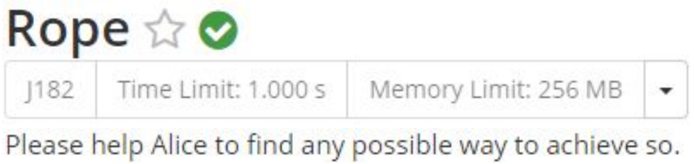
J161 | Time Limit: 1.000 s | Memory Limit: 256 MB

If there are multiple answers, you may output any one of them.


Card Game ☆ ✓

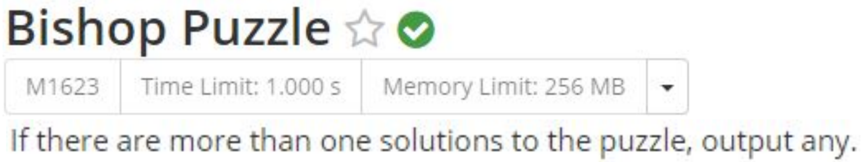
J172 | Time Limit: 1.000 s | Memory Limit: 256 MB

If there are several solutions, output any.


Rope ☆ ✓

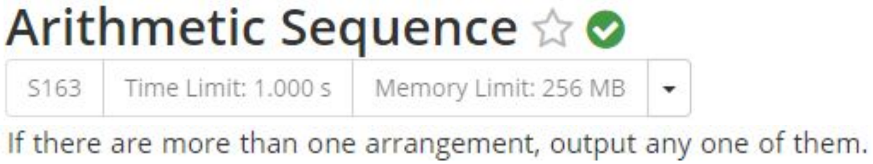
J182 | Time Limit: 1.000 s | Memory Limit: 256 MB

Please help Alice to find any possible way to achieve so.


Bishop Puzzle ☆ ✓

M1623 | Time Limit: 1.000 s | Memory Limit: 256 MB

If there are more than one solutions to the puzzle, output any.


Arithmetic Sequence ☆ ✓

S163 | Time Limit: 1.000 s | Memory Limit: 256 MB

If there are more than one arrangement, output any one of them.

Constructive Problems...

- Usually give some requirements / constraints to be fulfilled
- You should construct any arrangement that satisfies the given rules
  - Permutations
  - Sequences
  - Matrices
  - Placements
  - ...
- Usually interesting
- Often require more thinking than coding / knowledge of standard algorithms
- May have various correct solutions and “seemingly correct solutions”

An Example - CF1828A

Codeforces Round #873 (Div. 2) Problem A - Divisible Array

https://codeforces.com/contest/1828/problem/A

Given an integer N (≤ 200), find an integer array A of N elements such that:

- 1 ≤ A[i] ≤ 1000
- A[i] is divisible by i
- A[1] + A[2] + ... + A[N] is divisible by N

Sample Input N	Sample Output A
2	`[2, 4]`
3	`[1, 2, 3]`

An Example - CF1828A

Let’s start with cases we know the answer immediately.

When N is odd, a trivial integer array is

[1, 2, 3, 4, ..., N - 3, N - 2, N - 1, N]

Sum of A[i] in this construction = N (N + 1) / 2, which is divisible by N as (N + 1) is even.

Sample Input N	Sample Output A
2	[2, 4]
3	[1, 2, 3]

An Example - CF1828A

How about when N is even?

Sadly, the simplest construction [1, 2, …, N - 1, N] does not work as the sum = N (N + 1) / 2 is not divisible by N when N even.

However, as observed from the sample, we can simply multiply every elements by 2 to bring the sum up to N (N + 1), which is divisible by N.

Sample Input N	Sample Output A
2	`[2, 4]`
3	`[1, 2, 3]`

Let’s solve T222 together :)

The problem: given a paper strip consists of N (≤ 10^{18}) cells: 1, 2, ..., N, each step you can fold the paper strip into half. Construct a folding sequence of U and D such that two numbers A and B are **closest to each other** vertically.

Examples for N = 4, A = 1 and B = 2:

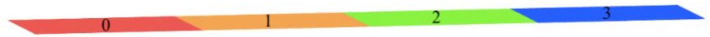


Figure 1: Initial state for $N = 4$

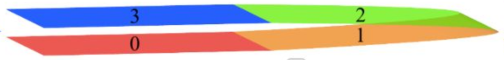


Figure 2: State after folding U

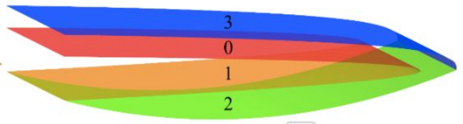


Figure 3: State after folding UD

Answer = UD with minimum distance 1

Imagine…

You have three hours left in the contest, only for this problem.

You code a plausible solution to the problem aiming to AC, but unfortunately you just keep getting WA for no apparent reason…

What would you do?

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You code a plausible solution to the problem aiming to AC, but unfortunately you just keep getting WA for no apparent reason…

What would you do?
Manually craft hack cases by writing on / folding paper

Imagine…

You have three hours left in the contest, only for this problem.

You code a plausible solution to the problem aiming to AC, but unfortunately you just keep getting WA for no apparent reason…

What would you do?
~~Manually craft hack cases by writing on / folding paper~~
End up 0 because it is too hard to thinking what folding is optimal :(
A quick ‘solution’ is to write a checker to get the answer by simulation

T222 Wormhole - Simulation

We treat the strips as a two-dimensional arrays, and simulate the folding process

Then, we can make a 'simple' brute-force solution by trying all the possibilities

```
typedef vector<vector<int>> strip;
strip fold_U(strip a){
    assert(a[0].size() % 2 == 0);
    int h = a.size(), w = a[0].size() / 2;
    vector b(h * 2, vector<int>(w));
    for(int i = 0; i < h; i++){
        for(int j = 0; j < w; j++){
            b[i][j] = a[h - 1 - i][w * 2 - 1 - j];
            b[h + i][j] = a[i][j];
        }
    }
    return b;
}
// similar for type D fold
```

T222 Wormhole - Simulation

```
int min_dist(int N, int A, int B, string S){
   strip s; s.push_back(vector<int>(N));
   iota(s[0].begin(), s[0].end(), 0);
   for (auto c : S) {
       if (c == 'D') s = fold_D(s);
       else s = fold_U(s);
   }
   int pos_a, pos_b;
   for (int i = 0; i < s.size(); i++) {
       if (s[i][0] == A) pos_a = i;
       if (s[i][1] == B) pos_b = i;
   }
   return abs(pos_a - pos_b);
}
```

```
pair<int, string> brute_force(int N, int A, int B){
   int k = 0; while (N % 2 == 0) ++k, N /= 2;
   int min_d = 1LL << 60; string folding;
   for (int msk = 0; msk < (1 << k); msk++) {
       string S = "";
       for (int j = 0; j < k; j++) {
           S.push_back(msk & (1 << j) ? 'U' : 'D');
       }
       if (min_dist(N, A, B, S) < min_d) {
           min_d = min_dist(N, A, B, S);
           folding = S;
       }
   }
   return {min_d, folding};
}
```

T222 Wormhole - Checking your solution

You can write a simple checker by simply comparing the answer given by the brute force solution and your solution

You can also check whether or not the folding sequence indeed is valid for that minimum distance

```
void stress_test(){
  int N = 8;
  for (int A = 0; A < N; A++) {
     for (int B = 0; B < N; B++) {
        if (brute_force(N, A, B).first
         != wormhole(N, A, B).first) {
          cout << "WA\n" << N << ' ' << A << ' ' << B;
          break;
        }
      }
   }
}

void correct_fold(int N, int A, int B, int D, string S){
   assert(D == min_dist(N, A, B, S));
}
```

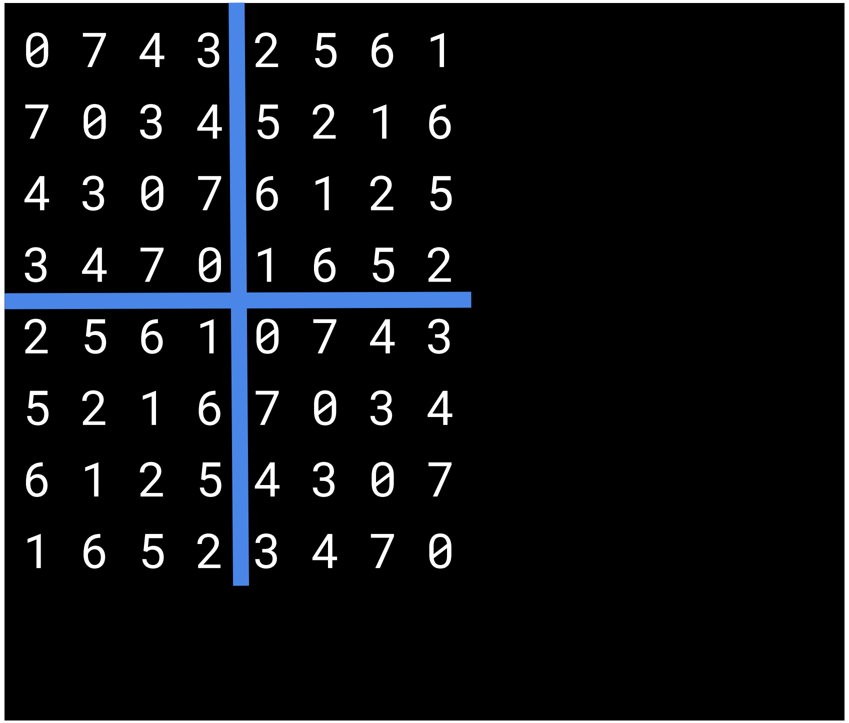
T222 Wormhole - Observation from small cases

We can output the answer of all the pairs (A, B) for a specific N (say 8)

If we try to look at the output, we can observe that it seems to follow some kind of pattern

Seems that we can cut it into 4 4 x 4 squares such that they are most likely the same
=> We can recursively solve the problem

You can get 80% by getting this correct :)



```
0 7 4 3 2 5 6 1
7 0 3 4 5 2 1 6
4 3 0 7 6 1 2 5
3 4 7 0 1 6 5 2
2 5 6 1 0 7 4 3
5 2 1 6 7 0 3 4
6 1 2 5 4 3 0 7
1 6 5 2 3 4 7 0
```

Binary vs Ternary

Given two binary strings A and B (Length of A, B ≤ **64**). Your target is to convert A into B using the following operations not more than **512** times:

- Choose a non-empty substring of $A[l, r] = A_l A_{l+1} \dots A_r$.
- Consider it as a ternary (base-3) number and convert it back to binary.
- Example: $(101)_3 = (1010)_2$, so you can transform 1**<u>101</u>**10 to 1**<u>1010</u>**10.
- A and B starts with 1.
- If impossible, report it.

Binary vs Ternary

Very often it’s hard to tackle the problem by solving small cases with programs…

- It’s hard to code a general exhaustion program
- It’s hard to observe patterns among all possibilities

We may try to work on special / small cases first and ‘build’ the solutions from them

Binary vs Ternary - Impossible Case

What if A = 1? Clearly we cannot change anything.
Therefore, the answer is YES if B = 1 also, and NO otherwise. Similar for B = 1.

So let's assume A and B both have at least two characters.

Binary vs Ternary - from 10 to anything

What if A = 10? Notice that

10 -> **11** -> 100

So by applying the operation twice, we produce a zero at the end.

We can convert 10 to 11 with cost 1, and 10 to 100 with cost 2.

What if A = 11? Notice that

11 -> 1**00** -> 10

So we can convert 11 to 10 with cost 2.

Binary vs Ternary - from 10 to anything

We now have three moves:

- Convert 10 to 11 with cost 1
- Convert 10 to 100 with cost 2
- Convert 11 to 10 with cost 2

Now, how can we solve the case when A = 10 and B is arbitrary?

Binary vs Ternary - from 10 to anything

We now have three moves:

- Convert 10 to 11 with cost 1
- Convert 10 to 100 with cost 2
- Convert 11 to 10 with cost 2

Now, how can we solve the case when A = 10 and B is arbitrary?

```
10 -> 100 -> 100..00 -> 110..00 -> 111..111 -> 110..0101
```

Binary vs Ternary - from anything to 10

So - how about converting *arbitrary* A to B = 10?

We now have *four* moves:

- Convert 10 to 11 with cost 1
- Convert 10 to 100 with cost 2
- Convert 11 to 10 with cost 2
- *Convert 00 to 0 with cost 1*

Binary vs Ternary - from anything to anything

So - how about converting *arbitrary* A to B = 10?

We now have *four* moves:

- Convert 10 to 11 with cost 1
- Convert 10 to 100 with cost 2
- Convert 11 to 10 with cost 2
- *Convert 00 to 0 with cost 1*

```
110..0101 -> 111..11 -> 100..00 -> 10
```

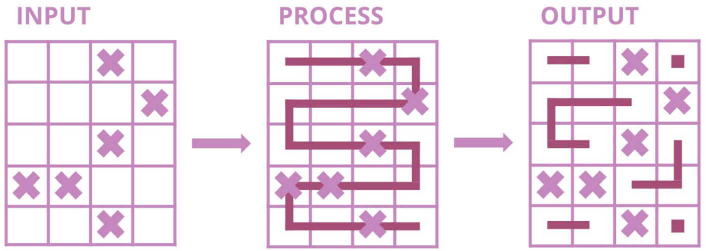
The problem is solved by simply combining two ‘special’ cases!

Techniques we learnt

- Generalise the problem from special case
- Observe patterns from small cases

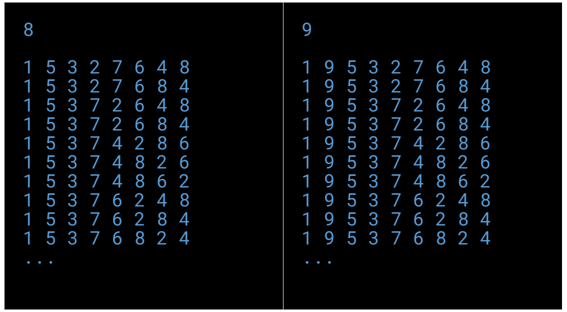
J182 - Rope

Generalise the problem from special case



S163 - Arithmetic Sequence

Observe patterns by brute-forcing small cases



```
8

1 5 3 2 7 6 4 8
1 5 3 2 7 6 8 4
1 5 3 7 2 6 4 8
1 5 3 7 2 6 8 4
1 5 3 7 4 2 8 6
1 5 3 7 4 8 2 6
1 5 3 7 4 8 6 2
1 5 3 7 6 2 4 8
1 5 3 7 6 2 8 4
1 5 3 7 6 8 2 4
...
```

```
9

1 9 5 3 2 7 6 4 8
1 9 5 3 2 7 6 8 4
1 9 5 3 7 2 6 4 8
1 9 5 3 7 2 6 8 4
1 9 5 3 7 4 2 8 6
1 9 5 3 7 4 8 2 6
1 9 5 3 7 4 8 6 2
1 9 5 3 7 6 2 4 8
1 9 5 3 7 6 2 8 4
1 9 5 3 7 6 8 2 4
...
```

Task: https://judge.hkoi.org/task/J182
Tutorial: https://assets.hkoi.org/training2018/J182.pdf

Task: https://judge.hkoi.org/task/S163
Tutorial: https://assets.hkoi.org/training2016/2016-sen3.pdf

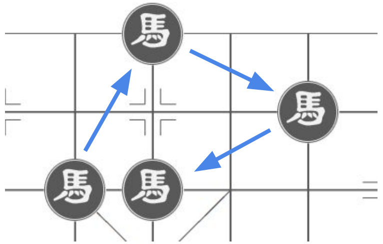
More Information: https://assets.hkoi.org/training2023/cast-i.pdf

Techniques we learnt

- Reducing the problem into smaller cases / lower dimensions
- Divide and Conquer

J193 - Hyper Knight II

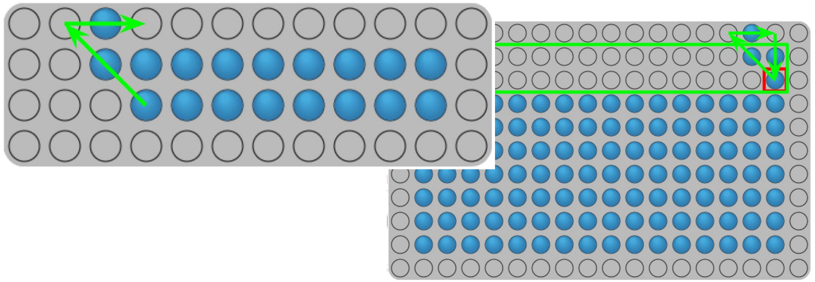
Repeating small cases of moving just a few steps



Task: https://judge.hkoi.org/task/J193
Tutorial: https://assets.hkoi.org/training2019/J193.pdf

S213 - Chinese Checkers

Reducing the problem to smaller problems



Task: https://judge.hkoi.org/task/S213
Tutorial: https://assets.hkoi.org/training2021/S213.pdf

Machine Building Tasks

- You are asked to generate a program in some artificial language that fulfills certain requirements
- You can only use certain set of instructions to build your program
- This type of constructive task has recently became more common
- Examples:
  - [IOI2012] Pebbling Odometer
  - [IOI2019] Vision Program
  - [IOI2021] Bit Shift Registers
  - [TFT2023] Complete the Sequence
  - [APIO2023] Alice, Bob and Circuit

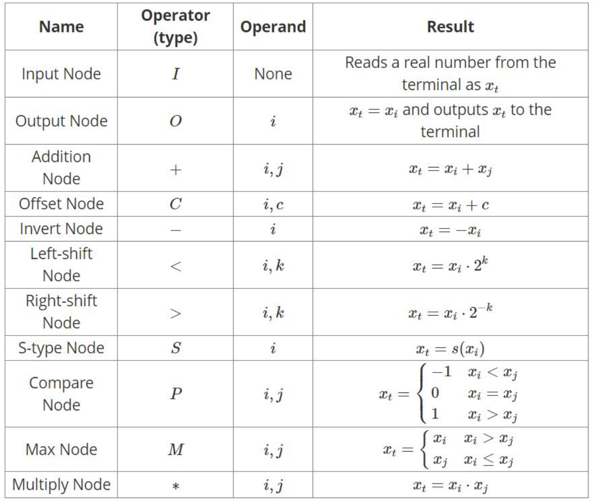
Machine Building Tasks

Some tips:

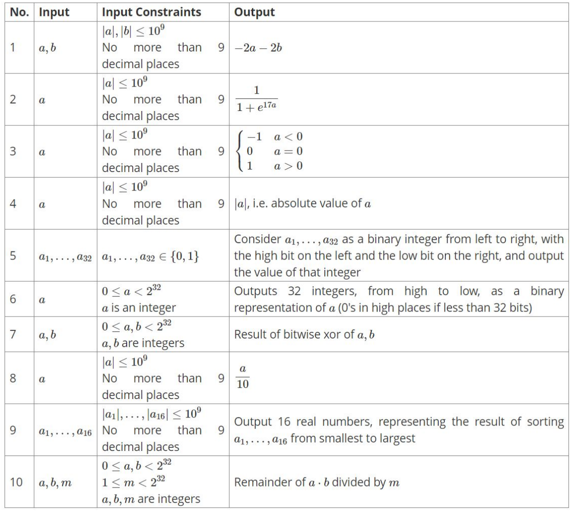
- The set of instructions given may be quite unfamiliar
  - Try solving the problem using familiar operations
  - See if you can construct the operations you want (eg addition) with the instructions they give you (eg a bunch of logic gates)
  - Even if you cannot construct that operation, your “solution” might provide some insight to a solution that uses the given instructions
- My code looks so confusing I don’t know what is going on anymore
  - It is very easy to get lost in your own code in these types of problems
  - A ***very good idea*** is to make sure that your code is well commented
  - Also make good use of things like functions and loops
    - Remember you are submitting a C++ file not a txt file

M23A1 The Great Wilderness Calculation

- Source: NOI 2016 曠野大計算
- Build a computer using some given operations, that is able to solve some computational tasks.



Name	Operator (type)	Operand	Result
Input Node	I	None	Reads a real number from the terminal as x_t
Output Node	O	i	$x_t = x_i$ and outputs x_t to the terminal
Addition Node	$+$	i, j	$x_t = x_i + x_j$
Offset Node	C	i, c	$x_t = x_i + c$
Invert Node	$-$	i	$x_t = -x_i$
Left-shift Node	$<$	i, k	$x_t = x_i \cdot 2^k$
Right-shift Node	$>$	i, k	$x_t = x_i \cdot 2^{-k}$
S-type Node	S	i	$x_t = s(x_i)$
Compare Node	P	i, j	$x_t = \begin{cases} -1 & x_i < x_j \\ 0 & x_i = x_j \\ 1 & x_i > x_j \end{cases}$
Max Node	M	i, j	$x_t = \begin{cases} x_i & x_i > x_j \\ x_j & x_i \le x_j \end{cases}$
Multiply Node	$*$	i, j	$x_t = x_i \cdot x_j$



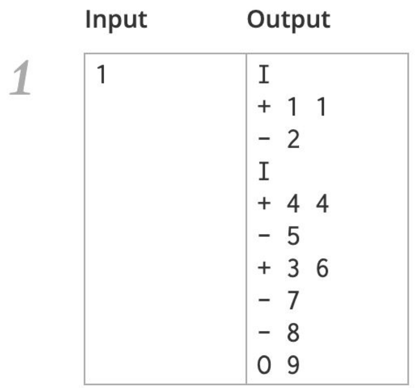
No.	Input	Input Constraints	Output
1	a, b	$\lvert a\rvert, \lvert b\rvert \le 10^9$ No more than 9 decimal places	$-2a - 2b$
2	a	$\lvert a\rvert \le 10^9$ No more than 9 decimal places	$\frac{1}{1+e^{17a}}$
3	a	$\lvert a\rvert \le 10^9$ No more than 9 decimal places	$\begin{cases} -1 & a < 0 \\ 0 & a = 0 \\ 1 & a > 0 \end{cases}$
4	a	$\lvert a\rvert \le 10^9$ No more than 9 decimal places	$\lvert a\rvert$, i.e. absolute value of a
5	$a_1, \ldots, a_{32}$	$a_1, \ldots, a_{32} \in \{0, 1\}$	Consider $a_1, \ldots, a_{32}$ as a binary integer from left to right, with the high bit on the left and the low bit on the right, and output the value of that integer
6	a	$0 \le a < 2^{32}$ a is an integer	Outputs 32 integers, from high to low, as a binary representation of a (0's in high places if less than 32 bits)
7	a, b	$0 \le a, b < 2^{32}$ a, b are integers	Result of bitwise xor of a, b
8	a	$\lvert a\rvert \le 10^9$ No more than 9 decimal places	$\frac{a}{10}$
9	$a_1, \ldots, a_{16}$	$\lvert a_1\rvert, \ldots, \lvert a_{16}\rvert \le 10^9$ No more than 9 decimal places	Output 16 real numbers, representing the result of sorting $a_1, \ldots, a_{16}$ from smallest to largest
10	a, b, m	$0 \le a, b < 2^{32}$ $1 \le m < 2^{32}$ a, b, m are integers	Remainder of $a \cdot b$ divided by m

M23A1 The Great Wilderness Calculation

- This problem is very complicated so we won't cover it here
  - If you are interested, you can have a look at https://assets.hkoi.org/training2023/cast-i.pdf

- Main objective now isn't how to solve this problem - but how can we simplify the implementation?

- Every computation node depends on previous ones
- We need to know the IDs of the nodes of our interest
- *How?* How can we find this not manually?



1

Input

```
1
```

Output

```
I
+ 1 1
- 2
I
+ 4 4
- 5
+ 3 6
- 7
- 8
O 9
```

M23A1 The Great Wilderness Calculation - Utility Functions

Instead of manually deducing the IDs, we can write utility functions to make our code looks tidier

The functions do the followings:

- Takes several (possibly none) node IDs as input
- Output the node information
- Return the ID of the newly created node

```
int called_cnt = 0;
int I(){
   cout << "I" << endl;
   return ++called_cnt;
}
int O(int i){
   cout << "O " << i << endl;
   return ++called_cnt;
}
… omitted …
int MAX(int i, int j){
   cout << "M " << i << ' ' << j << endl;
   return ++called_cnt;
}
int MUL(int i, int j){
   cout << "* " << i << ' ' << j << endl;
   return ++called_cnt;
}
```

M23A1 The Great Wilderness Calculation - Utility Functions

Then, using those utility functions, we can simply use the result from previous function calls as input

Let's use the sample as sample:

	Input	Output
1	1	I + 1 1 - 2 I + 4 4 - 5 + 3 6 - 7 - 8 O 9

```
void subtask_1(){
    int a = I();
    int c = INV(ADD(a, a)); // = -2a
    int b = I();
    int d = INV(ADD(b, b)); // = -2b
    int res = ADD(c, d); // = -2a - 2b
    O(INV(INV(res)));
}
```

M23A1 The Great Wilderness Calculation - Overloading Operators

Can we abuse the power of C++?

In C++, you can overload operators for self-defined struct - That means you can use operators like `+`, `*` for some data type you make

For example, we can declare a struct called `number`, treating it as a computational node

=> Our Target

```
void subtask_1(){
  number a, b; in >> a >> b;
  out << -((a + b) << 1);
}
```

M23A1 The Great Wilderness Calculation - Overloading Operators

In the struct `number`, we simply store the ID of that computation node

Then we can overload the operators in correspondence with the given functions

- for example, here we defined !x as S(x), a function given, for simplicity

We can then do computations as normal like in C++!

```cpp
struct number{
   int id;
   number operator+ (number y) {
       number z {ADD(id, y.id)};
       return z;
   }
   number operator- () {
       number z {INV(id)};
       return z;
   }
   number operator! () {
       number z {S(id)};
       return z;
   }
};
```

M23A1 The Great Wilderness Calculation - Overloading Operators

How about input and output? We cannot simply read from cin and write to cout

To resolve this, simply declare two more structs `input_device` and `output_device` and overload them also with `number`

```
struct input_device{
    input_device& operator>> (number& x) {
        x.id = I();
        return *this;
    }
} in;

struct output_device{
    output_device& operator<< (const number& x) {
        O(x.id);
        return *this;
    }
} out;
```

M23A1 The Great Wilderness Calculation - Overloading Operators

=> Final Product

Of course, it will already likely to be enough with utility functions

- You can strike a balance between time and simplicity

- However, always remember to think about *how* to implement before implementing something really complicated

```
void subtask_1(){
  number a, b; in >> a >> b;
  out << -((a + b) << 1);
}
```

Building common operations from boolean operators

Addition of 2 binary numbers:

- Adding the digits (and the carry) starting from the least significant bit

Assume A, B, C, D are N-bit integers

- Input: A, B
- Carry: C
- Output: D (= A + B)

```
for (int i = 0; i < N; i++) {
    bool b1 = A[i] && B[i];
    bool b2 = C[i] && (A[i] || B[i]);
    D[i] = b1 || b2;
    C[i + 1] = A[i] ^ B[i] ^ C[i];
}
```

Building common operations from boolean operators

A way to represent a negative N-bit integer: two's complement

- Flipping the sign of x: -x = ~x + 1
- You can think of it as

  $$2^N - x = ((2^N - 1) - x) + 1$$

- Note that most significant bit is 1 if and only if it is negative

Assume A, B are N-bit integers

- Input: A
- Output: B (= -A)

```
for (int i = 0; i < N; i++) {
   A[i] = !A[i];
}

bool c = true;
for (int i = 0; i < N; i++) {
   B[i] = A[i] ^ c;
   c = A[i] && c;
}
```

Building common operations from bitwise operators

Subtraction:

- Addition but with the sign of the second number flipped

Assume A, B, C, D are N-bit integers

- Input: A, B
- Output: C (= A - B)

See if you can think of more things to do :)

```
D := 1
B := ~B
B := B + D
C := A + B
```

Some more tips...

- Use some random ideas and carefully analyze why are they incorrect
- Be careful on small / special cases
- Be aware of some special constraints set in the task (if any)
- Double-check the cases you solve manually / the exhaustion program
- Don't think too much :)

Some more problems...

- Codeforces
  - Fraction
  - Lesha and array splitting
  - Dasha and Puzzle
  - Puzzling Language (April Fools Contest!!!)
  - Minimum Diameter Tree
  - Seating of Students
  - Construct a tree

- AtCoder
  - Four Coloring

- LS-PC Programming Challenge
  - Annoying Mathematics (2016)
  - Labyrinth (2018)
  - Monorail (2016)
  - Bob the Builder (2018)
  - Gravitational Tetris (2017)
  - Go (2018)
  - How to Get Rice (2021)
  - Lockout (2021)
  - Carpark (2022)
  - Delivery (2022)
  - Handful of Balls (2023)
  - Lift Problem (2023)

During the break...

A Practice Problem... :)

- Given a **16-bit** machine supporting operations (`<<, >>, |, &, ^, ~`):
  Calculate **min(A[0], A[1])** of two integers using least operations possible
  - Constraint: **$0 \le$ A[0], A[1] $\le 32767 = 2^{15} - 1$**
  - You cannot use conditional statements like if-else
  - You can use **A[i]** as registers, and store your answer back to **A[0]**

Finding minimum in 16-bit machine

Part 1: Subtract A[0] by A[1] and store the result in A[2] by A[2] = A[0] + ~A[1] + 1

- adding a constant is not supported

Part 2: Create a bitmask that is 65535 (11...11) when A[0] < A[1], and 0 if A[0] ≥ A[1]

- Right shift the A[2] so that only 0 or 1 remains, depends on sign of A[0] - A[1]
- Then repeatedly OR its left-shifts

Part 3: The answer is then simply

(A[0] & A[2]) | (A[1] & ~A[2])

In total: 18 operations used (Can you do better than this?)

```
// Part 1
A[4] := 1
A[2] := ~A[1]
A[2] := A[2] + A[0]
A[2] := A[2] + A[1]
A[2] := A[2] + A[4]
// Part 2
A[2] := A[2] >> 15
for i = 0 ... 3:
    A[3] := A[2] << 2^i
    A[2] := A[2] | A[3]
// Part 3
A[0] := A[0] & A[2]
A[2] := ~A[2]
A[1] := A[1] & A[2]
A[0] := A[0] | A[1]
```

Interactive Tasks

How Important?

- [IOI2013] Cave [IOI2014] Game [IOI2015] Scales
- [IOI2016] Unscrambling a Messy Bug [IOI2017] The Big Prize
- [IOI2018] Highway Tolls [IOI2018] Combo
- [IOI2019] Cycle* [IOI2019] Vision
- [IOI2020] Routers* [IOI2020] Mushrooms
- [IOI2022] Connected Towns* [IOI2022] Rarest Insects

- [NOI2019] I君的探險 [NOI2022] 樹上鄰域數點

- [TFT2012] Debug! [TFT2013] The Forgotten Triangle
- [TFT2016] Model Answer II [TFT2018] Cave Exploration
- [TFT2019] Liquid Layers [TFT2021] Re:Zero

*appeared in practice section

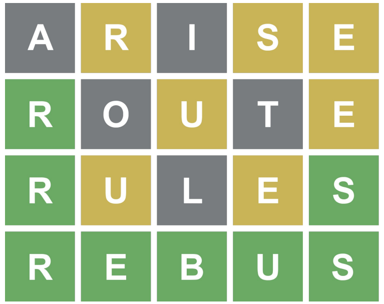
What's the difficulty?

- Unfamiliar style
- You may not be able to understand these problems during the contests,
  if you are the first time facing new types of tasks

- Feedback from inexperienced contestants after TFTs
  - 「唔知條題目講乜」 *"Don't know what it's talking about"*
  - 「睇唔明題目」 *"Don't understand the task"*
  - 「唔識用 grader」 *"Don't know how to use grader"*

Interactive task

- Your program will interact with the judging program
- You can consider it as: (suitable for most interactive tasks)
  - Your program asks some questions
  - The judging program answers your questions
  - Repeat the aboves until you can solve “something”
  - (Just like playing Wordle)
- Usually, there will be limits on number of questions asked
- Or, your score is determined by questions asked

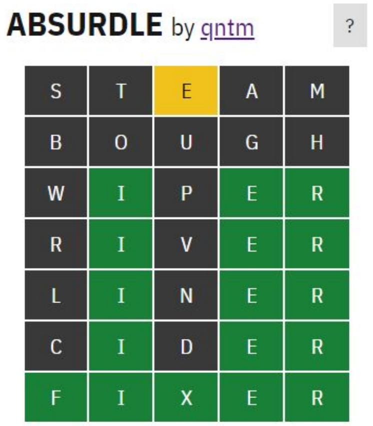


Types of Interactive tasks

Interaction method

- Standard I/O
- Grader
  - Without sample grader
  - With sample grader

Interactor behavior

- Pre-defined cases
  - E.g., Wordle
- Adaptive interactor
  - 

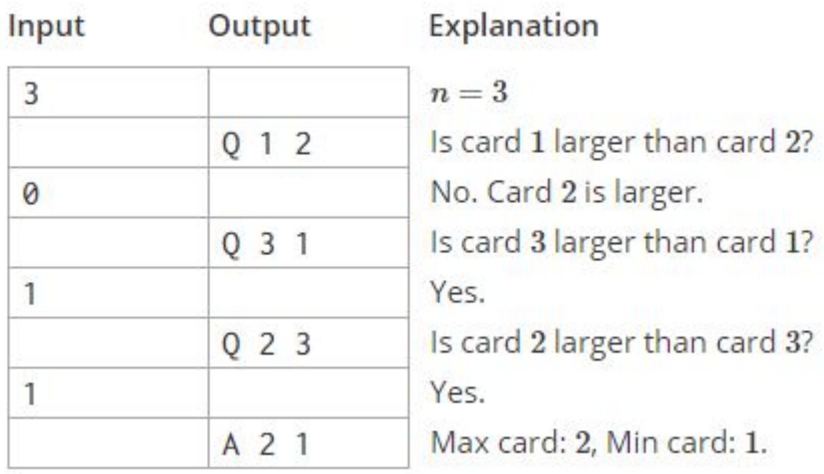
ABSURDLE by qntm ?

S	T	E	A	M
B	O	U	G	H
W	I	P	E	R
R	I	V	E	R
L	I	N	E	R
C	I	D	E	R
F	I	X	E	R

https://qntm.org/absurdle

Using standard I/O: M1431 Comparing Game

- **N** distinct cards not revealed to you
- Your goal: find where are the maximum and the minimum cards
- Question you may ask:
  - "Is card **X** larger than card **Y**?"
- Ask no more than ⌊1.5**N**⌋ questions



Input	Output	Explanation
3		$n = 3$
	Q 1 2	Is card 1 larger than card 2?
0		No. Card 2 is larger.
	Q 3 1	Is card 3 larger than card 1?
1		Yes.
	Q 2 3	Is card 2 larger than card 3?
1		Yes.
	A 2 1	Max card: 2, Min card: 1.

Using standard I/O: M1431 Comparing Game

How can our program asks questions?

Input	Output	Explanation
3		$n = 3$
	Q 1 2	Is card 1 larger than card 2?
0		No. Card 2 is larger.
	Q 3 1	Is card 3 larger than card 1?
1		Yes.
	Q 2 3	Is card 2 larger than card 3?
1		Yes.
	A 2 1	Max card: 2, Min card: 1.

```
cout << "Q " << i << " " << j << endl;
fflush(stdout); // IMPORTANT
cin >> result;
```

Actually std::endl will flush after putting endline character automatically.

Using standard I/O: M1431 Comparing Game

Sample partial solution in C++

Input	Output	Explanation
3		$n = 3$
	Q 1 2	Is card 1 larger than card 2?
0		No. Card 2 is larger.
	Q 3 1	Is card 3 larger than card 1?
1		Yes.
	Q 2 3	Is card 2 larger than card 3?
1		Yes.
	A 2 1	Max card: 2, Min card: 1.

```cpp
for (int i = 1; i <= N; i++)
  for (int j = 1; j <= N; j++) {
    counter = 0;
    if (i != j) {
      // vvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvvv
      cout << "Q " << i << " " << j << endl;
      fflush(stdout);
      cin >> result;
      // ^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
      if (result == 1)
        counter++;
    }
    if (counter == N - 1)
      bigIndex = i;
    if (counter == 0)
      smallIndex = i;
  }
}
```

Using standard I/O: M1431 Comparing Game

Recalling that...

- Ask no more than ⌊1.5**N**⌋ questions

We have asked **N(N-1)** questions :(

Some hints to the full solution:

- We can use 0.5**N** questions to split the cards into two groups...
- For **S** numbers, **S-1** comparison is sufficient to find the max/min number...

Grader

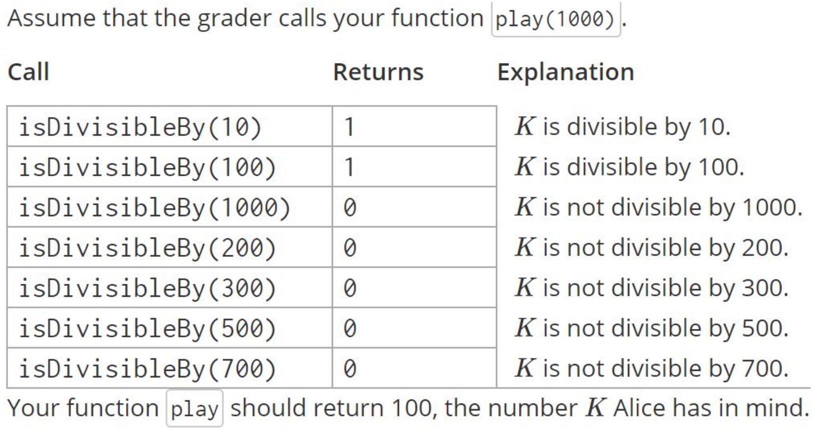
- The example just now performs interaction through standard I/O
  - cout / printf
  - cin / scanf
- Some interactive tasks are using another way
  - through the grader program
  - you will be given a template code
  - you will ask questions / get feedback by calling some given functions

Grader: I0501 Divisor Game

- An unknown integer **K** within the range [1, **N**]
- Your goal: find the value of **K**

- Question you may ask:
    - "Is the number **K** divisible by some integer **x**?"

- Ask minimal questions

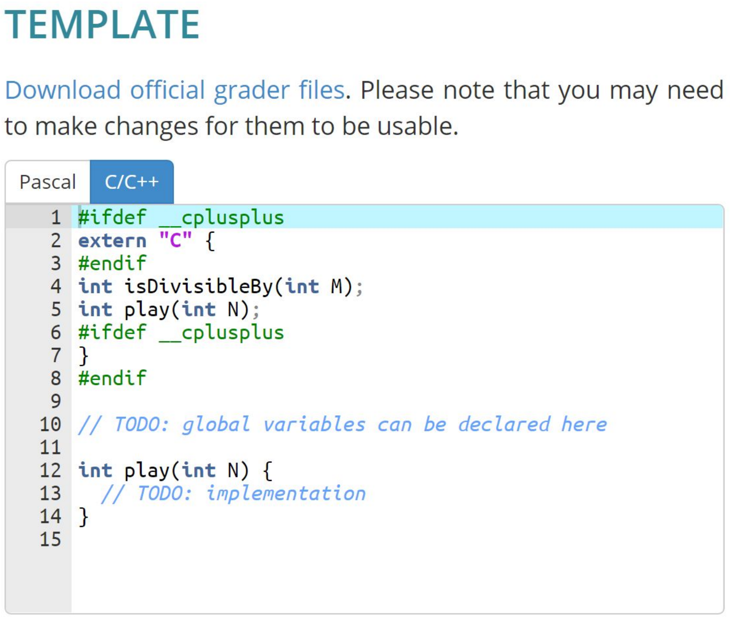

Assume that the grader calls your function `play(1000)`.

Call	Returns	Explanation
`isDivisibleBy(10)`	1	K is divisible by 10.
`isDivisibleBy(100)`	1	K is divisible by 100.
`isDivisibleBy(1000)`	0	K is not divisible by 1000.
`isDivisibleBy(200)`	0	K is not divisible by 200.
`isDivisibleBy(300)`	0	K is not divisible by 300.
`isDivisibleBy(500)`	0	K is not divisible by 500.
`isDivisibleBy(700)`	0	K is not divisible by 700.

Your function `play` should return 100, the number K Alice has in mind.

Grader: I0501 Divisor Game

- What is given?



TEMPLATE

Download official grader files. Please note that you may need to make changes for them to be usable.

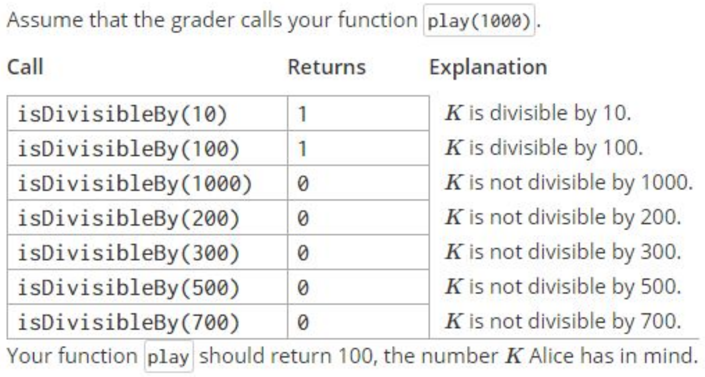
Pascal C/C++

```
#ifdef __cplusplus
extern "C" {
#endif
int isDivisibleBy(int M);
int play(int N);
#ifdef __cplusplus
}
#endif

// TODO: global variables can be declared here

int play(int N) {
  // TODO: implementation
}

```

Grader: I0501 Divisor Game

How can our program ask question?

- using grader functions



Assume that the grader calls your function `play(1000)`.

Call	Returns	Explanation
`isDivisibleBy(10)`	1	K is divisible by 10.
`isDivisibleBy(100)`	1	K is divisible by 100.
`isDivisibleBy(1000)`	0	K is not divisible by 1000.
`isDivisibleBy(200)`	0	K is not divisible by 200.
`isDivisibleBy(300)`	0	K is not divisible by 300.
`isDivisibleBy(500)`	0	K is not divisible by 500.
`isDivisibleBy(700)`	0	K is not divisible by 700.

Your function `play` should return 100, the number K Alice has in mind.

```
result = isDivisibleBy(x);
```

Grader: I0501 Divisor Game

- You cannot compile the program even if you have completed `play()`
    - it's because the main program is missing
- You cannot test the program
    - it's because the function `isDivisibleBy()` is not implemented
    - this function is implemented by the judging program
    - you are only required to implement `play()`

- So what can we do to test our program?

Grader: I0501 Divisor Game

So what can we do to test our program?

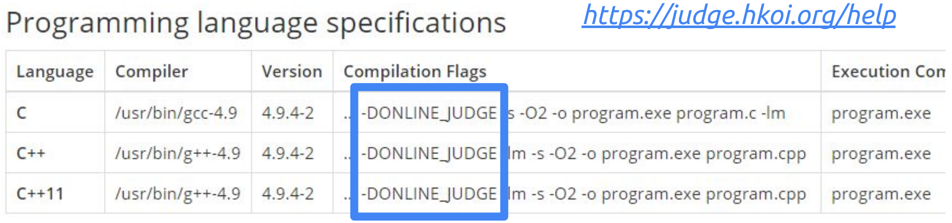
- We can implement the remaining functions
  - `int isDivisibleBy(int M)`
  - `int main()`
  - put them inside the same source code file
- Delete these parts before submitting
- Or you can use
  - `#ifndef ONLINE_JUDGE`
  - `#endif`

```
const int MAX = 1'000'000;
int secret, trials;

int isDivisibleBy(int M) {
  trials++;
  return secret % M == 0;
}

int main() {
  srand(time(0));
  for (int t = 0; t < 10; t++) {
    secret = rand() % MAX + 1;
    trials = 0;
    int guess = play(MAX);
    cout << "secret = " << secret << endl;
    cout << "guess  = " << guess  << endl;
    cout << "trials = " << trials << endl;
    cout << endl;
  }
}
```

Grader: I0501 Divisor Game

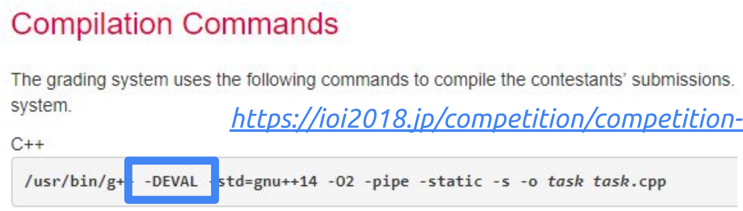
- ~~Delete these parts before submitting~~
- You can use
  - `#ifndef some_flag_here`
  - `#endif`

https://judge.hkoi.org/help



Programming language specifications

Language	Compiler	Version	Compilation Flags	Execution Con
C	/usr/bin/gcc-4.9	4.9.4-2	.. -DONLINE_JUDGE s -O2 -o program.exe program.c -lm	program.exe
C++	/usr/bin/g++-4.9	4.9.4-2	.. -DONLINE_JUDGE lm -s -O2 -o program.exe program.cpp	program.exe
C++11	/usr/bin/g++-4.9	4.9.4-2	.. -DONLINE_JUDGE lm -s -O2 -o program.exe program.cpp	program.exe



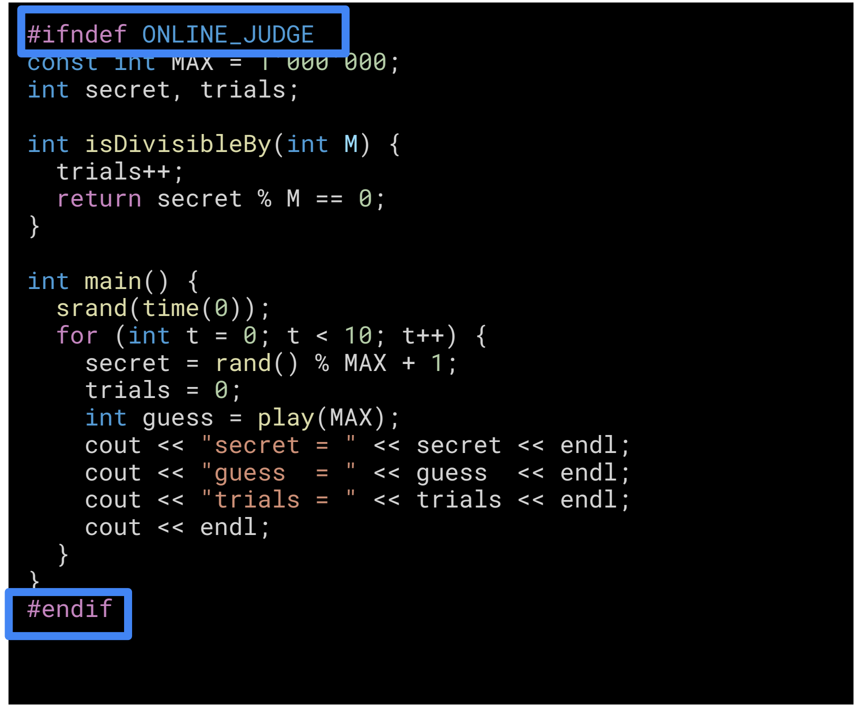
Compilation Commands

The grading system uses the following commands to compile the contestants' submissions. system.

https://ioi2018.jp/competition/competition-environment/

C++

```
/usr/bin/g+ -DEVAL std=gnu++14 -O2 -pipe -static -s -o task task.cpp
```



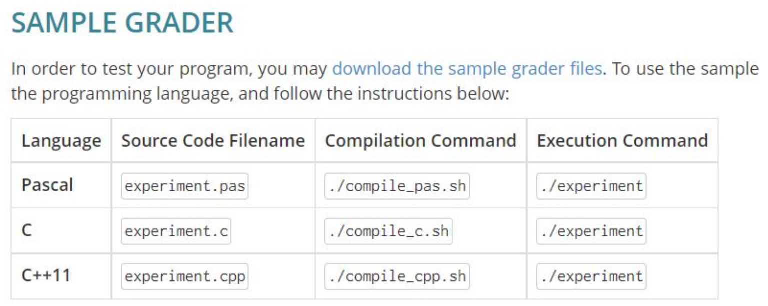
```
#ifndef ONLINE_JUDGE
const int MAX = 1 000 000;
int secret, trials;

int isDivisibleBy(int M) {
  trials++;
  return secret % M == 0;
}

int main() {
  srand(time(0));
  for (int t = 0; t < 10; t++) {
    secret = rand() % MAX + 1;
    trials = 0;
    int guess = play(MAX);
    cout << "secret = " << secret << endl;
    cout << "guess  = " << guess  << endl;
    cout << "trials = " << trials << endl;
    cout << endl;
  }
}
#endif
```

Sample grader: T193 Liquid Layers

Some problems (like T193) provide sample grader files for your testing

So you don't need to implement other functions by yourselves... hurray!!?

- Make sure that you know how to use them :(



SAMPLE GRADER

In order to test your program, you may download the sample grader files. To use the sample the programming language, and follow the instructions below:

Language	Source Code Filename	Compilation Command	Execution Command
Pascal	`experiment.pas`	`./compile_pas.sh`	`./experiment`
C	`experiment.c`	`./compile_c.sh`	`./experiment`
C++11	`experiment.cpp`	`./compile_cpp.sh`	`./experiment`

```
$ tree   # showing the unzipped folder
.
├── c
│   ├── compile_c.sh
│   ├── experiment.c
│   └── sample-grader.o
├── cpp
│   ├── compile_cpp.sh
│   ├── experiment.cpp
│   └── sample-grader.o
└── pas
    ├── compile_pas.sh
    ├── experiment.pas
    └── sample-grader.o

3 directories, 9 files

$ cd cpp    # moving to the cpp/ directory

$ ./compile_cpp.sh    # compile your C++ code

$ ls    # see the compiled executable "experiment"
compile_cpp.sh  experiment  experiment.cpp  sample-grader.o
```

Sample grader: T193 Liquid Layers

```
#ifndef __cplusplus
#include <stdbool.h>
#else
extern "C" {
#endif
  void pourLiquid(int index);
  int getReading();
  void answer(int order[]);

  void experiment(int N);
#ifdef __cplusplus
}
#endif

// TODO: global variables can be declared here

void experiment(int N) {
  // TODO: implementation
}
```

```
#include <bits/stdc++.h>   // don't forget the headers
using namespace std;

#ifndef __cplusplus
// ...
#endif

int order[105];  // you can declare global variables

void experiment(int N) {
  iota(order, order + N, 1);

  sort(order, order + N, [](int u, int v) {
    pourLiquid(u);
    pourLiquid(v);
    return getReading() == 0;
  });

  answer(order);
}
```

Sample grader: T193 Liquid Layers

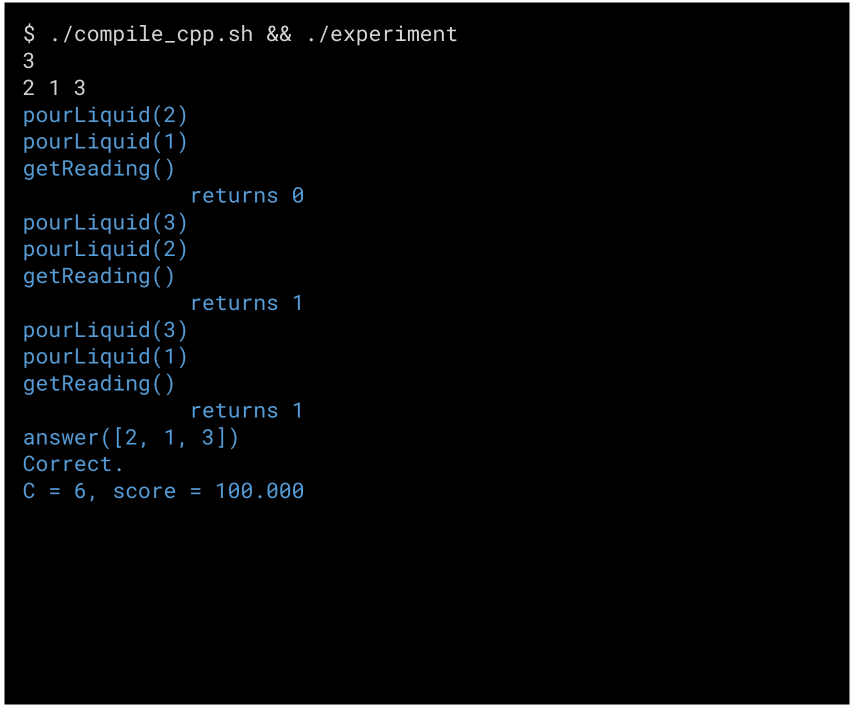
Read how to use the sample grader carefully

When testing your programs with the sample grader, your input should match the format and constraints from the task statement. Otherwise, unspecified behaviors may occur. The sample grader reads the input in the following format:

- line 1: N
- line 2: $order_0 \ order_1 \ \ldots \ order_{N-1}$, where
  - the liquid with label $order_0$ has the highest density.
  - the liquid with label $order_1$ has the second highest density.
  - ...
  - the liquid with label $order_{N-1}$ has the lowest density.

The labels should be distinct integers between 1 to N (inclusive).

If your program correctly finds out the order of the density, the sample grader outputs `Correct`, followed by the value of C and your score on that case. Otherwise, it outputs `Wrong Answer`, followed by a message suggesting what might have been done incorrectly. The sample grader also prints the function calls in the standard error stream.



```
$ ./compile_cpp.sh && ./experiment
3
2 1 3
pourLiquid(2)
pourLiquid(1)
getReading()
             returns 0
pourLiquid(3)
pourLiquid(2)
getReading()
             returns 1
pourLiquid(3)
pourLiquid(1)
getReading()
             returns 1
answer([2, 1, 3])
Correct.
C = 6, score = 100.000
```

Sample grader: T193 Liquid Layers

Learn how to use files and stream redirections to facilitate your testing

If your program correctly finds out the order of the density, the sample grader outputs `Correct`, followed by the value of C and your score on that case. Otherwise, it outputs `Wrong Answer`, followed by a message suggesting what might have been done incorrectly. The sample grader also prints the function calls in the standard error stream.

To read from file, you may use: `./experiment < input.txt`
To print the function calls to file, you may use: `./experiment 2> calls.txt`
To read from file and print the function calls to file, you may use: `./experiment < input.txt 2> calls.txt`

<	stdin	cin,scanf
>	stdout	cout,printf
2>	stderr	cerr,fprintf(stderr,

```
$ cat input.txt   # assume we've created this file
3
2 1 3

$ ./experiment < input.txt > output.txt 2> calls.txt

$ cat output.txt
Correct.
C = 6, score = 100.000

$ cat calls.txt
pourLiquid(2)
pourLiquid(1)
getReading()
             returns 0
pourLiquid(3)
pourLiquid(2)
getReading()
             returns 1
pourLiquid(3)
pourLiquid(1)
getReading()
             returns 1
answer([2, 1, 3])
```

Adaptive interactor

Some tasks use predefined cases to test your submission

- e.g. M1431, T182, T214
- this is not always the case

Some other tasks use “adaptive grader/interactor” to test your submission

- tries different strategies to exploit the weakness / worst case of your program
- randomized strategies likely won’t work

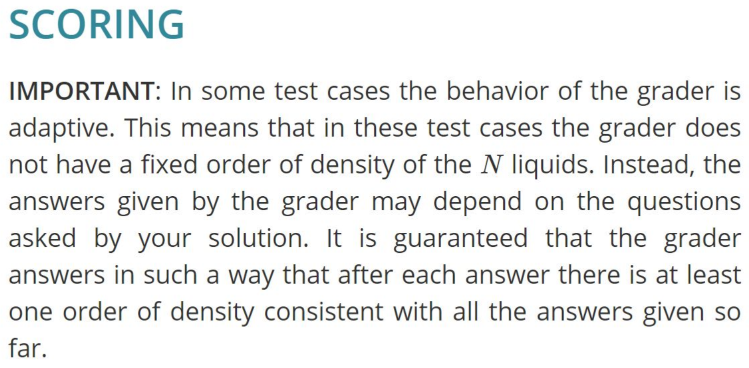
*I0501 also uses adaptive grader to judge without specify explicitly

Adaptive interactor: T193 Liquid Layers

Most of the time, this information is not very helpful in solving the tasks

However, please be reminded that:

- not to expect random solutions for the adaptive grading tasks



SCORING

IMPORTANT: In some test cases the behavior of the grader is adaptive. This means that in these test cases the grader does not have a fixed order of density of the N liquids. Instead, the answers given by the grader may depend on the questions asked by your solution. It is guaranteed that the grader answers in such a way that after each answer there is at least one order of density consistent with all the answers given so far.

https://judge.hkoi.org/task/T193

T214 Re: Zero - Modifying Sample Grader

Some problems also provide *modifiable* sample grader

- The given grader file is .cpp instead of .o
- Not the case for T193

You can in fact change the sample grader code to print debug message
For example, if you want to know the query made by the program

T214 Re: Zero - Modifying Sample Grader

```
std::vector<long long> surrender(long long D) {
 queries++;
 assert(0 <= D && D <= 1000000000000000LL);

 std::vector<long long> results(N);
 for (int i = 0; i < N; i++) {
   results[i] = Ps[i] + Px[i] * (D / Pt[i]);
 }

 return results;
}
```

→

```
#include <bits/stdc++.h>
using namespace std;
std::vector<long long> surrender(long long D) {
 queries++;
 cout << "Query " << queries << ": ";
 assert(0 <= D && D <= 1000000000000000LL);

 std::vector<long long> results(N);
 for (int i = 0; i < N; i++) {
   results[i] = Ps[i] + Px[i] * (D / Pt[i]);
 }
 for (auto x : results)
   cout << x << ' ';
 cout << endl;
 return results;
}
```

T214 Re: Zero - Storing Query Result

It is often a good idea to store down the information you have queried as most likely your score depends on the number of queries

A simple way is to self-define a query function so that if the question has been asked before, you will simply get the information from a map

This prevents you from accidentally query the same thing twice

```
#include <bits/stdc++.h>
using namespace std;

map<long long, vector<long long>> cache;
vector<long long> self_surrender(long long D)
{
 if (cache.count(D))
   return cache[D];
 else
   return cache[D] = surrender(D);
}
```

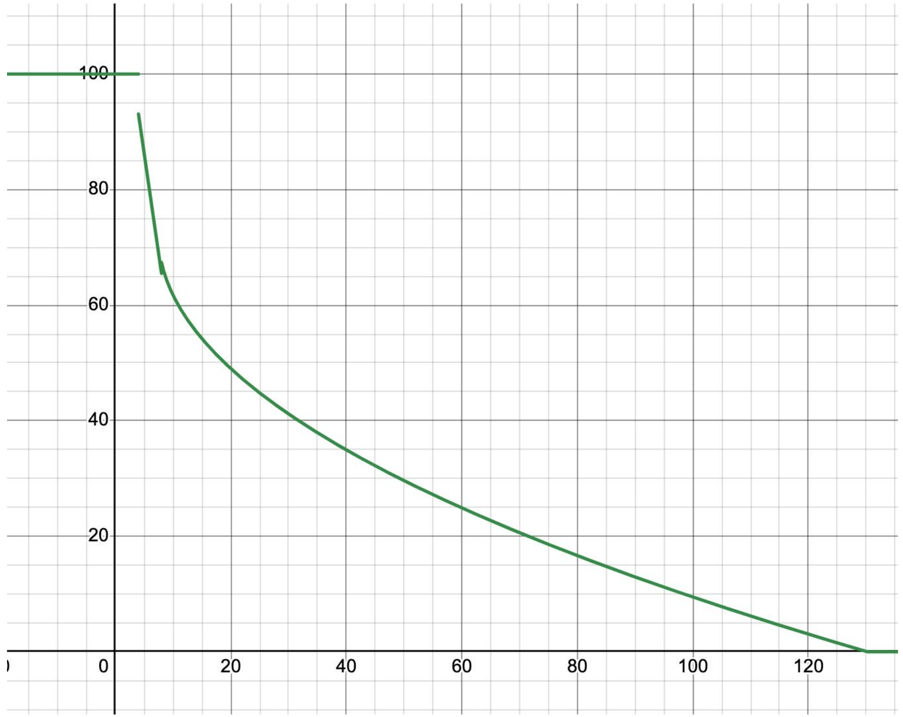
T214 Re: Zero - Scoring Curve

SCORING

If in any of the test cases, the calls to the procedure `surrender` do not conform to the rules mentioned above, or the return value of `analyze_monsters` is incorrect, the score of your solution will be 0. Otherwise, let W be the maximum number of calls to the procedure `surrender` among all test cases. Then, the score will be calculated according to the following table:

Condition	Score
$130 < W$	0
$8 \le W \le 130$	$72 - 6.5 \cdot \sqrt{W - 7.5}$
$4 \le W \le 7$	$100 - 6.9 \cdot (W - 3)$
$W \le 3$	100

Your score on this task is the lowest score you get among all test cases.

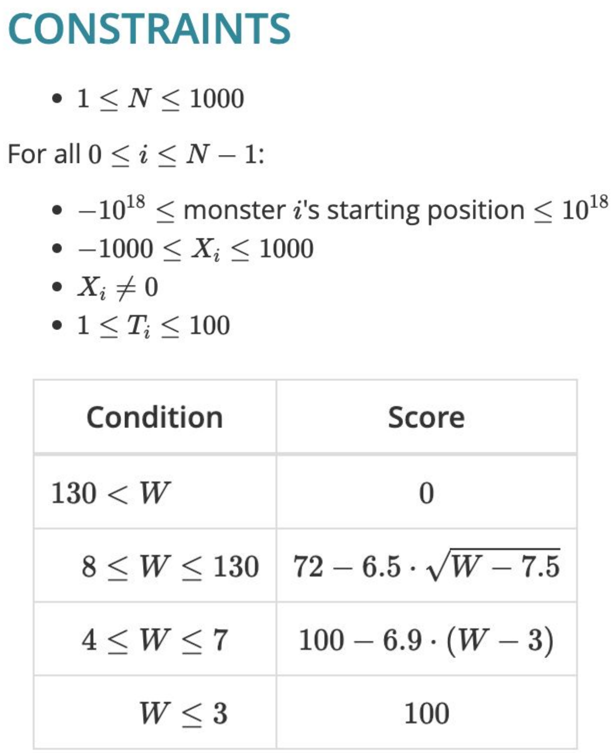


T214 Re: Zero - Scoring Curve

We can extract some useful information from constraints alone

- Can we find info for each monster separately?
  - Probably not as N can be 1000 > 130
- Is the monster's starting position relevant?
  - Probably not as it can be as large as 10^{18}
- Most relevant seems to be X_i and T_i

- What can we deduce from the scoring curve?
  - W = 130: linear on max{T_i}(?)
  - W = 7: some log
  - W = 3: constant



CONSTRAINTS

- $1 \leq N \leq 1000$

For all $0 \leq i \leq N-1$:

- $-10^{18} \leq$ monster i's starting position $\leq 10^{18}$
- $-1000 \leq X_i \leq 1000$
- $X_i \neq 0$
- $1 \leq T_i \leq 100$

Condition	Score
$130 < W$	0
$8 \leq W \leq 130$	$72 - 6.5 \cdot \sqrt{W - 7.5}$
$4 \leq W \leq 7$	$100 - 6.9 \cdot (W - 3)$
$W \leq 3$	100

A more typical example: I1722 Simurgh

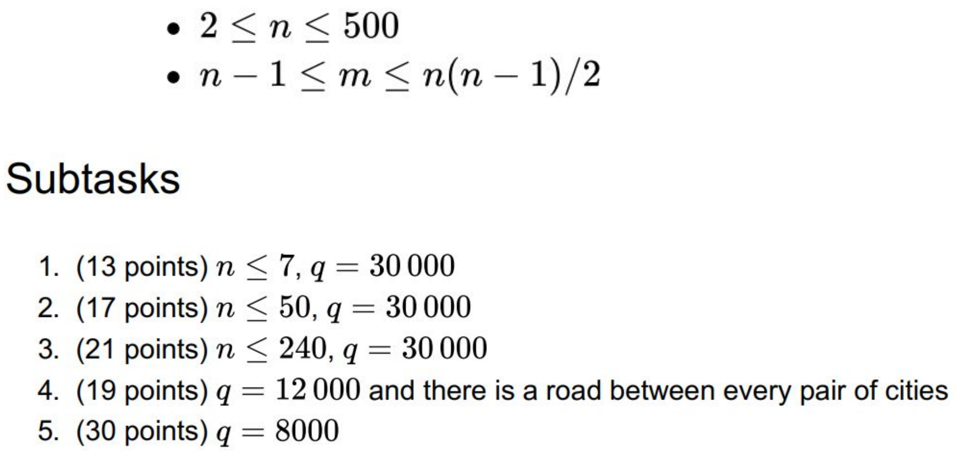
This problem does not have a scoring curve

- All or nothing for each subtask

It is a graph problem
What can we infer from the subtasks?

- Subtask 1: q ≈ 6n!
- Subtask 2: q ≈ $0.24n^3$ or $12n^2$
- Subtask 3: q ≈ $n^2 / 2$ ≈ m
- Subtask 4, 5: ???

Notice that the constant also matters - it tells you how many operations can be performed for each node (n), edge (m) etc



- $2 \leq n \leq 500$
- $n - 1 \leq m \leq n(n-1)/2$

Subtasks

1. (13 points) $n \leq 7$, $q = 30\,000$
2. (17 points) $n \leq 50$, $q = 30\,000$
3. (21 points) $n \leq 240$, $q = 30\,000$
4. (19 points) $q = 12\,000$ and there is a road between every pair of cities
5. (30 points) $q = 8000$

https://codeforces.com/blog/entry/88134?#comment-765758

Some more...

- Interactive Problems: Guide for Participants from Codeforces

Practice problems:

- 01084 Celebrity from HKOI Online Judge
- I1021 Memory available on HKOI Online Judge
- T054 Guess from HKOI Online Judge
- T134 The Forgotten Triangle from HKOI Online Judge
- some other problems... suggested by the Codeforces community
- Codeforces tasks with "interactive" tag

Minicomp

- Try to work on some special tasks!
  - Teams of 2, work collaboratively!
  - Duration: Around 1 hour (depends on lesson time)

- Balance (50%)
  - Constructive Algorithms
- Mineral Deposits (50%)
  - Interactive Task

- Team with highest score will get prizes

Minicomp Solution Session

- Balance
  - Source: CEOI 2023 Balance
  - Constructive Algorithms
- Mineral Deposits
  - Source: BalticOI 2023
  - Interactive Task
- We will go over some interesting insights gain from the question, not aiming to explain the whole solutions in details.

Minicomp Solution Session

- Reference
- Balance: https://www.ceoi2023.de/wp-content/uploads/2023/09/3-balance-spoiler.pdf

- You may refer to the above for further information.

M24A1
Balance

The Problem

Given N rows of S integers, with values in [1, T]. (S must be 2^k)

- You may arrange the S integers in each rows arbitrarily.
- Suppose value X appear in column C for col_C(X) times
  - Need to satisfy: **MAX(col_i(X)) - MIN(col_j(X)) <= 1 for all X**
  - In other words: maximum occurrence of X in a column and minimum occurrence of X in a column differ by at most one.
- Output any valid construction.

Common Tricks in Constructive Task

Output section: “It is guaranteed that such an assignment exists for each testcase.” / No instructions to output impossible.

- Does not equal to “An assignment exist for every input that satisfy the constraints”
- But you can assume it is and try to use it to your advantage
    - Can you quickly think of some counterexample?
    - If you can’t, then you should have a higher confidence that this assumption is correct

- The assumption is correct for this task.

Base Case

It is usually a good idea to start with small cases in constructive task.

- S = 2, number of submissions to each task is divisible by S.
- If a value V appears 2k times, it must appear k times in 1st and 2nd column.
    - How do we arrange for that?
    - Use our assumption: we replace V with k different values that appear 2 times each, if we find a solution for this -> we can replace them back with V to get a valid answer.

Simplified Problem

We now got N rows, 2 columns of values.

- The values is of 1..N, each appear exactly two times.
- We can swap the two values in each row.
  - Each value should only appear once in a column

Simplified Problem

We now got N rows, 2 columns of values.

- The values is of 1..N, each appear exactly two times.
- We can swap the two values in each row.
  - Each value should only appear once in a column
- We can model this as a bi-coloring graph problem.
  - Add an edge between two cells of each row.
  - Add an edge between two cells with the same value.
  - Find a coloring of nodes such that no edge connect two nodes with same colour.
    - It works because the graph is a bipartite graph with no odd cycle (Why?)
- At last, we can swap each row to make all cells with same colour ends up in the same column. This give us the answer to the problem. (Why?)

Base Case

It is usually a good idea to start with small cases in constructive task.

- S = 2, number of submissions to each task is divisible by S.
- If a value V appears 2k times, it must appear k times in 1st and 2nd column.
  - How do we arrange for that?
  - Use our assumption: we replace V with k different values that appear 2 times each, if we find a solution for this -> we can replace them back with V to get a valid answer.
    ^^^ We solved this ^^^
- How about when the number of submission of some task is odd?
  - The extra one submission can placed anywhere -> we can still use the bi-coloring algorithm to get a valid arrangement.

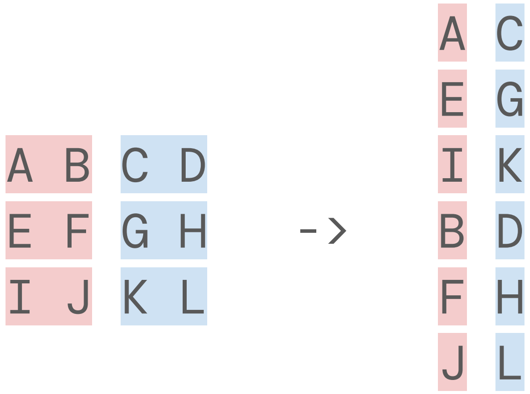
Divide and Conquer

To extend the solution to a more general case: observe that S must be 2^k

- This calls for thinking about Divide and Conquer
- Suppose we are handling a problem of S columns, how could we reduce it into a problem of S / 2 columns? (If S = 2, we can handle with our base case)
  - We want the frequencies that each values appear in the left part is nearly equal to that it appear in the right part. (Why?)
  - If we make that each value occurrences in left part and right part differ by at most 1 -> we can find a valid arrangement after recuringly do this.

Divide and Conquer

To make each value occurrences nearly equal in left and right part, we can reduce the problem as follow, suppose we have the following grid:



We can transform every subproblem into a S = 2 problem, solve it with our base case algorithm, to separate into left and right part.

Overview of the algorithm

```
Solve2(x, y): solve column x and y with bicoloring algorithm

// Solving columns of [L, R)
Solve(L, R):
    size := R - L;
    if (size == 2) Solve2(L, L + 1), return;
    V1 = {}, V2 = {}
    for i in [0, N): for j in [0..size / 2):
      V1.add((i, j)), V2.add((i, j + size / 2))
    result = Solve2(V1, V2)
    … Use result to remap values of the columns
    Solve(L, mid), Solve(mid, R);
```

M24A2
Mineral Deposits

The Problem

There are K special points on a 2D plane in the region where -B <= x, y <= B. The special points' coordinates are not given, and you need to find them out by queries.

- In each query, you can ask for information on D points (D <= 2000).
- The return values are the K * D Manhattan distances between the points asked and the special points.
    - The values are sorted in non-decreasing order, meaning you don't know which values is from which pair of points.
- You can ask at most W queries, and sum of D must be <= 20000.

To get 100 points, you can only ask at most 2 queries.

Producing Candidates Points

Querying {(b, b), (b, -b)} could trim down the number of possible points to a small amount of candidates.

- You can recover x + y from the result of point (x, y) and (b, b); and x - y from the result of point (x, y) and (b, -b).
  - Using x + y and x - y, you can easily get x and y separately.
- However, you don't know which results correspond to the same (x, y), and whether the results is with (b, b) or (b, -b).
  - You get 2k results for this query.
  - Just try every possibility of (2k) * (2k - 1) -> eliminate points that doesn't make sense (e.g. out of bounds, non-integral)
  - They are candidates for the actual answer (may not be in the actual set but it must contains all of the possible answer)

Validating Candidates Points

You now get ~(2k) * (2k - 1) points that may or may not be the answer, how to confirm their validity?

- The easiest solution is to query all the points one by one.
  - If smallest return value is 0 -> Point is in the actual answer
  - Else -> Point is not in the actual answer
- This will only get you through subtask 1?
- Can you use the information of the query to help you eliminate more points?

Validating Candidates Points

You now get ~(2k) * (2k - 1) points that may or may not be the answer, how to confirm their validity?

- Query all the points one by one.
  - Iff smallest return value is 0 -> Point is in the actual answer
  - We can loop through the points that are yet to be tested.
    - If distance between the current query point and the point iterated is not contained in the return values.
    - The point must be not valid.
  - Turns out you cannot really make cases that points are hard to be eliminated.
- This simple trick can get you through Subtask 1 - 4.

2 Queries

- With the number of queries permitted become very small, we cannot based on luck to trim down the number of candidates.
- Since we use 1 query to get the candidates points, it means that we only have one more query to use.
  - We need to handcraft the query so that we can test all candidates at the same time.
  - Each candidate should have a specialized test: iff a particular value V exists in the return values -> this candidate is valid.

2 Queries

- Each candidate should have a specialized test: iff a particular value V exists in the return values -> this candidate is valid.
- For subtask 5 (b <= 10^7), we can sort all candidates by x-coordinates.
  - We produce tests for each candidate (x, y) one by one.
  - The testing point are (x - V, y).
    - If V exists then, it must be because (x, y) is an answer.
    - (x - V, y) would produce some other distances value d[] with the remaining candidate -> we must make sure d[] is not used in previous tests (or it will make the previous tests not useful).
  - We must be able to find a useful V for each point to test.

2 Queries

- How to find the answer for full solution? When the b range is as large as the query-able range
- One way to do is to produce random points and try use them for tests on each candidates.
- Another way (deterministic way) is to try produce tests using (x + V, y), (x - V, y), (x, y + V), (x, y - V) with all points (x, y) and V from small to large.
  - With efficient breaks, you can do this naively within time limit.
  - Since the value of k is very small compared to the query range (~10^8): k^5 <= 10^8,
    - It is always possible to find a valid V for the test.