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Tarefa Básica I

1.a)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5 \quad D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$

$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = (-6) - (-2) = -4 \quad x = \frac{3}{5} \quad y = \frac{-4}{5}$$

$$V = \left\{ \left(\frac{3}{5}, \frac{-4}{5} \right) \right\}$$

b)
$$\begin{cases} 3x + y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = (-10) - 13 = -23$$

$$D_x = \begin{vmatrix} -1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} = (-22) - 1 = -23$$

$$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & 2 \end{vmatrix} = 32 - 65 = -33$$

$$Dz = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} \begin{vmatrix} 3 & -1 \\ 2 & 0 \\ 4 & -1 \end{vmatrix} = 6 - (-17) = 23$$

$$\begin{matrix} 0 & 4 & 2 \end{matrix}$$

$$\left. \begin{aligned} x &= \frac{-23}{-23} = 1 & y &= \frac{-23}{-23} = 1 & z &= \frac{23}{-23} = -1 \end{aligned} \right\} \text{ (a.)}$$

$$V = \{(-1, -1, 1)\}$$

$$2. \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \begin{vmatrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{vmatrix} = 61 - 31 = 30$$

$$Dy = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \begin{vmatrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{vmatrix} = 78 - 48 = 30$$

$$108 \quad 2 \quad -32$$

$$y = \frac{30}{30} \rightarrow \boxed{y = 1} \quad \boxed{\text{Alternative: A}}$$

$$3. \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases} \quad \begin{matrix} P = SS - VS + X \\ P = SS + VS + X \\ B = SS - X - X \end{matrix} \quad H$$

$$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -11 & 3 & -2 \\ 2 & 3 & -1 & 2 & 3 \end{vmatrix} \quad \begin{matrix} 2 & -33 & -6 \\ -1 & -44 & 9 \end{matrix}$$

$$D = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -11 & 3 & -2 \\ 2 & 3 & -1 & 2 & 3 \end{vmatrix} = (-36) - (-37) = 1$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ -2 & 1 & -11 & -2 & 1 \\ 1 & 3 & -1 & 1 & 3 \end{vmatrix} \quad \begin{matrix} 1 & -33 & 4 \\ -1 & -22 & -6 \\ -4 & -11 & -3 \end{matrix}$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ -2 & 1 & -11 & -2 & 1 \\ 1 & 3 & -1 & 1 & 3 \end{vmatrix} = (-29) - (-28) = -1$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix} \quad \begin{matrix} 2 & -22 & 3 \\ 2 & -6 & 6 \end{matrix}$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & -2 & -11 & 3 & -2 \\ 2 & 1 & -1 & 2 & 1 \end{vmatrix} = (-17) - (-18) = 1$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix} \quad \begin{matrix} 1 & -8 & 9 \end{matrix}$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 & 1 & 2 \\ 3 & 1 & -2 & 3 & 1 \\ 2 & 3 & 1 & 2 & 3 \end{vmatrix} = 2 - 2 = 0$$

$$x = \frac{-1}{1} = -1 \quad y = \frac{1}{1} = 1 \quad z = \frac{0}{1} = 0$$

$$a + b + c = 1 + (-1) + 0 = 0 \quad \Rightarrow \boxed{a + b + c = 0}$$

Alternativa C

$$4. \begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$$

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} = 1(-15) = -16$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} = (-30) - (-146) = 116$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} = 26 - (-54) = 80$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} = 3 - 99 = -96$$

$$x = \frac{116}{-16} = -7.25 \quad y = \frac{80}{-16} = -5 \quad z = \frac{-96}{-16} = 6$$

$$x + y + z = -7.25 - 5 + 6 = -6.25$$

Alternativa A

$$5. \begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 2(2 \cdot 1 - 1 \cdot 2) - 1(0 \cdot 1 - 3 \cdot 1) + 0(0 \cdot 3 - 6 \cdot 1) = 2(2 - 2) - 1(0 - 3) + 0(-3) = 0 + 3 + 0 = 3$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} = 5(2 \cdot 1 - 1 \cdot 2) - 1(3 \cdot 1 - 7 \cdot 1) + 0(3 \cdot 2 - 7 \cdot 1) = 5(2 - 2) - 1(3 - 7) + 0(6 - 7) = 0 + 4 + 0 = 4$$

$$D_y = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} = 2(3 \cdot 1 - 1 \cdot 7) - 5(0 \cdot 1 - 3 \cdot 1) + 0(0 \cdot 3 - 9 \cdot 1) = 2(3 - 7) - 5(0 - 3) + 0(-3) = 2(-4) - 5(-3) + 0 = -8 + 15 = 7$$

$$D_z = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} = 2(2 \cdot 7 - 3 \cdot 2) - 1(0 \cdot 7 - 21 \cdot 1) + 5(0 \cdot 2 - 6 \cdot 1) = 2(14 - 6) - 1(-21) + 5(-6) = 2(8) + 21 - 30 = 16 + 21 - 30 = 7$$

$$x = \frac{4}{3}$$

$$y = \frac{7}{3}$$

$$z = \frac{-5}{3}$$

Alternativa D

$$6. \left\{ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 2 & 1 & 0 & 7 \\ -1 & 2 & 2 & -1 \end{array} \right\} \begin{array}{l} x \\ y \\ z \end{array} = \begin{array}{l} 3 \\ 7 \\ -1 \end{array} \quad \begin{array}{l} 2 = x + x + x \\ 7 = x + y + z \\ -1 = x + y + z \end{array}$$

$$(1 \cdot x) + (0 \cdot y) + (0 \cdot z) = 3 \rightarrow \boxed{x = 3}$$

$$(2 \cdot x) + (1 \cdot y) + (0 \cdot z) = 7 \rightarrow 6 + y = 7 \rightarrow \boxed{y = 1}$$

$$(-1 \cdot x) + (2 \cdot y) + (2 \cdot z) = -1 \rightarrow (-3) + 2 + 2z = -1 \rightarrow 2z = 0 \rightarrow \boxed{z = 0}$$

Alternativa E

Tarefa Básica II

$$1. \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x \quad \quad - 5z = 3 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{\substack{-\frac{1}{2} \\ -\frac{1}{2}}} \sim \left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 0 & \frac{7}{2} & \frac{1}{2} & \frac{27}{2} \\ 0 & \frac{1}{2} & -\frac{7}{2} & \frac{11}{2} \end{array} \right) \xrightarrow{\frac{1}{7}}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & -3 & -5 \\ 0 & \frac{7}{2} & \frac{1}{2} & \frac{27}{2} \\ 0 & 0 & -\frac{25}{7} & \frac{25}{7} \end{array} \right)$$

$$\frac{-25}{7} \cdot z = \frac{25}{7} \rightarrow z = \frac{25}{7} \cdot \left(\begin{array}{c} -7 \\ 25 \end{array} \right) \rightarrow z = \left(\begin{array}{c} -25 \\ 25 \end{array} \right) \rightarrow \boxed{z = -1}$$

$$\frac{7}{2} \cdot y + \frac{1}{2} \cdot (-1) = \frac{27}{2} \rightarrow 7 \cdot y = \frac{27}{2} + \frac{1}{2} \rightarrow y = \frac{28}{2} \rightarrow \boxed{y = 4}$$

$$\begin{array}{cccccc} 2 & 2 & 2 & 2 & 2 & 2 \end{array}$$

$$x - 5 \cdot (-1) = 3 \rightarrow x = 3 - 5 \rightarrow \boxed{x = -2}$$

$$2. \begin{cases} x = 2x - y - z \\ 2y = 3z - x + 1 \\ x + y + z = 11 \end{cases} \rightarrow \begin{cases} x - 2x + y + z = 0 \\ 2y - 3z = -x + 1 \\ x + y + z = 11 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 1 \\ 1 & 1 & 1 & 11 \end{array} \right) \xrightarrow{-1} \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 2 & -3 & 1 \\ 0 & 3 & 1 & 11 \end{array} \right) \xrightarrow{-\frac{3}{2}}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 6 \\ 0 & 2 & -3 & 0 \\ 0 & 0 & \frac{11}{2} & 11 \end{array} \right)$$

$$\frac{11}{2} \cdot z = 11 \rightarrow z = \frac{11 \cdot 2}{11} \rightarrow z = 2$$

$$2y - 3 \cdot 2 = 0 \rightarrow 2y = 6 \rightarrow y = 3$$

$$x - 2 \cdot 3 = 0 \rightarrow x = 6$$

$$x + 2y + 3z = 6 + 6 + 6 \rightarrow \boxed{x + 2y + 3z = 18}$$

Alternativa B

$$3. \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6x + 3z = -12 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \xrightarrow{-2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \xrightarrow{2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 0 & -5 & -10 \end{array} \right)$$

$$-5Z = -10 \rightarrow \boxed{Z = 2}$$

Alternative D

$$4. \begin{cases} x = y + \frac{1}{5} \cdot z \\ 3y = z + \frac{1}{5} \cdot x \\ z + x + y = 68 \end{cases} \rightarrow \begin{cases} x - y - \frac{z}{5} = 0 \\ (-\frac{x}{5}) + 3y - z = 0 \\ x + y + z = 68 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & (-1) & (-\frac{1}{5}) & 0 \\ (-\frac{1}{5}) & 3 & (-1) & 0 \\ 1 & 1 & 1 & 68 \end{array} \right) \xrightarrow{\substack{R_2 + \frac{1}{5}R_1 \\ R_3 - R_1}} \sim \left(\begin{array}{ccc|c} 1 & (-1) & (-\frac{1}{5}) & 0 \\ 0 & \frac{14}{5} & \frac{4}{5} & 0 \\ 0 & 2 & \frac{6}{5} & 68 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & (-1) & (-\frac{1}{5}) & 0 \\ 0 & \frac{14}{5} & \frac{4}{5} & 0 \\ 0 & 0 & \frac{35}{5} & 68 \end{array} \right)$$

$$\frac{68}{35}, z = 68 \rightarrow z = \frac{68 \cdot 35}{68} \rightarrow z = 35$$

$$2y + \frac{6}{5} \cdot 35 = 68 \rightarrow 2y = 68 - 42 \rightarrow 2y = 26 \rightarrow y = 13$$

$$x + 13 + 35 = 68 \rightarrow x = 68 - 48 \rightarrow x = 20$$

$$x \rightarrow A_i = R\$ 20,00$$

$$y \rightarrow B_i = R\$ 13,00$$

$$z \rightarrow C_{i0} = R\$ 35,00$$

Alternative A

$$5. \quad A = \begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix} \quad x = \begin{vmatrix} x \\ y \\ z \end{vmatrix} = A \cdot x = \begin{vmatrix} 134 \\ 115 \\ 48 \end{vmatrix}$$

$$\begin{cases} 3Y + 4Z = 134 \\ X + 5Z = 115 \\ 2X + Y = 48 \end{cases}$$

$$\left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 2 & 1 & 0 & 48 \end{array} \right) \xrightarrow[-4]{-2} \left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 0 & 1 & -10 & -182 \end{array} \right) \xrightarrow{\cdot \frac{1}{3}}$$

$$\left(\begin{array}{ccc|c} 0 & 3 & 4 & 134 \\ 1 & 0 & 5 & 115 \\ 0 & 0 & -\frac{34}{3} & -\frac{680}{3} \end{array} \right)$$

$$\left(-\frac{34}{3} \right) \cdot Z = \left(-\frac{680}{3} \right) \rightarrow Z = \left(-\frac{680}{3} \right) \cdot \left(-\frac{3}{34} \right) \rightarrow Z = \frac{680}{34}$$

$$Z = 20$$

$$Y - 10 \cdot 20 = (-182) \rightarrow Y = (-182) + 200 \rightarrow Y = 18$$

$$X + 5 \cdot 20 = 115 \rightarrow X = 115 - 100 \rightarrow X = 15$$

$$20 + 18 + 15 = \boxed{58}$$

Alternativa A