Fatorial de um número natural

① a) $4! = 4\cdot3\cdot2\cdot1 = \boxed{24}$

b) $5!-6! = 5!-6\cdot5! = 5!\cdot(1-6) = 5!\cdot(-5) = \boxed{-600}$

c) $\dfrac{9!}{6!} = \dfrac{9\cdot8\cdot7\cdot\cancel{6!}}{\cancel{6!}} = 504$

d) $\dfrac{98!}{100!} = \dfrac{\cancel{98!}}{100\cdot99\cdot\cancel{98!}} = \dfrac{1}{9900}$

② $\dfrac{1}{n!} - \dfrac{n}{(n+1)!} \rightarrow \dfrac{(n+1)! - n\cdot n!}{n!\,(n+1)!}$

$\dfrac{(n+1)\cdot n! - n\cdot n!}{n!\,(n+1)!} \rightarrow \dfrac{\cancel{n!}(\cancel{n}+1-\cancel{n})}{\cancel{n!}\,(n+1)!} = \boxed{\dfrac{1}{(n+1)!}}$

③ $\dfrac{(n!)^2-(n-1)!\,n!}{(n-1)!\,n!} = \dfrac{(n(n-1)!)^2-(n-1)!\,n(n-1)!}{(n-1)!\,n(n-1)!} =$

$= \dfrac{\cancel{(n-1)!}\,[(n)^2-n]}{\cancel{(n-1)!}\,[n]} = \dfrac{\cancel{n}(n-1)}{\cancel{n}} = \boxed{n-1}$

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④ $\dfrac{(n+2)!\,(n-2)!}{(n+1)!\,(n-1)!} = 4$

$\dfrac{[(n+2)(n+1)n(n-1)(n-2)!]\,(n-2)!}{[(n+1)n(n-1)(n-2)!]\,[(n-1)(n-2)!]} = 4$

$\dfrac{n+2}{n-1} = 4 \rightarrow n+2 = 4n-4 \rightarrow 3n = 6 \rightarrow \boxed{n = 2}$

Portanto, n é par.

⑤ $\dfrac{(n+1)!-n!}{(n+1)!} = \dfrac{(n+1)n!-n!}{(n+1)n!} = \dfrac{n![(n+1)-1]}{n!(n+1)}$

$\dfrac{n}{n+1} = \dfrac{7}{n+1}$ logo, $\boxed{n = 7}$

⑥ $(n-1)!\,[(n+1)!-n!]$

$(n-1)!\,[(n+1)\cdot n!-n!]$

$(n-1)!\,n![(n+1)-1]$

$(n-1)!\,n!\,n$

$[n(n-1)!]\cdot n!$

$[n!]\,n!$

$\boxed{(n!)^2}$

DSTQQSS

(7) $\frac{n! + (n-1)!}{(n+1)! - n!} = \frac{n(n-1)! + (n-1)!}{(n+1)n(n-1)! - n(n-1)!}$

$\frac{(n-1)!\,[n+1]}{(n-1)!\,[(n+1)n-n]} = \frac{n+1}{n^2+n-n} = \frac{n+1}{n^2} = \frac{6}{25} = \frac{5+1}{5^2}$

Portanto, $\boxed{n=5}$

(8) $21! - 221$

$= 21 \cdot 20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4! - 221$

$20 \cdot 15 \cdot 10 \cdot 5 = 15000$

$$\begin{array}{r} \ldots 000 \\ -\ 221 \\ \hline 779 \end{array}$$

O algarismo das dezenas de $21! - 221$ é igual a 7.