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Matriz Inversa - tarefa básica

① $\begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = B \rightarrow \begin{bmatrix} 2 & 1 \\ -y & 3 \end{bmatrix} = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} = A \therefore x = 2 \text{ e } y = -5$

$2 - 5 = -3$

② $\begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix}$ $(3+k^2) - (1+3k) = k^2 - 3k + 2 = 0$

$\underline{1} + \underline{2} = 3$ e $\underline{1} \cdot \underline{2} = 2 \therefore \boxed{k = 1 \text{ ou } k = 2}$

③ $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \rightarrow B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$

$\det A = 12 - 10 = 2$

④ $\begin{vmatrix} x & 1 & 2 \\ 3 & 1 & +2 \\ 10 & 1 & \cancel{x} \end{vmatrix}$ $(x^2 + 6 + 20) - (20 + 2x + 3x) = x^2 - 5x + 6 \neq 0$

$\underline{2} + \underline{3} = 5$ Para $\det A \neq 0 \therefore \{x \neq 3 \text{ e } x \neq 2\}$

$\underline{2} \cdot \underline{3} = 6$

⑤ $\begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} = (1+2+4) - (2+2+2) = 1$ $\begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

$A + A^{-1} = \begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

⑥ $B = (X \cdot A)^t$

$B^t = ((X \cdot A)^t)^t$

$B^t = X \cdot A$

$B^t \cdot A^{-1} = X \cdot A \cdot A^{-1} \rightarrow \boxed{X = B^t \cdot A^{-1}}$

⑦ $A \cdot B = C \rightarrow \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$

$\det A = 4 \cdot 6 - 5 \cdot 5$
$= -1$

$\begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \xrightarrow{(\div -1)} \boxed{A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}}$

⑧ $\det A^{-1} = \dfrac{1}{\det A}$

$\det A = \begin{vmatrix} 2 & k \\ -2 & 1 \end{vmatrix} = 2 - (-2k) = 2 + 2k$

$2 + 2k \cdot (2 + 2k) = 1$

$4 + 4k + 4k + 4k^2 = 1$

$4k^2 + 8k + 3 = 0$

$\Delta = 8^2 - 4 \cdot 4 \cdot 3 = 16 \rightarrow k = \dfrac{-8 \pm 4}{8}$

$\rightarrow k_1 = -1/2$

$\rightarrow k_2 = -3/2$

$k_1 + k_2 = \left(-\dfrac{1}{2}\right) + \left(-\dfrac{3}{2}\right) = -\dfrac{4}{2} = \boxed{-2}$

⑨ a) $(A + B) \cdot (A - B) = \boxed{A^2 - AB + BA - B^2}$ $(AB \neq BA)$

b) $(A + B)^2 = A^2 + \underbrace{2 \cdot A \cdot B} + B^2 \rightarrow (AB = BA)$

c) $\dfrac{\det(A)}{\det(-A)} \rightarrow \det(-A) = (-1)^2 \cdot \det A = \det A \rightarrow \dfrac{\det A}{\det(-A)} = \boxed{1}$

d) $B = A^{-1} \rightarrow \det A \cdot \det B = 1 \rightarrow \boxed{\det B = \dfrac{1}{\det A}}$