Introduction

Lily is a student from the London School of Economics and Political Science studying Operations Research and Analytics for her master's degree, while her boyfriend David is learning computer science at Imperial College London. Due to the fact that they will return to China after graduating, so they want to have a 2-week trip traveling around some famous European cities. David is a fan of music, so he has a strong willingness to Vienna, and Lily really wants to visit Van Gogh Museum in Amsterdam. Based on their own interest, they select 9 different destinations finally, which are Vienna, Paris, Rome, Barcelona, Berlin, Amsterdam, Copenhagen, Zurich, and Budapest. They only want to travel by train and plane. However, Lily witnessed a plane crash at a young exposure, so she want to decrease the frequency of taking the plane. They could finish their final exams in late May as well as begin to prepare for their journey. They both have time in July and August, but they haven’t decided the exact time. Noticed that train and plane fares may vary depending on the month of booking. So how to organize the routes so that they could go to the famous tourist attractions only once with minimal cost and time?

Assumptions

1. Suppose it takes the same time and cost to get from city A to city B and from B to A.

2. We consider only two modes of transportation, train, and plane, even though there are alternative choices.

3. We ignore the effect of flight price fluctuations. We take the average fare between 8 AM and 8 PM from July 1 to July 7 and from August 1 to August 7 as the fare for the whole of July and August, without taking into account additional baggage charges.

Toy example

When planning a trip, the order of the destinations is different, resulting in a similar amount of time but different costs.

If we construct a traveling route starting from Hefei to 3 different destinations, which are Guangzhou, Shenzhen, and Foshan. It’s surprising to find that the airfare from Hefei to Shenzhen is twice the price to Guangzhou. So the best plan is to take a plane from Hefei to Guangzhou, then take a high-speed train to Shenzhen, Foshan, and finally take a plane back to Hefei from Foshan. However, if we fly through Shenzhen airport, it will cost nearly a thousand yuan more, even though the time we consume is almost the same.

Therefore, considering the way carefully before traveling is necessary, also finding an optimal plan in order to minimize our time and cost is significant.