$$\max_x \mathbb{E}_{s\in S}[\mathcal{Q}^s(x)] \tag{1}$$

where

$$\mathcal{Q}^s(x) = \sum_{i\in I^s}(\alpha_1 f_i^s + \alpha_2 g_i^s + \alpha_3 h_i^s + \alpha_4 w_i^s - \phi z_i^s)$$

$$\begin{aligned}
&\sum_{l\in L} x_{lk} \le \eta_k && k\in K && (2)\\
&\sum_{i\in I^s} y_{lki}^s \le x_{lk} && l\in L, k\in K, s\in S && (3)\\
&a_{1i}^s f_i^s \le \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s, \quad a_{2i}^s f_i^s \le \sum_{l\in L} c_{li} y_{l2i}^s && i\in I^s, s\in S && (4)\\
&a_{1i}^s g_i^s \le \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad a_{2i}^s g_i^s \le \sum_{l\in L} y_{l2i}^s && i\in I^s, s\in S && (5)\\
&g_i^s \le M\left(\sum_{l\in L}\sum_{k\in K} y_{lki}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s\right) && i\in I^s, s\in S && (6)\\
&h_i^s \le a_{1i}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad h_i^s \le a_{2i}^s - \sum_{l\in L} y_{l2i}^s && i\in I^s, s\in S && (7)\\
&\sum_{l\in L}\sum_{k\in K} y_{lki}^s h_i^s \le \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s && i\in I^s, s\in S && (8)\\
&w_i^s \le a_{1i}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad w_i^s \le a_{2i}^s - \sum_{l\in L} y_{l2i}^s && i\in I^s, s\in S && (9)\\
&w_i^s \le M\left(\sum_{l\in L}\sum_{k\in K} y_{lki}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s\right) && i\in I^s, s\in S && (10)\\
&\sum_{l\in L}\sum_{k\in K} y_{lki}^s + z_i^s \ge 1 && i\in I^s, s\in S && (11)\\
&f_i^s + g_i^s + h_i^s + w_i^s + z_i^s = 1 && i\in I^s, s\in S && (12)\\
&x_{lk}\in\mathbb{Z}^+, y_{lki}^s\in\{0,1\} && l\in L, k\in K, i\in I^s, s\in S && (13)\\
&f_i^s, g_i^s, h_i^s, w_i^s, z_i^s \in\{0,1\} && i\in I^s, s\in S. && (14)
\end{aligned}$$

1 Questions

Question 1. *The coefficients c_{li} are defined as fractionnal between 0 and 1 but used in the constraints defining late/intime ambulances. For this reason I believe that they are used as binary in the formulation. Is this true ?*

If the answer is no, then the following document is probably all wrong. if it is true, why are they defined as fractionnal.

Question 2. *Why is y^s_{lki} binary instead of being integer positive and do not correspond to the number of ambulances of type k dispatched from location l to demand point i in scenario s?*

Having y variables binary imply that constraints (5) correspond to the fact that sufficiently enough location send ambulances to demand point i in a given scenarion. It seems to me that the definition of the problem imply that sufficiently enough ambulances have to be sent whatever location they come from.

It seems to me that a trivial upper bound on these variable could be a^s_{lki} and a reinforcing constraints would be given by $\sum_{l \in L} y^s_{l1i} \leq a^s_{1i}$.

$\sum_{k \in K} \sum_{l \in L} y^s_{lki} \leq a^s_{1i} a^s_{2i}$

Question 3. *Given the definition of the problem, it seems to me that even if ALS can be sent instead of BLS, if a demand point needs 3 ALS and 2 BLS, sending only 3 ALS is not sufficient for a total coverage, sending 4 neither but 5 would be sufficient. Is that true ?*

If it is true, then Constraints (4) are not sufficient as it must also be ensured that over all types of vehicule, sufficiently enough ambulances are sent which is modelized by the following:

$$(a^s_{1i} + a^s_{2i}) f^s_i \leq \sum_{l \in L} \sum_{k \in K} c_{li} y^s_{lki} \quad \forall i \in I^s,\ s \in S$$

As this latter set of constraints dominates constraints

$$a^s_{1i} f^s_i \leq \sum_{l \in L} \sum_{k \in K} c_{li} y^s_{lki}$$

this part of constraints (4) could be removed from the model.

(This also happens whith constraints (5))

Question 4. *Why are Constraints* (6) *mandatory ?*

Constraints (12) imply that exactly one type of coverage is assigned to each demand point in each scenario. Obvisouly, it is preferable to have a total coverage than a total late coverage; for this reason, every objective function should have a higher coefficient on f variables than on g variables. Every optimal solution of such a program without Constraints (6) would satisfy Constraints (6).

Question 5. *Is it true that partial coverage can be rephrased to be: at least one vehicule of any type is dispatched, all dispatched vehicules are in time, and not all needed vehicules are dispatched ?*

If the answers to this question and the latter are true then, constraints (7) are not valid as they exclude the solutions where the a_{2i}^s ALS is dispatched but no BLS are dispatched and $a_1 i^s > 0$. To avoid this issue, it would be necessary to replace them by constraints of the form:

$$h_i^s \leq a_{1i}^s + a_{2i}^s - \sum_{l \in L} \sum_{k \in K} y_{lki}^s \quad \forall s \in S,\ i \in I^s$$

Moreover, constraints (8) ensure that $h_i^s = 1$ only when all dispatched vehicules are in time. This could be replaced by the following:

$$a_{ki}^s (1 - c_{li}) h_i^s + y_{lki}^s \leq a_{ki}^s \quad \forall s \in S, l \in L, i \in I^s, k \in K.$$

These are no more quadratic. Finally, to ensure that at least one vehicule arrives in time, one can add the following:

$$h_i^s \leq 1 - (1 - c_{li}) y_{lki} \quad \forall s \in S,\ i \in I^s,\ k \in K,\ l \in K$$

The same issue happens with constraints (9).

Question 6. *How do we ensure that if no vehicule is sent, then h is 0 ?*

Question 7. *How do we ensure that no vehicule is sent if not is needed ?*

2 Reformulation

With the precedent section in mind, if all answers have been yes, the following formulation should be valid.

$$\max_{x} \mathbb{E}_{s\in S}[\mathcal{Q}^s(x)] \tag{15}$$

where

$$\mathcal{Q}^s(x) = \sum_{i\in I^s}(\alpha_1 f_i^s + \alpha_2 g_i^s + \alpha_3 h_i^s + \alpha_4 w_i^s - \phi z_i^s)$$

$$\sum_{l\in L} x_{lk} \le \eta_k \qquad k \in K \tag{16}$$

$$\sum_{i\in I^s} y_{lki}^s \le x_{lk} \qquad l \in L, k \in K, s \in S \tag{17}$$

$$(a_{1i}^s + a_{2i}^s) f_i^s \le \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s, \quad a_{2i}^s f_i^s \le \sum_{l\in L} c_{li} y_{l2i}^s \qquad i \in I^s, s \in S \tag{18}$$

$$(a_{1i}^s + a_{2i}^s) g_i^s \le \sum_{l\in L}\sum_{k\in K} y_{lki}^s, \quad a_{2i}^s g_i^s \le \sum_{l\in L} y_{l2i}^s \qquad i \in I^s, s \in S \tag{19}$$

$$h_i^s \le a_{1i}^s + a_{2i}^s - \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s \qquad i \in I^s,\ s \in S \tag{20}$$

$$a_{ki}^s (1 - c_{li}) h_i^s + y_{lki}^s \le a_{ki}^s \qquad \forall i \in I^s, l \in L, s \in S, k \in K \tag{21}$$

$$h_i^s \le \sum_{l\in L}\sum_{k\in K} c_{li} y_{lki}^s \tag{22}$$

$$w_i^s \le a_{1i}^s + a_{2i}^s - \sum_{l\in L}\sum_{k\in K} y_{lki}^s \qquad i \in I^s,\ s \in S \tag{23}$$

$$w_i^s \le \sum_{l\in L}\sum_{k\in K} y_{lki} \qquad i \in I^s,\ s \in S \tag{24}$$

$$f_i^s + g_i^s + h_i^s + w_i^s + z_i^s = 1 \qquad i \in I^s, s \in S \tag{25}$$

$$x_{lk} \in \mathbb{Z}^+, y_{lki}^s \in \{0, a_{ki}^s\} \qquad l \in L, k \in K, i \in I^s, s \in S \tag{26}$$

$$f_i^s, g_i^s, h_i^s, w_i^s, z_i^s \in \{0,1\} \qquad i \in I^s, s \in S. \tag{27}$$

3 Surrogate model

$$\mathcal{G}^s(x) = \left[\sum_{l\in L}\sum_{k\in K}\sum_{i\in I^s}(\beta_1 u_{lki}^s + \beta_2 v_{lki}^s) - \sum_{k\in K}\sum_{i\in I^s}\phi\zeta_{ki}^s\right]$$

subject to

$$\sum_{l \in L} x_{lk} \leq \eta_k \qquad k \in K \tag{28}$$

$$\sum_{i \in I^s} (u^s_{lki} + v^s_{lki}) \leq x_{lk} \qquad l \in L, k \in K, s \in S \tag{29}$$

$$u^s_{lki} \leq c_{li} \qquad l \in L, i \in I^s, k \in K, s \in S \tag{30}$$

$$u^s_{lki} + v^s_{lki} \leq 1 \qquad l \in L, i \in I^s, k \in K, s \in S \tag{31}$$

$$a^s_{1i} = \sum_{l \in L} \sum_{k \in K} (u^s_{lki} + v^s_{lki} + \zeta^s_{ki}) \qquad i \in I^s, s \in S \tag{32}$$

$$a^s_{2i} = \sum_{l \in L} (u^s_{l2i} + v^s_{l2i} + \zeta^s_{2i}) \qquad i \in I^s, s \in S \tag{33}$$

$$\sum_{k \in K} u^s_{lki} \leq a^s_{i1}, \quad u^s_{l2i} \leq a^s_{i2} \qquad i \in I, l \in L, s \in S \tag{34}$$

$$\sum_{k \in K} v^s_{lki} \leq a^s_{i1}, \quad v^s_{l2i} \leq a^s_{i2} \qquad i \in I, l \in L, s \in S \tag{35}$$

$$x_{lk}, \zeta^s_{ki} \in \mathbb{Z}^+, u^s_{lki}, v^s_{lki} \in \{0, 1\} \qquad l \in L, k \in K, i \in I^s, s \in S$$

Question 8. *Constraints 31 imply that at most one ambulance can be sent to some location, for this reason, if a demand point needs more ambulances than there is locations it can not be covered. Shouldn't they be removed or the right hand side replaced by:*

- $u^s_{l1i} + v^s_{l1i} + u^s_{l2i} + v^s_{l2i} \leq a^s_{i1} + a^s_{i2}$
- $u^s_{l1i} + v^s_{l1i} \leq a^s_{i1}$

Question 9. *Constraints 32 should be replaced by* $a^s_{1i} + a^s_{2i} = \sum_{l \in L} \sum_{k \in K} (u^s_{lki} + v^s_{lki} + \zeta^s_{ki})$

Question 10. *Constraints 33 imply that ALS cannot replace BLS.*

Question 11. *In Constraints 34 35, the right hand side of the first ineq should be replaced by* $a^s_{i1} + a^s_{i2}$

Question 12. *Constraints 34 35 should be merged as the number of vehicule sent must be lower whatever they are late or not.*

4 Modification of the surrogate

$$\sum_{l\in L} x_{lk} \leq \eta_k \qquad k \in K \tag{36}$$

$$\sum_{i\in I^s} (u^s_{lki} + v^s_{lki}) \leq x_{lk} \qquad l \in L, k \in K, s \in S \tag{37}$$

$$u^s_{lki} \leq c_{li}(a^s_{1i} + a^s_{2i}) \qquad l \in L, i \in I^s, k \in K, s \in S \tag{38}$$

$$u^s_{l1i} + v^s_{l1i} + u^s_{l2i} + v^s_{l2i} \leq a^s_{1i} + a^s_{2i} \qquad l \in L, i \in I^s, k \in K, s \in S \tag{39}$$

$$u^s_{l1i} + v^s_{l1i} \leq a^s_{1i} \qquad l \in L, i \in I^s, k \in K, s \in S \tag{40}$$

$$a^s_{1i} + a^s_{2i} \leq \sum_{l\in L} \sum_{k\in K} (u^s_{lki} + v^s_{lki} + \zeta^s_{ki}) \qquad i \in I^s, s \in S \tag{41}$$

$$a^s_{2i} \leq \sum_{l\in L} (u^s_{l2i} + v^s_{l2i} + \zeta^s_{2i}) \qquad i \in I^s, s \in S \tag{42}$$

$$x_{lk}, \zeta^s_{ki} \in \mathbb{Z}^+, u^s_{lki}, v^s_{lki} \in \{0, 1\} \qquad l \in L, k \in K, i \in I^s, s \in S$$

Constraints 38 ensure the number of sent vehicule cannot exceed the number of needed vehicules (Whatever type they are).

Constraints 39 ensure that no more than needed BLS can be sent.

Constraints 40 ensures that if not sufficiently enought vehicule are sent, they should be compensated by a penality in the objective.

Constraints 41 ensures that the number of missing ALS is compensated by the penality in the objective.