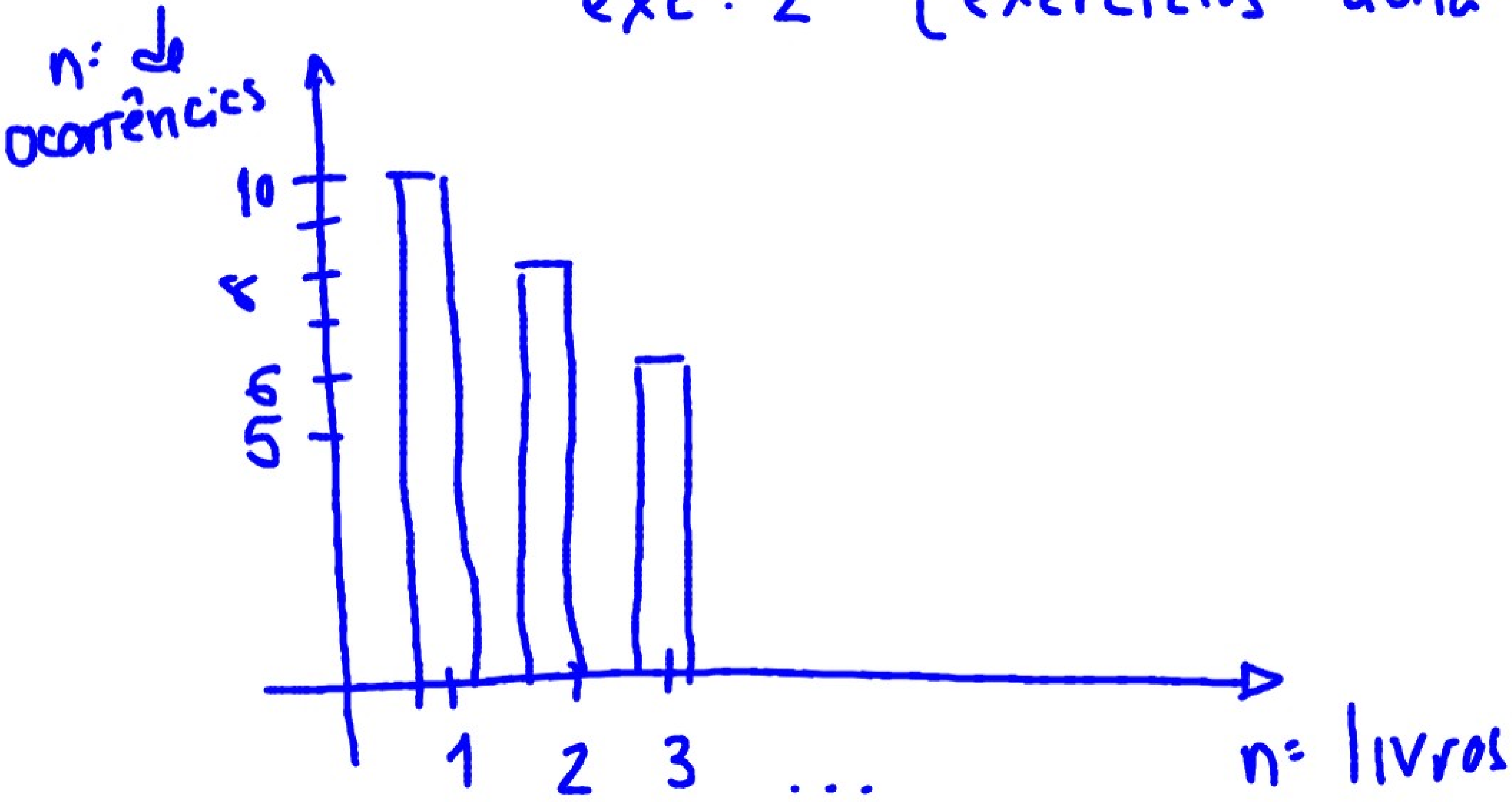
exe. 2 (exercícios aula 5)



$x_{20} = 3$

$x_{21} = 3$

$$\text{med} = \frac{x_{20} + x_{21}}{2} = \frac{3+3}{2} = 3$$

POTÊNCIAS

a^n , $a \equiv$ base , $n \equiv$ expoente

- $n \in \mathbb{N}$ (potências de expoente natural)

$n = 1, 2, 3, \ldots$

$$a^n = \underbrace{a \times a \times \ldots \times a}_{n \text{ vezes}}$$

ex: $8^4 = 8 \times 8 \times 8 \times 8$

$5^1 = 5$

PROPRIEDADES DAS POTÊNCIAS

- $a^n . a^m = a^{n+m}$

$$\underbrace{\underbrace{a \times a \times \ldots \times a}_{n \text{ vezes}} . \underbrace{a \times a \times \ldots \times a}_{m \text{ vezes}}}_{n+m \text{ vezes}} = a^{n+m}$$

- $\dfrac{a^n}{a^m} = a^{n-m}$

$$\frac{\overbrace{a \times \ldots \times a}^{n \text{ vezes}}}{\underbrace{a \times \ldots \times a}_{m \text{ vezes}}}$$

ex: $\frac{5^8}{5^3} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times 5 \times 5 \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5} \times \cancel{5}}$

$= 5 \times 5 \times 5 \times 5 \times 5 =$

$= 5^{5} = 5^{8-3}$

- $a^n . b^n = (ab)^n$

$\underbrace{a \times a \times \ldots \times a}_{n \text{ vezes}} . \underbrace{b \times b \times \ldots \times b}_{n \text{ vezes}} =$

$= \underbrace{ab \times ab \times \ldots \times ab}_{n \text{ vezes}} = (ab)^n$

- $$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$$

$$\frac{\overbrace{a \times a \times \dots \times a}^{n \text{ vezes}}}{\underbrace{b \times b \times \dots \times b}_{n \text{ vezes}}} = \frac{a}{b} \cdot \frac{\overbrace{a \times \dots \times a}^{n-1 \text{ vezes}}}{\underbrace{b \times \dots \times b}_{n-1 \text{ vezes}}} = \dots$$

$$= \underbrace{\frac{a}{b} \cdot \frac{a}{b} \cdot \dots \cdot \frac{a}{b}}_{n \text{ vezes}}$$

- $n \in \mathbb{Z}$ (potências de expoente inteiro)

$n = \ldots, -4, -3, \underleftarrow{-2, -1}_{?}, \underbrace{0}_{\checkmark}, \underbrace{1, 2, 3, \ldots}_{\checkmark}$

$\boxed{a^0 = 1}$ definição para que a propriedade seja válida

$$\frac{a^n}{a^m} = a^{n-m}$$

$$n=m : \quad \underbrace{\frac{a^n}{a^n}}_{=1} = a^{n-n} \iff 1 = a^0$$

$a^{-n} = ?$, $n \in \mathbb{N}$

$$\frac{a^n}{a^m} = a^{n-m}$$

Aplicar $\uparrow$ com $n = 0$:

$$\frac{a^0}{a^m} = a^{0-m} \iff \frac{1}{a^m} = a^{-m}$$

Logo $\boxed{a^{-n} = \frac{1}{a^n}}$, $n \in \mathbb{N}$

ex: $\underline{\underline{a^{-1}}} = \frac{1}{a^1} = \frac{1}{a} \rightarrow$ Inverso de a:

$$\frac{a}{1} \cdot \frac{1}{a} = \frac{a}{a} = 1$$

ex: $\frac{5^3}{5^8} = 5^{3-8} = 5^{-5} = \frac{1}{5^5}$

- $a^n \pm a^m \neq a^{\cdots}$

$a^n \pm b^n \neq (a \pm b)^n$

ex: $2^3 + 2^5 = 8 + 32 = 40$

ex: $x^3 + y^5$ ñ dá p/ simplificar

- ## Potência de potências

$$(a^n)^m = a^{nm} \qquad n, m \in \mathbb{Z}$$

$$(2^3)^4 = (\underbrace{2\times2\times2}_{3 \text{ vezes}})^4 =$$

$$= (2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)$$

$$= \underbrace{2\times2\times2\times\ldots\times2}_{\substack{12 \text{ vezes} \\ 3\times4}} = 2^{12} = 2^{3\times4}$$

ex.: Simplificar:

$$(x^3 \cdot y^2 \cdot z)^3 = (x^3)^3 \cdot (y^2)^3 \cdot z^3 =$$

$$a^n \cdot b^n = (ab)^n$$

$$a^n \cdot b^n \cdot c^n = (abc)^n$$

$$= x^{3\times 3} \cdot y^{2\times 3} \cdot z^3 =$$

$$(a^n)^m = a^{n.m}$$

$$= x^9 \cdot y^6 \cdot z^3$$

ex: $x^3 \cdot x^2 \cdot x^{-4} =$

$= x^{3+2+(-4)} = x^{5-4} = x^1 = x$

$a^n \cdot a^m = a^{n+m}$

ex: $x(x^2-1) + x^3 =$

$= \underline{x \cdot x^2} - x + x^3 =$

$= x^{1+2} - x + x^3 = x^3 - x + x^3 =$

$a^n \cdot a^m = a^{n+m}$

$= 2x^3 - x$

ex $\quad (x^{-2})^{-3} = x^{(-2)\cdot(-3)} = x^{6}$

$(a^n)^m = a^{nm}$

<u>ou</u>

$$(x^{-2})^{-3} \underset{a^{-n}=\frac{1}{a^n}}{=} \left(\frac{1}{x^2}\right)^{-3} \underset{a^{-n}=\frac{1}{a^n}}{=} \frac{1}{\left(\frac{1}{x^2}\right)^3} =$$

$$\underset{\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n}{=} \frac{1}{\frac{1^3}{(x^2)^3}} \underset{(a^n)^m = a^{nm}}{=} \frac{1}{\frac{1}{x^6}} =$$

$$= \frac{1}{\frac{1}{x^6}} = \frac{\frac{1}{1}}{\frac{1}{x^6}} = \frac{x^6}{1} = x^6$$

ex: $(x^2)^{-3} = x^{2(-3)} = \underline{\underline{x^{-6}}} = \underline{\underline{\frac{1}{x^6}}}$

ex: $(2x^3y^2+z)^0 (2x)^{-2} (3y)^3 \cdot x^5 =$

$= (2x)^{-2} \cdot (3y)^3 \cdot x^5 =$

$a^0 = 1$ $\qquad$ $a^n \cdot b^n = (ab)^n$

$$= 2^{-2} \cdot x^{-2} \cdot 3^{3} \cdot y^{3} \cdot x^{5} =$$

$$= \frac{1}{2^2} \cdot x^{-2+5} \cdot 27\, y^{3} = \frac{27}{4}\, x^{3}\, y^{3}$$

ex: $\dfrac{\overbrace{a^0}^{1} b^{-4} \cdot c^{-2}}{b^{-2} \cdot d^{-3}} \cdot \dfrac{a^{-1} \cdot b^{-2} \cdot d^{-4}}{c^2 \cdot d^{-1}} =$

$= b^{-4} \cdot c^{-2} \cdot b^2 \cdot d^3 \cdot a^{-1} \cdot b^{-2} \cdot d^{-4} \cdot c^{-2} \cdot d^1 =$

$= a^{-1} \cdot b^{-4+\cancel{2}-\cancel{2}} \cdot c^{-2-2} \cdot d^{\cancel{3}-\cancel{4}+\cancel{1}} = \rightarrow$

<u>Obs:</u>

- $\dfrac{1}{b^{-2}} = b^{-(-2)} = b^2$

- $\dfrac{1}{d^{-3}} = d^{-(-3)} = d^3$

$$= a^{-1} \cdot b^{-4} \cdot c^{-4} \cdot \underbrace{d^{0}}_{=1} =$$

$$= a^{-1} \cdot b^{-4} \cdot c^{-4} = \frac{1}{a \cdot b^{4} \cdot c^{4}}$$

ou

$$\frac{\overbrace{a^{0}}^{1} b^{-4} c^{-2}}{b^{-2} \cdot d^{-3}} \cdot \frac{a^{-1} \cdot b^{-2} \cdot d^{-4}}{c^{2} \cdot d^{-1}} =$$

$$= \frac{b^{-4} \cdot c^{-2} \cdot a^{-1} \cdot b^{-2} \cdot d^{-4}}{b^{-2} \cdot d^{-3} \cdot c^{2} \cdot d^{-1}} =$$

$$= \frac{a^{-1} \cdot b^{-4-2} \cdot c^{-2} \cdot d^{-4}}{b^{-2} \cdot c^{2} \cdot d^{-3-1}} =$$

$$= \frac{a^{-1} \cdot b^{-6} \cdot c^{-2} \cdot \cancel{d^{-4}}}{b^{-2} \cdot c^{2} \cdot \cancel{d^{-4}}} =$$

$$= a^{-1} \cdot b^{-6-(-2)} \cdot c^{-2-2} =$$

$$= a^{-1} \cdot b^{-4} \cdot c^{-4} = \frac{1}{a \cdot b^{4} \cdot c^{4}}$$

ex: Calcular:

$$2^3 \cdot \frac{1}{4^{-2}} \cdot \left(4^3\right)^{-2} \cdot \frac{5^{-3}}{3^2} \cdot \frac{1}{5^{-2}} =$$

$$= 8 \cdot \underbrace{4^2 \cdot 4^{-6}}_{4^{-4}} \cdot \frac{5^{-3}}{3^2} \cdot 5^2 =$$

$\frac{1}{9} \cdot 5^{-1}$

$$= 8 \cdot 4^{-4} \cdot \frac{5^{-3+2}}{9} = 8 \cdot \frac{1}{4^4} \cdot \frac{5^{-1}}{9} =$$

$$= 2 \cdot \cancel{4} \cdot \frac{1}{\cancel{4} \cdot 4^3} \cdot \frac{1}{9} \cdot \frac{1}{5} = \frac{2}{64 \cdot 45} = \frac{1}{32 \cdot 45} =$$

$$= \frac{1}{1440}$$

- **obs:**

$$\begin{array}{r|l} 50 & 2 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

$$50 = 2 \times 5^2$$

$$\begin{array}{r|l} 124 & 2 \\ 62 & 2 \\ 31 & 31 \\ 1 & \end{array}$$

$$124 = 2^2 \cdot 31$$

ex: Simplificar, usando factorização:

$$\frac{124}{50} = \frac{2^{\cancel{2}} . 31}{\cancel{2} . 5^2} = \frac{2 . 31}{5^2} = \frac{62}{25}$$

ex: $$\frac{2^3 . 3^4 . 5^2 . 7}{3^5 . 7^2} = 2^3 . 3^{4-5} . 5^2 . 7^{1-2} =$$

$$= 2^3 . 3^{-1} . 5^2 . 7^{-1} = \frac{2^3 . 5^2}{3 . 7} = \frac{200}{21}$$

CASOS NOTÁVEIS

- $(a+b)^2 = a^2 + 2ab + b^2$

$$(a+b)(a+b) = (a+b)a + (a+b).b =$$

$$= a^2 + ba + ab + b^2 =$$

$$= a^2 + ab + ab + b^2 =$$

$$= a^2 + 2ab + b^2$$

- $(a-b)^2 = a^2 - 2ab + b^2$

$$(a-b)(a-b) = (a-b)a - (a-b)b =$$

$$= a^2 - ba - (ab - b^2) =$$

$$= a^2 - ba - ab + b^2 =$$

$$= a^2 - ab - ab + b^2 =$$

$$= a^2 - 2ab + b^2$$

- $a^2 - b^2 = (a+b).(a-b)$

ex: $x^2 - 1 = 0 \iff$

$\iff x^2 - 1^2 = 0 \iff$

$\iff (x+1).(x-1) = 0 \iff$

$a = x$
$b = 1$

$\iff x+1=0 \ \vee \ x-1=0 \iff$

$\iff x=-1 \ \vee \ x=1$

$$(a+b)\cdot(a-b) = (a+b)\cdot a - (a+b)\cdot b =$$

$$= a^2 + ba - (ab + b^2) =$$

$$= a^2 + ba - ab - b^2 =$$

$$= a^2 + \cancel{ab} - \cancel{ab} - b^2 =$$

$$= a^2 - b^2$$

ex: Escrever como quadrados:

- $x^2 \underline{-} 2x + 1 = (x-1)^2$

  $a = x$, $b = 1$: $a^2 - 2ab + b^2 = x^2 - 2x1 + 1^2$ ✓

- $1 - x^4 = (\ldots + \ldots) \cdot (\ldots - \ldots)$

  $a^2 - b^2 = (a+b)(a-b)$

  $1 = a^2 \Rightarrow \underline{a = 1} \vee a = -1$, $b^2 = x^4 \Rightarrow b = x^2$

- $4y^2 - 16y + 16 = (2y-4)^2$

  $(a-b)^2 = a^2 - 2ab + b^2$

  $a^2 = 4y^2 \Rightarrow a = 2y$

  $b^2 = 16 \Rightarrow b = 4$

  $-2ab = -2 \cdot 2y \cdot 4 = -16y$ ✓

ex: Resolver $x^2 - 4x + 4 = 0$

(s/ fórmula resolvente).

Ideia: identificar no membro esquerdo um caso notável.

$$\underbrace{x^2 - 4x + 4}_{(a-b)^2 = a^2 - 2ab + b^2} = 0 \Leftrightarrow (x-2)^2 = 0 \Leftrightarrow$$

$a^2 = x^2 \Rightarrow a = x$

$b^2 = 4 \Rightarrow b = 2$

$$\Leftrightarrow \quad (x-2)\cdot(x-2)=0 \quad \Leftrightarrow \quad x-2=0 \quad \vee \quad x-2=0$$

$$\Leftrightarrow \quad x-2=0 \quad \Leftrightarrow \quad x=2$$

BASE DECIMAL

$$1234 = 1000 + 200 + 30 + 4 =$$

4 ↑ unidade

3 dezenas

2 centenas

1 milhar

$$= 1000 + 2\times 100 + 3\times 10 + 4\times 1$$

$$= 1\times 10^3 + 2\times 10^2 + 3\times 10^1 + 4\times \underbrace{10^0}_{=1}$$

$$10^3 = 10\times 10\times 10 = 1\,000$$

$$10^2 = 10\times 10 = 100$$

ex: $71.521 = 1.10^0 + 2.10^1 + 5.10^2 +$

$+ 1.10^3 + 7.10^4 =$

$= 1 + 20 + 500 + 1000 +$

$+ 70000$

$7 \to 10^4$, $1 \to 10^3$, $5 \to 10^2$, $2 \to 10^1$, $1 \to 10^0$

ex: 110101 em base binária

$110101 = 1\times 2^0 + 0\times 2^1 + 1\times 2^2 + 0\times 2^3 +$

$+ 1\times 2^4 + 1\times 2^5 =$

$= 1\times 1 + 0 + 1\times 4 + 0 + 1\times 16 + 1\times 32 =$

$= 1 + 4 + 16 + 32 = 53$

BASE BINÁRIA

É uma base em que qualquer número aparece escrito apenas c/ os números 0 e 1.

O valor de cada algarismo corresponde ao produto do algarismo pela potência 2^n onde n é a posição do algarismo qd lido da direita para a esquerda.

ex :

$$(\underbrace{1}_{2^3}\;\underbrace{0}_{2^2}\;\underbrace{1}_{2^1}\;\underbrace{1}_{2^0})_2 = 1 \times 2^0 + 1 \times 2^1 + 0 \times 2^2 + 1 \times 2^3 =$$

$$= 1 + 2 + 0 + 8 = 11$$

Dec: 1 , 10 , 100 , 1000 , 10 000

bin: 1 , 2 , 4 , 8 , 16 , 32 , ...

$(40)_2 = ?$

$40 = 8 + 32 = 2^3 + 2^5$

$(40)_2 = \underset{2^5}{\underline{1}}\ \underset{2^4}{\underline{0}}\ \underset{2^3}{\underline{1}}\ \underset{2^2}{\underline{0}}\ \underset{2^1}{\underline{0}}\ \underset{2^0}{\underline{0}}$