EXERCISES ABOUT FORMALIZATION IN PREDICATE LOGIC

Exercise A. Associate each sentence with the corresponding formula (Px: x is a painter, Lxy: x loves y, *a* is a constant for Alice, *b* is a constant for Bob)

Statement	Formula	Correspondence
A. Everyone is a painter	1. ∀x~Px	A with 5
B. Someone is a painter	2. ~∀xPx	
C. Not everyone is a painter	3. ∀xLxb	
D. No one is a painter	4. ∃xLax	
E. Everyone loves Bob	5. ∀xPx	
F. Alice loves someone	6. ∃xPx	
G. Everyone who is a painter loves Alice	7. (∀xLxa) → Pa	
H. Everyone who loves Alice is a painter	8. ∀x(Lxa → Px)	
I. If everyone loves Alice, then she is a painter	9. ∀x(Px → Lxa)	

Exercise B. Translate these observations into predicate calculus:

1. Only famous people may be rich
2. If anything is damaged, then everyone complains
3. Either all the gears are broken or a cylinder is missing
4. Some students are intelligent and hard working
5. Everything enjoyable is either illegal, immoral or fattening
6. Some medicines are dangerous if taken in excessive amounts
7. Some medicines are dangerous only if taken in excessive amounts
8. Any horse that is gentle has been well-trained
9. Only well-trained horses are gentle
10. If all ripe bananas are yellow, then some yellow things are ripe
11. No coat is waterproof unless it has been specially treated

Exercise C. Translate to Predicate Logic, indicating the constants and predicates that have been used:

1. All dogs bark
2. There is a dog who lives with Garfield that does not chase cats
3. Every person is chased by a dog
4. Every cat hates some animal
5. There is a cat who hates all the animals
6. Garfield is a cat and does not hate dogs
7. Some cats hate some mice
8. Tom hates Jerry
9. Every cat who hates Jerry also hates Speedy.
10. There is at least one cat who hates both Jerry and Speedy.

Exercise D. Formalize the following arguments in Predicate Logic:

1. Some famous people are rich and some rich people are not happy. Therefore, some famous people are not happy.

2. Miles lives in Oxford. Oxford is a city with high levels of air pollution. Therefore Miles lives in a city with high levels of air pollution.

3. Teachers are enthusiastic or fail. Not all teacher fail. Consequently, there are teachers that are enthusiastic.

4. No manager who is dictatorial or insensitive may be respected. We can find managers that are insensitive and also dictatorial. Therefore, no manager may be respected.

5. Bob is happy when all his friends like the film, but Bob is not happy now. So, it is the case that some friend of him does not like the film.

Exercise E. Interpret the expressions given for two different scenarios:

Predicates: Px: x is a prince, Dx: x is a dragon, Kxy: x kills y, Lxy: x is loved by y
Constants: a: Amisha , b: Buffy.

1. $\forall x\,(Px \wedge Lxa \rightarrow \exists y\,(Dy \wedge Kxy))$
2. $\exists x\,(Px \wedge Lxa \wedge Kxb \wedge Db)$

Predicates: Fxyt: x is fooled by y in period t; Px: x is a person; Tx: x is a time period

3. $\forall x\,(Px \rightarrow \exists t\,(Tt \wedge \forall y\,(Py \rightarrow Fxyt)))$
4. $\forall x\,(Px \rightarrow \forall t\,(Tt \rightarrow \exists y\,(Py \wedge Fxyt)))$
5. $\neg\exists x\,(Px \wedge \forall t\,(Tt \rightarrow \forall y\,(Py \rightarrow Fxyt)))$

Exercise F

Consider the following vocabulary:

- P_x: x is a political party.
- Q_x: x is a charismatic person.
- R_{xy}: x is the leader of y.
- S_x: x is self-confident.
- T_x: x achieves a power position.
- U_x: x is honest.
- *a*: constant that represents Barack Obama.
- *b*: constant that represents the Democrat Party.

1. Formalize the following assertions:
    - All political parties have at least one charismatic and self-confident leader.
    - It is necessary to be charismatic and self-confident to achieve a power position.
    - It is impossible to be honest and to achieve a power position.
    - Barack Obama is charismatic and leads the Democrat Party.

2. Give a natural language representation of the meaning of these formulas:
    - $\exists y \ (P_y \wedge \neg \exists x \ (U_x \wedge R_{xy}))$
    - $\forall x \ (Q_x \rightarrow (S_x \wedge U_x))$
    - $\forall x \forall y \ (P_y \wedge \neg Q_x \wedge R_{xy} \rightarrow U_x)$
    - $\neg \forall x \ (P_x \rightarrow (\exists y \ (R_{yx} \wedge Q_y) \wedge \exists y \ (R_{yx} \wedge \neg Q_y)))$

Exercise G

Consider the following vocabulary:

- E_x: x is an enterprise.
- M_x: x is multi-national.
- G_x: x is well-managed.
- B_x: x is profitable.
- P_x: x is a person.
- F_x: x is efficient.
- A_x: x has a PhD.
- T_{xy}: x works at y.
- *j*: constant that represents John.
- *k*: constant that represents Facebook.

1. Formalize the following assertions:
    - Any enterprise is well-managed if it has at least one efficient worker.
    - Everyone that has a PhD is efficient.
    - All the enterprises that are multi-national or well-managed are profitable.
    - It is necessary to have a PhD to work in a multi-national enterprise.
    - John works at Facebook.

2. Give a natural language representation of the meaning of these formulas:
    - $\exists x(E_x \wedge \forall y (P_y \wedge T_{yx} \rightarrow F_y))$
    - $\forall x (P_x \wedge T_{xk} \rightarrow A_x)$
    - $\forall x (E_x \wedge M_x \rightarrow \forall y (P_y \wedge T_{yx} \rightarrow F_y))$
    - $\neg\exists x (E_x \wedge \forall y (P_y \wedge T_{yx} \rightarrow F_y \wedge A_y))$
    - $\forall x (Ex \wedge \forall y (P_y \wedge T_{yx} \rightarrow F_y) \rightarrow B_x)$

Exercise H

Consider the following vocabulary:

- W_x: x is a writer
- B_x: x is a book
- S_x: x contains sex scenes
- Q_x: x is a best-seller
- P_x: x is a literary award
- A_{xy}: the writer x has won the literary award y
- G_{xy}: the writer x has written the book y
- Constants: c for George RR Martin, d for Game of Thrones

1. Formalize the following assertions:
    - There is at least one writer that has not written any book that is a best-seller.
    - There isn't any writer that has not won any literary prize.
    - A book is a best seller only if it contains sex scenes.
    - George RR Martin is a writer who wrote the best-seller book Game of Thrones.
    - There does not exist any writer that only writes best-sellers.

2. Give a natural language representation of the meaning of these formulas:
    - $\forall x \forall y (P_x \wedge W_y \wedge A_{yx} \rightarrow \forall z (B_z \wedge G_{yz} \rightarrow Q_z))$
    - $\neg \exists x (W_x \wedge \forall y (P_y \rightarrow A_{xy}))$
    - $\neg \exists x (B_x \wedge S_x \wedge Q_x)$
    - $\exists x (W_x \wedge \forall y (B_y \wedge Q_y \rightarrow G_{xy}))$
    - $\exists x \exists y (W_x \wedge P_y \wedge A_{xy} \wedge \neg \exists z (B_z \wedge Q_z \wedge G_{xz}))$

Exercise I

Consider the following vocabulary:

- D_x: x is a doctor
- P_x: x is a patient
- R_x: x is rich (poor=not rich)
- E_x: x is an expert
- $T_{x,y}$: x takes care of y (x treats y, y is treated by x)

1. Formalize the following statements:

    - Each doctor takes care of at least one patient.
    - There is at least one doctor that only takes care of rich patients.
    - There is at least one poor patient that is not treated by any doctor.
    - Rich patients are only treated by expert doctors.

2. Give a natural language description of the meaning of these formulas:

    - $\exists x\ (D_x \wedge R_x \wedge \forall y\ (P_y \wedge T_{xy} \rightarrow \neg R_y))$
    - $\neg\exists x\ (P_x \wedge R_x \wedge \forall y\ (D_y \rightarrow \neg T_{yx}))$
    - $\forall x\ (D_x \wedge E_x \rightarrow \forall y\ (P_y \rightarrow T_{xy}))$
    - $\forall x\ (D_x \rightarrow (P_x \rightarrow \neg\exists y\ (D_y \wedge T_{yx})))$