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Heuristic Approaches for a Scheduling Problem in the Plastic Molding Department of an Audio Company

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Abstract

A production scheduling problem for making plastic molds of hi-fi models is considered. The objective is to minimize the total machine makespan in the presence of due dates, variable lot size, multiple machine types, sequence dependent, machine dependent setup times, and inventory limits. Goal programming and load balancing are applied to select the set of machine types and assign mold types to machines, resulting in a set of single-machine scheduling problems. A mixed-integer program (MIP) is formulated for the general problem but could solve only small instances. A single-machine scheduling heuristic is designed to adopt a production sequence from a travelling salesman solution. The start time of every cycle is determined by a simplified MIP. Production cycles are defined to equalize the stockout times of mold types. A post-processing step reduces the number of setups in the last cycle. Results using real-life data are promising. Characteristics giving rise to high machine utilization are discussed.

Key Words: makespan, lot sizing, machine dependent and sequence dependent setup times, due dates, inventory limits

Introduction

An audio company with one of its factories based in mainland China manufactures various audio products for its overseas market. One operation of a hi-fi model involves making its plastic components. The largest two plastic components on a hi-fi are the bottom frame and the front cover. Other components include the speaker and the small functional buttons. To make a plastic component, a specific mold must be produced on the molding machine. A typical hi-fi model may require between 30 to 40 different types of plastic molds. Since there is a one-to-one relationship between plastic components and mold types, these two terms are used synonymously in the paper.

This paper considers the plastic mold production of two major hi-fi models, denoted by models A and B, involving a total of 77 types of plastic molds. Each plastic mold requires a specific material, color and a single operation on a molding machine. To make a particular type of plastic mold, three setup procedures are involved: metal mold setup, color cleansing and material melting. The molding machine needs to be setup with the required metal mold

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and if the previous mold on the machine is of a different color, cleansing needs to be done. At the same time, the plastic material is to be melted before injecting into the metal mold. Different machine types could make the same mold but would incur different setup times (machine dependent setup time). In practice, at most two machine types are assigned to make one type of plastic mold.

The objective is to assign components (molds) to machines, determine the best sequence and lot sizes in order to minimize the machine makespan on each machine. (Costs are not considered in our case as management finds it difficult to quantify the unit cost of setup or holding inventory.) The (machine dependent) setup time of the metal mold ranges between 40 minutes to 3 hours. Longer times are associated with machines of larger tonnage. The color cleansing process (if necessary) follows after setting up of the metal mold. This sequence dependent cleansing time ranges from 10 minutes (for light colors) to 12 hours (if the previous color is darker than the current one). Similarly, if the plastic material used is different from the preceding one, the melting time of the new material may take up to 3 hours, depending on the material type. Hence, the overall setup time of a plastic mold is expressed as follows:

$$\begin{aligned}\text{setup time} = \max\{&\text{setup time for metal mold}\\ &+ \text{color cleansing time, material melting time}\}\end{aligned} \tag{1}$$

The demand for hi-fi models during the year is derived from the master production schedule (MPS). As the plastic molding department in the factory is a supporting department to the assembly line, it has to supply a given number of plastic molds to the assembly line on each working day of the week. Hence, the number of due dates in a week may vary between 4 to 6. Inventory limits for finished molds are specified to be within the demand on the following 3 (minimum) to 10 (maximum) working days. Hence from the MPS and the bill of materials, the daily demand for each component and its inventory limits can be derived. The current scheduling practice in the department is based on experience where machines are either dedicated to make molds of the same color or the same material. (In this way, at least one component of the sequence dependent setup time could be eliminated.)

Previous literature on scheduling molding processes include Van Wassenhove and De Bodt (1983) which considered a medium-term production smoothing problem in the injection molding department of a company producing visual communication aids and stationery products. The objective is to minimize the total costs of setup and inventory holding. Approximations were taken to reduce the complex problem to a set of one machine capacitated dynamic lot sizing problems to be solved by procedures of Lambrecht and Vanderveken (1979), and Dixon and Silver (1981). Veleris and Park (1998) examined a productivity improvement problem in the urethane parts molding department in a manufacturer of solids handling systems and accessories. In using a combined approach of line balancing, scheduling and mold mix optimization (by a linear programming model), higher throughput rate, better resource utilization and reduced parts inventory were achieved. Nagarur, Vrat, and Duongsuwan (1997) considered an injection molding problem of PVC pipe fittings involving 200 products, 140 moulds and 14 machines. The problem was divided into subproblems based on mold-machine compatibility and load balancing. Each subproblem was solved as a single-machine, multi-product, lot-sizing problem by a goal programming formulation to minimize total costs of inventory, shortages and machine utilization. Job sequencing was based on a heuristic incorporating a job flexibility index into the LPT rule and load balancing. Kwak, Freeman, and Schniederjans (1989) examined a blow molding operation producing plastic bottles. The policy of longer production run was tested against the short-run EOQ system, the advantages of the long-run policy in decreasing the setup costs were however outweighed by the higher storage costs. A decision support system developed could somehow reduce the hourly cost of assigning jobs to machines.

The scheduling problem addressed here (following after the component-machine assignment) is a capacitated lot sizing problem with sequence dependent setup times, due dates and inventory limits. When inventory limits are not considered and the unit processing time is identical for all jobs, the time horizon can be divided into small time periods and the problem becomes a discrete lot sizing and scheduling problem (DLSP). Salomon et al. (1997) transformed the DLSP into a travelling salesman problem with time windows, which was solved by a dynamic programming approach. Jordan and Drexl (1998) analysed the equivalence relationship between DLSP and the batch sequencing problem and presented a branch-and-bound algorithm to solve the latter problem. Laguna (1999) considered a similar problem as in this paper but the setup time was carried out in the overtime period and the objective is to minimize the total costs associated with overtime and inventory holding. The problem was decomposed into two subproblems: a production and inventory problem solved by a linear program; and a travelling salesman problem handled the sequencing decisions. The search for optimal schedules was controlled by a short-term memory tabu search with an embedded linear program used to evaluate each move. Miller et al. (1999) proposed a hybrid genetic search algorithm for minimizing the total costs on a single-machine scheduling problem with due dates, lot-sizing and sequence dependent setup times. (Our problem has further complexities due to inventory limits and allowing for multiple setups for a mold type during the planning horizon.) Hasse (1994) reviewed a variety of single-level lot sizing, scheduling problems and their solution techniques. For the multi-level version where the product structure is composed of multi-levels of components, Kimms (1997) presented an introduction and review.

This study involves scheduling 77 mold types of two hi-fi models on 14 compatible machine types, with 13 and 9 associated colors and plastic materials, respectively. The objective is to find a production plan that meets due date demand, inventory requirements and minimizes the makespan on all machines. The available machine capacity is assumed to be 23.5 hours on Monday to Saturday and half a day (11.75 hours) on Sunday, after accounting for the planned maintenance and average downtime due to breakdowns. The inventory level of each mold type at the end of a day must fall within specified ranges given by the demand on the following 3 to 10 working days.

In Section 1, we present a mixed integer programming formulation of the general production scheduling and lot sizing problem. This NP-hard problem can be tackled by a hierarchical approach of first selecting the machine types (Section 2), then assigning the mold types to machines (Section 3), followed by scheduling on single machines (Section 4). The methodology is illustrated in an example using real-life data (Section 5). A summary and comments on infeasibilities and characteristics leading to high machine utilization are given at the end (Section 6).

1. Mixed integer programming formulation

The objective of the problem is to minimize the makespan on all selected machines. Since a machine could be idle due to changeover or forced to be idle when the inventory upper limits of molds assigned are reached, the sum of makespan reflects the amount of resources being dedicated towards producing the two hi-fi models. The following formulation incorporates a mold-machine assignment problem, sequencing and lot sizing problem. Each mold type is associated with a specific plastic component.

Decision variables

P_{kjt}	Production quantity of mold type k on machine j on day t
I_{kt}	Inventory level for mold type k at the end of day t
x_{lkjt}	This variable equals 1 if mold type k is produced immediately after mold type l ($\neq k$) on machine j on day t, and zero otherwise
w_{lkjt}	This variable equals 1 if mold type l is the last one to be produced on machine j on or before day $t-1$, followed by mold type k ($\neq l$) on day t, and zero otherwise
y_{ljt}	This variable equals 1 if mold type l is the last one to be produced on or before day t, and zero otherwise
z_{kjt}	This variable equals 1 if production of mold type k is linked on machine j on day $t-1$ (or before if there is forced idleness) and day t, and zero otherwise
C_{kjt}	Time index indicating the production of mold type k on machine j on day t

Parameters

K	Total number of mold types
J	Total number of machines
T	Duration of planning period (days)
k	Mold type index ($k = 0$ and $k = K + 1$ are dummy indices indicating the start and end of a day's production respectively)
j	Machine index
t	Day index
s_{lkj}	(Sequence and machine dependent) setup time of mold type k on machine j when its immediate predecessor is mold type l
A_t	Available machine capacity (hours) on day t
γ_k	Processing time of a mold of type k
B_{kt}	Upper bound on the number of molds of type k produced on day $t = A_t/\gamma_k$
d_{kt}	Demand for mold type k on day t
L_{kt}	Lower inventory limit of mold type k on day t
U_{kt}	Upper inventory limit of mold type k on day t
$M(k)$	Set of compatible machines for mold type k

Mixed integer program

$$\text{Minimize} \quad Z = \sum_j (C_{K+1,j,T} - C_{K+1,j,0})$$

subject to:

Production time capacity constraints

$$\sum_{\{k \neq 0, K+1 \mid j \in M(k)\}} \left[\left(\sum_{\{l \neq k, K+1 \mid j \in M(l)\}} s_{lkj} \cdot w_{lkjt} + \sum_{\{l \neq 0, k, K+1 \mid j \in M(l)\}} s_{lkj} \cdot x_{lkjt} \right) + \gamma_k \cdot p_{kjt} \right] \leq \mathrm{A}_t, \quad \forall j, t \tag{2}$$

Inventory balance constraints

$$I_{k,t-1} + \sum_{j \in M(k)} p_{kjt} - I_{kt} = d_{kt}, \quad \forall k (\neq 0, K+1), t \tag{3}$$

Inventory limit constraints

$$L_{kt} \leq I_{kt} \leq U_{kt}, \quad \forall k (\neq 0, K+1), t \tag{4}$$

Setup constraints

$$\sum_{l \neq k, K+1} B_{kt} \cdot x_{lkjt} - p_{kjt} \geq 0, \quad \forall k (\neq 0, K+1), j \in M(k), t \tag{5}$$

$$p_{kjt} - \sum_{l \neq k, K+1} x_{lkjt} \geq 0, \quad \forall k (\neq 0, K+1), j \in M(k), t \tag{6}$$

Linked production constraints

$$\sum_{\{k \mid j \in M(k)\}} z_{kjt} \leq 1, \quad \forall j, t \tag{7}$$

Predecessor assignment constraints

$$\sum_{\{l \neq k, K+1 \mid j \in M(l)\}} x_{lkjt} \leq 1, \quad \forall k (\neq 0, K+1), j \in M(k), t \tag{8}$$

TSP constraints

$$\sum_{\{l \neq 0 \mid j \in M(l)\}} x_{0ljt} = 1, \quad \forall j, t \tag{9}$$

$$\sum_{\{l \neq K+1 \mid j \in M(l)\}} x_{l,K+1,j,t} = 1, \quad \forall j, t \tag{10}$$

$$\sum_{\{l \neq k, K+1 \mid j \in M(l)\}} x_{lkjt} - \sum_{\{l \neq 0, k \mid j \in M(l)\}} x_{kljt} = 0, \quad \forall k (\neq 0, K+1), j \in M(k), t \tag{11}$$

Identification constraints of last mold

$$y_{ljt} - x_{l,K+1,j,t} \geq 0, \quad \forall l (\neq 0, K+1), j \in M(l), t \tag{12}$$

$$\sum_{\{l \neq K+1 \mid j \in M(l)\}} y_{ljt} = 1, \quad \forall j, t \tag{13}$$

$$y_{0j0} = 1; \quad y_{lj0} = 0, \quad \forall l (\neq 0, K+1), j \in M(l), t \tag{14}$$

$$y_{l,j,t-1} - y_{ljt} + x_{0,K+1,j,t} \leq 1, \quad \forall l (\neq K+1), j \in M(l), t \tag{15}$$

$$-y_{l,j,t-1} + y_{ljt} + x_{0,K+1,j,t} \leq 1, \quad \forall l (\neq K+1), j \in M(l), t \tag{16}$$

Day-to-day linkage constraints

$$y_{l,j,t-1} + x_{0ljt} - z_{ljt} \leq 1, \quad \forall l (\neq 0, K+1), j \in M(l), t \tag{17}$$

$$y_{l,j,t-1} + x_{0ljt} - 2z_{ljt} \geq 0, \quad \forall l (\neq 0, K+1), j \in M(l), t \tag{18}$$

$$y_{l,j,t-1} + x_{0kjt} - z_{kjt} - w_{lkjt} \leq 1, \quad \forall l (\neq K+1), k (\neq 0, K+1), j \in M(l) \cap M(k), t \tag{19}$$

$$y_{l,j,t-1} + x_{0kjt} - 2w_{lkjt} \geq 0, \quad \forall l (\neq k, K+1), k (\neq 0, K+1), j \in M(l) \cap M(k), t \tag{20}$$

Subtour breaking constraints

$$C_{K+1,j,t} - C_{0jt} \geq 0, \quad \forall j, t; \quad C_{K+1,j,T} \leq T \tag{21}$$

$$C_{0jt} - C_{K+1,j,t-1} - \sum_k \frac{\gamma_k}{A_t} \cdot p_{kjt} \geq 0, \quad \forall j, t \tag{22}$$

$$C_{0jt} + (t-1) x_{0,K+1,j,t} - \sum_k \frac{\gamma_k}{A_t} \cdot p_{kjt} \geq t - 1, \quad \forall j, t \tag{23}$$

$$C_{0jt} - (T - t + 1) \cdot x_{0,K+1,j,t} - \sum_k \frac{\gamma_k}{A_t} \cdot p_{kjt} \leq t - 1, \quad \forall j, t \tag{24}$$

$$C_{0jt} - C_{kjt} + z_{kjt} + \left(1 + \frac{s_{lkj}}{A_t}\right) \cdot w_{lkjt} \leq 1, \quad \forall l (\neq k, K+1), k (\neq 0, K+1), j, t > 1 \tag{25}$$

$$-C_{kjt} + C_{ljt} + \left(1 + \frac{s_{lkj}}{A_t}\right) \cdot x_{lkjt} \leq 1, \quad \forall l (\neq 0, k, K+1), k (\neq 0), j, t \tag{26}$$

$$p_{kjt}, I_{kt}, C_{kjt} \geq 0, \quad \forall k, j, t$$

$$x_{lkjt}, w_{lkjt}, y_{ljt}, z_{kjt} = 0, 1$$

The objective function consists of the sum of makespan on all machines where each is expressed in terms of the difference between the start time index and the end time index. (Further explanations on the construction of the time indices will be given in the following.) Constraint set (2) describes the daily machine utilization, which is made up of the setup times (of the first and subsequent molds) and the processing times. Constraint set (3) models the inventory balance. Bounds on inventory levels are enforced by constraint (4). Constraint (5) provides the upper bound for the daily production quantity and constraint (6) relates the production with the setup variable, accounting for all possible predecessor mold types. Constraint sets (7) and (8) respectively allow at most one mold to be linked at the start of a day and at most one predecessor of a mold. Constraint sets (9) and (10) indicate exactly one starting mold and one ending mold (including dummy indices) respectively in a day's production. The number of predecessor, if any, for a mold type must equal the number of successor as described by constraint set (11). Constraint set (12) identifies the last mold type (including dummy index) to be produced in a day. Constraint set (13) restricts exactly one mold type to be the last one in a day and constraint set (14) gives the initial conditions on day 0. Constraint sets (15) and (16) are formulated to retain the identity of the most recent mold type produced on day $t-1$, if there is no production on day t (i.e., $x_{0,K+1,j,t}=1$). Constraint set (17) forces the linking variable z_{ljt} to be one if the most recent mold type produced coincides with the first one produced on the current day. Otherwise, this variable is set to zero by constraint set (18). Similarly, constraint sets (19) and (20) impose the same set of conditions on the variable w_{lkjt} which retains the identity of the two different mold types produced at the start of day t and its immediate predecessor (completed on or before day $t-1$).

Constraint set (21) describes the relationship between the time indices indicating the start and end production in a day, and restricts the last one to fall within the planning horizon. The remaining equations handle the construction of the time indices, which have units expressed in days. The time indices are introduced to eliminate subtour formation in the sequencing decisions. They are constructed such that the sequence of mold types produced has increasing time indices, where each consecutive pair differs by the sequence dependent setup time. Constraint set (22) moves the processing time variables in a day before the start time index (C_{0jt}) such that the subsequent time indices on that day can be formulated as linear constraints in terms of the sequencing variables $\{w_{lkjt}, x_{lkjt}\}$. Without loss of generality, all time indices take on values within the interval $[0, T]$. If production occurs on day t (i.e., $x_{0,K+1,k,j}=0$), constraint sets (23) and (24) force the start time index to take on the value $t-1$ plus the processing time on day t. (Sequence dependent setup times will be added in subsequently.) Constraints (25) and (26) set values for the time indices of the first and subsequent mold types produced respectively. If mold type l immediately precedes mold type $k(w_{lkjt}=1$ or $x_{lkjt}=1)$, the corresponding time indices in constraints (25) and (26), respectively, would differ by the setup time s_{lkjt}. Otherwise, these constraints become redundant. In this way, the sequencing decisions of the molds with consideration of the production quantity and the daily machine capacity are formulated by constraint sets (21)–(26). Note that by constraint sets (22)–(24), the time index $C_{K+1,j,0}$ in the objective function is pushed to the actual start time production (on some day t) by the minimization criterion. Similarly by constraint sets (21), (22) and (26), the last time index $C_{K+1,j,T}$ is set to the production end time. Hence, the makespan of each machine is contained in the objective function.

Monma and Potts (1989) analysed the complexity of scheduling problems with batch setup times. In the single-machine maximum completion time problem with sequence dependent setup times, an optimal ordering of the batches is obtained from the solution of the travelling salesman problem (TSP). So the problem is NP-hard when in addition, the number of batches becomes variable. (In our case, a batch corresponds to a lot of a mold type of variable size.) They have also shown that on two identical parallel machines with variable number of batches, the maximum completion time problem remains NP-hard with or without job preemption. The problem addressed in this paper has further considerations of due dates and inventory limits. If it is solvable by a polynomial algorithm, the special case of relaxing the due dates and inventory limits, which is the problem in Monma and Potts (1989), would also be solvable by a polynomial algorithm. As the special case is proved to be NP-hard, so is our problem. This necessitated the need for heuristics to be developed.

To obtain a feasible solution to the above formulation, a certain level of initial inventory is expected. This could be achieved by incorporating a secondary goal of minimizing the initial inventory $\{I_{k0}\}$ into the objective function. The model was first tested on single-machine problems involving up to 3 mold types and a 9-day planning horizon using Extended Lindo 5.3. Optimality was reached after hours of computational time and this would become increasingly unacceptable for larger problems. For the current problem of 77 mold types, each with 2 compatible machine types and a 10-day planning period, the number of integer variables could involve up to 247, 470 in the MIP formulation. By ignoring the sequencing decisions, the problem can be simplified to a production and inventory problem that provides a lower bound for the optimal solution. However, the deviation from optimality increases with the increase in number of mold types of different colors/material, or the duration of the planning horizon.

Problem decomposition becomes essential to solve such a complex problem as was adopted in Laguna (1999) and Nagarur, Vrat, and Duongsuwan (1997). Similarly, a hierarchical approach has often been applied in production planning for jobs classified as product families as reviewed in Bitran and Tirupati (1993). In the following sections, we adopt such an approach to decompose the problem into a series of subproblems. The subproblems aim at reducing components of setup time, balancing the load on machines, and determining the production sequence and (variable) lot sizes.

2. Selection of machine types

A set of machine types is to be selected to cover all the required mold types. The determination of the smallest such set is to be solved as a *set covering problem*. (Naturally it seems that the more machine types available, the less time would be wasted in changeover to different molds. However as there exists upper inventory limit for each mold type, idle time would be enforced once a mold type reaches the upper limit and if no other mold type is assigned on the same machine. Hence, using the smallest possible number of machine types increases machine utilization.) As the objective of the problem is to minimize the sum of makespan on all machines, the current stage aims at reducing the machine dependent setup time component—the metal mold changing time, which is unavoidable irrespective of the sequencing decisions. A machine of larger tonnage takes longer in changing the metal mold. The goal program in the following would consider the two goals of (i) selecting the smallest set of machine types, and (ii) minimizing the setup time incurred in changing the metal mold. Constraints include at least one compatible machine type for producing each mold type (constraint (27)), and selection of a machine type if it is used in production (constraint (28)).

Decision variables

X_{ik} This variable equals 1 if machine type i is used in producing mold type k, and zero otherwise

α_i This variable equals 1 if machine type i is selected, and zero otherwise

Parameters

$N(k)$ Set of compatible machine types for mold type k

h_i Metal mold changing time on machine type i

Goal program

$$\text{Minimize} \quad Z = P_1 \sum_i \alpha_i + P_2 \sum_k \sum_{i \in N(k)} h_i \cdot X_{ik}$$

$$\text{subject to:} \quad \sum_{i \in N(k)} X_{ik} \geq 1, \quad \forall k \tag{27}$$

$$\sum_{\{k \mid i \in N(k)\}} X_{ik} \leq K \cdot \alpha_i, \quad \forall i \tag{28}$$

$$X_{ik}, \alpha_i = 0, 1$$

where P_1 and P_2 are weights associated with the priority goals. Any ambiguous assignment (when more than one X_{ik} in constraint (27) equal one) from the goal program is resolved in the following by assigning the mold type to the machine type with molds of similar color or material, whichever incurs the smaller changeover time. From the results of the goal program, let $N_s(k)$ denote the set of selected machine type for mold type k and $\kappa(i)$, the set of assigned molds on machine type i. Further, let c_{lk} and m_{lk} represent the color cleansing time and material melting time respectively, when the production changes over from mold type l to k. Mold type k is assigned to the machine type $i^*(\in N_s(k))$ in which it incurs the smallest changeover time given by the following:

$$\text{smallest changeover time incurred by mold type } k$$
$$= \min_{i \in N_s(k)} \left\{ \min_{l \in \kappa(i)} \{\max\{h_i + c_{lk}, m_{lk}\}\} \right\} \tag{29}$$

Note that h_i has the same value for all machine type $i \in N_s(k)$ due to the ambiguous assignment of mold type k. An example is shown in figure 1 for mold type A12 which could be processed on either machine type 80 or 140 (tonnage indicator).

Mold type A12 would be assigned to machine type 80 that consists of molds of the same color and same material. This will eliminate the sequence dependent setup time (on color and material).

3. Mold-machine assignment

The subsequent stages involve calculating the number of machines by type (selected in Section 2) and assigning mold types to specific machines before scheduling on the single

	Qualified machine type					
	80			140		
Unassigned mold type	Molds assigned	Material code	Color code	Molds assigned	Material code	Color code
A12	B7	T450(A)	BLK	B26	DEL	25477
(material code: PMMA;	B8	T450(A)	BLK	A32	A	17939
color code: CLEAR)	⋮	⋮	⋮	A33	A	17939
	A11	PMMA	CLEAR	B36	A	17939
	A13	PMMA	CLEAR	B37	A	17939
	A14	PMMA	CLEAR			
	A31	PMMA	CLEAR			

Figure 1. Assignment of mold type A12 in ambiguous situation.

machines. The problem formulated in this section is solved for each selected machine type.

The minimum number of machines required by type is simply estimated by dividing the sum of processing times of the assigned molds by the machine capacity in the planning horizon, and rounding it up to the nearest integer.

The assignment of mold types to specific machines is based on load balancing. Consider first the machine types with molds of homogeneous color and material. We assume no splitting of a mold type on different machines, unless its total processing time exceeds the machine capacity. Suppose machine type i requires I machines to process all the assigned mold types in set $\kappa(i)$ (Section 2), the following binary integer program determines the mold-machine assignment for machine type i.

Decision variables

Y_{kj} This variable equals 1 if mold type $k(\in \kappa(i))$ is assigned to the jth machine of type i, and zero otherwise

Z Maximum makespan among all I machines of type i

Parameters

Γ_k Total processing time of mold type $k = \gamma_k \sum_t d_{kt}$

Binary integer program

$$
\begin{aligned}
\text{Minimize} \quad & Z \\
\text{subject to:} \quad & \sum_k (\Gamma_k + h_i) Y_{kj} \le Z, \quad \forall j = 1, \ldots, I \\
& \sum_j Y_{kj} = 1, \quad \forall k \in \kappa(i) \\
& Y_{kj} = 0, 1
\end{aligned}
$$

Note that the formulation needs only consider the machine dependent setup time component h_i as the material and color for mold types in this group of machines are the same.

For machine types with heterogeneous mold material or color, some preprocessing is carried out to group mold types of the same material or color (whichever incurs a smaller changeover time as determined by constraint (29)) into one block. Mold type, which does not result in time savings in grouping, or has no common material or color with others, would stand alone as one block. The load balancing formulation is then applied on these blocks.

4. Single-machine scheduling

On obtaining the specific mold-machine assignment, the problem is reduced to a set of single-machine, multi-product lot-sizing problem. A general description of heuristics on scheduling single or parallel machines could be found in Morton and Pentico (1993).

In the following subsections, we propose heuristics to solve the scheduling and lot-sizing problem with inventory limits and due dates. First, the starting time of production is determined (Subsection 4.1). In order to find a production sequence (Subsection 4.2) for a machine of type i with n molds assigned, we solve a TSP with a distance matrix $\Delta = \{\Delta_{lk} \mid 1 \leq l, k \leq n\}$ where Δ_{lk} is the changeover time from mold type l to $k(\neq l)$, given by $\Delta_{lk} = \max\{h_i + c_{lk}, m_{lk}\}$. We then re-index the mold types from 1 to n according to the TSP optimal sequence. In every production cycle, all the n mold types in the TSP optimal sequence will be produced. The (variable) cycle length and the lot size (by mold type) in each cycle are determined subsequently (in Subsections 4.3 and 4.4 respectively). Finally, a post-processing procedure (Subsection 4.5) is applied to transfer quantities and possibly eliminate lots in the last cycle.

4.1. Starting time

If there were no upper inventory limit, then the objective of minimizing machine makespan would encourage the machine to start early and process continuously in order to accumulate sufficient molds to meet their due dates. The choice of the starting time will affect the number of changeovers, the amount of forced idle time on the machine and the feasibility of the schedule. When the initial inventory is high, starting early would lead to the upper limit being reached quickly. The machine will changeover to another mold type frequently as soon as one meets its upper inventory limit. On the other hand, starting late when the initial inventory is low may lead to violation of due date constraints or lower inventory limits. The starting time in every cycle is to be determined by a simplified version of the MIP (Section 1) that ignores the sequencing decision variables and setup times.

Decision variables

- p'_{kt} Production quantity of mold type k on day t
- I_{kt} Inventory level for mold type k at the end of day t
- δ_t This variable equals 1 if the machine is in production on day t, and zero otherwise
- C'_t Time index indicating the production end time on day t (C'_0 is a dummy index indicating the production start time)

Parameters

T, A_t, γ_k, L_{kt} and U_{kt} are notations adopted from the MIP (Section 1)
B'_t Upper bound on the number of molds produced on day $t = \max_k\{A_t/\gamma_k\}$

Simplified MIP

$$
\begin{aligned}
\text{Minimize}\quad & Z = C'_T - C'_0 \\
\text{subject to:}\quad & \sum_k \gamma_k \cdot p'_{kt} \le A_t, \quad \forall t \\
& I_{k,t-1} + p'_{kt} - I_{kt} = d_{kt}, \quad \forall k, t(\neq 0) \\
& L_{kt} \le I_{kt} \le U_{kt}, \quad \forall k, t \\
& B'_t \cdot \delta_t - \sum_k p'_{kt} \ge 0, \quad \forall t(\neq 0) \\
& C'_t - C'_{t-1} - \sum_k \frac{\gamma_k}{A_t} \cdot p'_{kt} \ge 0, \quad \forall t(\neq 0) \\
& C'_t - (t-1) \cdot \delta_t - \sum_k \frac{\gamma_k}{A_t} \cdot p'_{kt} \ge 0, \quad \forall t(\neq 0) \\
& C'_t + (T - t + 1) \cdot \delta_t - \sum_k \frac{\gamma_k}{A_t} \cdot p'_{kt} \le T, \quad \forall t(\neq 0) \\
& p'_{kt}, I_{kt}, C'_t \ge 0, \quad \forall k, t \\
& \delta_t = 0, 1
\end{aligned}
$$

This simplified formulation considers daily machine capacity, inventory balance, inventory limits, and machine in production or idle state. The production end time in a day is formulated by the time indices. The index C'_0 will give the earliest starting time that achieves the minimum makespan ($C'_T - C'_0$) in the simplified problem. The simplified MIP will be rerun in every cycle with updated parameters to generate the starting time for production.

4.2. *Production sequence*

On a machine with n mold types, the production sequence is based on the cyclic TSP sequence $(1, 2, \ldots, n)$ and the stockout time, defined as the day in which the current inventory of a mold type first falls below its lower inventory limit. In the first production cycle, we identify the mold type, say k, which has the earliest stockout time. We delay its production till the latest possible time while producing the other $n - 1$ mold types in the sequence of $(k+1, k+2, \ldots, n, 1, \ldots, k-1)$. (In case of ties, choose k such that the setup time $\Delta_{k,k+1}$ is the largest among the arcs of the TSP sequence. This amount of time will be saved in the first cycle.) The objective is to maximize production while equalizing the stockout time of the first $n-1$ mold types at the end of the cycle. Mold type k will be produced enough to fulfil its lower inventory limit. In the next cycle, mold type k will be produced first, followed by mold type $k+1, \ldots, n, 1, 2, \ldots, k-1$ in sequence, again leaving the last mold type $(k-1)$ to start at the latest possible time. This procedure repeats until the inventory level for each mold type satisfies the demand in the remaining planning horizon. A post-processing procedure (Subsection 4.5) will be performed to eliminate unnecessary setups in the last cycle.

4.3. *Cycle length*

Once the starting time of the cycle, denoted by t_s, is determined by the simplified MIP (Subsection 4.1), the cycle length is defined as the time elapsed till the earliest stockout time among the n mold types. Let k denote the mold type with the earliest stockout time on day t_r. Then the cycle length is given by $t_r + 1 - t_s$. If there is no production of mold type k during the cycle, its inventory will fall below the lower inventory limit (I_{kt_r}) by some quantity Q. Within this cycle, the $n - 1$ mold types ($k + 1, k + 2, \ldots, n, 1, \ldots, k - 1$) will be produced in quantities equalizing their stockout times, while Q units of mold type k will be produced on day t_r to fulfil its lower inventory limit. The next cycle will naturally start with mold type k and ends with mold type $k - 1$. The procedure starting from Subsection 4.1 repeats. (The TSP sequence in Subsection 4.2 remains the same as before.) In the last cycle, the cycle length will be shorter than expected when the inventory of each mold type is built up to a level equal to the remaining demand.

4.4. *Lot size determination*

As the cycle length varies in each cycle, the production quantity of each mold type, i.e. *lot size*, is determined based on the criteria of equalizing stockout times. (This concept also appeared in Bitran and Tirupati (1993, pp. 533–554) when determining the family run quantities among a family of items.) The choice of the starting time (determined in Subsection 4.1) avoids forced idle time during the cycle, and enables the cycle length to be maximized to produce the $n - 1$ mold types in lot sizes resulting in the same stockout time.

From Subsection 4.2, the sequence of mold types is identified for production in a cycle on the current machine (say of type j). Assume this sequence is $(1, \ldots, n)$ and its starting day is t_s (Subsection 4.1). From the previous subsection, let Q denote the quantity that the last mold type will fall short of its lower inventory limit on its stockout day t_r, if there were no production in this cycle. The lot size $\{q_l^*\}$ for mold types ($l = 1, \ldots, n$) in the current cycle is determined by the following procedure.

Algorithm for determining the lot size in a cycle with sequence $(1, \ldots, n)$:

Step 1. Solve for $q_1, q_2, \ldots, q_{n-1}$ in the simultaneous equations:

$$\begin{cases} \dfrac{q_l + I_{l,t_s-1} - \sum_{\tau=t_s}^{t_r} d_{l\tau}}{\bar{d}_l} = \dfrac{q_{l+1} + I_{l+1,t_s-1} - \sum_{\tau=t_s}^{t_r} d_{l+1,\tau}}{\bar{d}_{l+l}}, \quad l = 1, \ldots, n-2; \\ q_n = Q; \\ \displaystyle\sum_{l=1}^{n} (s_{l-1,l,j} + q_l \cdot \gamma_l) = t_r + 1 - t_s \end{cases} \tag{30}$$

where

$$\bar{d}_l = \text{average daily demand in the remaining period} = \sum_{\tau=t_r+1}^{T} d_{l_\tau} \bigg/ (T - t_r).$$

Step 2.

$$q_l^* = \min\left\{q_l, \sum_{\tau=t_s}^{T} d_{l\tau} - I_{l,t_s-1}\right\} \quad \forall l = l, \ldots, n$$

Step 1 evaluates the production quantities of the first $n-1$ mold types by equalizing their stockout times at the end of the cycle on day t_r. The last mold type is delayed till the latest time, just enough for it to produce Q units. The total setup and production time is limited to the duration of the cycle ($= t_r + 1 - t_s$). The lot size determined in Step 2 also considers the last cycle when one only needs to produce the extra amount required for the remaining period, i.e. $\sum_{\tau=t_s}^{T} d_{l\tau} - I_{l,t_s-1}, l = 1, \ldots, n$.

4.5. Post-processing for the last cycle

The lot size in the last cycle is usually small. In order to reduce the number of setups, lot sizes in the last cycle are merged with those in the previous cycle. A post-processing step is applied to eliminate the last lot of a mold type by shifting its production quantities to the previous cycle. As a tradeoff, this affects the lot size of other mold type(s) in the last two cycles.

An example with two production cycles in Table 1 is used to illustrate the concept. The TSP production sequence is M1-M2-M3-M4-M5. The first cycle starts on day 1 and ends on day 3. The last cycle starts on day 4. Mold type M5 is the last one to be processed in the first cycle and accordingly, the first one to start in the next cycle. The figures inside Table 1 represent the lot sizes. Those in the parentheses indicate the minimum lot size needed to be produced on that day to allow sufficient inventory to survive till the next cycle without violating the lower inventory limit. To eliminate setups in the last cycle, start from

Table 1. A 2-cycle production example illustrating the post-processing procedure. (TSP optimal sequence: M1-M2-M3-M4-M5.)

	Day					
Mold type	1	2	3	4	5	
M1	801 (250)			199		
M2	1166 (858)	251 (0)			341	
M3		1417 (408)			341	
M4		174	1429		397[a]	Makespan = 4.65 days
M5			500	1500		

[a]Quantities to be transferred to the previous cycle when eliminating the last lot of M4.
--- Mold types and their quantities affected during the post-processing procedure.
Figures in the parenthesis indicate the minimum lot size required to survive, if there is no production till the next cycle.

Table 2. Resulting schedule after post-processing once. (TSP optimal sequence: M1-M2-M3-M4-M5.)

	Day					
Mold type	1	2	3	4	5	
M1	801 (250)			199		
M2	1166 (858)	251 (0)			341	
M3		1020 (408)			738[a]	Makespan = 4.61 days
M4		571	1429			
M5			500	1500		

[a]Quantities to be transferred to the previous cycle when eliminating the last lot of M3.
--- Mold types and their quantities affected during the post-processing procedure.
Figures in the parenthesis indicate the minimum lot size required to survive, if there is no production till the next cycle.

the last mold type M4 and transfer the production quantities to the previous cycle. As a tradeoff, the lot size of the predecessor mold type M3 needs to be reduced in the previous cycle (on day 2) by transferring certain quantities to its last lot, ensuring that the minimum lot size for M3 in the previous cycle is not violated. The exchange rate between M4 and M3 is inversely proportional to their processing times (1 : 1 in this example). The resulting schedule is shown in Table 2, in which the last lot of M4 can be totally eliminated, thus reducing the setup time. Now the last mold type becomes M3. The same procedure is applied to merge the last lot of M3 with the lot in the previous cycle and hence affecting the lot size of its predecessor mold types. The immediate predecessor M2 is first chosen for exchange. Table 3 shows the result that M2 eliminates its production on day 2 (by transferring it to the last cycle on day 5) and reaches its minimum lot size on day 1. Hence, M1 is the next

Table 3. Resulting schedule after post-processing twice. (TSP optimal sequence: M1-M2-M3-M4-M5.)

	Day					
Mold type	1	2	3	4	5	
M1	460 (250)			491	49	
M2	858 (858)				900	
M3	401	1357				Makespan = 4.57 days
M4		571	1429			
M5			500	1500		

mold type affected. By transferring certain quantities of M1 from the first cycle to the last, the last lot of M3 could be entirely eliminated. No more reduction in setup in the last cycle (for M2) is possible in Table 3, as the lower inventory limit of the predecessor mold type (M1) would be violated.

5. Computational results

We apply the approach in Sections 2–4 to the 77 types of plastic molds (A1, ..., A37, B1, ..., B40) for the two hi-fi models (A and B). Real molding information (Tables 4 and 5) for the 77 plastic mold types is obtained from the company. A planning period of 10 days is considered and Table 6 shows the daily demand data. The daily machine capacity is assumed to be 23.5 hours and on Sunday, it is a half of that after accounting for the planned maintenance time. The three types of setup time range from 40 min.–3 hrs. for metal mold setup, 10 min.–12 hrs. for color cleansing, and 0–3 hrs. for material melting. All the MIPs and goal program in Sections 1–4 are run on Extended Lindo 5.3. The scheduling heuristic in Section 4 is performed on Excel spreadsheet with the Solver function. On applying the machine selection approach (Section 2), 7 machine types of tonnage 550, 350, 280, 170, 140, 80 and 30 are selected out of a total of 14 types. After load balancing (Section 3), the 77 mold types are assigned on 15 machines of the 7 selected types with detailed results shown in Appendix A. The 15 subproblems of single-machine scheduling (Section 4) yields the performance measures summarized in Table 8. An example is shown in Table 7 for machine type 140 with 5 assigned mold types: B36, B37, A32, A33 and B26. The earliest starting time is day 1 (Subsection 4.1) and the earliest stockout time in the first cycle occurs on day 3 for B26. Hence, the first cycle length is 3 days. In the second cycle, the earliest stockout time occurs later than day 5. However, the demand required for the remaining period could be produced without exhausting the second cycle. Hence, the production finishes early on day 5. Here, one post-processing step has been performed to eliminate the lot of A33 in the last cycle and no more improvement could be made. The resulting schedule is shown in Table 7.

The solution from the MIP (Section 1) for these single-machine problems could provide an assessment of the heuristic solution quality. However, apart from the single mold problem (machine type 550), the MIP generates lower bound, LB, equal to the sum of processing times, which is not very informative. In problems with more than 2 mold types, no feasible solution can be found from the MIP even after long computational hours. Another tighter lower bound, LB_1, is given by assuming that at least one cycle is required, i.e.

$$\begin{aligned}\mathrm{LB}_1 = &\text{ sum of processing times} + \text{length of TSP sequence} \\ &+ \min_k\{s_{0kj} - s_{k-1,k,j}\}\end{aligned} \tag{31}$$

where the TSP sequence is labelled as $(1, \dots, n)$ and the length corresponds to the sum of setup times of mold types in this tour. s_{0kj} is the initial setup time of mold type k on the current machine, say of type j. This lower bound could be weak when the initial inventory

Table 4. Molding information for plastic components of model A (37 plastic mold types).

Plastic mold type	Material code	Color code	Compatible machine types	No. of molds produced per pallet	Processing time per pallet (sec.)	No. of molds required per model
A1	4241	BLK	350/450	1	59	1
A2	4241	BLK	350/280	1	55	1
A3	4241	BLK	280/350	1	50	1
A4	4241	BLK	280/350	1	50	1
A5	4241	BLK	280/350	1	50	1
A6	T450(A)	BLK	315/350	1	56	1
A7	T450(A)	BLK	130/170	1	40	1
A8	T450(A)	BLK	220/170	1	44	1
A9	T450(A)	BLK	220/170	1	44	1
A10	T450(A)	BLK	220/170	2	38	2
A11	PMMA	CLEAR	80/80	2	32	1
A12	PMMA	CLEAR	80/140	2	49	1
A13	PMMA	CLEAR	80/80	1	40	1
A14	PMMA	CLEAR	80/80	1	40	1
A15	A	18397	30/75	2	22	1
A16	A	18397	30/75	2	24	1
A17	A	18397	75/80	2	22	2
A18	A	18397	75/80	2	26	1
A19	A	18397	75/80	2	26	1
A20	A	18397	75/80	2	24	1
A21	A38R	TH968	30/75	2	22	1
A22	A	18397	30/75	2	24	1
A23	A	18397	75/80	2	24	1
A24	PMMA	53S804	30/75	2	24	1
A25	A	18397	75/80	8	35	10
A26	A	18397	75/80	2	30	1
A27	A	18397	30/75	1	23	1
A28	PMMA	53S942	30/75	4	17	3
A29	T450(A)	BLK	30/75	2	20	1
A30	PC	26382	80/80	2	45	1
A31	PMMA	CLEAR	80/80	2	250	1
A32	A	17939	140/130	1	42	1
A33	A	17939	140/130	1	42	1
A34	A	17939	30/75	2	21	1
A35	PMMA	20110	30/75	2	31	1
A36	PC	25068	75/80	4	24	1
A37	KJT(A)	135285	30/22	4	18	1

Table 5. Molding information for plastic components of model B (40 plastic mold types).

Plastic mold type	Material code	Color code	Compatible machine types	No. of molds produced per pallet	Processing time per pallet (sec.)	No. of molds required per model
B1	4241	BLK	450/350	1	68	1
B2	4241	BLK	350/450	1	60	1
B3	4241	BLK	280/350	1	52	1
B4	4241	BLK	280/350	1	52	1
B5	4241	BLK	280/350	1	50	1
B6	T450(A)	BLK	315/350	1	55	1
B7	T450(A)	BLK	80/140	1	32	1
B8	T450(A)	BLK	80/140	1	32	1
B9	T450(A)	BLK	80/140	1	29	1
B10	SAN	25468	80/75	2	32	1
B11	SAN	25468	80/140	2	32	1
B12	SAN	25468	75/80	2	32	1
B13	T450(A)	BLK	75/80	2	24	1
B14	T450(A)	BLK	75/80	2	24	1
B15	T450(A)	BLK	75/80	2	24	1
B16	T450(A)	BLK	75/80	2	20	1
B17	T450(A)	BLK	75/80	4	25	2
B18	T450(A)	BLK	80/140	8	26	6
B19	T450(A)	BLK	75/80	2	23	1
B20	T450(A)	BLK	30/75	2	20	1
B21	T450(A)	BLK	75/80	2	20	1
B22	T450(A)	BLK	75/80	2	20	1
B23	T450(A)	BLK	75/80	2	23	1
B24	DEL	NAT	75/80	2	21	1
B25	DEL	NAT	75/80	2	24	1
B26	DEL	25477	140/120	2	22	1
B27	DEL	NAT	75/80	2	20	1
B28	T450(A)	BLK	30/75	4	20	2
B29	PMMA	53S942	30/75	4	22	3
B30	PMMA	53S942	30/75	4	22	7
B31	4241	BLK	280/315	1	46	2
B32	4241	BLK	450/550	1	80	2
B33	4241	BLK	280/315	1	45	1
B34	4241	BLK	280/315	1	45	1
B35	4241	BLK	30/75	2	20	2
B36	A	17939	140/130	1	42	1
B37	A	17939	140/130	1	42	1
B38	A	17939	30/75	2	21	1
B39	PMMA	20110	30/30	2	31	1
B40	4241	BLK	30/75	1	23	1

Table 6. Daily demand for models A and B.

	Day									
Daily demand	1	2	3	4	5	6	7	8	9	10
Model A	409	409	408	408	408	408	0	450	450	450
Model B	500	500	500	500	500	500	0	500	500	500

Table 7. Production schedule for machine type 140. (TSP sequence: B36-B37-A32-A33-B26.)

		Day									
Mold type		1	2	3	4	5	6	7	8	9	10
B36		1307			693						
B37		365	939		585	111					
A32			858			900					
A33			46	1712							
B26				500	1500						
Time usage (sec.)		84600 (23.5 hrs)	84600	84600	84600	46072					
Demand											
Model A		409	409	408	408	408	408	0	450	450	450
Model B		500	500	500	500	500	500	0	500	500	500
Inventory											
B36	2500	3307	2807	2307	2500	2000	1500	1500	1000	500	0
B37	2500	2365	2804	2304	2389	2000	1500	1500	1000	500	0
A32	2042	1633	2082	1674	1266	1758	1350	1350	900	450	0
A33	2042	1633	1270	2574	2166	1758	1350	1350	900	450	0
B26	2500	2000	1500	1500	2500	2000	1500	1500	1000	500	0
Lower inventory limit											
Model A		1225	1224	1224	1266	1308	1350	1350	900	450	0
Model B		1500	1500	1500	1500	1500	1500	1500	1000	500	0
Upper inventory limit											
Model A		3391	2982	2574	2166	1758	1350	1350	900	450	0
Model B		4000	3500	3000	2500	2000	1500	1500	1000	500	0

Setup time = 0.55 day.
Total process. time = 3.99 days.
Makespan = 4.54 days.
LB = 4.33 days.

Table 8. Performance measures on the 15 single-machine scheduling problems.

Machine type (no.)	No. of mold types	Initial inventory required (days of demand)	Setup time (days)	Idle time (days)	No. of post-processing steps	Processing time (days)	Makespan (days)	LB (days)	% diff.	% Machine utilization
550	1	4	0.13	0	0	4.73	4.86[a]	4.86	0	97.37
350 (#1)	3	4	0.64	0	1	4.57	5.21	4.89	6.52	87.74
350 (#2)	2	3	0.62	0	1	4.54	5.16	4.73	9.03	88.03
280 (#1)	4	5	0.51	0	1	4.26	4.77	4.60	3.7	89.30
280 (#2)	3	4	0.47	0	1	4.55	5.02	4.78	4.98	90.60
280 (#3)	3	5	0.23	0	0	4.30	4.53[a]	4.54	0	94.84
170	4	4	0.51	0	2	4.25	4.76	4.51	5.67	89.27
140	5	5	0.55	0	1	3.99	4.54	4.33	4.91	87.83
80 (#1)	4	5	0.17	0	0	4.77	4.94[a]	4.94	0	96.55
80 (#2)	9	5	0.89	0	6	2.83	3.72	3.55	4.79	75.99
80 (#3)	7	3	1.49	0	2	3.98	5.47	4.28	27.86	72.76
80 (#4)	6	3	1.06	0	1	3.79	4.86	4.05	19.96	78.10
80 (#5)	7	3	1.36	0	5	3.79	5.15	4.09	26.00	73.59
30 (#1)	9	5	0.91	0	5	3.06	3.97	3.67	8.11	77.14
30 (#2)	10	5	0.70	0	6	3.13	3.83	3.66	4.66	81.83
							Average:		8.41	85.40

[a]Optimal as found by MIP (Section 1) or objective value coincides with LB.

level is low (Table 8), as the stockout times occur earlier and the number of cycles and setups required increases.

Results in Table 8 show that 10 out of 15 machines have utilization over 80% with an average of 85.4%. This has improved over the current utilization of about 80% by manual scheduling. Three problems are verified to be optimal. A certain minimum level of initial inventory is required, otherwise the problem becomes infeasible due to violation of the lower inventory limit. Apart for problems with very low initial inventory level (= lower inventory limit of the following 3 days' demand), the other problems have makespan within 10% from the lower bound LB. For all problems, the average % difference from LB is 8.41%. All machines start early with no forced idle time necessary. In each post-processing step, the number of setups in the last cycle is reduced by one. The number of post-processing steps engaged in improving the solution increases with the number of mold types on the machine. The special characteristics leading to low utilization (less than 80%) of some machines are large number of mold types assigned and the non-homogeneity of material/color. Besides, in other tested problems, when processing time of the daily demand is small as compared with the daily machine capacity, the inventory is built up quickly and the upper inventory limit will be reached frequently, resulting in changeover to another mold or forced idle time. Consequently, in order to achieve high machine utilization, it is advised not to assign too many mold types on a machine and those assigned are preferred to have processing time for its daily demand occupying a large portion of the daily machine capacity.

6. Summary and conclusion

We presented a decomposition approach for a real plastic mold production problem involving machine type selection, mold-machine assignment and single-machine scheduling. Few papers have addressed the modeling of this entire process. The problem is further complicated by considerations of machine dependent and sequence dependent setup times, due dates, lot-sizing and inventory limits. We propose first to select the smallest set of compatible machine types and make assignment of blocks of mold (homogeneous color/material) onto machines based on load balancing. A travelling salesman solution is adopted as the production sequence on the single machines. The starting time for every cycle is determined by a simplified MIP and the end time is defined as the earliest stockout time. Lot-sizing is based on equalizing the stockout times of molds at the end of a cycle. A post-processing step is further proposed to reduce the number of lots in the last cycle. Results on real-life data for a 10-day planning horizon achieve averages of 85% machine utilization and less than 9% deviation from a lower bound on makespan. A MIP is formulated for the general problem (Section 1), but it could only generate weak lower bound. Future research could consider establishing tighter lower bound for this NP-hard single-machine scheduling problem.

In the situation of infeasibilities which occur usually in violation of some lower inventory limit on a machine, this gives the signal of insufficient machine capacity. The planner is advised to attempt the following alternatives: increase the initial inventory or to start production as early as possible. The relaxation of the lower inventory limit could serve as a short-term solution, more appropriate if it is near the end of the demand period. Another alternative is to shift some production of mold type(s) involved to a underutilized compatible machine. (This should be tested out carefully as it may also lead to infeasibilities on this machine.) Ultimately, when the planner encounters unavoidable delay in meeting a due date after seeking all possible alternatives to increase the capacity, negotiation between related departments, such as production, sales etc. and the customer will be made. The production sequence will be changed according to their compromised priorities of products to be delivered first.

It is observed that high machine utilization is achieved on machines with not too many assigned mold types, molds of homogeneous colors/material, and comparable processing time of the daily demand with respect to the machine capacity. The first and second factor reduces the number of machine setups and sequence dependent setups respectively. The third factor (processing time of the daily demand), when it is too small, would result in forced idle time as the upper inventory limit is reached quickly. When it is too large, the lower inventory limit and the due dates would be violated. Future effort will attempt to incorporate the sequence dependent setup times into the mold-machine assignment problem. Alternatively, the problem treating the upper inventory limits as soft constraints, rather than hard constraints, could be considered in future research.

Appendix A. Mold-machine assignment (Section 3)

Machine type 550 (metal mold setup: 3 hrs.)

Mold type	Total process. time (hrs.)	Material code	Color code
B32	200	4241	BLK
Total	200		

Machine type 170 (metal mold setup: 1.5 hrs.)

Mold type	Total process. time (hrs.)	Material code	Color code
A7	42.24	T450(A)	BLK
A8	46.47	T450(A)	BLK
A9	46.47	T450(A)	BLK
A10	40.13	T450(A)	BLK
Total	175.31		

Machine type 350 (metal mold setup: 2.5 hrs.)

Mold type	Total process. time (hrs.)	Material code	Color code
	Machine no. 1		
A1	62.28	4241	BLK
A6	59.11	4241	BLK
B6	68.75	4241	BLK
Total	109.14		
	Machine no. 2		
B1	85	4241	BLK
B2	75	4241	BLK
Total	160		

Machine type 280 (metal mold setup: 2 hrs.)

Mold type	Total process. time (hrs.)	Material code	Color code
	Machine no. 1		
A2	58.06	4241	BLK
A3	52.78	4241	BLK
A4	52.78	4241	BLK
A5	52.78	4241	BLK
Total	216.39		
	Machine no. 2		
B3	65.0	4241	BLK
B4	65.0	4241	BLK
B5	62.5	4241	BLK
Total	192.5		
	Machine no. 3		
B31	115.00	4241	BLK
B33	56.25	4241	BLK
B34	56.25	4241	BLK
Total	227.50		

Machine type 140 (metal mold setup: 1 hr.)

Mold type	Total process. time (hrs.)	Material code	Color code
B36	52.50	A	17939
B37	52.50	A	17939
A32	44.33	A	17939
A33	44.33	A	17939
B26	13.75	DEL	25477
Total	207.42		

Machine type 80 (metal mold setup: 1 hr.)

Mold type	Total process. time (hrs.)	Material code	Color code
	Machine no. 1		
A31	131.94	PMMA	Clear
A13	42.22	PMMA	Clear
A14	42.22	PMMA	Clear
A12	25.86	PMMA	Clear
Total	242.25		
	Machine no. 2		
B10	20.00	SAN	25468
B11	20.00	SAN	25468
B12	20.00	SAN	25468
B24	13.13	DEL	NAT
B25	15.00	DEL	NAT
B27	12.50	DEL	NAT

(*Continued on next page*.)

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Mold type	Total process. time (hrs.)	Material code	Color code
A30	23.75	PC	26382
A36	6.33	PC	25068
A11	16.89	PMMA	Clear
Total	147.60		
	Machine no. 3		
A25	4.62	A	18397
A17	11.61	A	18397
A26	15.83	A	18397
A18	13.72	A	18397
A19	13.72	A	18397
A20	12.67	A	18397
A23	12.67	A	18397
Total	84.84		
	Machine no. 4		
B7	40.00	T450(A)	BLK
B9	36.25	T450(A)	BLK
B13	15.00	T450(A)	BLK
B14	15.00	T450(A)	BLK
B15	15.00	T450(A)	BLK
B16	12.50	T450(A)	BLK
Total	133.75		
	Machine no. 5		
B8	40.00	T450(A)	BLK
B18	4.06	T450(A)	BLK
B17	7.81	T450(A)	BLK
B19	14.38	T450(A)	BLK
B23	14.38	T450(A)	BLK
B21	12.50	T450(A)	BLK
B22	12.50	T450(A)	BLK
Total	105.63		

Machine type 30 (metal mold setup: 2/3 hr.)

Mold type	Total process. time (hrs.)	Material code	Color code
	Machine no. 1		
A24	12.67	PMMA	53S804
A28	13.46	PMMA	53S942
B29	20.63	PMMA	53S942
B30	20.63	PMMA	53S942
A35	16.36	PMMA	20110
B39	19.38	PMMA	20110
A34	11.08	A	17939
B38	13.13	A	17939
A37	4.75	KJT(A)	135285
Total	159.57		

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Mold type	Total process. time (hrs.)	Material code	Color code
	Machine no. 2		
A29	12.67	T450(A)	BLK
B20	12.50	T450(A)	BLK
B28	6.25	T450(A)	BLK
B35	12.50	4241	BLK
B40	28.75	4241	BLK
A15	11.61	A	18397
A16	12.67	A	18397
A22	12.67	A	18397
A27	24.28	A	18397
A21	11.61	A38R	TH968
Total	145.50		

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