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CT11350

Tarefa Básica I

1. $\begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases}$

$$D = \begin{vmatrix} a & 4 \\ 1 & 2 \end{vmatrix} = 2a - 4 \quad D_x = \begin{vmatrix} 1 & 4 \\ b & 2 \end{vmatrix} = 2 - 4b$$

$$x = \frac{2 - 4b}{2a - 4}$$

$$\boxed{\begin{array}{l} S.P. \rightarrow N=0 \\ D=0 \end{array}}$$

$$\hookrightarrow 2 - 4b = 0 \rightarrow 2 = 4b \rightarrow b = \frac{1}{2}$$

$$2a - 4 = 0 \rightarrow 2a = 4 \rightarrow \boxed{a=2}$$

Alternativa A

2. $\begin{cases} x + Ky = 1 \\ Kx + y = 1 - K \end{cases}$

$$\begin{matrix} \leftrightarrow & \begin{pmatrix} 1 & K & | & 1 \\ K & 1 & | & 1-K \end{pmatrix} & \rightarrow & \begin{pmatrix} 1 & K & | & 1 \\ 0 & 1-K^2 & | & 1-2K \end{pmatrix} \end{matrix}$$

$$(1-K^2)y = 1-2K \rightarrow y = \frac{1-2K}{1-K^2}$$

$$S.P. \rightarrow N=0 \text{ e } D=0$$

$$1-2K=0 \rightarrow 1=2K \rightarrow K=\frac{1}{2}$$

$$1-K^2=0 \rightarrow 1=K^2 \rightarrow K=\sqrt{1} \rightarrow K=\pm 1$$

K não tem único valor

$SPD \Leftrightarrow D \neq 0$

$$1 - K^2 \neq 0 \Rightarrow K^2 \neq 1 \Rightarrow K \neq \sqrt{1} \Rightarrow K \neq \pm 1$$

$$SI \rightarrow D = 0 \text{ e } N \neq 0$$

$$1 - 2K \neq 0 \Leftrightarrow 1 \neq 2K \Rightarrow K \neq \frac{1}{2}$$

$$1 - K^2 = 0 \Rightarrow 1 = K^2 \Rightarrow K = \sqrt{1} \Rightarrow K = \pm 1$$

Alternativa D)

$$3.a) \begin{cases} x + 2y + cz = 1 \\ y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$$

$$\left| \begin{array}{ccc|c} -3 & 1 & 2 & c \\ 1 & 0 & 1 & 1 \\ 1 & 3 & 2 & 2 \end{array} \right| \xrightarrow{R_1 \rightarrow R_1 + 3R_2} \left| \begin{array}{ccc|c} 0 & 1 & 2 & c+3 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 4 & 2-3c \end{array} \right| \xrightarrow{R_3 \rightarrow R_3 - R_2} \left| \begin{array}{ccc|c} 0 & 1 & 2 & c+3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 6-3c & 1-2c \end{array} \right|$$

$$(D = 6 - 3c)$$

b) $SPD \Leftrightarrow D \neq 0$

$$\left| \begin{array}{ccc|c} -3 & 1 & 2 & c \\ 1 & 0 & 1 & 1 \\ 1 & 3 & 2 & 2 \end{array} \right| \xrightarrow{R_1 \rightarrow R_1 + 3R_2} \left| \begin{array}{ccc|c} 0 & 1 & 2 & c+3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 4 & 2-3c \end{array} \right| \xrightarrow{R_3 \rightarrow R_3 - 4R_2} \left| \begin{array}{ccc|c} 0 & 1 & 2 & c+3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 6-3c & 1-2c \end{array} \right|$$

$$(6-3c)y=4 \Rightarrow y = \frac{4}{6-3c} \quad 6-3c \neq 0$$

$$6 \neq 3c$$

$$[c \in \mathbb{R} - \{2\}]$$

$$4. \begin{cases} x - y = k \\ 12y - ky + z = 1 \\ 36x + kz = 2 \end{cases}$$

$$\xrightarrow{-36 -12} \left(\begin{array}{ccc|c} 1 & -1 & 0 & k \\ 12 & -k & 1 & 1 \\ 36 & 0 & k+2 & 2 \end{array} \right) \xrightarrow{\frac{k-12}{36}} \left(\begin{array}{ccc|c} 1 & -1 & 0 & k \\ 0 & 12-k & 1 & 1-12k \\ 0 & 36 & k+2-36k & 2 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & k \\ 0 & 0 & \frac{k^2-12k+36}{36} & \frac{18k^2+k+6}{18} \\ 0 & 36 & k & 12-36k \end{array} \right)$$

$$\left(\frac{k^2-12k+36}{36} \right) y = \frac{18k^2+k+6}{18} \Rightarrow y = \frac{18k^2+k+6}{18} \cdot \frac{36}{k^2-12k+36}$$

$$y = \frac{36k^2+2k+12}{k^2-12k+36}$$

$$k^2 - 12k + 36 - a = 1 \quad b = -12 \quad c = 36$$

$$\Delta = (-12)^2 - 4 \cdot 1 \cdot 36$$

$$\Delta = 144 = 144$$

$$1 = 0$$

$$k \neq \frac{12 \pm \sqrt{0}}{2 \cdot 1} \Rightarrow k \neq \frac{12}{2} \Rightarrow \boxed{k \neq 6}$$

Alternativa C

$$5. \begin{cases} x - y + z = 6 \\ 2y + y - z = -3 \\ x + 2y - z = -5 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} = 4 - 1 = 3 \quad D_x = \begin{vmatrix} 6 & -1 & 1 \\ -3 & 1 & -1 \\ -5 & 2 & -1 \end{vmatrix} = (-17) - (-20) = 3$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 2 & -3 & -1 \\ 1 & -5 & -1 \end{vmatrix} = (-13) - (-10) = -3 \quad D_z = \begin{vmatrix} 1 & -1 & 6 \\ 2 & 1 & -3 \\ 1 & 2 & -5 \end{vmatrix} = 22 - 10 = 12$$

$$x = \frac{3}{3} = 1 \quad y = \frac{-3}{-3} = -1 \quad z = \frac{12}{3} = 4$$

$$x \cdot y \cdot z = 1 \cdot (-1) \cdot 4 \Rightarrow \boxed{y \cdot y \cdot z = -4} \quad \boxed{\text{Alternative B}}$$

$$6. \begin{cases} x + y + z = K \\ Ky + y + z = 1 \\ x + y - z = K \end{cases} \quad \boxed{\text{Alternative D}}$$

$$\xrightarrow[-1-K]{\text{L}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & K \\ K & 1 & 1 & 1 \\ 1 & 1 & -1 & K \end{array} \right) \xrightarrow{\text{L}} \left(\begin{array}{ccc|c} 1 & 1 & 1 & K \\ 0 & 1-K & 1-K & 1-K^2 \\ 0 & 0 & -2 & 0 \end{array} \right)$$

$$-2z = 0 \Rightarrow z = 0$$

$$(1-K)y = 1 - K^2 \Rightarrow y = \frac{1 - K^2}{1 - K} \quad \text{SPT} \Rightarrow N=0 \text{ e } D=0$$

$$1 - K^2 = 0 \Rightarrow 1 = K^2 \Rightarrow K = \sqrt{1} \Rightarrow K = \pm 1$$

$$1 - K = 0 \Rightarrow 1 = K \Rightarrow K = 1$$

$$7. \begin{cases} x + y + z = 11 \\ mx - 2y + 4z = 5 \Rightarrow D = 0 \\ m^2x + 4y + 16z = 25 \end{cases}$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ m & -2 & 5 \\ m^2 & 4 & 25 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ m & -2 \end{vmatrix} = (-50 + 5m^2 + 4m) - (-2m^2 + 20 + 25m)$$

$$7m^2 - 3m - 70 \stackrel{?}{=} m^2 - 7m - 10 \quad a=1 \quad b=-3 \quad c=-10$$

$$\Delta = (-3)^2 - 4 \cdot 1 \cdot (-10)$$

$$\Delta = 9 + 40$$

$$\Delta = 49$$

Alternative B

$$m = \frac{3 \pm \sqrt{49}}{2 \cdot 1} = \frac{3 \pm 7}{2} \quad \begin{array}{l} \rightarrow m' = 5 \\ \rightarrow m'' = -2 \end{array}$$

$$\text{Soma dos valores: } 5 + (-2) = 3$$

Tarefa Básica II

$$1. \begin{vmatrix} 1 & ? & 0 \\ ? & 1 & ? \\ 0 & ? & 1 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = K \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

$$\begin{vmatrix} x+7y \\ 7x+y \\ 7y+x \end{vmatrix} = \begin{vmatrix} Kx \\ Ky \\ Kz \end{vmatrix}$$

$$\begin{cases} x+7y = Kx \\ 7x+y = Ky \\ 7y+x = Kz \end{cases} \Rightarrow \begin{cases} x-Kx+7y=0 \\ 7x+y-Ky=0 \\ 7y+x-Kz=0 \end{cases}$$

$$D = \begin{vmatrix} 1-K & 7 & 0 \\ 7 & 1-K & 0 \\ 0 & 0 & 1-K \end{vmatrix} = (1-K)^2 - 49 = 4K^2 - 2K - 49$$

$$D \neq 0 \Rightarrow D = 0$$

$$(1-K)^2 - 49 = 0 \Rightarrow 1-K = \sqrt{49} \Rightarrow K = 7+1 \Rightarrow K = 8$$

(Alternative D)

$$V = KxV - KV^2 + KV^2 - KV^3 + V^3 - 2(V^2 - KV^2) - 4(V^3 - KV^3) - 6(V^4 - KV^4)$$

$$2. \begin{cases} 3x - 4y - z = 0 & Dx = 0 \\ 2y - x + 3z = 0 & Dy = 0 \\ x + y - 0 & Dz = 0 \end{cases}$$

$$D = \begin{vmatrix} 3 & 4 & -1 & | & 3 & 4 \\ 2 & -1 & 3 & | & 2 & -1 \\ 1 & 1 & 0 & | & 1 & 1 \end{vmatrix} = 10 - 10 = 0$$

SPJ $\rightarrow D=0$ e $N=0$

[Alternative D]

$$3. \begin{cases} K + y + z = 0 & Dx = 0 \\ Ky + 3y + 4z = 0 & Dy = 0 \\ y + Ky + 3z = 0 & Dz = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 & | & 1 & 1 \\ K & 3 & 4 & | & K & 3 \\ 1 & K & 3 & | & 1 & K \end{vmatrix} = 13 + K^2 - (3 + K) \rightarrow K^2 - 7K + 10$$

O \rightarrow SPJ $\rightarrow N=0$ e $D=0$

$$K^2 - 7K + 10 = 0$$

$$\Delta K^2 - 7K + 10 = 0 \quad a=1 \quad b=-7 \quad c=10$$

$$\Delta = (-7)^2 - 4 \cdot 1 \cdot 10$$

$$\Delta = 49 - 40$$

$$\Delta = 9$$

$$K = \frac{7 \pm \sqrt{9}}{2 \cdot 1} = \frac{7 \pm 3}{2} \quad \begin{array}{l} \Delta K' = 5 \\ \Delta K'' = 2 \end{array}$$

$$\text{Soma de valores: } 5 + 2 = \boxed{7}$$

[Alternative D]

$$4. \begin{cases} x + kz = 0 \\ ky + y = 0 \\ x + ky = 0 \end{cases} \quad \begin{array}{l} Dx = 0 \\ Dy = 0 \\ Dz = 0 \end{array}$$

$$D = \begin{vmatrix} 1 & 0 & k \\ k & 1 & 0 \\ 1 & k & 0 \end{vmatrix} = 1 \cdot 1 \cdot k - k \cdot k \cdot 1 = k^3 - k$$

$$k^3 - k \neq 0 \Rightarrow k(k^2 - 1) \neq 0 \Rightarrow k \neq 0$$

$$k^2 - 1 \neq 0 \Rightarrow k \neq \sqrt{1} \Rightarrow k \neq \pm 1$$

Alternative A

$$5. \begin{cases} -x + 2y - 3 = 0 \\ 3x - y + 3 = 0 \Rightarrow \\ 2x - 4y + 6 = 0 \end{cases} \quad \begin{cases} -x + 2y = 3 \\ 3x - y = -3 \\ 2x - 4y = -6 \end{cases}$$

$$-x + 2y = 3 \Rightarrow -x = 3 - 2y \Rightarrow x = 2y - 3$$

$$3x - y = -3 \Rightarrow 3(2y - 3) - y = -3 \Rightarrow 6y - 9 - y = -3 \\ 5y = 6 \Rightarrow y = \frac{6}{5}$$

$$x = 2 \cdot \frac{6}{5} - 3 \Rightarrow x = \frac{12}{5} - 3 \Rightarrow x = -\frac{3}{5}$$

Alternative B | SPD $\Rightarrow D \neq 0$