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Tarefa Básica

1. $A = \begin{vmatrix} x & 1 \\ 5 & 3 \end{vmatrix}$ $B = \begin{vmatrix} 3 & -1 \\ y & 2 \end{vmatrix}$

$A \cdot A^{-1} = I_n \rightarrow B \cdot A = I_n$

$\begin{vmatrix} 3 & -1 \\ y & 2 \end{vmatrix} \cdot \begin{vmatrix} x & 1 \\ 5 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$\begin{cases} 3x - 5 = 1 \\ xy + 10 = 0 \end{cases}$ $\begin{cases} 3 - 3 = 0 \\ y + 6 = 1 \end{cases}$

$3x - 5 = 1 \rightarrow x = \frac{6}{3} \rightarrow x = 2$

$y + 6 = 1 \rightarrow y = -5$

$x + y \rightarrow 2 + (-5) = \boxed{-3}$ $\boxed{\text{Alternativa C}}$

2. $A = \begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix}$ → não tem inversa se seu determinante é igual a zero.

$1 \quad 3k \quad 0$

$A = \begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix} = 3 + k^2 - 1 - 3k \rightarrow k^2 - 3k + 2 = 0$

$3 \quad 0 \quad k^2$

$\Delta = (-3)^2 - 4 \cdot 1 \cdot 2$

$\Delta = 9 - 8$

$\Delta = 1$

$K = \frac{3 \pm \sqrt{1}}{2 \cdot 1} = \frac{3 \pm 1}{2} \rightarrow K' = 2 \quad K'' = 1$

$\boxed{\text{Alternativa C}}$

3. $A = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} \quad A \cdot A^{-1} = I_n$

det $A = 12 - 10 = 2 \rightarrow \neq 0$ é invertível

$B = A^{-1} = \begin{vmatrix} 4 & -5 \\ -2 & 3 \end{vmatrix} \div 2 = \begin{vmatrix} 2 & \frac{-5}{2} \\ -1 & \frac{3}{2} \end{vmatrix}$ $\boxed{\text{Alternativa C}}$

4. $\begin{array}{ccc|cc} & & 20 & 2X & 3X \\ x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \\ & & x^2 & 20 & 6 \end{array} = x^2 + 2 \cdot 6 - 20 - 5X \rightarrow x^2 - 5x + 6 \neq 0$

$\rightarrow$ é invertível se det $\neq 0$

$\Delta = (-5)^2 - 4 \cdot 1 \cdot 6$
$\Delta = 25 - 24$
$\Delta = 1$

$x \neq \frac{5 \pm \sqrt{1}}{2 \cdot 1} \neq \frac{5 \pm 1}{2}$ $\rightarrow x' \neq 3$, $\rightarrow x'' \neq 2$

$\{x \neq 3 \text{ e } x \neq 2\}$

$\boxed{\text{Alternativa A}}$

5.

$$A = \begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix}\begin{matrix} -1 & -1 \\ 2 & 1 \\ 1 & 1 \end{matrix} = 7 - 6 = 1$$

$$A = \begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} \qquad A' = \begin{vmatrix} (-1-(-2)) & (-2-(-2)) & (2-1) \\ (1-2) & (1-2) & (1-(-1)) \\ (2-2) & (2-4) & (-1-(-2)) \end{vmatrix}$$

$$A' = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{vmatrix} \qquad \bar{A} = (A')^t = \begin{vmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{vmatrix}$$

$$A^{-1} = \frac{\bar{A}}{1} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{vmatrix}$$

Alternativa B

$$A + A^{-1} = \begin{vmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{vmatrix}$$

6. $(X \cdot A)^t = B \Rightarrow ((X \cdot A)^t)^t = B^t \Rightarrow X \cdot A = B^t$

$X \cdot A \cdot A^{-1} = B^t \cdot A^{-1} \Rightarrow \boxed{X = B^t \cdot A^{-1}}$

Alternativa B

7. $B = \begin{vmatrix} x \\ y \end{vmatrix} \qquad C = \begin{vmatrix} 4x + 5y \\ 5x + 6y \end{vmatrix}$

$$A \cdot \begin{vmatrix} x \\ y \end{vmatrix} = \begin{vmatrix} 4x + 5y \\ 5x + 6y \end{vmatrix} \Rightarrow A = \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} = 24 - 25 = -1$$

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$A^{-1} = \begin{vmatrix} 6 & -5 \\ -5 & 4 \end{vmatrix} \div -1 \rightarrow A^{-1} = \begin{vmatrix} -6 & 5 \\ 5 & -4 \end{vmatrix}$ $\boxed{\text{Alternativa D}}$

8. $A = \begin{vmatrix} 2 & K \\ -2 & 1 \end{vmatrix} = 2 - (-2K) \rightarrow 2 + 2K$

$\det A \cdot \det A^{-1} = 1 \rightarrow (2+2K) \cdot (2+2K) = 1$

$4 + 4K + 4K + 4K^2 = 1 \rightarrow 4K^2 + 8K + 3 = 0$

$\Delta = 8^2 - 4 \cdot 4 \cdot 3$

$\Delta = 64 - 48$

$\Delta = 16$

$K = \frac{(-8) \pm \sqrt{16}}{2 \cdot 4} = \frac{(-8) \pm 4}{8}$ $\rightarrow K' = \frac{-4}{8} = \frac{-1}{2}$; $\rightarrow K'' = \frac{-12}{8} = \frac{-3}{2}$

soma dos valores de K: $\frac{-1}{2} + \left(\frac{-3}{2}\right) = \frac{-4}{2} = \boxed{-2}$

$\boxed{\text{Alternativa B}}$

9. a) $(A+B) \cdot (A-B) = \boxed{A^2 - AB + BA - B^2}$

b) $(A+B)^2 = A^2 + \underbrace{2AB}_{\rightarrow AB = BA} + B^2$

c) $\frac{\det A}{\det(-A)} \rightarrow \frac{\det A}{\det A} = \boxed{1}$

$\hookrightarrow \det(-A) = (-1)^2 \cdot \det A = \det A$

d) $B = A^{-1} \rightarrow \det A \cdot \det B = 1 \rightarrow \boxed{\det B = \frac{1}{\det A}}$