

# 4.2 - 2020.11.27 Aula 2 - exercicios

$$\frac{\frac{3 - \frac{1}{2}x}{\frac{1}{1} \text{ (2)} \frac{1}{1}}}{-\frac{3x}{\frac{1}{1} \text{ (5)}} + \frac{1}{5}} = \frac{\frac{2 - \frac{1}{3}}{\frac{1}{1} \text{ (3)}}}{3} \quad \Leftrightarrow$$

$$\Leftrightarrow \frac{\frac{\frac{6}{2} - \frac{x}{2}}{-\frac{15x}{5} + \frac{1}{5}}}{3} = \frac{\frac{6}{3} - \frac{1}{3}}{3} \quad \Leftrightarrow$$

$$\Leftrightarrow \left( \frac{\frac{6-x}{2}}{-\frac{15x+1}{5}} \right) = \left( \frac{\frac{5}{3}}{\frac{3}{1}} \right)$$

$$\frac{(6-x)5}{2(-15x+1)} = \frac{5}{9} \quad \Leftrightarrow$$

NOTA:

$$\frac{a}{b} = \frac{c}{d}$$

ssr

$$ad = bc$$

$$\Leftrightarrow \frac{30-5x}{-30x+2} = \frac{5}{9} \quad \Leftrightarrow$$

$$\Leftrightarrow (30-5x)9 = 5(-30x+2) \quad \Leftrightarrow$$

$$\Leftrightarrow 270 - 45x = -150x + 10 \quad \Leftrightarrow$$

$$\Leftrightarrow 150x - 45x = 10 - 270 \quad \Leftrightarrow$$

$$\Leftrightarrow \underline{105} X = -260 \Leftrightarrow$$

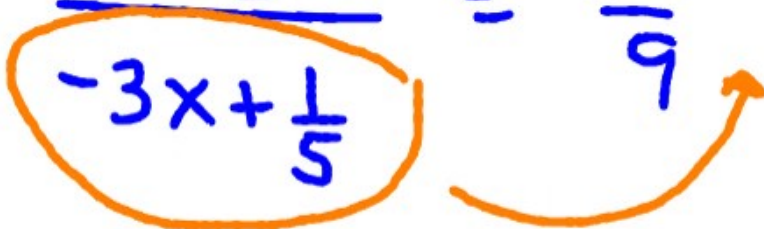

$$\Leftrightarrow X = -\frac{260}{105} = -\frac{52}{21}$$

4.2

Outra maneira

$$\frac{3 - \frac{1}{2}x}{-3x + \frac{1}{5}} = \frac{2 - \frac{1}{3}}{3} \quad \Leftrightarrow$$

$$\Leftrightarrow \frac{3 - \frac{1}{2}x}{-3x + \frac{1}{5}} = \frac{\frac{6}{3} - \frac{1}{3}}{3} = \frac{\frac{5}{3}}{\frac{3}{1}} = \frac{5}{9}$$

$$\Leftrightarrow \frac{3 - \frac{1}{2}x}{-3x + \frac{1}{5}} = \frac{5}{9} \quad \Leftrightarrow$$


$$\Leftrightarrow 3 - \frac{1}{2}x = \frac{5}{9} \left( -3x + \frac{1}{5} \right) \Leftrightarrow$$

$$\Leftrightarrow 3 - \frac{1}{2}x = - \frac{5 \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3}} x + \frac{\cancel{5}}{9 \cdot \cancel{5}} \Leftrightarrow$$

$$\Leftrightarrow 3 - \frac{1}{2}x = -\frac{5}{3}x + \frac{1}{9} \Leftrightarrow$$

$$\Leftrightarrow \underset{(3)}{-\frac{1}{2}x} + \underset{(2)}{\frac{5}{3}x} = \frac{1}{9} - \underset{(9)}{3} \Leftrightarrow$$

$$\Leftrightarrow -\frac{3x}{6} + \frac{10x}{6} = \frac{1}{9} - \frac{27}{9} \Leftrightarrow \dots$$

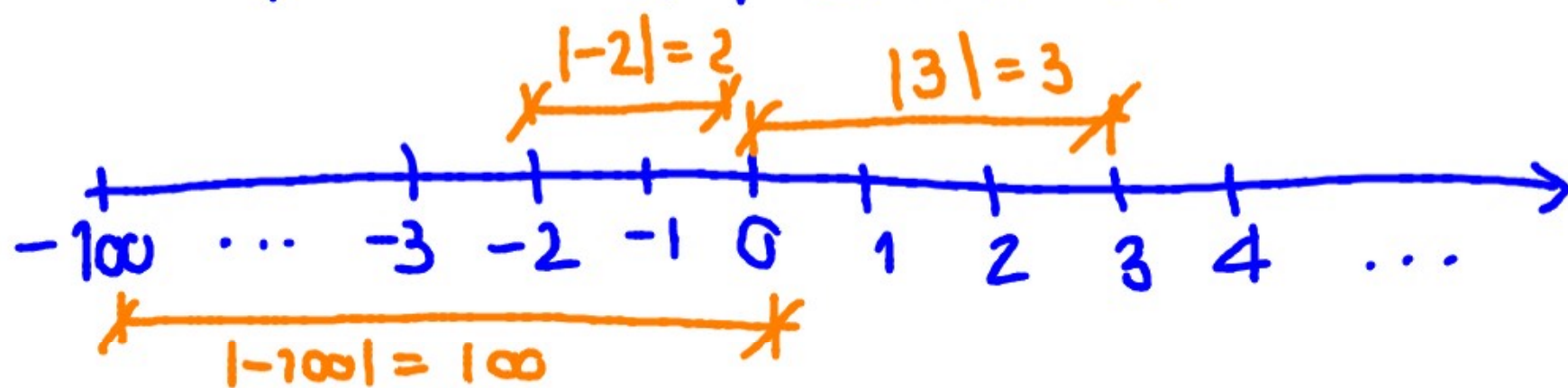


4.3  $\frac{2}{3-x} = \frac{5}{2} \Leftrightarrow 2 = \frac{5}{2} (3-x) \Leftrightarrow$

$\Leftrightarrow 4 = 5(3-x) \Leftrightarrow \frac{4}{5} = 3-x$

# MÓDULO OU VALOR ABSOLUTO

NOTAÇÃO :  $|x| \equiv$  módulo de  $x$



A cada número está associado  
o seu módulo

## Propriedades :

$$\bullet \quad |x| \geq 0 \quad \forall x$$

$$\bullet \quad |x| = x \quad \text{se} \quad x \geq 0$$

$$\bullet \quad |x| = -x \quad \text{se} \quad x < 0$$

ex:  $| -2 | = 2$

$| -2 | = -(-2) = 2$

$$\bullet \quad |x| = 0 \Leftrightarrow x = 0$$



$$\text{ex : } |-121| = 121$$

$$|3\pi| = 3\pi \quad \pi \approx 3,14$$

$$\left|\frac{5}{2}\right| = \frac{5}{2}$$

$$|(-2) \cdot 3| = \underset{-+ = -}{|-6|} = 6$$

$$\left|\frac{-1}{-2}\right| = \left|\frac{1}{2}\right| = \frac{1}{2}$$

$$|2-2| = |0| = 0$$

$$|x| = \begin{cases} x, & \text{se } x \geq 0 \\ -x, & \text{se } x < 0 \end{cases}$$

ex :  $|2x| = \begin{cases} 2x, & \text{se } 2x \geq 0 \\ -2x, & \text{se } 2x < 0 \end{cases}$

CA :  $\underline{2x \geq 0} \Leftrightarrow x \geq \frac{0}{2} = 0$

$\underline{2x < 0} \Leftrightarrow x < \frac{0}{2} = 0$

$$|2x| = \begin{cases} 2x & , x \geq 0 \\ -2x & , x < 0 \end{cases}$$

ex:  $|x-1| = \begin{cases} x-1 & , \text{ se } \underline{x-1} \geq 0 \\ -(\underline{x-1}) & , \text{ se } \underline{x-1} < 0 \end{cases}$

$$= \begin{cases} x-1 & , \text{ se } x \geq 1 \\ -x+1 & , \text{ se } x < 1 \end{cases}$$

$$\text{ex: } \left| \frac{x+1}{3} \right| = \begin{cases} \frac{x+1}{3} , & \text{se } \frac{x+1}{3} \geq 0 \\ -\frac{x+1}{3} , & \text{se } \frac{x+1}{3} < 0 \end{cases}$$

$$= \begin{cases} \frac{x+1}{3} , & \text{se } x+1 \geq 0 \\ -\frac{x+1}{3} , & \text{se } x+1 < 0 \end{cases}$$

$$= \begin{cases} \frac{x+1}{3} , & \text{se } x \geq -1 \\ -\frac{x+1}{3} , & \text{se } x < -1 \end{cases}$$

ex:  $|x| = 3$



$$|x| = 3 \Leftrightarrow x = 3 \quad \underline{\vee} \quad x = -3$$

$\vee \equiv \text{OU}$

$$|x| = \begin{cases} x & , \text{ se } x \geq 0 \\ -x & , \text{ se } x < 0 \end{cases}$$



Se  $x \geq 0$  :

$$|x|=3 \Leftrightarrow x=3 \geq 0 \quad \checkmark$$

Logo  $x=3$  é solução

Se  $x < 0$  :

$$|x|=3 \Leftrightarrow \underline{-1}x = 3 \Leftrightarrow x = \frac{3}{-1} = -3$$

$x = -3 < 0$  , logo  $x = -3$  é solução

Conclusão :  $|x|=3 \Leftrightarrow x=3 \vee x=-3$

ex:  $\underbrace{\left| \frac{-x + \frac{5}{2}}{\frac{2}{3}} \right|}_{\geq 0} = -1$  impossível

ex:  $2|x+3| - 2 = 5$

$$|x+3| = \begin{cases} x+3 & , \text{ se } x+3 \geq 0 \\ -1(x+3) & , \text{ se } x+3 < 0 \end{cases}$$

$$= \begin{cases} x+3 & , \text{ se } \boxed{x \geq -3} \\ -x-3 & , \text{ se } x < -3 \end{cases}$$

Se  $x \geq -3$  :

$$2|x+3|-2=5 \Leftrightarrow 2(x+3)-\underline{2}=5 \Leftrightarrow$$

$$\Leftrightarrow \underline{2(x+3)} = 5+2=7 \Leftrightarrow x+3 = \frac{7}{2} \Leftrightarrow$$

$$\Leftrightarrow x = \frac{7}{2} - \frac{3}{1}_{(2)} = \frac{7}{2} - \frac{6}{2} = \frac{1}{2} > -3$$

Logo  $x = \frac{1}{2}$  é solucció

Se  $x < -3$ :

$$2|x+3| - 2 = 5 \Leftrightarrow 2(-x-3) - 2 = 5 \Leftrightarrow$$

$$\Leftrightarrow 2(-x-3) = 5+2=7 \Leftrightarrow$$

$$\Leftrightarrow -2x - 6 = 7 \Leftrightarrow -2x = 7+6=13 \Leftrightarrow$$

$$\Leftrightarrow x = \frac{13}{-2} = -\frac{13}{2} = -6,5 < -3 \quad \checkmark$$

Logo  $x = -\frac{13}{2}$   
é solução  $\parallel \frac{13}{2} = \frac{12}{2} + \frac{1}{2} = 6 + \frac{1}{2} = 6,5$

Conclusi

$$2|x+3|-2=5 \quad (\Rightarrow) \quad x = \frac{1}{2} \vee x = -\frac{13}{2}$$

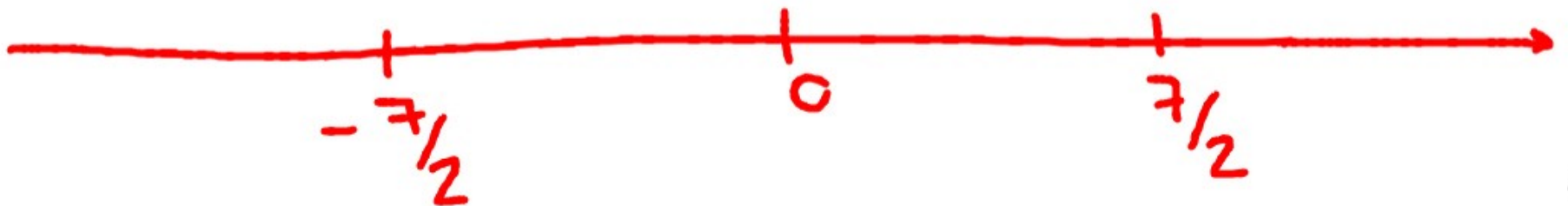


Outra maneira,

$$2|x+3| - 2 = 5 \Leftrightarrow$$

$$\Leftrightarrow 2|x+3| = 7 \Leftrightarrow |x+3| = \frac{7}{2}$$

$$|y| = \frac{7}{2} \Leftrightarrow y = \frac{7}{2} \vee y = -\frac{7}{2}$$




$$\Leftrightarrow \quad \underline{X+3} = \underline{\frac{7}{2}} \quad \vee \quad \underline{X+3} = \underline{-\frac{7}{2}} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad X = \frac{7}{2} - 3_{(2)} \quad \vee \quad X = -\frac{7}{2} - 3_{(2)} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad X = \frac{7}{2} - \frac{6}{2} \quad \vee \quad X = -\frac{7}{2} - \frac{6}{2} \quad \Leftrightarrow$$

$$\Leftrightarrow \quad X = \frac{1}{2} \quad \vee \quad X = -\frac{13}{2}$$

$$\text{ex: } -7 + 2|3x+1| = \frac{1}{2} - \frac{7}{2} \Leftrightarrow$$

$$\Leftrightarrow \underline{-7} + 2|3x+1| = -\frac{6}{2} = -3 \Leftrightarrow$$


$$\Leftrightarrow \underline{2|3x+1|} = -3 + 7 = 4 \Leftrightarrow$$


$$\Leftrightarrow |3x+1| = \frac{4}{2} = 2 \Leftrightarrow$$

$$\Leftrightarrow 3x+1=2 \quad \vee \quad 3x+1=-2 \Leftrightarrow$$

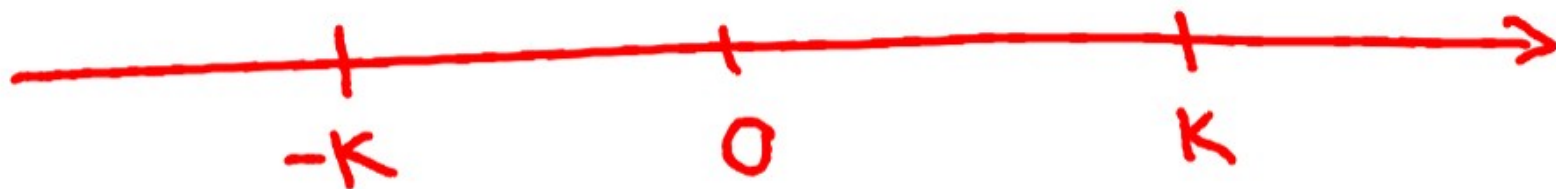
$$\Leftrightarrow 3x + \underline{1} = 2 \quad \vee \quad 3x + \underline{1} = -2 \quad (\Leftrightarrow)$$

$$\Leftrightarrow \underline{3x} = 2 - 1 = 1 \quad \vee \quad \underline{3x} = -2 - 1 = -3 \quad (\Leftrightarrow)$$

$$\Leftrightarrow x = \frac{1}{3} \quad \vee \quad x = -\frac{3}{3} = -1$$

ex:  $|x+1| = |x| \Leftrightarrow |y| = k$

$\underbrace{x+1}_y = \underbrace{|x|}_k$



$$\Leftrightarrow y = k \vee y = -k \Leftrightarrow$$

$$\Leftrightarrow \underline{x+1 = |x|} \vee \underline{x+1 = -1|x|} \Leftrightarrow$$

$$\Leftrightarrow \left( x = x+1 \vee x = -(x+1) \right) \vee$$

$$|x| = k \Leftrightarrow$$

$$\Leftrightarrow x = k \vee x = -k$$

$$|x| = -(x+1) \Leftrightarrow$$



$$\Leftrightarrow \left( \cancel{x} = \cancel{x} + 1 \vee \underbrace{x = -x - 1} \right) \vee \\ \vee \left( x = -\underbrace{(x+1)} \vee \cancel{x} = \cancel{x} + 1 \right) \Leftrightarrow$$

$$|x| = k \Leftrightarrow x = k \vee x = -k$$

$$\Leftrightarrow \left( \underbrace{0 = 1}_{\text{imp}} \vee 2x = -1 \right) \vee \\ \vee \left( x = -x - 1 \vee \underbrace{0 = 1}_{\text{imp}} \right) \Leftrightarrow$$

$$\Leftrightarrow 2x = -1 \vee 2x = -1 \Leftrightarrow \underbrace{2x = -1} \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{1}{2} //$$

## PROPRIEDADES

$$\cdot |x| \geq 0 \quad \forall x \in \mathbb{R}$$

$$\cdot |a| \cdot |b| = |ab|$$

$$\text{ex: } |-2 \times 3| = |-2| \times |3| = 2 \times 3 = 6$$

$$|2x| = 2|x|$$

$$\cdot \frac{|a|}{|b|} = \left| \frac{a}{b} \right|$$

$$\text{ex: } \left| \frac{-2}{3} \right| = \frac{|-2|}{|3|} = \frac{2}{3}$$

ex: Outra maneira

$$|x+1| = |x| \cdot 1 \quad (*)$$

Se  $x \neq 0$

$$\text{Se } x=0 : |0+1| = |0| \quad (*) \quad 1=0$$

imp

Logo  $x=c \neq 0$  é solução

Assim, podemos assumir que  $x \neq 0$

$$|x+1| = |x| \cdot 1 \quad \Leftrightarrow \quad \frac{|x+1|}{|x|} = 1 \quad \Leftrightarrow$$

pq  
 $x \neq 0$

$\frac{|a|}{|b|} = \left| \frac{a}{b} \right|$

$$\Leftrightarrow \left| \frac{x+1}{x} \right| = 1 \quad \Leftrightarrow \quad \frac{x+1}{\textcircled{x}} = 1 \quad \vee \quad \frac{x+1}{\textcircled{x}} = -1$$

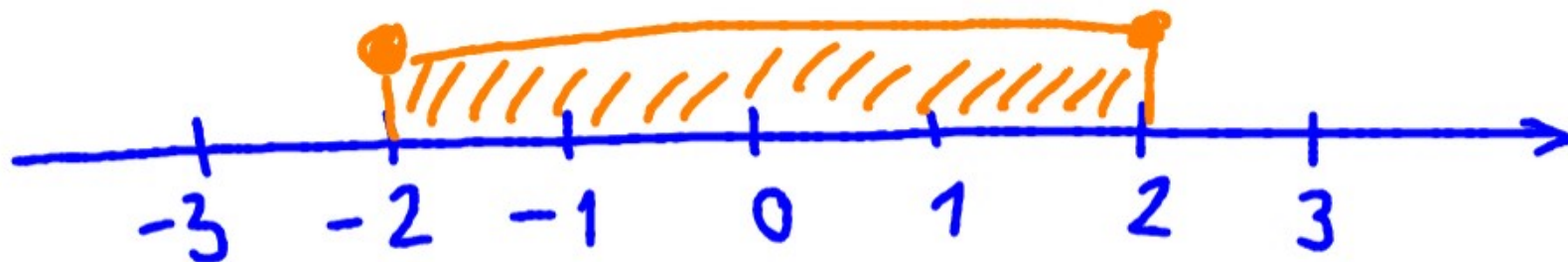
$$\Leftrightarrow \cancel{x+1} = \cancel{x} \quad \vee \quad x + \underbrace{1 = -x} \quad \Leftrightarrow$$

$$\Leftrightarrow \underbrace{1=0}_{\text{imp}} \quad \vee \quad 2x = -1 \quad \Leftrightarrow \quad \underline{2x = -1} \quad \Leftrightarrow$$

$$\Leftrightarrow x = -\frac{1}{2} //$$

## Inequações

ex:  $|x| \leq 2$



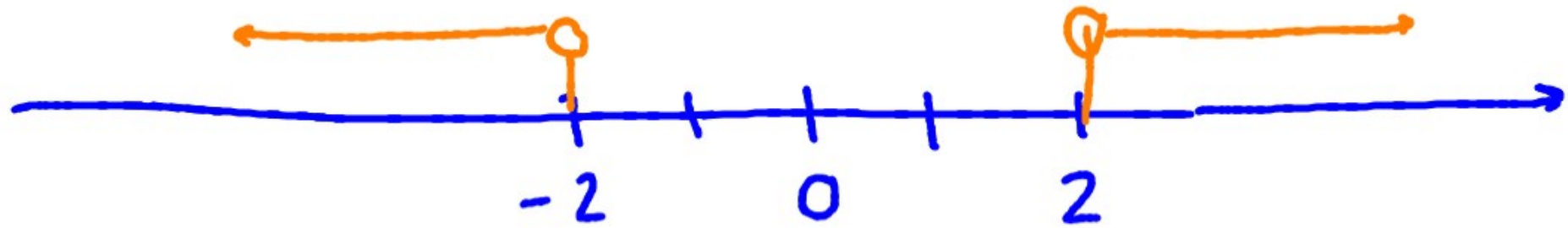
$$|x| \leq 2 \Leftrightarrow -2 \leq x \leq 2 \quad (\Leftrightarrow)$$

$$\Leftrightarrow x \leq 2 \quad \wedge \quad x \geq -2$$

$\underbrace{\quad}_{\wedge \text{ "e"}}$



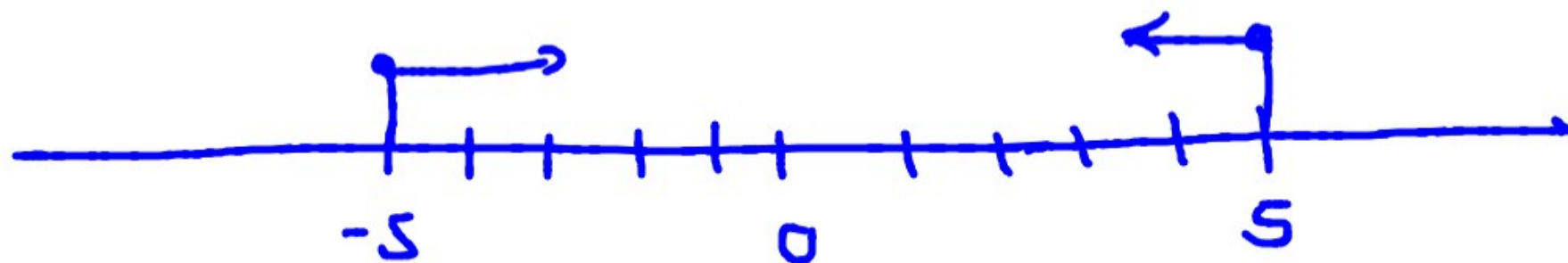
ex:  $|x| > 2$



$$|x| > 2 \Leftrightarrow x > 2 \vee x < -2$$

ou

ex:  $|x - 3| \leq 5$



$$|x - 3| \leq 5 \Leftrightarrow -5 \leq x - 3 \leq 5 \quad (=)$$

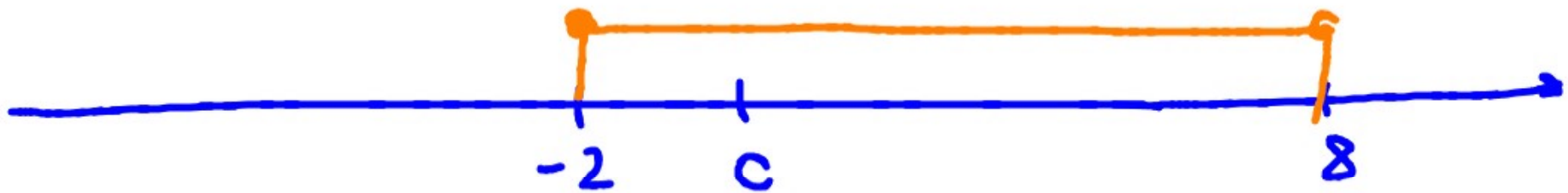
$$(\Rightarrow) \quad \underline{x - 3} \leq 5 \quad \wedge \quad \underline{x - 3} \geq -5 \quad (\Rightarrow)$$

$$\Leftrightarrow x - \cancel{3} + \cancel{3} \leq 5 + 3 \quad \wedge \quad x - 3 + 3 \geq -5 + 3$$

$$\Leftrightarrow x \leq 8 \quad \wedge \quad x \geq -2$$

$$\Leftrightarrow -2 \leq x \leq 8 \quad \Leftrightarrow$$

$$(\Rightarrow) x \in [-2, 8]$$



$\in$  "pertence"

$\notin$  "não pertence"

ex:  $2|x-2| > \underline{|x|} \cdot 1$

↖  
se  $|x| \neq 0$ , ou seja  
se  $x \neq 0$

Se  $x=0$ ,  $2|0-2| > |0| \Leftrightarrow$   
 $\Leftrightarrow 2|-2| > 0 \quad (\Rightarrow) 4 > 0 \quad \checkmark$

Logo  $x=0$  É SOLUÇÃO

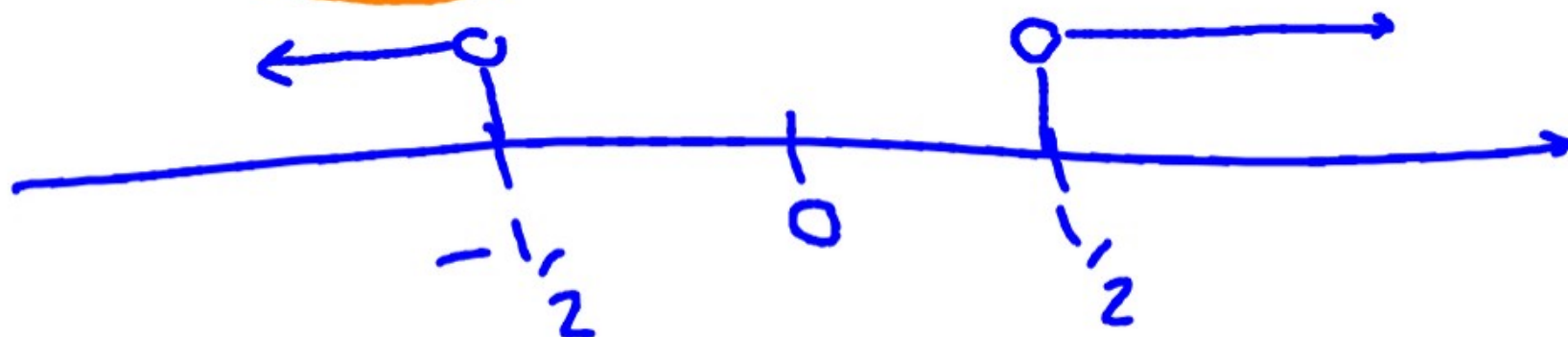
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Se  $x \neq 0$ ,  $2 \frac{|x-2|}{|x|} > 1 \Leftrightarrow$   
 $|x| > 0$


$$\Leftrightarrow 2 \left| \frac{x-2}{x} \right| > 1 \Leftrightarrow \left| \frac{x-2}{x} \right| > \frac{1}{2}$$





$$\Leftrightarrow \frac{x-2}{x} > \frac{1}{2} \vee \frac{x-2}{x} < -\frac{1}{2}$$

$$\bullet \quad \frac{x-2}{x} > \frac{1}{2} \Leftrightarrow \frac{x}{x} - \frac{2}{x} > \frac{1}{2} \Leftrightarrow$$

$$\Leftrightarrow 1 - \frac{2}{x} > \frac{1}{2} \Leftrightarrow -\frac{2}{x} > \frac{1}{2} - 1$$


$$\Leftrightarrow -\frac{2}{x} > -\frac{1}{2} \Leftrightarrow \frac{2}{x} < \frac{1}{2} \Leftrightarrow$$


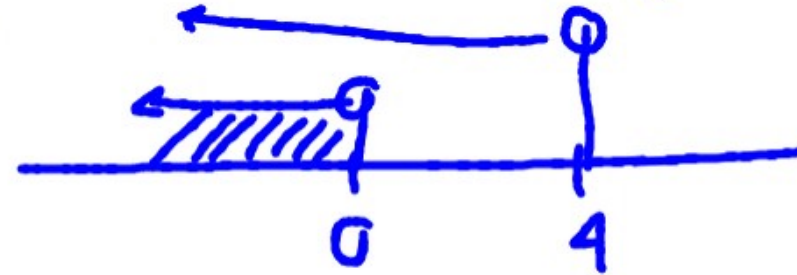
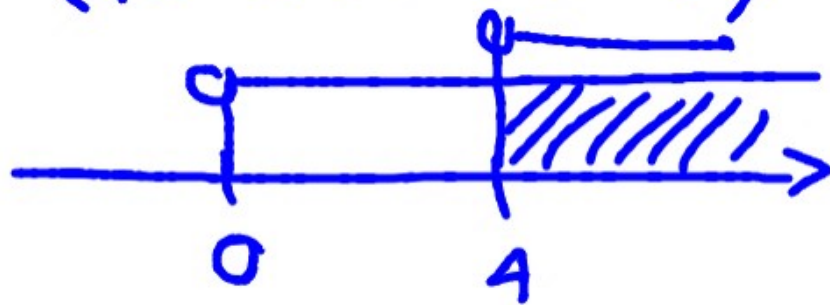
$$\Leftrightarrow \left( x > 0 \wedge 2 < \frac{1}{2}x \right) \vee$$


$$\vee \left( x < 0 \wedge 2 > \frac{1}{2}x \right) \Leftrightarrow$$


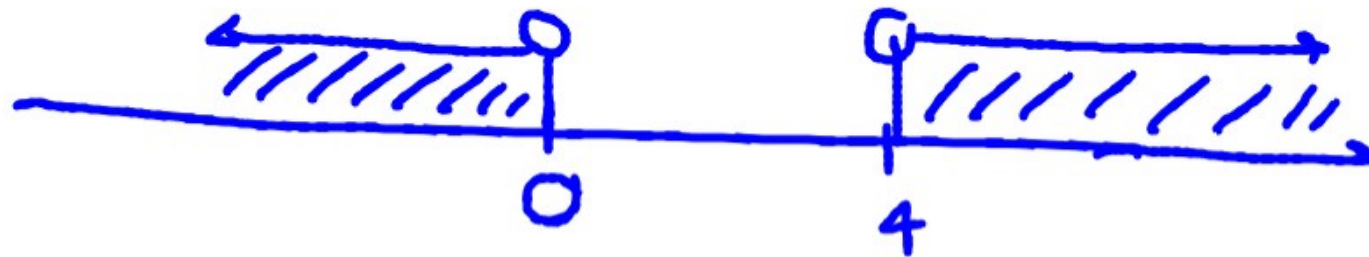


$$\Leftrightarrow (x > 0 \wedge 4 < x) \vee (x < 0 \wedge 4 > x) \Leftrightarrow$$

$$\Leftrightarrow (x > 0 \wedge x > 4) \vee (x < 0 \wedge x < 4)$$



$$\Leftrightarrow x > 4 \vee x < 0 \Leftrightarrow$$



$$\Leftrightarrow x \in ]-\infty, 0[ \cup ]4, +\infty[$$

## PROPORCIONALIDADE DIRETA

Venda de livros todos a  
10 €.

Se comprar 1 livro, pago 10€

" " 2 livros, " 20€

" " 3 " , " 30€

" " n " , n.10

razão de proporcionalidade = 10

$$\frac{\cancel{1} 10}{\cancel{1}} = 10$$

• Quanto pago por 7 livros?

Regra de 3 simples:

$$\begin{array}{cc} 1 & 10 \\ 7 & x \end{array}$$

$$1 \cdot x = 7 \cdot 10 \Leftrightarrow x = 70$$

$$\begin{array}{rcl} 10 & - & 50 \\ 3 & - & X \end{array}$$

$$10x = 150 \quad (=)$$

$$\Rightarrow x = \frac{150}{10} = 15$$

