$$\max_x \mathbb{E}_{s\in S}[\mathcal{Q}^s(x)] \tag{1}$$

where

$$\mathcal{Q}^s(x) = \sum_{i\in I^s}(\alpha_1 f_i^s + \alpha_2 g_i^s + \alpha_3 h_i^s + \alpha_4 w_i^s - \phi z_i^s)$$

$$\sum_{l\in L} x_{lk} \leq \eta_k \qquad k \in K \tag{2}$$

$$\sum_{i\in I^s} y_{lki}^s \leq x_{lk} \qquad l \in L, k \in K, s \in S \tag{3}$$

$$a_{1i}^s f_i^s \leq \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s, \quad a_{2i}^s f_i^s \leq \sum_{l\in L} c_{li} y_{l2i}^s \qquad i \in I^s, s \in S \tag{4}$$

$$a_{1i}^s g_i^s \leq \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad a_{2i}^s g_i^s \leq \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{5}$$

$$g_i^s \leq M\left(\sum_{l\in L}\sum_{k\in K} y_{lki}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s\right) \qquad i \in I^s, s \in S \tag{6}$$

$$h_i^s \leq a_{1i}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad h_i^s \leq a_{2i}^s - \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{7}$$

$$\sum_{l\in L}\sum_{k\in K} y_{lki}^s h_i^s \leq \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s \qquad i \in I^s, s \in S \tag{8}$$

$$w_i^s \leq a_{1i}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad w_i^s \leq a_{2i}^s - \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{9}$$

$$w_i^s \leq M\left(\sum_{l\in L}\sum_{k\in K} y_{lki}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s\right) \qquad i \in I^s, s \in S \tag{10}$$

$$\sum_{l\in L}\sum_{k\in K} y_{lki}^s + z_i^s \geq 1 \qquad i \in I^s, s \in S \tag{11}$$

$$f_i^s + g_i^s + h_i^s + w_i^s + z_i^s = 1 \qquad i \in I^s, s \in S \tag{12}$$

$$x_{lk} \in \mathbb{Z}^+, y_{lki}^s \in \{0,1\} \qquad l \in L, k \in K, i \in I^s, s \in S \tag{13}$$

$$f_i^s, g_i^s, h_i^s, w_i^s, z_i^s \in \{0,1\} \qquad i \in I^s, s \in S. \tag{14}$$

1 Questions

Question 1. *The coefficients c_{li} are defined as fractionnal between 0 and 1 but used in the constraints defining late/intime ambulances. For this reason I believe that they are used as binary in the formulation. Is this true ?*

If the answer is no, then the following document is probably all wrong. if it is true, why are they defined as fractionnal.

Question 2. *Why is y^s_{lki} binary instead of being integer positive and do not correspond to the number of ambulances of type k dispatched from location l to demand point i in scenario s?*

Having y variables binary imply that constraints (5) correspond to the fact that sufficiently enough location send ambulances to demand point i in a given scenarion. It seems to me that the definition of the problem imply that sufficiently enough ambulances have to be sent whatever location they come from.

It seems to me that a trivial upper bound on these variable could be a^s_{lki} and a reinforcing constraints would be given by $\sum_{l\in L} y^s_{l1i} \leq a^s_{1i}$.

$\sum_{k\in K}\sum_{l\in L} y^s_{lki} \leq a^s_{1i}a^s_{2i}$

Question 3. *Given the definition of the problem, it seems to me that even if ALS can be sent instead of BLS, if a demand point needs 3 ALS and 2 BLS, sending only 3 ALS is not sufficient for a total coverage, sending 4 neither but 5 would be sufficient. Is that true ?*

If it is true, then Constraints (4) are not sufficient as it must also be ensured that over all types of vehicule, sufficiently enough ambulances are sent which is modelized by the following:

$$(a^s_{1i} + a^s_{2i})f^s_i \leq \sum_{l\in L}\sum_{k\in K} c_{li}y^s_{lki} \quad \forall i \in I^s,\ s \in S$$

As this latter set of constraints dominates constraints

$$a^s_{1i}f^s_i \leq \sum_{l\in L}\sum_{k\in K} c_{li}y^s_{lki}$$

this part of constraints (4) could be removed from the model.

(This also happens whith constraints (5))

Question 4. *Why are Constraints* (6) *mandatory ?*

Constraints (12) imply that exactly one type of coverage is assigned to each demand point in each scenario. Obvisouly, it is preferable to have a total coverage than a total late coverage; for this reason, every objective function should have a higher coefficient on f variables than on g variables. Every optimal solution of such a program without Constraints (6) would satisfy Constraints (6).

Question 5. *Is it true that partial coverage can be rephrased to be: at least one vehicule of any type is dispatched, all dispatched vehicules are in time, and not all needed vehicules are dispatched ?*

If the answers to this question and the latter are true then, constraints (7) are not valid as they exclude the solutions where the a_{2i}^s ALS is dispatched but no BLS are dispatched and $a_1 i^s > 0$. To avoid this issue, it would be necessary to replace them by constraints of the form:

$$h_i^s \leq a_{1i}^s + a_{2i}^s - \sum_{l \in L} \sum_{k \in K} y_{lki}^s \quad \forall s \in S,\ i \in I^s$$

Moreover, constraints (8) ensure that $h_i^s = 1$ only when all dispatched vehicules are in time. This could be replaced by the following:

$$a_{ki}^s(1 - c_{li})h_i^s + y_{lki}^s \leq a_{ki}^s \quad \forall s \in S, l \in L, i \in I^s, k \in K.$$

These are no more quadratic. Finally, to ensure that at least one vehicule arrives in time, one can add the following:

$$h_i^s \leq 1 - (1 - c_{li})y_{lki} \quad \forall s \in S,\ i \in I^s,\ k \in K,\ l \in K$$

The same issue happens with constraints (9).

2 Reformulation

With the precedent section in mind, if all answers have been yes, the following formulation should be valid.

$$\max_x \mathbb{E}_{s \in S}[\mathcal{Q}^s(x)] \tag{15}$$

where

$$\mathcal{Q}^s(x) = \sum_{i \in I^s} (\alpha_1 f_i^s + \alpha_2 g_i^s + \alpha_3 h_i^s + \alpha_4 w_i^s - \phi z_i^s)$$

$$\sum_{l \in L} x_{lk} \leq \eta_k \qquad k \in K \qquad (16)$$

$$\sum_{i \in I^s} y_{lki}^s \leq x_{lk} \qquad l \in L, k \in K, s \in S \qquad (17)$$

$$(a_{1i}^s + a_{2i}^s) f_i^s \leq \sum_{l \in L} \sum_{k \in K} c_{li} y_{lki}^s, \quad a_{2i}^s f_i^s \leq \sum_{l \in L} c_{li} y_{l2i}^s \qquad i \in I^s, s \in S \qquad (18)$$

$$(a_{1i}^s + a_{2i}^s) g_i^s \leq \sum_{l \in L} \sum_{k \in K} y_{lki}^s, \quad a_{2i}^s g_i^s \leq \sum_{l \in L} y_{l2i}^s \qquad i \in I^s, s \in S \qquad (19)$$

$$h_i^s \leq a_{1i}^s + a_{2i}^s - \sum_{l \in L} \sum_{k \in K} c_{li} y_{lki}^s \qquad i \in I^s,\ s \in S \qquad (20)$$

$$a_{ki}^s (1 - c_{li}) h_i^s + y_{lki}^s \leq a_{ki}^s \qquad \forall i \in I^s, l \in L, s \in S, k \in K \qquad (21)$$

$$h_i^s \leq 1 - \frac{1 - c_{li}}{a_{lki}^S} y_{lki}^S \qquad i \in I^s,\ s \in S,\ l \in L,\ k \in K \qquad (22)$$

$$w_i^s \leq a_{1i}^s + a_{2i}^s - \sum_{l \in L} \sum_{k \in K} y_{lki}^s \qquad i \in I^s,\ s \in S \qquad (23)$$

$$w_i^s \leq \sum_{l \in L} y_{lki} \qquad i \in I^s,\ s \in S \qquad (24)$$

$$f_i^s + g_i^s + h_i^s + w_i^s + z_i^s = 1 \qquad i \in I^s, s \in S \qquad (25)$$

$$x_{lk} \in \mathbb{Z}^+, y_{lki}^s \in \{0, a_{ki}^s\} \qquad l \in L, k \in K, i \in I^s, s \in S \qquad (26)$$

$$f_i^s, g_i^s, h_i^s, w_i^s, z_i^s \in \{0, 1\} \qquad i \in I^s, s \in S. \qquad (27)$$

- It is unclear whether inequalities (22) should be for all $k \in K$ without summing over K or not. It depends if it possible that a point with no needed vehicule of type 1 can be partial covered by a vehicule of type 1.