Reprova

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Exercício 1

Considere as bases do R-espaço vetorial R^3 , A = {(4, 2, 0),(1, -1, 1),(5, 3, 3)} e
B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base $M_{B \to A}$ e $M_{A \to B}$.
Escreva também os vetores abaixo nas bases indicadas:

•v = $(0, 1, 2)_A$ em B
•v = $(1, 3, 1)_B$ em A

Resposta:

A)(4,2,0),(1,-1,1),(5,3,3)
B)(1,-2,1),(1,5,2),(1,0,1)

•$M_{A \to B}(1, -2, 1)$

$$\left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 1 \end{array}\right] 1) l_2 + l_3 \to l_2 \left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & -1 \\ 0 & 1 & 3 & 1 \end{array}\right] 2) l_2 \leftrightarrow l_1 \left[\begin{array}{ccc|c} 2 & 0 & 6 & -1 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 1 \end{array}\right] 3) l_2 - 2 \cdot l_1 \to l_2$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & -1 \\ 0 & 1 & -7 & 1 \\ 0 & 1 & 3 & 1 \end{array}\right] \quad 4) l_2 - l_3 \to l_3 \left[\begin{array}{ccc|c} 2 & 0 & 6 & -1 \\ 0 & 4 & -7 & 3 \\ 0 & 0 & 10 & -2 \end{array}\right] \quad 5) 10 \cdot l_2 - 7 \cdot l_3 \to l_2$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & -1 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array}\right] \quad 6) 10 \cdot l_1 - 6 \cdot l_3 \to l_1 \left[\begin{array}{ccc|c} 20 & 0 & 0 & 2 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array}\right] \quad 7) l_1/20 \to l_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 1 & \frac{-2}{10} \end{array}\right] \quad 8) l_2/10 \to l_2 \qquad \mathbf{x} = \frac{2}{20} = \frac{1}{10}, \quad \mathbf{y} = \frac{16}{10} = \frac{8}{5}, \quad \mathbf{z} = \frac{-2}{10} = \frac{1}{5}$$

•(1,5,2)

$$\left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 5 \\ 0 & 1 & 3 & 2 \end{array}\right] \quad 1) l_2 \leftarrow l_2 + l_3 \left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 2 \\ 0 & 1 & 3 & 2 \end{array}\right] \quad 2) l_1 \leftrightarrow l_2$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 7 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 2 \end{array}\right] \quad 3) l_2 \leftarrow l_2 - 2 \cdot l_1 \left[\begin{array}{ccc|c} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 1 & 3 & 2 \end{array}\right] \quad 4) l_3 \leftarrow l_3 - l_2$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 10 & 15 \end{array}\right] \quad 5) l_3 \leftarrow l_3 \div 10 \left[\begin{array}{ccc|c} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{15}{10} \end{array}\right] \quad simp. \tfrac{15}{10} por 5$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right] \quad 6) l_2 \leftarrow l_2 + 7 \cdot l_3 \left[\begin{array}{ccc|c} 2 & 0 & 6 & 7 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right] \quad 7) l_l \leftarrow l_1 - 6 \cdot l_3$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right] \quad 8) l_1 \leftarrow l_1 \div 2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array}\right]$$

• (1,0,1)

$$\left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 3 & 1 \end{array}\right] \quad 1) l_2 \leftarrow l_2 + l_3 \left[\begin{array}{ccc|c} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 1 \\ 0 & 1 & 3 & 1 \end{array}\right] \quad 2) l_1 \leftrightarrow l_2 \left[\begin{array}{ccc|c} 2 & 0 & 6 & 1 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 1 \end{array}\right] \quad 3) l_2 \leftarrow l_2 - 2 \cdot l_1$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & 1 \\ 0 & 1 & 3 & 1 \end{array}\right] \quad 4) l_3 \leftarrow l_3 - l_2 \left[\begin{array}{ccc|c} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 10 & 2 \end{array}\right] \quad 5) l_3 \leftarrow l_3 \div 10$$

$$\left[\begin{array}{ccc|c} [2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{array}\right] \quad 6) l_1 \leftarrow l_1 - 6 \cdot l_3 \left[\begin{array}{ccc|c} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{array}\right] \quad 7) l_2 \leftarrow l_2 + 7 \cdot l_3$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{10} \end{array}\right] \quad 8) l_1 \leftarrow l_1 \div 2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-1}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{10} \end{array}\right] \quad simp. \tfrac{2}{10} por 2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-1}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right]$$

$\bullet M_{B\to A}(4,2,0)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad l_3 \leftarrow 5\cdot l_3 - 2\cdot l_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{array}\right] \quad l_1 \leftarrow 5\cdot l_1 - l_2$$

$$\left[\begin{array}{ccc|c} 7 & 0 & 5 & 18 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & 4 \end{array}\right] \quad l_1 \leftarrow l_1 - l_3 \left[\begin{array}{ccc|c} -2 & 0 & 0 & 22 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{array}\right] \quad l_2 \leftarrow l_2 - l_1$$

$$\left[\begin{array}{ccc|c} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 9 & 0 & 5 & -4 \end{array}\right] \quad l_3 \leftarrow 2\cdot l_3 + 9\cdot l_1 \left[\begin{array}{ccc|c} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{array}\right] \quad l_1 \leftarrow l_1 \div 2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 19 \end{array}\right] \quad l_2 \leftarrow l_2 \div 2 \qquad \mathbf{x} = -11, \quad \mathbf{y} = -4, \quad \mathbf{z} = 19$$

• **(1,-1,1)**

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 1 & 2 & 1 & 1 \end{array}\right] \quad 1) l_2 l_3 \leftarrow l_3 - l_1 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{array}\right] \quad 2) l_2 \leftrightarrow l_3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -2 & 5 & 0 & -1 \end{array}\right] \quad 3) l_1 \leftrightarrow l_3 \left[\begin{array}{ccc|c} -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \quad 4) l_1 \leftarrow -l_1$$

$$\left[\begin{array}{ccc|c} 2 & -5 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \quad 5) l_1 \leftarrow l_1 + l_2 \left[\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \quad 6) l_1 \leftarrow l_1 \div 2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \quad 7) l_3 \leftarrow l_3 - l_2 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{array}\right] \quad 8) l_3 \leftarrow l_3 - l_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

$\mathbf{x} = \frac{1}{2}, \quad \mathbf{y} = 0, \quad \mathbf{z} = \frac{1}{2}$

•(5,3,3)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 2 & 5 & 0 & 3 \\ 1 & 2 & 1 & 3 \end{array}\right] \quad 1) l_3 \leftarrow 5 \cdot l_3 - 2 \cdot l_2 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{array}\right] \quad 2) l_1 \leftarrow l_1 - l_2$$

$$\left[\begin{array}{ccc|c} 7 & 0 & 5 & 22 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{array}\right] \quad 3) l_1 \leftarrow l_1 - l_3 \left[\begin{array}{ccc|c} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 9 & 0 & 5 & 9 \end{array}\right] \quad 4) l_2 \leftarrow l_2 - l_1$$

$$\left[\begin{array}{ccc|c} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{array}\right] \quad 5) l_3 \leftarrow 2 \cdot l_3 + 9 \cdot l_1 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{array}\right] \quad 6) l_1 \leftarrow l_1 \div (-2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 10 & 135 \end{array}\right] \quad 7) l_2 \leftarrow l_2 \div 5 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{27}{2} \end{array}\right] \quad l_3 \leftarrow l_3 \div 10$$

$\mathbf{x} = \frac{-13}{2}, \quad \mathbf{y} = -2, \quad \mathbf{z} = \frac{27}{2}$

Exercício 2

Considere o conjunto $S = \{$(1, 1, 1, 1, 1),(2, 0, -1, 1, 3),(3, 1, 0, 2, 4),(2, 2, 5, 8, -1),(0, 1, 0, 2, 3)$\}$.

(a) S é li ou ld?
(b) S forma uma base do R-espaço vetorial R^5?

Resposta :

$a_1 = (1, 1, 1, 1, 1)$
$a_2 = (2, 0, -1, 1, 3)$
$a_3 = (2, 1, 0, 2, 4)$
$a_4 = (2, 2, 5, 8, -1)$
$a_5 = (0, 1, 0, 2, 3)$

$a_1 + 2a_2 + 3a_3 + 2a_4 = 0$
$a_1 + a_3 + 2a_4 + a_5 = 0$
$a_1 - a_2 + 5a_4 = 0$
$a_1 + a_2 + 2a_3 + 8a_4 + 2a_5 = 0$
$a_1 + 3a_2 + 4a_3 - a_4 + 3a_3 = 0$

(a$_1+2a_2+3a_3+2a_4=0, a_1+a_3+2a_4+a_5=0, a_1-a_2+5a_4=0, a_1+a_2+2a_3+8a_4+2a_3=0, a_1+3a_2+4a_3-a_4+3a_5=0)$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] \quad l_2 \to l_2 - 1 \cdot l_1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] \quad l_3 \to l_3 - 1 \cdot l_1$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] \quad l_4 \to l_4 - 1 \cdot l_1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] \quad l_5 \to l_5 - 1 \cdot l_1$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] \quad l_2 \to l_2 \cdot (\tfrac{-1}{2}) \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0\frac{-1}{2} & 0 & \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] \quad l_3 \to l_3 \cdot (\tfrac{-1}{3})$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] \quad l_4 \to l_4 \cdot (-1) \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -6 & -2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] \quad l_3 \to l_3 - 1 \cdot l_2$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 1 & 1 & -6 & -2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] \quad l_4 \to l_2 \cdot l_4 - 1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -6 & \frac{-3}{2} & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] \quad l_5 \to l_2 \cdot l_5 - 1$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -6 & \frac{-3}{2} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right] \quad l_3 \to l_3 \cdot (-1) \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & -6 & \frac{-3}{2} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right] \quad l_3 \to l_4 \cdot (\tfrac{-1}{6})$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & -3 & \frac{7}{2} & 0 \end{array}\right] \quad l_5 \to l_5 \cdot (\tfrac{-1}{3}) \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 1 & \frac{-7}{6} & 0 \end{array}\right] \quad l_5 \to l_3 \cdot l_5 - 1$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & \frac{-2}{3} & 0 \end{array}\right] \quad l_4 \to l_4 \cdot \tfrac{4}{3} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{-2}{3} & 0 \end{array}\right] \quad l_5 \to l_5 \cdot (\tfrac{-3}{2})$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad l_5 \to l_4 \cdot l_5 - 1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{-1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Exercício 3

Considere o conjunto $W = \{(x, y, z, w, t, u) | x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R6$. Mostre que o conjunto W é um subespaço do R-espaço vetorial $R6$.

w = x - t
y - x + t - z = 0 $\to y = z - t + x$
$x + z - t + x + x - t + z + t + u = 0 \to 3x + 2z - t + u = 0 \to u = t - 2z - 3x$
$W = \{(x, z - t + x, z, x - t, t, t - 2z - 3x) | x, z, t \in R)\}$

1)$0 \in W para x = 0 z = 0 t = 0 : (0, 0 - 0 + 0, 0, 0 - 0, 0, 0 - 20 - 30) = (0, 0, 0, 0, 0, 0)$
$Logo, 0 \in W$

2) V U $\in W \to V + U \in W$:
V = (x1, z1-t1 + x1, z1, x1 - t1, t1, t1 - 2z1 - 3x1)

U = (x2, z2 - t2 + x2, z2, x2 - t2, t2, t2 - 2z2 - 3x2)

V + U = (x1, z1 -t1 + x1, z1, x1 - t1, t1, t1 - 2z1 - 3x1) + (x2, z2 - t2 + X2, z2, x2 - t2, t2, +2 - 2z2 - 3x2)

V + U = (x1 + x2, z1 - t1 + x1 + z2 - t2 + x2, z1 +Z2, x1 - t1 + x2-t2, t1 + t2, t1 - 2z1 - 3x1 + t2 - 2z2 - 3x2)

Logo V + U $\in W$

$3) a \in R, V \in W \rightarrow aV \in W:$

$V = (x, z - t + x, z, x - t, t, t - 2z - 3x)$

$a \cdot V = a \cdot (x, z - t + x, z, x - t, t, t - 2z - 3x)$

$a \cdot V = (ax, az - at + ax, az, ax - at, at, at - 2az - 3ax)$

$Logo, aV \in W$. Portanto, W é um subespaço vetorial de R6.

•O conjunto W = {(x, y, z)|x, y $\in R \wedge$ x z = 1 $\wedge$ y + x = 0} é um subespaço vetorial de R3? Esboce graficamente W.

$a) 0 \in W - (0, 0, 0)$

$x + y = 0$
$y + x = 0$
$x + z = 1$
supondo que$x = 1, y = -1$
$0 + (1, -1, z)$
$0 + 1 = 1$
$0 + (-1) = -1$

Logo, W não é subespaço vetorial de R3

•Invente seu subespaço vetorial em qualquer Rn com n $\geq$ 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova.

$S = \{(x, y) \in R2, y = 2x\})$

$i) 0 \in S = (0, 0)$
$x = 0$, então (0,0) $\in S$

$ii)$ Seja v$_1 = (x_1, 2x_1), ev_2 = (x_2, 2x_2)$

$v_1 + v_2 \in S \rightarrow (x_1, 2x_1) + (x_2 + 2x_2) = (x_1 + x_2, 2x_1 + 2x_2) = (x_1 + x_2, 2(x_1 + x_2)) \in S.$

$iii) \alpha v_1 \in S$

$\alpha(x_1, 2x_1) = (2x_1, \alpha(2x_1)) = (\alpha x_1, 2(x_1)) \in S$

Portanto, S é um subespaço vetorial de R2

Exercício 4

Mostre que o conjunto {(1, 1, 1, 1, 0, 1, 1),(1, 0, 1, 1, 1, 1, 0),(2, 2, 1, 1, 1, 1, 1), (1, 0, 0, 1, 2, 1, 1),(2, 0, 2, 0, 2, 0, 2),(1, 1, 1, 1, 1, 1, 1),(3, 0, 2, 0, 2, 1, 2)} forma uma base para o R-espaço vetorial R7. Escreva o vetor (0, 1, 1, 1, 1, 0, 1) nesta base.

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 1 & 0 & 2 & 0 & 0 & -1 & 0 & y \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & z \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & w \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 1 & -2 & 1 & 1 & 0 & -2 & -1 & u \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & t \end{array}\right] \quad 1) l_1 \leftarrow -1 \cdot l_1$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y \cdot (-x) \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & z \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & w \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & u \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & t \end{array}\right] \quad 2) l_2 \leftarrow l_2 - 1 \cdot l_1$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & w \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & u \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & t \end{array}\right] \quad 3) l_3 \leftarrow l_3 - 1\cdot l_1$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & w\cdot(-x) \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & u \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & t \end{array}\right] \quad 4) l-4 \leftarrow l_4 - 1\cdot l_1$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & w\cdot(-x) \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & t \end{array}\right] \quad 5) l_6 \leftarrow l_6 - 1\cdot l_1$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & w\cdot(-x) \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & t\cdot(-x) \end{array}\right] \quad 6) l_7 \leftarrow l_7 - 1\cdot l_1$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 1 & \frac{3}{2} & 1 & 1 & 0 & \frac{3}{2} & \frac{w\cdot(-x)}{2} \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & t\cdot(-x) \end{array}\right] \quad 7) l_4 \leftarrow l_4 \cdot \tfrac{1}{2}$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 0 & \frac{-5}{2} & 0 & -1 & 0 & \frac{-3}{2} & \frac{w\cdot(-2x)\cdot y}{2} \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & v \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & t\cdot(-x) \end{array}\right] \quad 8) l_4 \leftarrow l_4 - 1\cdot l_2$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 0 & \frac{-5}{2} & 0 & -1 & 0 & \frac{-3}{2} & \frac{w\cdot(-2x)\cdot y}{2} \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & v\cdot(-x) \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & t\cdot(-x) \end{array}\right] \quad 9) l_5 \leftarrow l_5 - 1\cdot l_2$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & z\cdot(-x) \\ 0 & 0 & \frac{-5}{2} & 0 & -1 & 0 & \frac{-3}{2} & \frac{w\cdot(-2x)\cdot y}{2} \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & v\cdot(-x) \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & t\cdot(-2x)\cdot y \end{array}\right] \quad 10) l_7 \leftarrow l_7 - 1\cdot l_2$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & \frac{-5}{2} & 0 & -1 & 0 & \frac{-3}{2} & \frac{w\cdot(-2x)\cdot y}{2} \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & v\cdot(-x) \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & t\cdot(-2x)\cdot y \end{array}\right] \quad 11) l_3 \leftarrow l_3 \cdot \tfrac{1}{3}$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 1 & 0 & \frac{2}{5} & 0 & \frac{3}{5} & \frac{w\cdot 2x\cdot y}{5} \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & v\cdot(-x) \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & t\cdot(-2x)\cdot y \end{array}\right] \quad 12) l_4 \leftarrow l_4 \cdot (\tfrac{-2}{5})$$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 1 & 0 & \frac{2}{5} & 0 & \frac{3}{5} & \frac{w\cdot 2x\cdot y}{5} \\
0 & 0 & 1 & \frac{-1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & v\cdot y\cdot x\cdot\frac{1}{5} \\
0 & 0 & 3 & 2 & 2 & 0 & 2 & u\cdot(-x) \\
0 & 0 & -1 & 1 & 2 & 2 & 2 & t\cdot(-2x)\cdot y
\end{array}\right] \quad 13) l_5 \leftarrow l_5 \cdot (\tfrac{-1}{5})$$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 1 & 0 & \frac{2}{5} & 0 & \frac{3}{5} & \frac{w\cdot 2x\cdot y}{5} \\
0 & 0 & 1 & \frac{-1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & v\cdot y\cdot x\cdot\frac{1}{5} \\
0 & 0 & 1 & \frac{2}{5} & \frac{2}{3} & 0 & \frac{2}{3} & u\cdot(-x)\cdot\frac{1}{3} \\
0 & 0 & -1 & 1 & 2 & 2 & 2 & t\cdot(-2x)\cdot y
\end{array}\right] \quad 14) l_6 \leftarrow l_6 \cdot \tfrac{1}{3}$$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 1 & 0 & \frac{2}{5} & 0 & \frac{3}{5} & \frac{w\cdot 2x\cdot y}{5} \\
0 & 0 & 1 & \frac{-1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & v\cdot y\cdot x\cdot\frac{1}{5} \\
0 & 0 & 1 & \frac{2}{5} & \frac{2}{3} & 0 & \frac{2}{3} & u\cdot(-x)\cdot\frac{1}{3} \\
0 & 0 & 1 & -1 & -2 & -2 & -2 & -t\cdot 2x\cdot(-y)
\end{array}\right] \quad 15) l_7 \leftarrow l_7 \cdot (-1)$$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 1 & \frac{-1}{3} & \frac{-14}{15} & 0 & \frac{-16}{15} & \frac{-w\cdot 2\cdot x^2\cdot y\cdot z}{15} \\
0 & 0 & 1 & \frac{-1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & v\cdot y\cdot x\cdot\frac{1}{5} \\
0 & 0 & 1 & \frac{2}{5} & \frac{2}{3} & 0 & \frac{2}{3} & u\cdot(-x)\cdot\frac{1}{3} \\
0 & 0 & 1 & -1 & -2 & -2 & -2 & -t\cdot 2x\cdot(-y)
\end{array}\right] \quad 16) l_4 \leftarrow l_4 - 1 \cdot l_3$$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 1 & \frac{-1}{3} & \frac{-14}{15} & 0 & \frac{-16}{15} & \frac{-w\cdot 2\cdot x^2\cdot y\cdot z}{15} \\
0 & 0 & 0 & \frac{-8}{15} & \frac{-4}{3} & \frac{1}{5} & \frac{-22}{15} & \frac{-v\cdot y\cdot x^2\cdot z}{15} \\
0 & 0 & 1 & \frac{2}{5} & \frac{2}{3} & 0 & \frac{2}{3} & u\cdot(-x)\cdot\frac{1}{3} \\
0 & 0 & 1 & -1 & -2 & -2 & -2 & -t\cdot 2x\cdot(-y)
\end{array}\right] \quad 17) l_5 \leftarrow l_5 - 1 \cdot l_3$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 1 & \frac{-1}{3} & \frac{-14}{15} & 0 & \frac{-16}{15} & \frac{-w\cdot 2\cdot x^2\cdot y\cdot z}{15} \\ 0 & 0 & 0 & \frac{-8}{15} & \frac{-4}{3} & \frac{1}{5} & \frac{-22}{15} & \frac{-v\cdot y\cdot x^2\cdot z}{15} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{-2}{3} & 0 & -1 & \frac{u\cdot x^2\cdot z}{9} \\ 0 & 0 & 1 & -1 & -2 & -2 & -2 & -t\cdot 2x\cdot(-y) \end{array}\right] \quad 18) l_6 \leftarrow l_6 - 1\cdot l_3$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 1 & \frac{-1}{3} & \frac{-14}{15} & 0 & \frac{-16}{15} & \frac{-w\cdot 2\cdot x^2\cdot y\cdot z}{15} \\ 0 & 0 & 0 & \frac{-8}{15} & \frac{-4}{3} & \frac{1}{5} & \frac{-22}{15} & \frac{-v\cdot y\cdot x^2\cdot z}{15} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{-2}{3} & 0 & -1 & \frac{u\cdot x^2\cdot z}{9} \\ 0 & 0 & 0 & \frac{-4}{3} & \frac{-10}{3} & -2 & \frac{-11}{3} & \frac{-t\cdot 2x^2\cdot y\cdot z}{3} \end{array}\right] \quad 19) l_7 \leftarrow l_7 - 1\cdot l_3$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & \frac{-8}{15} & \frac{-4}{3} & \frac{1}{5} & \frac{-22}{15} & \frac{-v\cdot y\cdot x^2\cdot z}{15} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{-2}{3} & 0 & -1 & \frac{u\cdot x^2\cdot z}{9} \\ 0 & 0 & 0 & \frac{-4}{3} & \frac{-10}{3} & -2 & \frac{-11}{3} & \frac{-t\cdot 2x^2\cdot y\cdot z}{3} \end{array}\right] \quad 20) l_4 \leftarrow l_4\cdot(-3)$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{8} & \frac{11}{4} & \frac{v\cdot y\cdot x^2\cdot z}{8} \\ 0 & 0 & 0 & \frac{1}{3} & \frac{-2}{3} & 0 & -1 & \frac{u\cdot x^2\cdot z}{9} \\ 0 & 0 & 0 & \frac{-4}{3} & \frac{-10}{3} & -2 & \frac{-11}{3} & \frac{-t\cdot 2x^2\cdot y\cdot z}{3} \end{array}\right] \quad 21) l_5 \leftarrow l_5 \frac{-15}{8}$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{8} & \frac{11}{4} & \frac{v\cdot y\cdot x^2\cdot z}{8} \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & \frac{u\cdot x^2\cdot z}{3} \\ 0 & 0 & 0 & \frac{-4}{3} & \frac{-10}{3} & -2 & \frac{-11}{3} & \frac{-t\cdot 2x^2\cdot y\cdot z}{3} \end{array}\right] \quad 22) l_6 \leftarrow l_6\cdot 3$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{-3}{8} & \frac{11}{4} & \frac{v\cdot y\cdot x^2\cdot z}{8} \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & \frac{u\cdot x^2\cdot z}{3} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{3}{2} & \frac{11}{4} & \frac{t\cdot x^2\cdot y\cdot z}{2} \end{array}\right] \quad 23) l_7 \leftarrow l_7\cdot(\tfrac{-3}{4})$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-3}{10} & \frac{-3}{8} & \frac{-9}{20} & \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{y\cdot 2x^2\cdot w\cdot z}{5} \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & \frac{u\cdot x^2\cdot z}{3} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{3}{2} & \frac{11}{4} & \frac{t\cdot x^2\cdot y\cdot z}{2} \end{array}\right] \quad 24) l_5 \leftarrow l_5 - 1\cdot l_4$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-3}{10} & \frac{-3}{8} & \frac{-9}{20} & \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{y\cdot 2x^2\cdot w\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-24}{5} & 0 & \frac{-31}{5} & \frac{u\cdot x^2\cdot z}{3} - \frac{2\cdot x^2\cdot w\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 1 & \frac{5}{2} & \frac{3}{2} & \frac{11}{4} & \frac{t\cdot x^2\cdot y\cdot z}{2} \end{array}\right] \quad 25) l_6 \leftarrow l_6 - 1\cdot l_4$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-3}{10} & \frac{-3}{8} & \frac{-9}{20} & \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{y\cdot 2x^2\cdot w\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-24}{5} & 0 & \frac{-31}{5} & \frac{u\cdot x^2\cdot z}{3} - \frac{2\cdot x^2\cdot w\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-3}{10} & \frac{3}{2} & \frac{-9}{20} & \frac{t\cdot x^2\cdot y\cdot z}{2} - \frac{2x^2\cdot w\cdot y\cdot z}{5} \end{array}\right] \quad 26) l_7 \leftarrow l_7 - 1\cdot l_4$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8} + \frac{y\cdot 4x^2\cdot w\cdot z}{3} \\ 0 & 0 & 0 & 0 & \frac{-24}{5} & 0 & \frac{-31}{5} & \frac{u\cdot x^2\cdot z}{3} - \frac{2\cdot x^2\cdot w\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & \frac{-3}{10} & \frac{3}{2} & \frac{-9}{20} & \frac{t\cdot x^2\cdot y\cdot z}{2} - \frac{2x^2\cdot w\cdot y\cdot z}{5} \end{array}\right] \quad 27) l_5 \leftarrow l_5\cdot\tfrac{-10}{3}$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{y\cdot 4x^2\cdot w\cdot z}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{-5}{4} & \frac{-5}{24} & \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}-\frac{2x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8} \\ 0 & 0 & 0 & 0 & \frac{-3}{10} & \frac{3}{2} & \frac{-9}{20} & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{2x^2\cdot w\cdot y\cdot z}{5} \end{array}\right] \quad 28)l_6 \leftarrow l_6-1\cdot l_5$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{y\cdot 4x^2\cdot w\cdot z}{3} \\ 0 & 0 & 0 & 0 & 0 & \frac{-5}{4} & \frac{-5}{24} & \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}-\frac{2x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8} \\ 0 & 0 & 0 & 0 & 0 & \frac{-25}{4} & 0 & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{2x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{4x^2\cdot y\cdot w\cdot z}{3} \end{array}\right] \quad 29)l_7 \leftarrow l_7-1\cdot l_5$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{y\cdot 4x^2\cdot w\cdot z}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{6} & \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}-\frac{2x^2\cdot w\cdot y\cdot z}{5}+\frac{v\cdot y\cdot x^2\cdot z}{10} \\ 0 & 0 & 0 & 0 & 0 & \frac{-25}{4} & 0 & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{2x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{4x^2\cdot y\cdot w\cdot z}{3} \end{array}\right] \quad 30)l_6 \leftarrow l_6\cdot(\tfrac{-4}{5})$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{y\cdot 4x^2\cdot w\cdot z}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{6} & \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}-\frac{2x^2\cdot w\cdot y\cdot z}{5}+\frac{v\cdot y\cdot x^2\cdot z}{10} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{2x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \end{array}\right] \quad 31)l_7 \leftarrow l_7\cdot(\tfrac{-4}{25})$$

$$\left[\begin{array}{ccccccc|c} 1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\ 0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8}+\frac{y\cdot 4x^2\cdot w\cdot z}{3} \\ 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{6} & \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}-\frac{2x^2\cdot w\cdot y\cdot z}{5}+\frac{v\cdot y\cdot x^2\cdot z}{10} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{6} & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{v\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}+\frac{u\cdot y\cdot x^2\cdot z}{10} \end{array}\right]$$

$32)l_7 \leftarrow l_7\cdot -1\cdot l_6$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y \cdot (-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8} + \frac{y\cdot 4x^2\cdot w\cdot z}{3} \\
0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{6} & \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} - \frac{2x^2\cdot w\cdot y\cdot z}{5} + \frac{v\cdot y\cdot x^2\cdot z}{10} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2} - \frac{4x^2\cdot w\cdot y\cdot z}{5} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} - \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$33) l_7 \leftarrow l_7 \cdot (-6)$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y \cdot (-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & \frac{3}{2} & \frac{v\cdot y\cdot x^2\cdot z}{8} + \frac{y\cdot 4x^2\cdot w\cdot z}{3} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-6x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5} + \frac{v\cdot y\cdot x^2\cdot z}{10} - \frac{tx^2\cdot y\cdot z}{12} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2} - \frac{4x^2\cdot w\cdot y\cdot z}{5} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} - \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$34) l_6 \leftarrow l_6 - \frac{-1}{6} \cdot l_7$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y \cdot (-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & \frac{16}{5} & \frac{w\cdot 2x^2\cdot y\cdot z}{5} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{v\cdot y^2\cdot x^2\cdot z-v\cdot yx^2\cdot z}{8} - \frac{ux^2\cdot z}{3} - \frac{y\cdot 3x^2+z}{4} - \frac{4x^2\cdot w\cdot y\cdot z}{5} - \frac{y\cdot 16x^2\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-6x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5} + \frac{v\cdot y\cdot x^2\cdot z}{10} - \frac{tx^2\cdot y\cdot z}{12} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2} - \frac{4x^2\cdot w\cdot y\cdot z}{5} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} - \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$35) l_5 \leftarrow l_5 - \frac{3}{2} \cdot l_7$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y \cdot (-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & \frac{5}{3} & \frac{z\cdot(-x)}{3} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & 0 & \frac{w\cdot(-2x^2)\cdot y\cdot z-8x^2\cdot t\cdot y\cdot z}{5} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{-16x^2\cdot y\cdot w\cdot z}{75} - \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{v\cdot y^2\cdot x^2\cdot z-v\cdot yx^2\cdot z}{8} - \frac{ux^2\cdot z}{3} - \frac{y\cdot 3x^2+z}{4} - \frac{4x^2\cdot w\cdot y\cdot z}{5} - \frac{y\cdot 16x^2\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-6x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5} + \frac{v\cdot y\cdot x^2\cdot z}{10} - \frac{tx^2\cdot y\cdot z}{12} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2} - \frac{4x^2\cdot w\cdot y\cdot z}{5} - \frac{v\cdot y\cdot x^2\cdot z}{8} - \frac{16x^2\cdot y\cdot w\cdot z}{75} - \frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$36) l_4 \leftarrow l_4 - \frac{16}{5} \cdot l_7$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & y\cdot(-x) \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & 0 & \frac{-z\cdot x-u\cdot x^2\cdot z-4x^2\cdot y\cdot w\cdot z}{3}-\frac{5x^2\cdot t\cdot y\cdot z}{6}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & 0 & \frac{w\cdot(-2x^2)\cdot y\cdot z-8x^2\cdot t\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{-16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{v\cdot y^2\cdot x^2\cdot z-v\cdot yx^2\cdot z}{8}-\frac{ux^2\cdot z}{3}-\frac{y\cdot 3x^2+z}{4}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{y\cdot 16x^2\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-6x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5}+\frac{v\cdot y\cdot x^2\cdot z}{10}-\frac{tx^2\cdot y\cdot z}{12}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$37) l_3 \leftarrow l_3 - \frac{5}{3} \cdot l_7$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & -3 & -x \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & -y\cdot x-\frac{3y\cdot x^2\cdot+z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & 0 & \frac{-z\cdot x-u\cdot x^2\cdot z-4x^2\cdot y\cdot w\cdot z}{3}-\frac{5x^2\cdot t\cdot y\cdot z}{6}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & 0 & \frac{w\cdot(-2x^2)\cdot y\cdot z-8x^2\cdot t\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{-16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{v\cdot y^2\cdot x^2\cdot z-v\cdot yx^2\cdot z}{8}-\frac{ux^2\cdot z}{3}-\frac{y\cdot 3x^2+z}{4}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{y\cdot 16x^2\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-6x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5}+\frac{v\cdot y\cdot x^2\cdot z}{10}-\frac{tx^2\cdot y\cdot z}{12}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$38) l_2 \leftarrow l_2 - 3 \cdot l_7$

$$\left[\begin{array}{ccccccc|c}
1 & -1 & -2 & -1 & -2 & -1 & 0 & -x+\frac{3x^2\cdot t\cdot y\cdot z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & -y\cdot x-\frac{3y\cdot x^2\cdot+z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & 0 & \frac{-z\cdot x-u\cdot x^2\cdot z-4x^2\cdot y\cdot w\cdot z}{3}-\frac{5x^2\cdot t\cdot y\cdot z}{6}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & 0 & \frac{w\cdot(-2x^2)\cdot y\cdot z-8x^2\cdot t\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{-16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3} \\
0 & 0 & 0 & 0 & 1 & \frac{5}{4} & 0 & \frac{v\cdot y^2\cdot x^2\cdot z-v\cdot yx^2\cdot z}{8}-\frac{ux^2\cdot z}{3}-\frac{y\cdot 3x^2+z}{4}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{y\cdot 16x^2\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{-6x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5}+\frac{v\cdot y\cdot x^2\cdot z}{10}-\frac{tx^2\cdot y\cdot z}{12}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{t\cdot x^2\cdot y\cdot z}{2}-\frac{4x^2\cdot w\cdot y\cdot z}{5}-\frac{v\cdot y\cdot x^2\cdot z}{8}-\frac{16x^2\cdot y\cdot w\cdot z}{75}-\frac{u\cdot x^2\cdot z+4x^2\cdot y\cdot w\cdot z}{3}
\end{array}\right]$$

$39) l_1 \leftarrow l_1 + 3 \cdot l_7$

OBS - A partir daqui coloquei apenas as linhas que foram sendo modificadas, por conta do tamanho das contas

$$\left[\begin{array}{ccccccc||}
1 & -1 & -2 & -1 & -2 & -1 & 0 \\
0 & 1 & 4 & 1 & 2 & 0 & 0 \\
0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & 0 \\
0 & 0 & 0 & 1 & \frac{14}{5} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right] 40) l_5 \leftarrow l_5 - \frac{5}{4} \cdot l_6$$

$L5 - \frac{v \cdot y^2 \cdot z - v \cdot 2x^2 \cdot v \cdot y \cdot z}{8} - \frac{ux^2 \cdot z}{3} + \frac{y \cdot 3x^2 \cdot t \cdot z + 6x^2 \cdot w \cdot y \cdot z + 3x^2 \cdot v \cdot y \cdot z}{4} - \frac{4x^2 \cdot w \cdot y \cdot z}{5} + \frac{16x^2 \cdot y^2 \cdot w \cdot z - 16x^2 \cdot y \cdot z \cdot w}{75} + \frac{u \cdot y \cdot x^2 \cdot z}{10} - \frac{t \cdot x^2 \cdot y \cdot z}{12}$

$$\left[\begin{array}{ccccccc} 1 & -1 & -2 & -1 & -2 & 0 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{14}{5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] 41) l_1 \leftarrow l_1 + 1 \cdot l_5$$

$L1 \to x + \frac{3x^2 \cdot t \cdot y \cdot z}{2} + (\frac{-10x^2 \cdot w \cdot y \cdot z - 3x^2 \cdot v \cdot y \cdot z}{5}) - \frac{v \cdot y \cdot x^2 \cdot y \cdot w \cdot z}{4} - \frac{32x^2 \cdot y \cdot w \cdot z}{75} - \frac{u \cdot x^2 \cdot z + 4x^2 \cdot y \cdot w \cdot z}{3} + \frac{u \cdot y \cdot x^2 \cdot z}{10} - \frac{t \cdot x^2 \cdot y \cdot z}{12}$

$$\left[\begin{array}{ccccccc} 1 & -1 & -2 & -1 & -2 & 0 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{4}{3} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] 42) l_4 \leftarrow l_4 - \frac{14}{5} \cdot l_5$$

$L4 \to \frac{-6w \cdot x^2 \cdot y \cdot z 8x^2 \cdot t \cdot y \cdot z}{5} - \frac{v \cdot y \cdot x^2 \cdot z}{8} + (\frac{-32x^2 \cdot y \cdot w \cdot z + 64 \cdot w \cdot x^2 \cdot z}{75}) + \frac{-2x^2 \cdot u \cdot z - 4x^2 \cdot y \cdot w \cdot z}{3} - \frac{7(v \cdot y^2 \cdot z - 2x^2 \cdot v \cdot y \cdot z}{20} - \frac{3x^2 \cdot y \cdot t \cdot z + 6w \cdot x^2 \cdot y \cdot z + 3x^2}{4} + \frac{u \cdot y \cdot x^2 \cdot z}{10} - \frac{t \cdot x^2 \cdot y \cdot z}{12}$

$$\left[\begin{array}{ccccccc} 1 & -1 & -2 & -1 & -2 & 0 & 0 \\ 0 & 1 & 4 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] 43) l_3 \leftarrow l_3 - \frac{4}{3} \cdot l_5$$

$L3 \to \frac{-z \cdot x + 4x^2 \cdot y \cdot w \cdot z}{3} + (\frac{-5x^2 \cdot t \cdot y \cdot z - v \cdot y^2 \cdot z + 2x^2 \cdot v \cdot y \cdot z}{6}) - \frac{8x^2 \cdot w \cdot y \cdot z}{5} - \frac{v \cdot y \cdot x^2 \cdot z}{8} + (\frac{32x^2 \cdot y \cdot w \cdot z + 64x^2 \cdot z \cdot w}{75}) - \frac{3x^2 \cdot y \cdot t \cdot z + 6x^2 \cdot w \cdot y \cdot z + 3x^2 \cdot v \cdot y \cdot z}{4} + \frac{u \cdot y \cdot x^2 \cdot z}{10} - \frac{t \cdot x^2 \cdot y \cdot z}{12}$

$$\left[\begin{array}{ccccccc} 1 & -1 & -2 & -1 & -2 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] 44) l_2 \leftarrow l_2 - 2 \cdot l_5$$

$L2 \to -y \cdot - \frac{3y \cdot x^2 \cdot t \cdot z}{2} - \frac{8x^2 \cdot w \cdot y \cdot z}{5} - \frac{v \cdot y \cdot x^2 \cdot z}{8} + (\frac{32y \cdot x^2 \cdot w \cdot z + 64x^2 \cdot z \cdot w}{75}) + (\frac{-2x^2 \cdot u \cdot z - 4y \cdot x^2 \cdot w \cdot z}{3}) +$

$(\frac{-v\cdot y^2-x^2\cdot v\cdot y\cdot z-3y\cdot x^2\cdot t\cdot z-6x^2\cdot w\cdot y\cdot z}{4}) + \frac{u\cdot y\cdot x^2\cdot z}{10} - \frac{t\cdot x^2\cdot y\cdot z}{12}$

$$\begin{bmatrix} 1 & -1 & -2 & -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 45) l_1 \leftarrow l_1 + 2 \cdot l_5$$

$L1 \rightarrow -x+\frac{3x^2\cdot t\cdot y\cdot z}{2}+(\frac{-14x^2\cdot w\cdot y\cdot z+3x^2\cdot v\cdot y\cdot z}{5}+\frac{v\cdot y^2\cdot z-v\cdot y^2\cdot x^2\cdot w\cdot z-5x^2\cdot v\cdot y\cdot z-3x^2\cdot y\cdot t\cdot z-6x^2\cdot w\cdot y\cdot z}{4}+(\frac{-48x^2\cdot y\cdot w\cdot z+64x^2\cdot z\cdot w}{75})+ (\frac{-2x^2\cdot u\cdot z-4x^2\cdot y\cdot w\cdot z}{3}) - \frac{t\cdot x^2\cdot y\cdot z}{6}$

$$\begin{bmatrix} 1 & -1 & -2 & -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 46) l_3 \leftarrow l_3 \frac{-1}{3} \cdot l_4$$

$L3 \rightarrow \frac{z\cdot x-2x^2\cdot u\cdot z+8x^2\cdot y\cdot w\cdot z}{3}+\frac{v\cdot y^2\cdot z-6x\cdot x^2\cdot y\cdot t-2x^2\cdot u\cdot y\cdot x}{6}+(\frac{-8x^2\cdot w\cdot y\cdot z+u\cdot y\cdot x^2\cdot z}{5})+(\frac{-3x^2-4z\cdot x^2\cdot y\cdot v-6x^2\cdot y\cdot t\cdot z-12z\cdot x^2\cdot y\cdot w}{4})- \frac{2(64z\cdot x^2\cdot w+32z\cdot x^2\cdot y\cdot w}{75} + \frac{6x^2\cdot w\cdot y\cdot z-8x^2-t-z}{15} - \frac{7(v\cdot y^2\cdot z-2x^2\cdot v\cdot y\cdot z)}{20}$

$$\begin{bmatrix} 1 & -1 & -2 & -1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 47) l_2 \leftarrow l_2 - 1 \cdot l_4$$

$L2 \rightarrow -y\cdot\frac{3y\cdot x^2\cdot t\cdot z}{2}+(\frac{-2y\cdot x^2\cdot z\cdot w-8x^2-t-z+u\cdot y\cdot x^2\cdot z}{5})+(\frac{-v\cdot y^2-3x^2}{4})-\frac{2(64x^2\cdot z\cdot w+32y\cdot x^2\cdot z\cdot w)}{75}-\frac{2(2x^2\cdot z\cdot u-4y\cdot x^2\cdot z\cdot w)}{3}- \frac{t\cdot x^2\cdot y\cdot z}{6} - \frac{7(v\cdot y^2-2x^2\cdot v\cdot y\cdot y\cdot z)}{20}$

$$\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 48) l_1 \leftarrow l_1 + 1 \cdot l_4$$

$L1 \rightarrow -x+\frac{3x^2\cdot t\cdot y\cdot z}{2}+\frac{8x^2-20x^2\cdot yz\cdot w-3x^2\cdot v\cdot y\cdot z+t+z}{5}+(\frac{-3x^2+v\cdot y^2\cdot z-v\cdot y^2\cdot x^2\cdot w\cdot z-5x^2zcdotv\cdot y\cdot z-6x^2\cdot y\cdot t\cdot z-12x^2\cdot y\cdot z}{4})+$

$(\frac{-128x^2\cdot z\cdot w-80x^2\cdot y\cdot w\cdot z}{75}) - \frac{2(2x^2\cdot v\cdot z-4x^2\cdot y\cdot w\cdot z)}{3} - \frac{t\cdot x^2\cdot y\cdot z}{6} - \frac{v\cdot x^2\cdot z}{8} - \frac{7(v\cdot y^2-2x^2\cdot v\cdot y\cdot z)}{20} + \frac{u\cdot y\cdot x^2\cdot z}{10} - \frac{t\cdot x^2\cdot y\cdot z}{12}$

$$\begin{bmatrix} 1 & -1 & -2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 49) l_2 \leftarrow l_2 - 4 \cdot l_3$$

$L2 \rightarrow -y \cdot x - \frac{3y\cdot x^2\cdot t\cdot z}{2} + (\frac{-10y\cdot x^2\cdot z\cdot w+8x^2+t+z-2\cdot u\cdot y\cdot x^2\cdot z}{5}) + (\frac{-v\cdot y^2+4y\cdot x^2\cdot z\cdot v+6y\cdot x^2\cdot t\cdot z+12y\cdot x^2\cdot z\cdot w}{4}) - \frac{4(64x^2\cdot z\cdot w+32y\cdot x^2\cdot z\cdot w)}{75} + (\frac{4x\cdot z+4x^2\cdot z\cdot u-24y\cdot x^2\cdot z\cdot w}{3}) + \frac{v\cdot y^2\cdot z-7y\cdot x^2\cdot t\cdot z-2x^2\cdot u\cdot y\cdot z}{6} + (\frac{-7(v\cdot y^2-2x^2\cdot v\cdot y\cdot z)-7(v\cdot y^2\cdot z-2x^2\cdot u\cdot y\cdot z)}{20} + \frac{6x^2\cdot w\cdot y\cdot z-8x^2-t-z}{15}$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 50) l_1 \leftarrow l_1 + 2 \cdot l_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} 51) l_2 \leftarrow l_2 + 1 \cdot l_3$$