Brief history of Logic as a Tool for Knowledge representation and Reasoning in AI

Logics for AI 2023-24

Toni Moreno

Artificial Intelligence

- **Main aim**: understanding intelligent behaviour and replicating it in computational systems.
- Two basic approaches:
  - Reactive, non-symbolic systems, based on the direct acquisition of data from the environment and immediate rule-based reaction to it.
  - Symbolic, deliberative systems, based on the *Hypothesis of the Physical Symbols System*: "Intelligent behaviour can only be obtained in symbol-processing systems".

Knowledge Representation and Reasoning

- In any deliberative intelligent system we need some kind of formal language with which we can:
  - Represent knowledge

    Knowledge that we have about the domain, about the world, about other entities in the world, about our beliefs/desires/goals/intentions/plans …
  - Reason about that knowledge

    Is something true or false in the world? Is a certain piece of data compatible with my general domain knowledge? How is the world going to change if I execute a certain plan?

Mathematical Logic

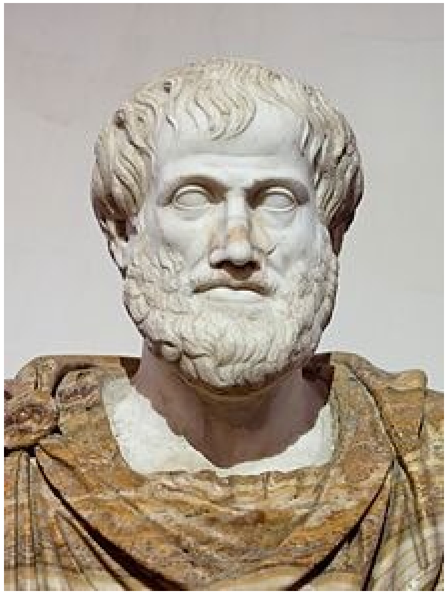
- Branch of Mathematics that studies reasoning processes.
- Pattern of a reasoning argument :
  - Set of (N>=0) *premises*, which are the "input facts"
  - One *conclusion*, which is supposed to "follow" from the premises
- An argument is *valid* if, whenever all the premises are true, the conclusion must also be true (i.e., there is no way in which the premises can be true and the conclusion false). In that case, the conclusion is a logical consequence of the premises. Otherwise, the argument is *invalid*.

Logic Formalisms

- A logic must provide a *formal language* in which we can represent the premises and the conclusions of an argument.
- A logic must provide mechanisms that permit
  - To find out whether a given conclusion can be logically deduced from a given set of premises (*validation* procedure).
  - To discover information that can be logically deduced from a given set of premises (*reasoning, inference* methods).
- There is always a trade-off between the expressivity of the representation language and the complexity of the validation/inference procedures.

Early Origins of Mathematical Logic

- Greek philosophers, in particular Aristotle (IV century B.C.)
- Discussions about the validity of arguments
- They already noticed that the validity of an argument depends on the *form* of the premises and the conclusion, rather than on their actual factual content



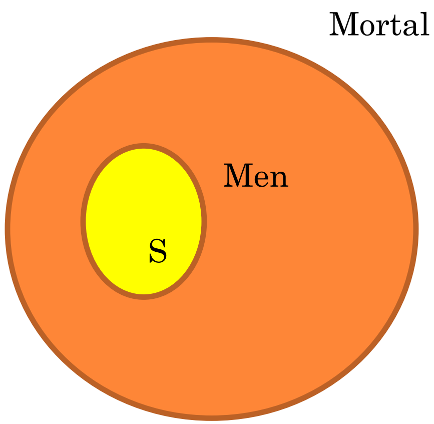
Valid argument

- All men are mortal, and Socrates is a man. Therefore, Socrates is mortal

- Valid argument form:

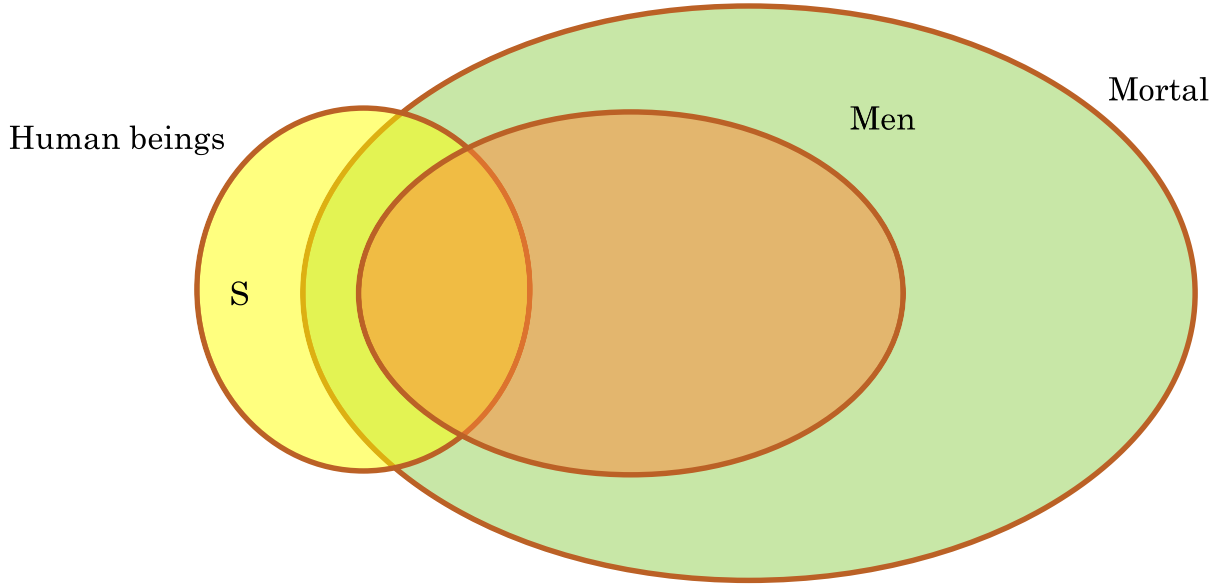
All As are Bs, and X is an A.

Therefore, X is a B.



Invalid Argument

- Some human beings are men, and all men are mortal. Socrates is a human being; therefore, he is mortal.



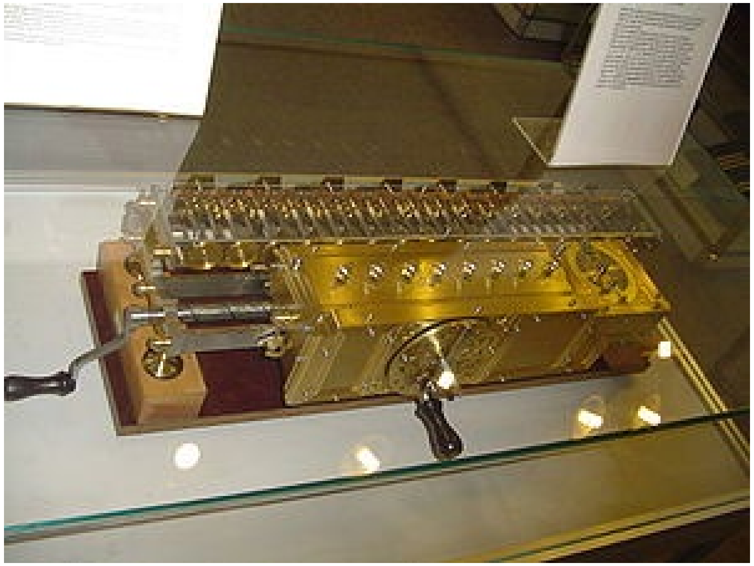
- Invalid argument form: Some As are Bs, and all Bs are Cs. X is an A, therefore X is a C.

RAMON LLULL (1232-1315)

- *Ars Magna*: logical machine, in which theological theories and subjects are represented with geometrical forms and, by moving some levers, propositions stopped in front of appropriate "truth" or "falsehood" positions.

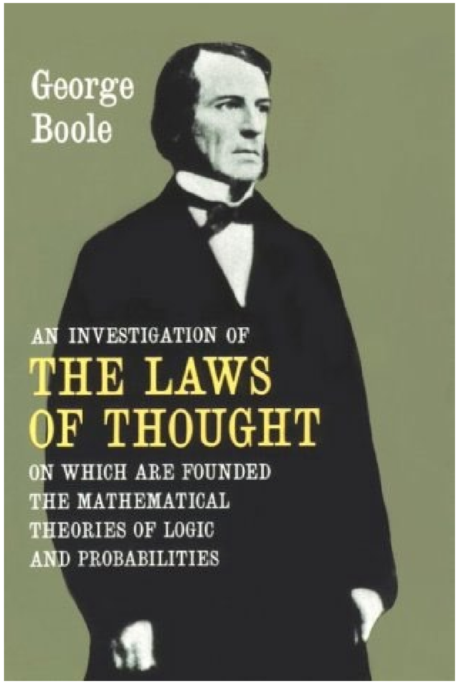
LEIBNITZ (1646-1716)

- Idea of expressing arguments in a *formal* language, so that they can be manipulated (e.g. validated) in a purely mechanical way
- *Leibnitz machine*: could make additions, substractions, products, divisions and square roots



GEORGE BOOLE (1815-1864)

- Definition of a set of (algebraic) rules that must be followed to infer correct conclusions from a set of premises



Ex.—"No men are heroes but those who unite self-denial to courage."

Let x = "men," y = "heroes," z = "those who practise self-denial," w, "those who possess courage."

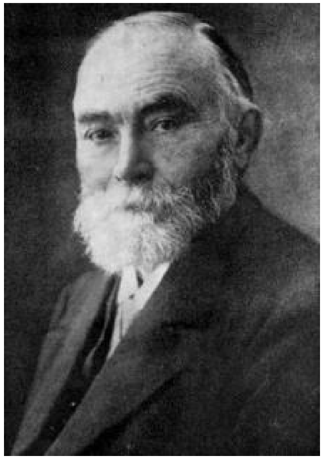
The assertion really is, that "men who do not possess courage and practise self-denial are not heroes."

Hence we have

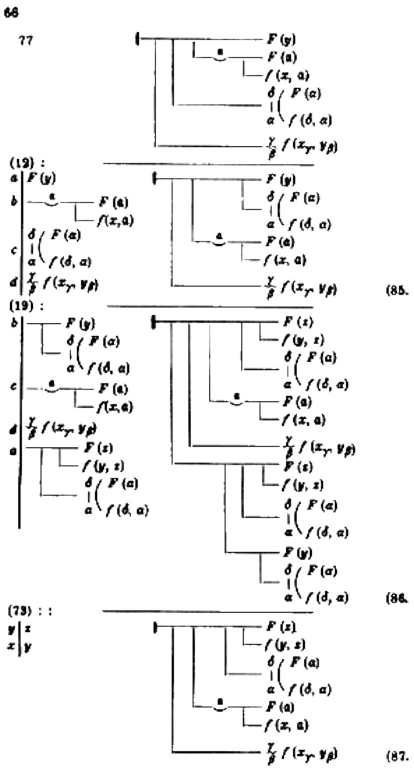
$$x(1 - zw) = v(1 - y)$$

for the equation required.

Gottlob Frege (1848-1925)



- Begriffsschrift (concept script): “A formula language, modeled on that of aritmethic, of pure thought”




BEGRIFFSSCHRIFT,
EINE DER ARITHMETISCHEN NACHGEBILDETE
FORMELSPRACHE
DES REINEN DENKENS.
Dr. GOTTLOB FREGE.
HALLE a/S.
VERLAG VON LOUIS NEBERT.
1879.

John Alan Robinson (1928-2016)

- Resolution: method that can automatically validate whether an argument is valid or not
- Transform the premises and the conclusions into a predefined "normal form" (a certain syntactic structure)
- Apply a single logical rule

$$\frac{\begin{array}{l}\{\phi_1, \dots, \chi, \dots, \phi_m\} \\ \{\psi_1, \dots, \neg\chi, \dots, \psi_n\}\end{array}}{\{\phi_1, \dots, \phi_m, \psi_1, \dots, \psi_n\}}$$

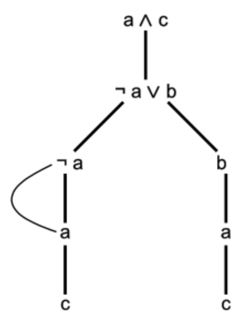
Basic Logic Systems (I)

- Propositional Logic
  - Two valued. Only deals with propositions and basic logical operators (conjunction, disjunction, negation, implication)

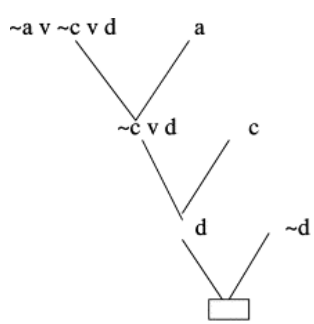
1.	Law of Identity	$A = A$ $\overline{A} = \overline{A}$
2.	Commutative Law	$A \cdot B = B \cdot A$ $A + B = B + A$
3.	Associative Law	$A \cdot (B \cdot C) = A \cdot B \cdot C$ $A + (B + C) = A + B + C$
4.	Idempotent Law	$A \cdot A = A$ $A + A = A$
5.	Double Negative Law	$\overline{\overline{A}} = A$
6.	Complementary Law	$A \cdot \overline{A} = 0$ $A + \overline{A} = 1$
7.	Law of Intersection	$A \cdot 1 = A$ $A \cdot 0 = 0$
8.	Law of Union	$A + 1 = 1$ $A + 0 = A$
9.	DeMorgan's Theorem	$\overline{AB} = \overline{A} + \overline{B}$ $\overline{A + B} = \overline{A}\,\overline{B}$
10.	Distributive Law	$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$ $A + (BC) = (A + B) \cdot (A + C)$
11.	Law of Absorption	$A \cdot (A + B) = A$ $A + (AB) = A$
12.	Law of Common Identities	$A \cdot (\overline{A} + B) = AB$ $A + (\overline{A}B) = A + B$

1 $(\mathbf{p} \wedge \mathbf{q}) \vee (\mathbf{q} \wedge \mathbf{r}) \vdash \mathbf{q} \wedge (\mathbf{p} \vee \mathbf{r})$

1	$(p \wedge q) \vee (q \wedge r)$	**Axiom**
2	$p \wedge q$	**Assumption**
3	p	$\wedge$ **E** 2('And' Elimination on line 2)
4	q	$\wedge$ **E** 2
5	$p \vee r$	$\vee$ **I** 3('Or' Introduction on line 3)
6	$q \wedge (p \vee r)$	$\wedge$ **I** 4,5('And' Introduction on lines 4 and 5)
7	$q \wedge r$	**Assumption**
8	q	$\wedge$ **E** 7
9	r	$\wedge$ **E** 7
10	$p \vee r$	$\vee$ **I** 9
11	$q \wedge (p \vee r)$	$\wedge$ **I** 8,10
12	$q \wedge (p \vee r)$	$\vee$ **E** 1,2-6,7-11('Or' Elimination on line 1 with 2-6 and 7-11 as evidence)



	P	Q	R	S	(P v	Q =>	R) &	(R =>	S) =>	(~S	=>	~P)
0)	T	T	T	T	T	T	T	T	T	F	T	F
1)	T	T	T	F	T	T	F	F	T	T	F	F
2)	T	T	F	T	T	F	F	T	T	F	T	F
3)	T	T	F	F	T	F	F	T	T	T	F	F
4)	T	F	T	T	T	T	T	T	T	F	T	F
5)	T	F	T	F	T	T	F	F	T	T	F	F
6)	T	F	F	T	T	F	F	T	T	F	T	F
7)	T	F	F	F	T	F	F	T	T	T	F	F
8)	F	T	T	T	T	T	T	T	T	F	T	T
9)	F	T	T	F	T	T	F	F	T	T	T	T
10)	F	T	F	T	T	F	F	T	T	F	T	T
11)	F	T	F	F	T	F	F	T	T	T	T	T
12)	F	F	T	T	F	T	T	T	T	F	T	T
13)	F	F	T	F	F	T	F	F	T	T	T	T
14)	F	F	F	T	F	T	T	T	T	F	T	T
15)	F	F	F	F	F	T	T	T	T	T	T	T
					^ 1	^ 2	^ 4	^ 3	^ 8	^ 5	^ 7	^ 6



$$\mathfrak{A} \rightarrow \mathfrak{A}$$

$$\frac{\Gamma \rightarrow \Theta}{\mathfrak{D}, \Gamma \rightarrow \Theta} \qquad \frac{\Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, \mathfrak{D}}$$

$$\frac{\mathfrak{D}, \mathfrak{D}, \Gamma \rightarrow \Theta}{\mathfrak{D}, \Gamma \rightarrow \Theta} \qquad \frac{\Gamma \rightarrow \Theta, \mathfrak{D}, \mathfrak{D}}{\Gamma \rightarrow \Theta, \mathfrak{D}}$$

$$\frac{\Delta, \mathfrak{D}, \mathfrak{E}, \Gamma \rightarrow \Theta}{\Delta, \mathfrak{E}, \mathfrak{D}, \Gamma \rightarrow \Theta} \qquad \frac{\Gamma \rightarrow \Theta, \mathfrak{E}, \mathfrak{D}, \Lambda}{\Gamma \rightarrow \Theta, \mathfrak{D}, \mathfrak{E}, \Lambda}$$

$$\frac{\mathfrak{A}, \Gamma \rightarrow \Theta, \mathfrak{B}}{\Gamma \rightarrow \Theta, \mathfrak{A} \supset \mathfrak{B}} \qquad \frac{\Gamma \rightarrow \Theta, \mathfrak{A} \quad \mathfrak{B}, \Delta \rightarrow \Lambda}{\mathfrak{A} \supset \mathfrak{B}, \Gamma, \Delta \rightarrow \Theta, \Lambda}$$

$$\frac{\Gamma \rightarrow \Theta, \mathfrak{A}}{\neg \mathfrak{A}, \Gamma \rightarrow \Theta} \qquad \frac{\mathfrak{A}, \Gamma \rightarrow \Theta}{\Gamma \rightarrow \Theta, \neg \mathfrak{A}}$$

$$\frac{\mathfrak{A}, \Gamma \rightarrow \Theta}{\mathfrak{A} \,\&\, \mathfrak{B}, \Gamma \rightarrow \Theta} \qquad \boxed{\frac{\Gamma \rightarrow \Theta, \mathfrak{D} \quad \mathfrak{D}, \Delta \rightarrow \Lambda}{\Gamma, \Delta \rightarrow \Theta, \Lambda}} \qquad \frac{\Gamma \rightarrow \Theta, \mathfrak{A}}{\Gamma \rightarrow \Theta, \mathfrak{A} \vee \mathfrak{B}}$$

$$\frac{\mathfrak{B}, \Gamma \rightarrow \Theta}{\mathfrak{A} \,\&\, \mathfrak{B}, \Gamma \rightarrow \Theta} \qquad \frac{\Gamma \rightarrow \Theta, \mathfrak{B}}{\Gamma \rightarrow \Theta, \mathfrak{A} \vee \mathfrak{B}}$$

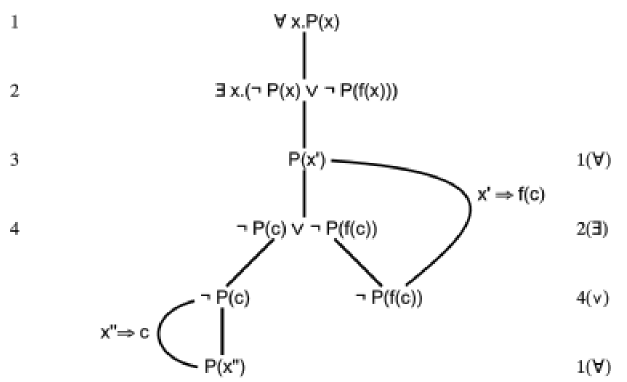
$$\frac{\Gamma \rightarrow \Theta, \mathfrak{A} \quad \Gamma \rightarrow \Theta, \mathfrak{B}}{\Gamma \rightarrow \Theta, \mathfrak{A} \,\&\, \mathfrak{B}} \qquad \frac{\mathfrak{A}, \Gamma \rightarrow \Theta \quad \mathfrak{B}, \Gamma \rightarrow \Theta}{\mathfrak{A} \vee \mathfrak{B}, \Gamma \rightarrow \Theta}$$

$$\frac{\Gamma \rightarrow \Theta, \mathfrak{F}a}{\Gamma \rightarrow \Theta, \exists \mathfrak{x}\, \mathfrak{F}\mathfrak{x}} \qquad \frac{\mathfrak{F}a, \Gamma \rightarrow \Theta}{\forall \mathfrak{x}\, \mathfrak{F}\mathfrak{x}, \Gamma \rightarrow \Theta}$$

$$\frac{\Gamma \rightarrow \Theta, \mathfrak{F}a}{\Gamma \rightarrow \Theta, \forall \mathfrak{x}\, \mathfrak{F}\mathfrak{x}} \qquad \frac{\mathfrak{F}a, \Gamma \rightarrow \Theta}{\exists \mathfrak{x}\, \mathfrak{F}\mathfrak{x}, \Gamma \rightarrow \Theta}$$

Basic Logic Systems (II)

- Predicate Logic (First-Order Logic)
  - Two valued. Adds n-ary predicates and (existential and universal) quantifiers over variables.



(1)	$1 \vdash$	$R_{o\iota\iota} A_\iota A \wedge R[S_{\iota\iota} A]A \wedge \forall x_\iota \forall y_\iota [R\,xy \supset Ry.S\,x]$ $\wedge \forall x \forall y.R[S\,x]y \supset R[Sy]x$	Hyp
(2)	$1 \vdash$	$R_{o\iota\iota} A_\iota A$	RuleP: 1
(3)	$1 \vdash$	$\forall x_\iota \forall y_\iota . R_{o\iota\iota} xy \supset Ry.S_{\iota\iota} x$	RuleP: 1
(4)	$1 \vdash$	$\forall y_\iota . R_{o\iota\iota} A_\iota y \supset Ry.S_{\iota\iota} A$	UI: A_ι 3
(5)	$1 \vdash$	$R_{o\iota\iota} A_\iota [S_{\iota\iota} A] \supset R[SA].SA$	UI: $[S_{\iota\iota} A_\iota]$ 4
(6)	$1 \vdash$	$R_{o\iota\iota} A_\iota A \supset RA.S_{\iota\iota} A$	UI: A_ι 4
(7)	$1 \vdash$	$R_{o\iota\iota} [S_{\iota\iota} A_\iota].SA$	RuleP: 2 5 6
(8)	$\vdash$	$R_{o\iota\iota} A_\iota A \wedge R[S_{\iota\iota} A]A \wedge \forall x_\iota \forall y_\iota [R\,xy \supset Ry.S\,x]$ $\wedge \forall x \forall y [R[S\,x]y \supset R[Sy]x] \supset R[SA].SA$	Deduct: 7

$$\frac{\{\phi_1, \ldots, \phi, \ldots, \phi_m\} \qquad \{\psi_1, \ldots, \neg\psi, \ldots, \psi_n\}}{\{\phi_1\tau, \ldots, \phi_m\tau, \psi_1, \ldots, \psi_n\}\sigma}$$

where τ is a variable renaming on $\{\phi_1, \ldots, \phi, \ldots, \phi_m\}$

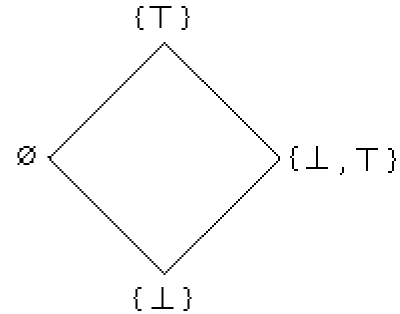
where $\sigma = mgu(\phi\tau, \psi)$

BASIC LOGIC SYSTEMS (III)

- Logical formalisms with more than 2 truth values
  - Fuzzy logic: extension of classical logic that allows to manage truth values between 0 and 1.
  - Multi-valued logics: extensions of classical logic that allow more than 2 truth values

p	q	p OR q	p AND q	$p = q$
True	True	True	True	True
True	False	True	False	False
True	Unknown	True	Unknown	Unknown
False	True	True	False	False
False	False	False	False	True
False	Unknown	Unknown	False	Unknown
Unknown	True	True	Unknown	Unknown
Unknown	False	Unknown	False	Unknown
Unknown	Unknown	Unknown	Unknown	Unknown

p	NOT p
True	False
False	True
Unknown	Unknown



Basic Logic Systems (IV)

- Logic Programming
  - Represent the domain knowledge with facts and rules, so that we can make directly queries on the consequences of this knowledge.

```
/* game-of-thrones-0.pl */

/* general family relationships */
/* usage: parent(X,Y) means child X has parent Y, or a parent of X is Y. */

mother(X,Y)        :- parent(X,Y), female(Y).
father(X,Y)        :- parent(X,Y), male(Y).
siblings(X,Y)      :- parent(X,Z), parent(Y,Z), X \= Y.
brother_of(X,Y)    :- siblings(X,Y), male(Y).
sister_of(X,Y)     :- siblings(X,Y), female(Y).
grandparent(X,Y)   :- parent(X,Z), parent(Z,Y).
grandmother(X,Y)   :- grandparent(X,Y), female(Y).
grandfather(X,Y)   :- grandparent(X,Y), male(Y).
ancestor(X,Y)      :- parent(X,Y).
ancestor(X,Y)      :- parent(X,Z), ancestor(Z,Y).

/* facts from the family tree of the Stark family */
parent(rickard_stark,     edwyle_stark).
parent(ned_stark,         rickard_stark).          /* ned = eddard */
parent(jon_snow,          ned_stark).
parent(rob_stark,         ned_stark).
parent(sansa_stark,       ned_stark).
parent(arya_stark,        ned_stark).
parent(bran_stark,        ned_stark).          /* bran = brandon jnr */
parent(rickon_stark,      ned_stark).
parent(rob_stark,         catelyn_tully).
parent(sansa_stark,       catelyn_tully).
parent(arya_stark,        catelyn_tully).
parent(bran_stark,        catelyn_tully).
parent(rickon_stark,      catelyn_tully).

/* females among Stark family */
female(catelyn_tully).
female(sansa_stark).
female(arya_stark).

/* males among Stark family */
male(edwyle_stark).
male(rickard_stark).
male(ned_stark).
male(jon_snow).
male(rob_stark).
male(brandon_stark).
male(rickon_stark).
```

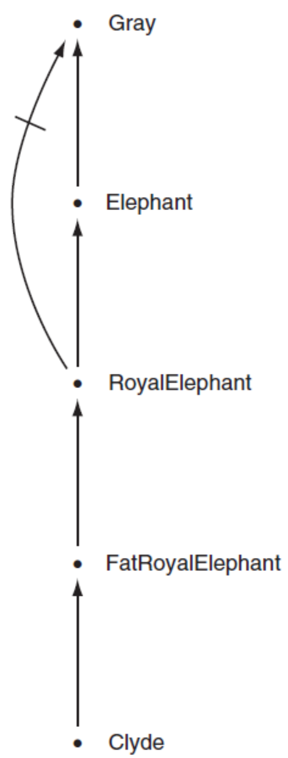
BASIC LOGIC SYSTEMS (V)

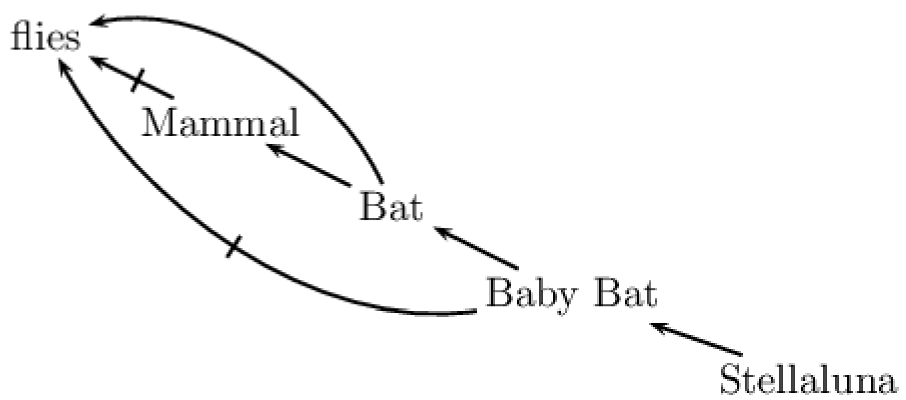
- Description Logics
  - Family of representation languages, which form the basic foundation of modern ontological languages.

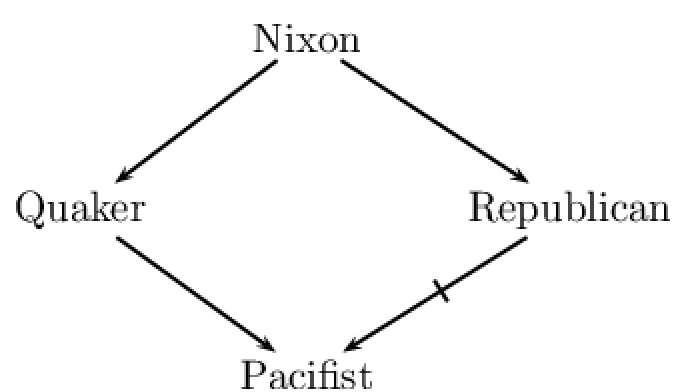
Axiom	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
sameClassAs	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
samePropertyAs	$P_1 \equiv P_2$	cost $\equiv$ price
disjointWith	$C_1 \sqsubseteq \neg C_2$	Male $\sqsubseteq$ $\neg$Female
sameIndividualAs	$\{i_1\} \equiv \{i_2\}$	{President_Bush} $\equiv$ {G_W_Bush}
differentIndividualFrom	$\{i_1\} \sqsubseteq \neg\{i_2\}$	{john} $\sqsubseteq$ $\neg${peter}
inverseOf	$P_1 \equiv P_2^-$	hasChild $\equiv$ hasParent$^-$
transitiveProperty	$P^+ \sqsubseteq P$	ancestor$^+$ $\sqsubseteq$ ancestor
uniqueProperty	$\top \sqsubseteq \leqslant 1\, P.\top$	$\top \sqsubseteq \leqslant 1$ hasMother.$\top$
unambiguousProperty	$\top \sqsubseteq \leqslant 1\, P^-.\top$	$\top \sqsubseteq \leqslant 1$ isMotherOf$^-$.$\top$
range	$\top \sqsubseteq \forall P.C$	$\top \sqsubseteq \forall$hasParent.Human
domain	$\top \sqsubseteq \forall P^-.C$	$\top \sqsubseteq \forall$hasParent$^-$.Human
i type C	$i : C$	john : Man
$i_1\ P\ i_2$	$\langle i_1, i_2 \rangle : P$	$\langle$john, peter$\rangle$: hasParent

BASIC LOGIC SYSTEMS (VI)

- Inheritance networks
  - Management of generic domain knowledge







Basic Logic Systems (VII)

- Defeasible logics
  - Non-monotonic logics, in which the addition of new premises may prevent the proof of a previous conclusion.

$$D = \left\{ \frac{Bird(X) : Flies(X)}{Flies(X)} \right\}$$

$$W = \{Bird(Condor), Bird(Penguin), \neg Flies(Penguin), Flies(Eagle)\}$$

Closed World Assumption, Circumscription, Default Logic, Autoepistemic Logic

Basic Logic Systems (VIII)

- Modal Logics
  - Deal with different modes (or ways) in which propositions may be true or false
  - Example: Temporal Logic (F: future, P:past, X:next, U:until, G. always)
  - Doxastic logics, epistemic logics, dynamic logics, …

Axioms:

$(LAX1)$	propositional tautologies
$(LAX2)$	$\neg X\phi \leftrightarrow X\neg\phi$
$(LAX3)$	$X(\phi \rightarrow \psi) \rightarrow (X\phi \rightarrow X\psi)$
$(LAX4)$	$G(\phi \rightarrow \psi) \rightarrow (G\phi \rightarrow G\psi)$
$(LAX5)$	$G\phi \rightarrow (\phi \wedge XG\phi)$
$(LAX6)$	$G(\phi \rightarrow X\phi) \rightarrow (\phi \rightarrow G\phi)$
$(LAX7)$	$(\phi U\psi) \rightarrow F\psi$
$(LAX8)$	$(\phi U\psi) \leftrightarrow (\psi \vee (\phi \wedge X(\phi U\psi)))$

Inference Rules:

$(LIR1)$	From $\vdash \phi \rightarrow \psi$ and $\vdash \phi$ infer $\vdash \psi$
$(LIR2)$	From $\vdash \phi$ infer $\vdash G\psi$

Basic Logic Systems (IX)

- Infinitary logics
  - It is possible to represent infinitely long formulas and infinitely long proofs
- Higher-order logics, categorical logics, probabilistic logics, quantum logics, …

$\forall P \, \forall x (x \in P \lor x \notin P)$ Second-order logic

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