

The manuscript presents a generalization of the logarithmic vanishing theorem on weakly 1-complete Kähler manifolds, building upon the work of Huang, Liu, Wan, and Yang, who established similar results for compact Kähler manifolds. The main theorem asserts the vanishing of certain cohomology groups under specific conditions, extending the well-established results in the compact setting to the broader class of weakly 1-complete manifolds.

While this extension is mathematically correct, the methodology closely follows the analytical techniques used by Huang et al., particularly their L^2 -methods and the use of approximation theorems. In fact, the paper essentially adapts their arguments to weakly 1-complete Kähler manifolds without encountering significant new challenges or introducing substantial innovations in the techniques used.

Historically, vanishing theorems have played a crucial role in both algebraic geometry and complex analysis, dating back to the classical Akizuki-Kodaira-Nakano vanishing theorems on compact Kähler manifolds, which have had far-reaching applications. The generalization of these results to non-compact and weakly 1-complete settings, pioneered by researchers like Norimatsu, Esnault, and Viehweg, has continued this trend. These generalizations aim to explore cohomological behavior in broader geometric contexts, and the work of Huang et al. represents a key development in extending these ideas to compact Kähler manifolds.

The authors of the current manuscript attempt to contribute to this ongoing line of research by moving from compact Kähler to weakly 1-complete Kähler manifolds. However, the transition appears relatively straightforward, with no significant additional difficulties encountered. The core obstacle of constructing suitable Hermitian metrics and employing Poincaré-type metrics remains largely unchanged from the previous results in the compact case. As a result, the overall contribution, while correct, seems to lack the level of novelty and difficulty that would typically be expected for publication in *Differential Geometry and its Applications*.

In conclusion, although the paper provides a technically sound extension of an existing result, it does not introduce substantial new methods or insights beyond what has already been established by Huang, Liu, Wan, and Yang. For this reason, I am uncertain whether the contribution is strong enough to meet the standards of this journal.