Coeficientes Binomiais

TRIÂNGULO DE PASCAL / TARTAGLIA

01. $\binom{8}{3} = \dfrac{8!}{3!5!} = \dfrac{8 \cdot 7 \cdot \cancel{6} \cdot \cancel{5!}}{\cancel{6} \cdot \cancel{5!}} = \boxed{56}$

02. $\binom{200}{198} = \dfrac{200!}{198!2!} = \dfrac{\overset{100}{\cancel{200}} \cdot 199 \cdot \cancel{198!}}{\cancel{2} \cdot 1 \cdot \cancel{198!}} = \boxed{19900}$

03. $\binom{n-1}{2} = \binom{n+1}{4}$ $\quad n > 0$

$2 + 4 = 6$

complementares: $n \cancel{-1} + n \cancel{+1} \leq 6$

$2n \leq 6 \therefore n \leq 3$

$S = \{n \in \mathbb{N} \mid 0 < n \leq 3\}$

$\boxed{S = \{1, 2, 3\}}$

04. $\binom{n}{p} + \binom{n}{p+1} = \binom{n+1}{p+1}$

complementar

$14 + 7 = 21$

$\binom{20}{13} + \binom{20}{14} = \boxed{\binom{21}{14}} = \binom{21}{7}$

05. $\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \cdots + \binom{n}{n}$ → linha → coluna

No triângulo de Pascal / Tartaglia, a soma dos elementos na linha n, será 2^n

06.

a) $\sum_{p=0}^{10} \binom{10}{p} = 2^n = 2^{10} = \boxed{1024}$

b) $\sum_{p=0}^{9} \binom{10}{p} = 2^{10} - 1 = 1024 - 1 = \boxed{1023}$

c) $\sum_{p=2}^{9} \binom{9}{p} = 2^9 - 1 - 9 = 512 - 10 = \boxed{502}$

d) $\sum_{p=4}^{10} \binom{p}{4} = \binom{n+1}{k+1} = \binom{11}{5} = \dfrac{11!}{5!\,6!} = \dfrac{11 \cdot \cancel{10}^{2} \cdot \cancel{9}^{3} \cdot \cancel{8}^{2} \cdot 7 \cdot \cancel{6!}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1 \cdot \cancel{6!}} =$

$= 11 \cdot 3 \cdot 2 \cdot 7 = \boxed{462}$

→ complementar de $\binom{11}{5}$.

e) $\sum_{p=5}^{10} \binom{p}{5} = \binom{n+1}{k+1} = \binom{11}{6} = \boxed{462}$

07. $\sum_{k=0}^{m} \binom{m}{k} = 512$ soma da linha $= 2^m$

$512 = 2^9$

$\boxed{m = 9}$

$$\begin{array}{r|l} 512 & 2 \\ 256 & 2 \\ 128 & 2 \\ 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$512 = 2^9$