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UPskill

Radicals – exercises

1. $3\sqrt{12}$ # $6\sqrt{3}$

2. $6\sqrt{128}$ # $48\sqrt{2}$

3. $7\sqrt{128}$ # $56\sqrt{2}$

4. $-8\sqrt{392}$ # $-112\sqrt{2}$

5. $-7\sqrt{63}$ # $-21\sqrt{7}$

6. $\sqrt{192n}$ # $8\sqrt{3n}$

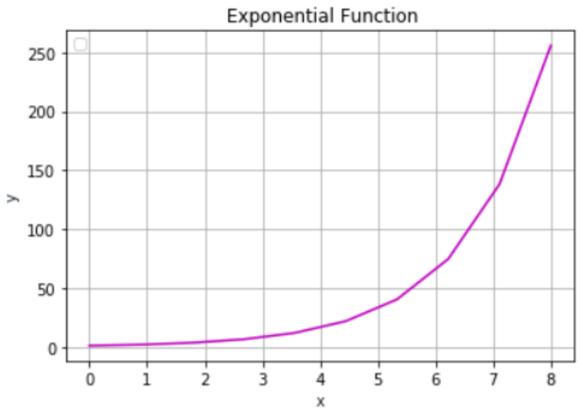
7. $\sqrt{343b}$ # $7\sqrt{7b}$

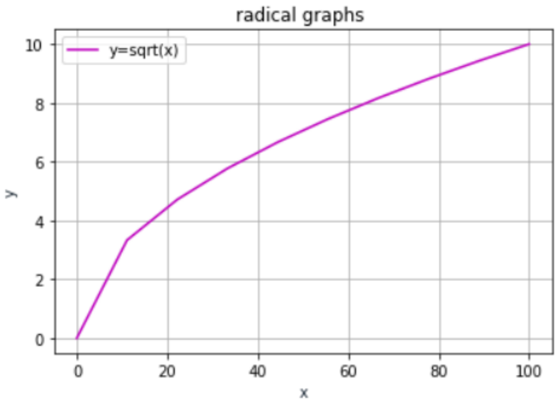
8. $\sqrt{196v^2}$ # 14v

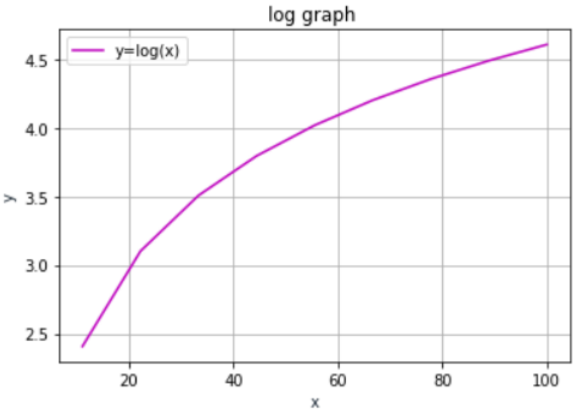
9. $\sqrt{100n^3}$ # $10\text{n}\sqrt{n}$

10. $\sqrt{200a^3}$ # $10\text{a}\sqrt{2a}$

Exponents, logarithms and radicals





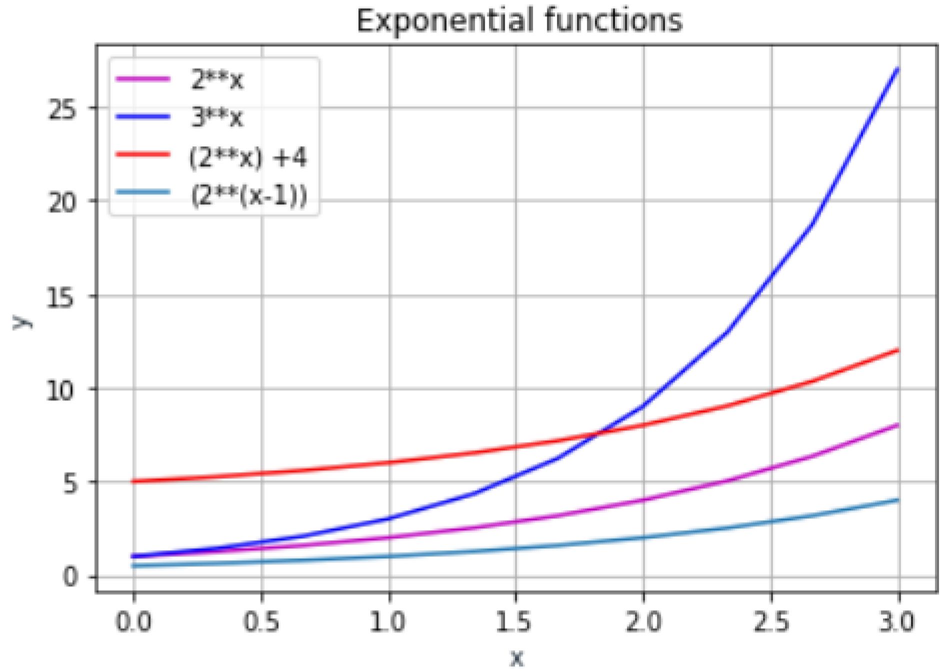



```
x = np.linspace(0,8,10)
y1 = 2**x -3
plt.plot(x, y1, ‘-m’)
plt.show()
```

```
x = np.linspace(0,100,10)
y1 = np.sqrt(x)
plt.plot(x, y1,'-m', label='y=sqrt(x)')
plt.show()
```

```
x = np.linspace(0,100,10)
y1 = np.log(x)
plt.plot(x, y1,'-m', label='y=log(x)’)
plt.show()
```

Multiple functions for comparisons

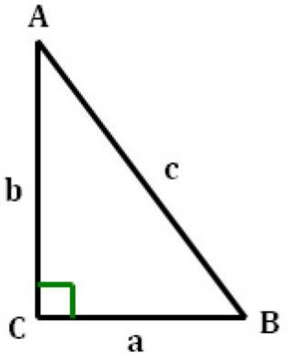



```
import matplotlib.pyplot as plt
import numpy as np
x = np.linspace(0,3,10)
y1 = 2**x
y2 = 3**x
y3 = (2**x) +4
y4 = (2**(x-1))

plt.plot(x,y1, 'm', label= '2**x')
plt.plot(x, y2, '-b', label=
'3**x')
plt.plot(x, y3, '-r', label=
'(2**x) +4')
plt.plot(x, y4, label= '(2**(x-
1))')
plt.title(‘ Exponential function')
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc='upper left')
plt.grid()
plt.show()
```

<Trigonometry>

Trigonometric

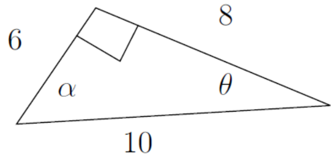


SOH-CAH-TOA

$$\sin = \frac{opposite}{hypoteneuse} \qquad \sin A = \frac{a}{c} \qquad \sin B = \frac{b}{c}$$

$$\cos = \frac{adjacent}{hypoteneuse} \qquad \cos A = \frac{b}{c} \qquad \cos B = \frac{a}{c}$$

$$\tan = \frac{opposite}{adjacent} \qquad \tan A = \frac{a}{b} \qquad \tan B = \frac{b}{a}$$



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$

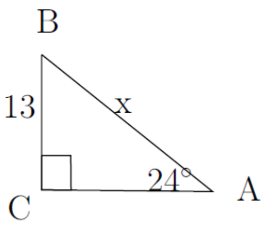
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{6}{8} = \frac{3}{4}$$

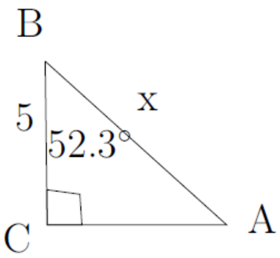
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{10} = \frac{4}{5}$$

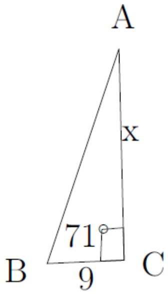
$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{10} = \frac{3}{5}$$

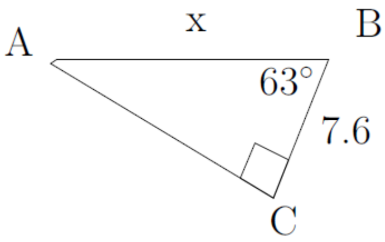
$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{6} = \frac{4}{3}$$

Trigonometry – examples

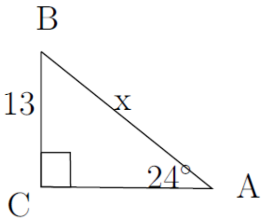






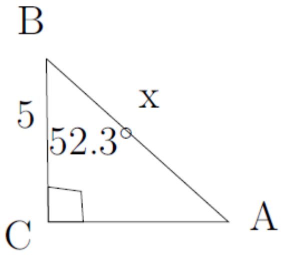


Trigonometry – examples



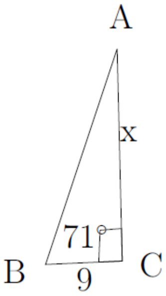
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

#31.96



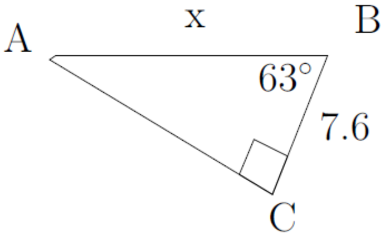
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

#8.17



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

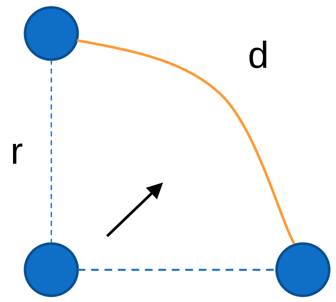
#26.1



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

#16.7

Radians and Degrees



Observer's perspective

Degrees to radians

360°= 2π radians
180°= π radians

Examples

$$45° * \frac{\pi}{180} \text{ rad} = \frac{\pi}{4}$$

$$150° * \frac{\pi}{180} \text{ rad} = \frac{5\pi}{6}$$

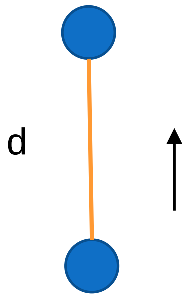
Radians to degrees

360°= 2π radians
180°= π radians

Examples

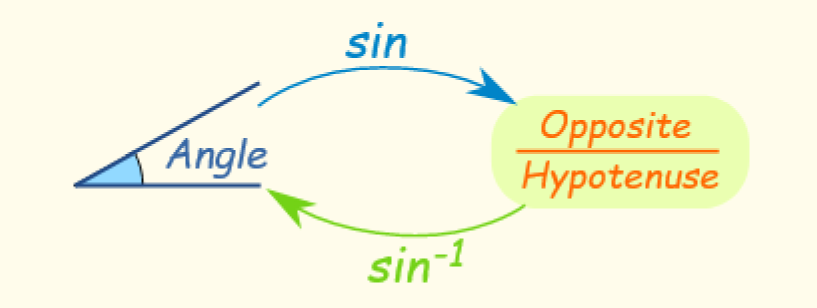
$$\frac{\pi}{4} \text{ rad} * \frac{180}{\pi \text{ rad}} = 45°$$

$$60° * \frac{180}{\pi \text{ rad}} = \frac{\pi}{3}$$



Driver's perspective

Trigonometric



$$\sin^{-1}\left(\frac{\text{opposite}}{\text{hypotenuse}}\right)=\theta \quad \cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)=\theta \quad \tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)=\theta$$

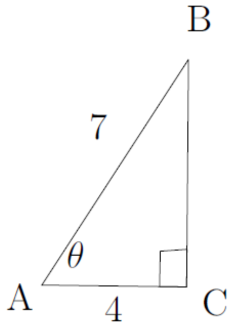
$$\sin \mathrm{A}=0.5$$
$$\sin^{-1}(0.5)=A$$
$$30^\circ=A$$

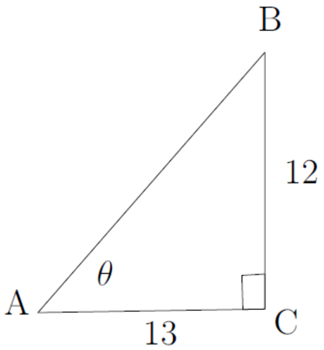
```
x= np.arcsin(0.5)
y = np.rad2deg(x)
```

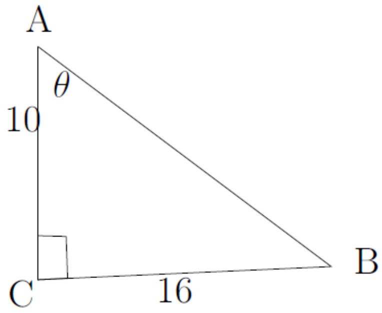
$$\cos \mathrm{B}=0.667$$
$$\cos^{-1}(0.667)=B$$
$$48^\circ=B$$

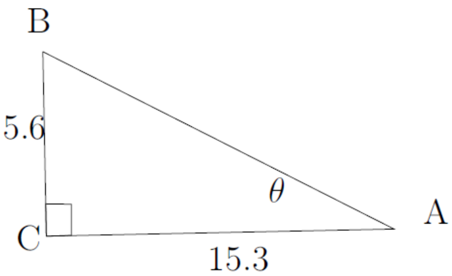
```
x= np.arccos(0.667)
y = np.rad2deg(x)
```

Exercises









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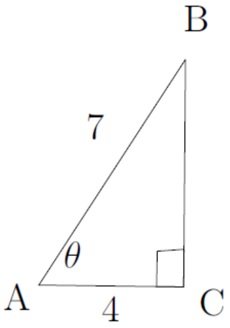
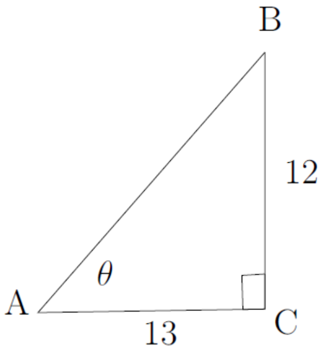
Exercises



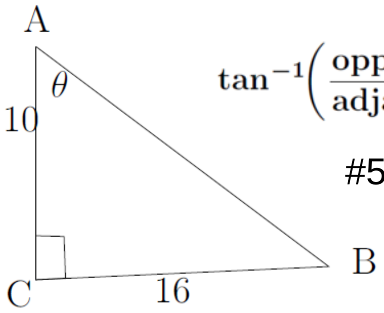
$$\cos^{-1}\left(\frac{\text{adjacent}}{\text{hypotenuse}}\right)$$

#55.2



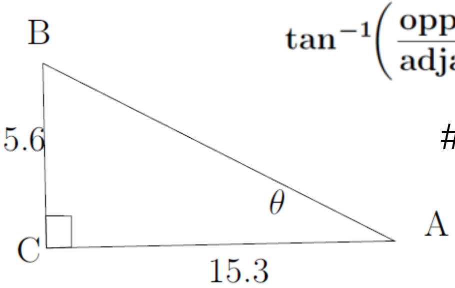
$$\tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$$

#42.7



$$\tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$$

#57.99



$$\tan^{-1}\left(\frac{\text{opposite}}{\text{adjacent}}\right)$$

#20.1

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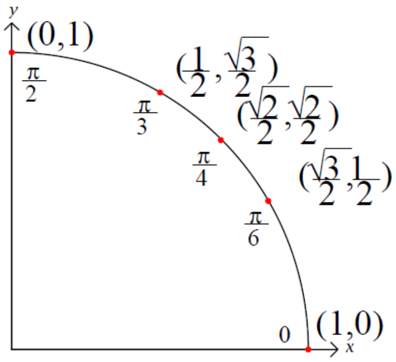
Most used angles in math located in the first quadrant

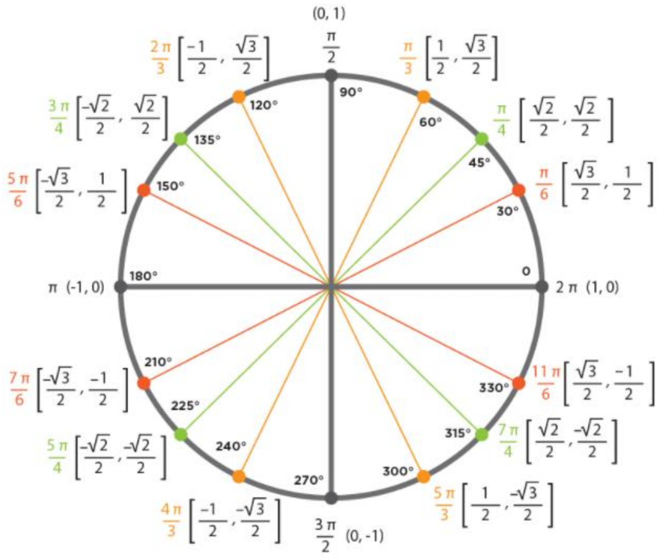


θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined

```
import numpy as np
numpy.set_printoptions(precision=2)
numpy.set_printoptions(formatter={"float":'{:0.2f}'.format})
a = np.array([0,30,45,60,90])

print('Sine of different angles:')
# Convert to radians by multiplying to pi/180
print(np.sin(a*np.pi/180), "\n")

print("Cosine values for angles in array:")
print(np.cos(a*np.pi/180), "\n")

print('Tangent values for given angles:')
print(np.tan(a*np.pi/180))
```

```
Sine of different angles:
[0.00 0.50 0.71 0.87 1.00]

Cosine values for angles in array:
[1.00 0.87 0.71 0.50 0.00]

Tangent values for given angles:
[0.00 0.58 1.00 1.73 16331239353195370.00]
```

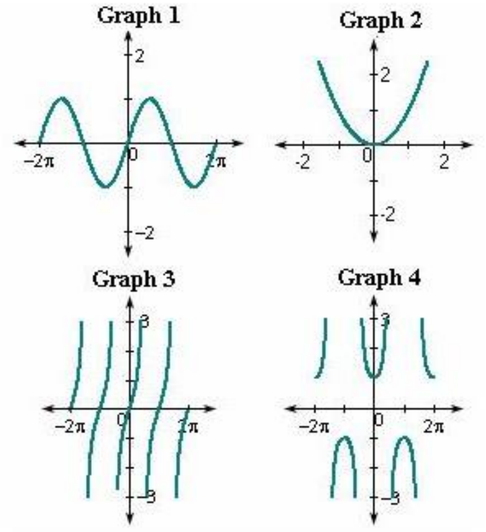
Trigonometric relation in the circle

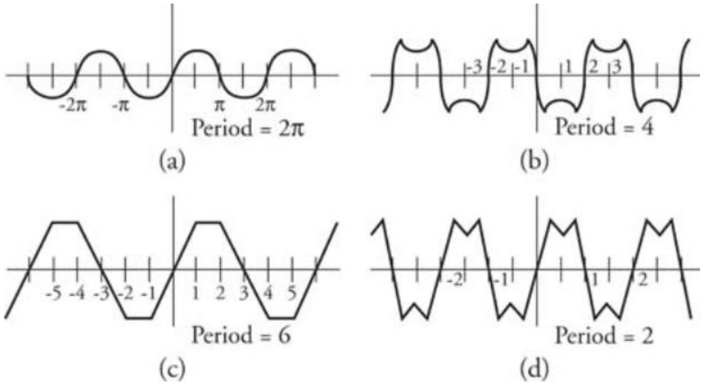


θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

Periodic functions definition

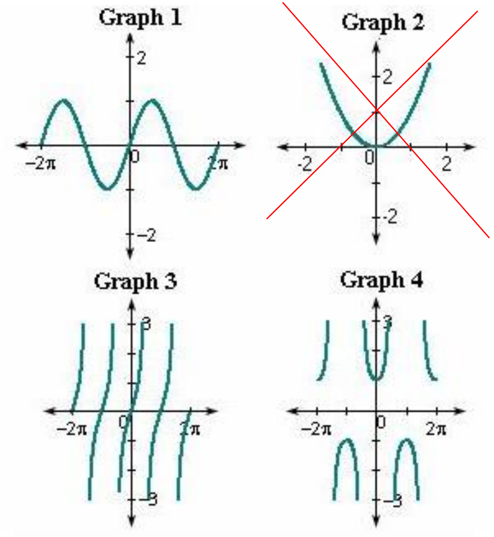
A **periodic function** occurs when a specific horizontal shift, *P*, results in the original function; where *f* (*x* + *P*) = *f* (*x*) for all values of *x*. When this occurs we call the horizontal shift the **period** of the function.

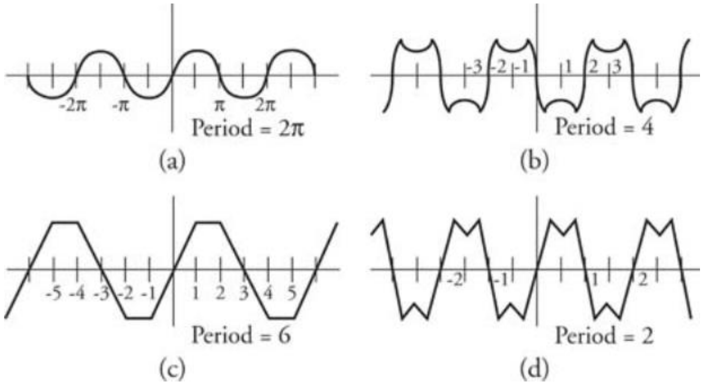




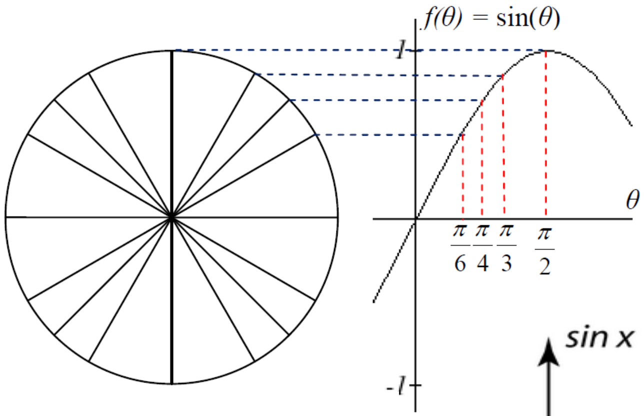
Periodic functions definition

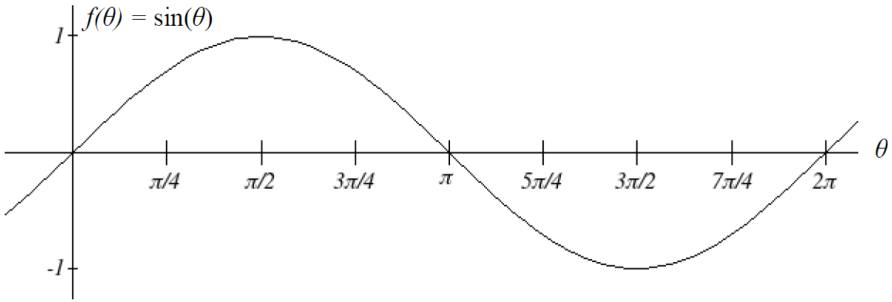
A **periodic function** occurs when a specific horizontal shift, *P*, results in the original function; where $f(x + P) = f(x)$ for all values of *x*. When this occurs we call the horizontal shift the **period** of the function.



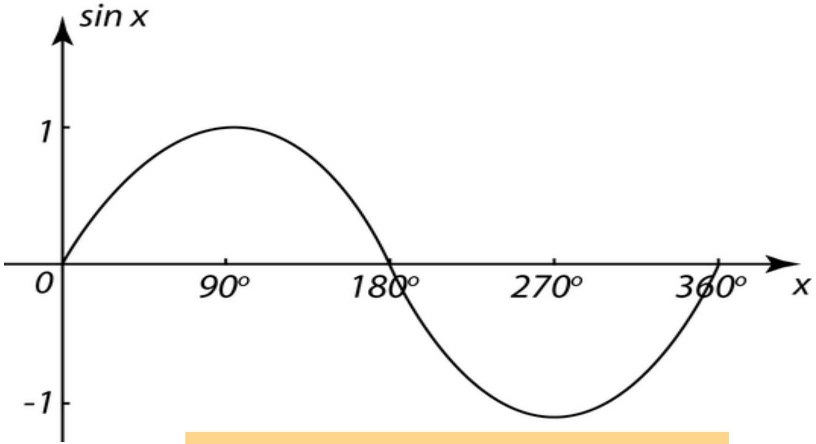


Periodic functions





radians representation



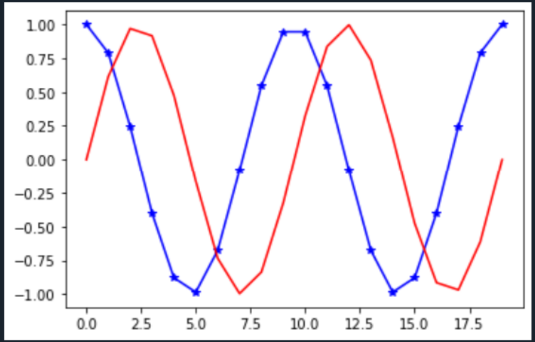
Degrees representation

C:\Users\Josifefe\Desktop\python\pythoncode5\readFile\test.py

temp.py readJson.py readfile.py test.py

```python
# x1 = np.array([[-1, 3],[4,2]])
# x2 = np.array([[1, 2],[3,4]])
# x4 = x1+x2
# x5 = x1*x2
# x3 =  np.dot(x1,x2)
# print(x3)
# print(x4)
# print(x5)

import matplotlib.pyplot as plt

values = np.linspace(-(2*np.pi),2*np.pi, 20)
cos_values = np.cos(values)
sin_values = np.sin(values)

plt.plot(cos_values, color = 'blue', marker= '*')
plt.plot(sin_values, color = 'red')
plt.show()
```

145 %



Explorador de variáveis Ajuda Figuras Arquivos

Console 1/A

```
Desktop/python/pythoncode5/readFile/
test.py', wdir='C:/Users/Josifefe/
Desktop/python/pythoncode5/readFile')
2020-11-02 2020-11-03 2020-11-04
731 1
[[2. 2. 2. 2. 2. 2.]
 [2. 2. 2. 2. 2. 2.]]

In [156]:
```

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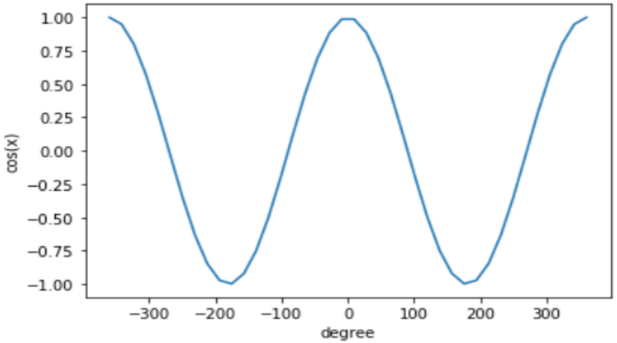
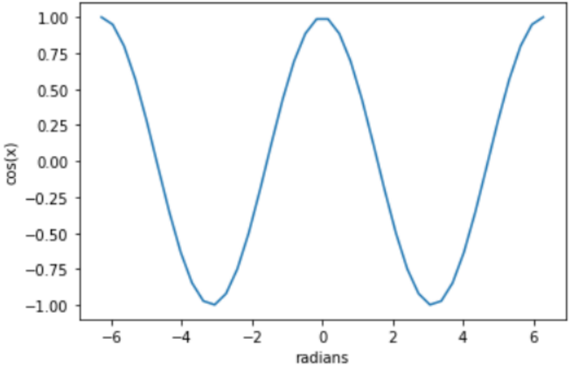
cos (x) in degrees




```
import numpy as np
import matplotlib.pyplot as plt
x = np.linspace(-2*np.pi, 2*np.pi, 40)
x1 = np.rad2deg(x)
y = np.cos(x)
plt.plot(x1,y)
plt.xlabel('degree')
plt.ylabel('cos(x)')
```

cos (x) in radians




```
import numpy as np
import matplotlib.pyplot as plt
x = np.linspace(-2*np.pi, 2*np.pi, 40)
y = np.cos(x)
plt.plot(x,y)
plt.xlabel('radians')
plt.ylabel('cos(x)')
```

Periodic functions

- Amplitude
- Period
- Phase shift

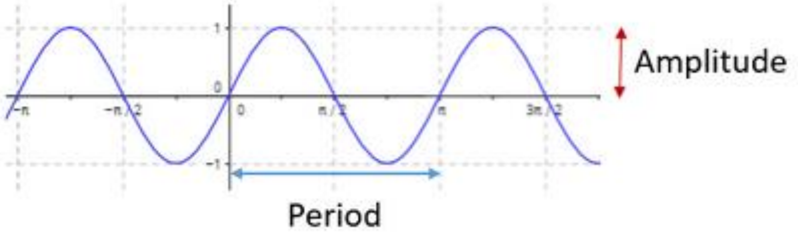
$$y = A\sin\left[B(x-C)\right]+D$$

$|A|$ is the amplitude

The period is $\frac{2\pi}{B}$

Phase (horizontal) shift is C

Vertical shift is D



The same applies for the Cosine Function.

y= $4+\frac{1}{2}\sin(x)$

y= $-4\,cos(3x\ -\pi)$

Periodic functions – exercises spyder

y= $4 + \frac{1}{2}\sin(x)$	Radians	degrees	samples
y= $-4\,cos(3x\ -\pi)$			

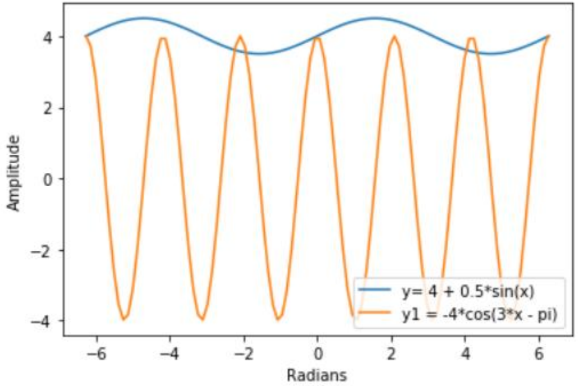
What is the period of these functions?
What is the amplitude?
How many samples (minimum amount) is needed to see the smoothing charts ?

Periodic functions – exercises spyder

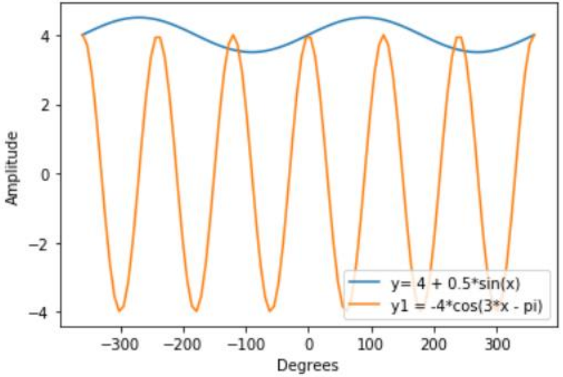
$$y = 4 + \frac{1}{2}\sin(x)$$

$$y = -4\,cos(3x\ -\pi)$$

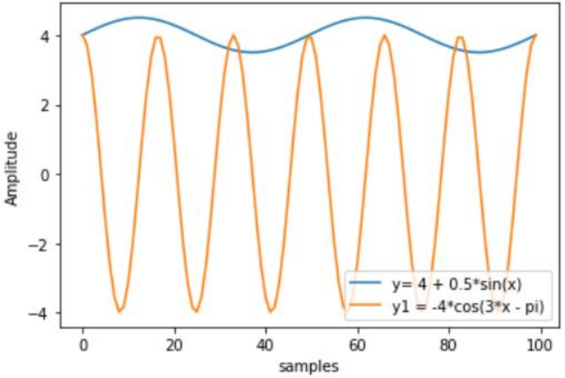
Radians



degrees



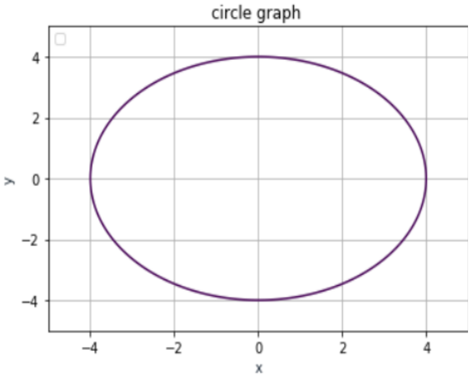
Samples

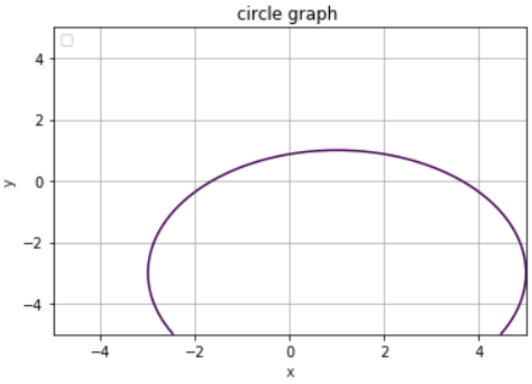


What is not a function ?

Circle equation

X^2 +Y^2 = 16
(X-1)**2+(Y+3)**2 =16






```
import matplotlib.pyplot as plt
import numpy as np
x = np.linspace(-5,5,100)
y = np.linspace(-5,5,100)

X, Y = np.meshgrid(x,y)
f = X**2+Y**2 -16
fig, ax = plt.subplots()
ax.contour(X,Y,f,[0])
plt.title('circle graph ')
plt.xlabel('x', color='#1C2836')
plt.ylabel('y', color='#1C2833')
plt.legend(loc='upper left')
plt.grid()
plt.show()
```

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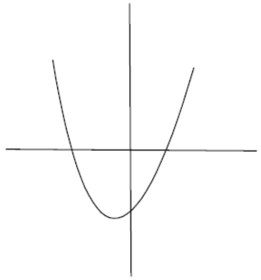
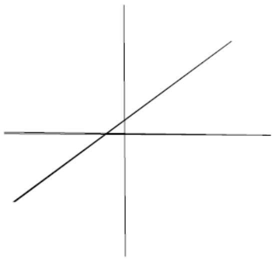
What are functions here ?

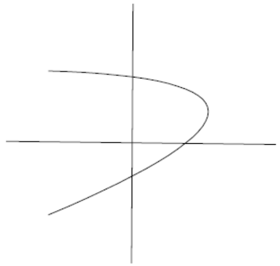
a)



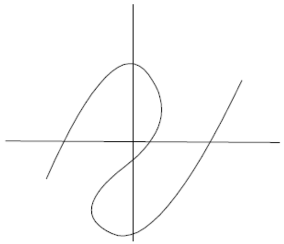
c)



b)

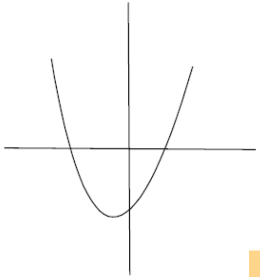


d)



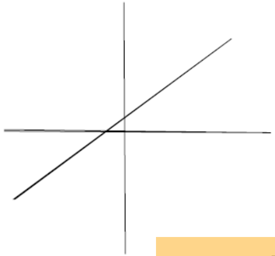
What are functions here ?

a)



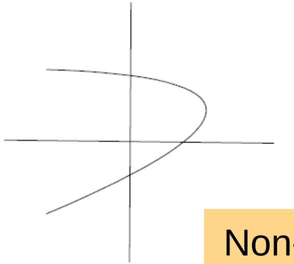
Function

c)



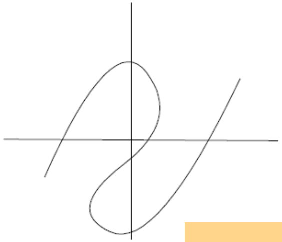
Function

b)



Non-Function

d)



Non-Function

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“

- *Make it work*
- *Make it Right*
- *Make it Fast*

O futuro profissional começa aqui

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