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CT11350

Tarefa Básica I

$$1. \begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases}$$

$$D = \begin{vmatrix} a & 4 \\ 1 & 2 \end{vmatrix} = 2a - 4 \quad Dx = \begin{vmatrix} 1 & 4 \\ b & 2 \end{vmatrix} = 2 - 4b$$

$$x = \frac{2 - 4b}{2a - 4}$$

$$\boxed{\text{S.P. } \rightarrow \begin{matrix} N=0 \\ D=0 \end{matrix}}$$

$$\rightarrow 2 - 4b = 0 \rightarrow 2 = 4b \rightarrow b = \frac{1}{2}$$

$$2a - 4 = 0 \rightarrow 2a = 4 \rightarrow \boxed{a = 2}$$

Alternativa A

$$2. \begin{cases} x + Ky = 1 \\ Kx + y = 1 - K \end{cases}$$

$$\rightarrow \begin{pmatrix} 1 & K & | & 1 \\ K & 1 & | & 1 - K \end{pmatrix} \rightarrow \begin{pmatrix} 1 & K & | & 1 \\ 0 & 1 - K^2 & | & 1 - 2K \end{pmatrix}$$

$$(1 - K^2)y = 1 - 2K \rightarrow y = \frac{1 - 2K}{1 - K^2}$$

$$\text{S.P. } \rightarrow N=0 \text{ e } D=0$$

$$1 - 2K = 0 \rightarrow 1 = 2K \rightarrow K = \frac{1}{2}$$

$$1 - K^2 = 0 \rightarrow 1 = K^2 \rightarrow K = \sqrt{1} \rightarrow K = \pm 1$$

K não tem único valor

$$SPD \Rightarrow D \neq 0$$

$$1 - K^2 \neq 0 \Rightarrow K^2 \neq 1 \Rightarrow K \neq \sqrt{1} \Rightarrow K \neq \pm 1$$

$$SI \Rightarrow D = 0 \text{ e } N \neq 0$$

$$1 - 2K \neq 0 \Rightarrow 1 \neq 2K \Rightarrow K \neq \frac{1}{2}$$

$$1 - K^2 = 0 \Rightarrow 1 = K^2 \Rightarrow K = \sqrt{1} \Rightarrow K = \pm 1$$

Alternativa D

$$3.a) \begin{cases} x + 2y + Cz = 1 \\ y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$$

$$\begin{array}{c} -3 \\ \downarrow \\ \begin{vmatrix} 1 & 2 & C \\ 0 & 1 & 1 \\ 3 & 2 & 2 \end{vmatrix} \end{array} \xrightarrow{4} \begin{array}{c} \begin{vmatrix} 1 & 2 & C \\ 0 & 1 & 1 \\ 0 & -4 & 2-3C \end{vmatrix} \end{array} \xrightarrow{-} \begin{array}{c} \begin{vmatrix} 1 & 2 & C \\ 0 & 1 & 1 \\ 0 & 0 & 6-3C \end{vmatrix} \end{array}$$

$$D = 6 - 3C$$

$$b) SPD \Rightarrow D \neq 0$$

$$\begin{array}{c} -3 \\ \downarrow \\ \begin{vmatrix} 1 & 2 & C \\ 0 & 1 & 1 \\ 3 & 2 & 2 \end{vmatrix} \end{array} \xrightarrow{4} \begin{array}{c} \begin{vmatrix} 1 & 2 & C \\ 0 & 1 & 1 \\ 0 & -4 & 2-3C \end{vmatrix} \end{array} \xrightarrow{-} \begin{array}{c} \begin{vmatrix} 1 & 2 & C \\ 0 & 1 & 1 \\ 0 & 0 & 6-3C \end{vmatrix} \end{array}$$

$$(6-3C)y = 4 \Rightarrow y = \frac{4}{6-3C} \quad \begin{matrix} 6-3C \neq 0 \\ 6 \neq 3C \end{matrix}$$

$$C \in \mathbb{R} - \{2\}$$

$$C \neq 2$$

$$4. \begin{cases} x - y = K \\ 12y - Ky + z = 1 \\ 36x + Kz = 2 \end{cases}$$

$$\begin{pmatrix} -36 & -12 & 0 & 1 & K \\ 0 & 12-K & 1 & 1 & 1 \\ 0 & 36 & 0 & K & 2 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 0 & 12-K & 1 & 1 & 1 \\ -36 & -12 & 0 & 1 & K \\ 0 & 36 & 0 & K & 2 \end{pmatrix} \xrightarrow{R_1 \cdot \frac{K-12}{36}} \begin{pmatrix} 0 & 1 & \frac{K-12}{36} & \frac{K-12}{36} & \frac{K-12}{36} \\ -36 & -12 & 0 & 1 & K \\ 0 & 36 & 0 & K & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & \frac{K^2-12K+36}{36} & \frac{18K^2+K+6}{18} & \frac{K^2-12K+36}{36} \\ -36 & -12 & 0 & 1 & K \\ 0 & 36 & 0 & K & 2 \end{pmatrix}$$

$$\left(\frac{K^2-12K+36}{36} \right) y = \frac{18K^2+K+6}{18} \rightarrow y = \frac{18K^2+K+6}{18} \cdot \frac{36}{K^2-12K+36} = \frac{36K^2+2K+12}{K^2-12K+36}$$

$$y = \frac{36K^2+2K+12}{K^2-12K+36}$$

$$K^2-12K+36 \quad a=1 \quad b=-12 \quad c=36$$

$$\Delta = (-12)^2 - 4 \cdot 1 \cdot 36$$

$$\Delta = 144 - 144$$

$$\Delta = 0$$

$$K \neq \frac{12 \pm \sqrt{0}}{2 \cdot 1} \rightarrow K \neq \frac{12}{2} \rightarrow \boxed{K \neq 6}$$

Alternativa E

$$5. \begin{cases} x - y + z = 6 \\ 2x + y - z = -3 \\ x + 2y - z = -5 \end{cases}$$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & -1 \end{vmatrix} = 4 - 1 = 3 \quad Dx = \begin{vmatrix} 6 & -1 & 1 \\ -3 & 1 & -1 \\ -5 & 2 & -1 \end{vmatrix} = (-17) - (-20) = 3$$

$$Dy = \begin{vmatrix} 1 & 6 & 1 \\ 2 & -3 & -1 \\ 1 & -5 & -1 \end{vmatrix} = (-13) - (-10) = -3 \quad Dz = \begin{vmatrix} 1 & -1 & 6 \\ 2 & 1 & -3 \\ 1 & 2 & -5 \end{vmatrix} = 22 - 10 = 12$$

$$x = \frac{3}{3} = 1 \quad y = \frac{-3}{3} = -1 \quad z = \frac{12}{3} = 4$$

$$x \cdot y \cdot z = 1 \cdot (-1) \cdot 4 \rightarrow \boxed{y \cdot y \cdot z = -4} \quad \boxed{\text{Alternative B}}$$

$$6. \begin{cases} x + y + z = K \\ Ky + y + z = 1 \\ x + y - z = K \end{cases} \quad \boxed{\text{Alternative D}}$$

$$\begin{matrix} -1 & -K \\ \begin{matrix} 1 \\ 0 \\ 0 \end{matrix} \end{matrix} \begin{pmatrix} 1 & 1 & 1 & | & K \\ K & 1 & 1 & | & 1 \\ 1 & 1 & -1 & | & K \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 1 & 1 & | & K \\ 0 & 1-K & 1-K & | & 1-K^2 \\ 0 & 0 & -2 & | & 0 \end{pmatrix}$$

$$-2z = 0 \rightarrow z = 0$$

$$(1-K)y = 1-K^2 \rightarrow y = \frac{1-K^2}{1-K} \quad \text{SPT} \rightarrow K=0 \text{ e } D=0$$

$$1-K^2=0 \rightarrow 1=K^2 \rightarrow K=\sqrt{1} \rightarrow K=\pm 1$$

$$1-K=0 \rightarrow 1=K \rightarrow K=1$$

$$7. \begin{cases} x + y + z = 11 \\ mx - 2y + 4z = 5 \\ m^2x + 4y + 16z = 25 \end{cases} \rightarrow D=0$$

$$D_z = \begin{vmatrix} 1 & 1 & 1 \\ m & -2 & 5 \\ m^2 & 4 & 25 \end{vmatrix} \begin{vmatrix} 1 & 1 \\ m & -2 \\ m^2 & 4 \end{vmatrix} = (-50 + 5m^2 + 4m) - (-2m^2 + 20 + 25m)$$

$$7m^2 - 3m - 70 \rightarrow m^2 - 7m - 10 \quad a=1 \quad b=-3 \quad c=-10$$

$$\Delta = (-3)^2 - 4 \cdot 1 \cdot (-10)$$

$$\Delta = 9 + 40$$

$$\Delta = 49$$

Alternative B

$$m = \frac{3 \pm \sqrt{49}}{2 \cdot 1} = \frac{3 \pm 7}{2} \rightarrow \begin{cases} m' = 5 \\ m'' = -2 \end{cases}$$

$$\text{Soma dos valores: } 5 + (-2) = 3$$

Tarefa Básica Π

$$1. \begin{vmatrix} 1 & 7 \\ 7 & 1 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \end{vmatrix} = K \begin{vmatrix} x \\ y \end{vmatrix}$$

$$\begin{vmatrix} x+7y \\ 7x+y \end{vmatrix} = \begin{vmatrix} Kx \\ Ky \end{vmatrix}$$

$$\begin{cases} x+7y = Ky \\ 7x+y = Ky \end{cases} \rightarrow \begin{cases} x-Kx+7y=0 \\ 7x+y-Ky=0 \end{cases}$$

$$D = \begin{vmatrix} 1-K & 7 \\ 7 & 1-K \end{vmatrix} = (1-K)^2 - 49 = 0$$

$$D \neq 0 \rightarrow D = 0$$

$$(1-K)^2 - 49 = 0 \rightarrow 1-K = \sqrt{49} \rightarrow K = 7+1 \rightarrow \boxed{K=8}$$

[Alternative D]

$$y = \frac{Ky - 7Ky}{-49yy} \rightarrow y = \frac{7Ky - Ky}{49yy} \rightarrow y = \frac{6Ky}{49yy}$$

$$y = \frac{Ky - 7Ky}{-49yy} \rightarrow y = \frac{7Ky - Ky}{49yy} \rightarrow y = \frac{6Ky}{49yy}$$

$$2. \begin{cases} 3x - 4y - z = 0 & Dx = 0 \\ 2x - y + 3z = 0 & Dy = 0 \\ x + y = 0 & Dz = 0 \end{cases} \quad A$$

$$D = \begin{vmatrix} 3 & 4 & -1 & 3 & 4 \\ 2 & -1 & 3 & 2 & -1 \\ 1 & 1 & 0 & 1 & 1 \end{vmatrix} = 10 - 10 = 0$$

$$SPI \rightarrow D=0 \quad C=N=0$$

Alternative D

$$3. \begin{cases} x + y + z = 0 & Dx = 0 \\ ky + 3y + 4z = 0 & Dy = 0 \\ x + ky + 3z = 0 & Dz = 0 \end{cases}$$

$$D = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ k & 3 & 4 & k & 3 \\ 1 & k & 3 & 1 & k \end{vmatrix} = 13 + k^2 - (3 + 7k) \rightarrow k^2 - 7k + 10$$

$$0 \rightarrow SPI \rightarrow N=0 \quad e \quad D=0$$

$$k^2 - 7k + 10$$

$$\Delta k^2 - 7k + 10 = 0 \quad a=1 \quad b=-7 \quad c=10$$

$$\Delta = (-7)^2 - 4 \cdot 1 \cdot 10$$

$$\Delta = 49 - 40$$

$$\Delta = 9$$

$$k = \frac{7 \pm \sqrt{9}}{2 \cdot 1} = \frac{7 \pm 3}{2} \quad \begin{cases} k' = 5 \\ k'' = 2 \end{cases}$$

Alternative D

$$\text{Soma de valores: } 5 + 2 = 7$$

$$4. \begin{cases} x + kz = 0 \\ ky + z = 0 \\ x + ky = 0 \end{cases} \quad \begin{cases} Dx = 0 - x - x \\ Dy = 0 - y - y \\ Dz = 0 - x - x \end{cases}$$

$$D = \begin{vmatrix} 1 & 0 & k \\ k & 1 & 0 \\ 1 & k & 0 \end{vmatrix} = k^3 - k$$

$$k^3 - k \neq 0 \rightarrow k(k^2 - 1) \neq 0 \rightarrow k \neq 0$$

$$k^2 - 1 \neq 0 \rightarrow k \neq \pm 1$$

Alternativa A

$$5. \begin{cases} -x + 2y - 3 = 0 \\ 3x - y + 3 = 0 \\ 2x - 4y + 6 = 0 \end{cases} \quad \begin{cases} -x + 2y = 3 \\ 3x - y = -3 \\ 2x - 4y = -6 \end{cases}$$

$$-x + 2y = 3 \rightarrow -x = 3 - 2y \rightarrow x = 2y - 3$$

$$3x - y = -3 \rightarrow 3(2y - 3) - y = -3 \rightarrow 6y - 9 - y = -3 \rightarrow 5y = 6 \rightarrow y = \frac{6}{5}$$

$$x = 2 \cdot \frac{6}{5} - 3 \rightarrow x = \frac{12}{5} - 3 \rightarrow x = -\frac{3}{5}$$

Alternativa B

$$SPD \rightarrow D \neq 0$$