EXERCISES ABOUT FORMALIZATION IN PREDICATE LOGIC

Exercise A. Associate each sentence with the corresponding formula (Px: x is a painter, Lxy: x loves y, *a* is a constant for Alice, *b* is a constant for Bob)

Statement	CP1	Correspondence
A. Everyone is a painter	1. $\forall x \neg Px$	A - 5
B. Someone is a painter	2. $\neg \forall x Px$	B – 6
C. Not everyone is a painter	3. $\forall x Lxb$	C – 2
D. No one is a painter	4. $\exists x Lax$	D – 1
E. Everyone loves Bob	5. $\forall x Px$	E – 3
F. Alice loves someone	6. $\exists x Px$	F – 4
G. Everyone who is a painter loves Alice	7. $\forall x(Lxa) \rightarrow Pa$	G – 9
H. Everyone who loves Alice is a painter	8. $\forall x(Lxa \rightarrow Px)$	H – 8
I. If everyone loves Alice, then she is a painter	9. $\forall x(Px \rightarrow Lxa)$	I – 7

Exercise B. Translate these observations into predicate calculus:

1. Only famous people may be rich
2. If anything is damaged, then everyone complains
3. Either all the gears are broken or a cylinder is missing
4. Some students are intelligent and hard working
5. Everything enjoyable is either illegal, immoral or fattening
6. Some medicines are dangerous if taken in excessive amounts
7. Some medicines are dangerous only if taken in excessive amounts
8. Any horse that is gentle has been well-trained
9. Only well-trained horses are gentle
10. If all ripe bananas are yellow, then some yellow things are ripe
11. No coat is waterproof unless it has been specially treated

1. $\forall x\ (Rx \rightarrow Fx)$
2. $(\exists x\ Dx) \rightarrow \forall y\ (Py \rightarrow Cy)$
3. $\forall x\ (Gx \rightarrow Bx) \vee \exists x (Cx \wedge Mx)$
4. $\exists x\ (Sx \wedge Ix \wedge Hx)$
5. $\forall x\ (Ex \rightarrow \neg Lx \vee \neg Mx \vee Fx)$
6. $\exists x\ (Mx \wedge (Tx \rightarrow Dx))$
7. $\exists x\ (Mx \wedge (Dx \rightarrow Tx))$
8. $\forall x\ ((Hx \wedge Gx) \rightarrow Wx)$
9. $\forall x\ ((Hx \wedge Gx) \rightarrow Wx)$
10. $\forall x\ ((Bx \wedge Rx) \rightarrow Yx) \rightarrow \exists y\ (Yy \wedge Ry)$
11. Four possible formalizations: $\forall x\ (Cx \rightarrow (\neg Wx \vee Sx))$ or

    $\forall x\ (Cx \rightarrow (\neg Sx \rightarrow \neg Wx))$ or $\forall x\ ((Cx \wedge Wx) \rightarrow Sx)$

    or $\neg \exists x\ (Cx \wedge Wx \wedge -Sx)$

Exercise C. Translate to Predicate Logic, indicating the constants and predicates that have been used:

1. All dogs bark
2. There is a dog who lives with Garfield that does not chase cats
3. Every person is chased by a dog
4. Every cat hates some animal
5. There is a cat who hates all the animals
6. Garfield is a cat and does not hate dogs
7. Some cats hate some mice
8. Tom hates Jerry
9. Every cat who hates Jerry also hates Speedy.
10. There is at least one cat who hates both Jerry and Speedy.

Dx: x is a dog, Bx: x barks, Cx: x is a cat, Px: x is a person, Mx: x is a mouse, Ax: x is an animal, Lxy: x lives with y, Sxy: x chases y, Hxy: x hates y
J:Jerry, s:Speedy, t:Tom, g:Garfield

1. All dogs bark.	$\forall x\ (Dx \rightarrow Bx)$
2. There is a dog that lives with Garfield that does not chase cats.	$\exists x\ (Dx \wedge Lxg \wedge \forall y\ (Cy \rightarrow \neg Sxy))$
3. Every person is chased by a dog	$\forall x(Px \rightarrow \exists y\ (Dy \wedge Syx))$
4. Every cat hates some animal	$\forall x(Cx \rightarrow \exists y\ (Ay \wedge Hxy))$
5. There is a cat who hates all the animals	$\exists x\ (Cx \wedge \forall y\ (Ay \rightarrow Hxy))$
6. Garfield is a cat and does not hate dogs	$Cg \wedge \forall x\ (Dx \rightarrow \neg Hgx)$
7. Some cats hate some mice	$\exists x\ (Cx \wedge \exists y\ (My \wedge Hxy))$
8. Tom hates Jerry	Htj
9. Every cat who hates Jerry also hates Speedy.	$\forall x\ (Cx \wedge Hxj \rightarrow Hxs)$
10. There is at least one cat who hates both Jerry and Speedy.	$\exists x\ (Cx \wedge Hxj \wedge Hxs)$

Exercise D. Formalize the following arguments in Predicate Logic:

1. Some famous people are rich and some rich people are not happy. Therefore, some famous people are not happy.
2. Miles lives in Oxford. Oxford is a city with high levels of air pollution. Therefore Miles lives in a city with high levels of air pollution.
3. Teachers are enthusiastic or fail. Not all teacher fail. Consequently, there are teachers that are enthusiastic.
4. No manager who is dictatorial or insensitive may be respected. We can find managers that are insensitive and also dictatorial. Therefore, no manager may be respected.
5. Bob is happy when all his friends like the film, but Bob is not happy now. So, it is the case that some friend of him does not like the film.

1. $\exists x\ (Fx \land Rx)$
   $\exists y\ (Ry \land \neg Hy)$
   $\therefore\ \exists z\ (Fz \land \neg Hz)$

2. Lmo (m: Miles ; o: Oxford)
   $Co \land Po$
   $\therefore\ \exists x\ (Cx \land Px \land Lmx)$

3. $\forall x\ (Tx \rightarrow (Ex \lor Fx))$
   $\neg \forall x\ (Tx \rightarrow Fx)$
   $\therefore\ \exists x\ (Tx \land Ex)$

4. $\neg\exists x\ (Mx \land (Dx \lor Ix) \land Rx)$
   $\exists x\ (Mx \land Ix \land Dx)$
   $\therefore\ \neg \exists x\ (Mx \land Rx)$

5. $\forall x\ (Fxb \rightarrow Lxf) \rightarrow Hb$ (b: bob; f:a certain film)
   $\neg Hb$
   $\therefore\ \exists x\ (Fxb \land \neg Lxf)$

Exercise E. Interpret the expressions given for two different scenarios:

Predicates: Px: x is a prince, Dx: x is a dragon, Kxy: x kills y, Lxy: x is loved by y
Constants: a: Amisha , b: Buffy.

1. $\forall x\ (Px \land Lxa \rightarrow \exists y\ (Dy \land Kxy))$
2. $\exists x\ (Px \land Lxa \land Kxb \land Db)$

Predicates: Fxyt: x is fooled by y in period t; Px: x is a person; Tx: x is a time period

3. $\forall x\ (Px \rightarrow \exists t\ (Tt \land \forall y\ (Py \rightarrow Fxyt)))$
4. $\forall x\ (Px \rightarrow \forall t\ (Tt \rightarrow \exists y\ (Py \land Fxyt)))$
5. $\neg \exists x\ (Px \land \forall t\ (Tt \rightarrow \forall y\ (Py \rightarrow Fxyt)))$

1. All princes loved by Amisha are dragon-killers.
2. Amisha loves a prince that killed Buffy, the dragon.
3. Any person is fooled by everybody during some time.
4. Everybody is always fooled by someone.
5. No one is always fooled by everybody.

Exercise F

Consider the following vocabulary:

- P_x: x is a political party.
- Q_x: x is a charismatic person.
- R_{xy}: x is the leader of y.
- S_x: x is self-confident.
- T_x: x achieves a power position.
- U_x: x is honest.
- *a*: constant that represents Barack Obama.
- *b*: constant that represents the Democrat Party.

1. Formalize the following assertions:
   - All political parties have at least one charismatic and self-confident leader.
   - It is necessary to be charismatic and self-confident to achieve a power position.
   - It is impossible to be honest and to achieve a power position.
   - Barack Obama is charismatic and leads the Democrat Party.

   $\forall x\ (Px \rightarrow \exists y\ (Ryx \wedge Qy \wedge Sy))$

   $\forall x\ (Tx \rightarrow Qx \wedge Sx)$

   $\neg\exists x\ (Ux \wedge Tx)$ or $\forall x\ (\neg Ux \vee \neg Tx)$ or $\forall x\ (Ux \rightarrow \neg Tx)$ or $\forall x\ (Tx \rightarrow \neg Ux)$

   $Qa \wedge Rab\ [\ \wedge Pb]$

2. Give a natural language representation of the meaning of these formulas:
   - $\exists y\ \ (P_y \wedge \neg\exists x\ (U_x \wedge R_{xy}))$
   - $\forall x\ (Q_x \rightarrow (S_x \wedge U_x))$
   - $\forall x \forall y\ (P_y \wedge \neg Q_x \wedge R_{xy} \rightarrow U_x)$
   - $\neg\forall x\ (P_x \rightarrow (\exists y\ (R_{yx} \wedge Q_y) \wedge \exists y\ (R_{yx} \wedge \neg Q_y)))$

   There exists at least one party without honest leaders.
   All charismatic people are honest and self-confident (It is necessary to be honest and self-confident to be charismatic).
   All the dull (uncharismatic) leaders of political parties are honest.
   It is not the case that all parties have charismatic and dull leaders.

Exercise G

Consider the following vocabulary:

- E_x: x is an enterprise.
- M_x: x is multi-national.
- G_x: x is well-managed.
- B_x: x is profitable.
- P_x: x is a person.
- F_x: x is efficient.
- A_x: x has a PhD.
- $T_{x,y}$: x works at y.
- *j*: constant that represents John.
- *k*: constant that represents Facebook.

1. Formalize the following assertions:
    - Any enterprise is well-managed if it has at least one efficient worker.
      $\forall x\ (E_x \wedge \exists y\ (P_y \wedge T_{y,x} \wedge F_y) \rightarrow G_x)$
    - Everyone that has a PhD is efficient.
      $\forall x\ (P_x \wedge A_x \rightarrow F_x)$
    - All the enterprises that are multi-national or well-managed are profitable.
      $\forall x\ (E_x \wedge (M_x \vee G_x) \rightarrow B_x)$
    - It is necessary to have a PhD to work in a multi-national enterprise.
      $\forall x\ (P_x \wedge \exists y\ (E_y \wedge M_y \wedge T_{x,y}) \rightarrow A_x)$
    - John works at Facebook.
      $T_{j,k}\ [\ \wedge P_j \wedge E_k\ ]$

2. Give a natural language representation of the meaning of these formulas:
    - $\exists x(E_x \wedge \forall y\ (P_y \wedge T_{yx} \rightarrow F_y))$
      There are enterprises that only have efficient workers.
    - $\forall x\ (P_x \wedge T_{xk} \rightarrow A_x)$
      It is necessary to have a PhD to work at Facebook.
    - $\forall x\ (E_x \wedge M_x \rightarrow \forall y\ (P_y \wedge T_{yx} \rightarrow F_y))$
      Multi-national enterprises only contract efficient people.
    - $\neg\exists x\ (E_x \wedge \forall y\ (P_y \wedge T_{yx} \rightarrow F_y \wedge A_y))$
      There does not exist any company all whose workers are efficient doctors.
    - $\forall x\ (Ex \wedge \forall y\ (P_y \wedge T_{yx} \rightarrow F_y) \rightarrow B_x)$
      If all the workers of a company are efficient then it is profitable.

Exercise H

Consider the following vocabulary:

- W_x: x is a writer
- B_x: x is a book
- S_x: x contains sex scenes
- Q_x: x is a best-seller
- P_x: x is a literary award
- A_{xy}: the writer x has won the literary award y
- G_{xy}: the writer x has written the book y
- Constants: c for George RR Martin, d for Game of Thrones

3. Formalize the following assertions:
    - There is at least one writer that has not written any book that is a best-seller.

      $\exists x\ (W_x \wedge \forall y\ (B_y \wedge G_{xy} \rightarrow \neg Q_y))$
    - There isn't any writer that has not won any literary prize.

      $\forall x\ (W_x \rightarrow \exists y\ (P_y \wedge A_{xy}))$
    - A book is a best seller only if it contains sex scenes.

      $\forall x\ (B_x \wedge Q_x \rightarrow S_x)$
    - George RR Martin is a writer who wrote the best-seller book Game of Thrones.

      $W_c \wedge B_d \wedge Q_d \wedge G_{cd}$
    - There does not exist any writer that only writes best-sellers.

      $\neg\exists x\ (W_x \wedge \forall y\ (B_y \wedge G_{xy} \rightarrow Q_y))$

4. Give a natural language representation of the meaning of these formulas:
    - $\forall x\forall y\ (P_x \wedge W_y \wedge A_{yx} \rightarrow \forall z\ (B_z \wedge G_{yz} \rightarrow Q_z))$

      All the books written by award-winning writers are best-sellers.
    - $\neg\exists x\ (W_x \wedge \forall y\ (P_y \rightarrow A_{xy}))$

      There isn't any writer that has won all the literary awards.
    - $\neg\exists x\ (B_x \wedge S_x \wedge Q_x)$

      Best-seller books do not contain sex scenes.
    - $\exists x\ (W_x \wedge \forall y\ (B_y \wedge Q_y \rightarrow G_{xy}))$

      There exists a writer that has written all the best-selling books.
    - $\exists x\exists y\ (W_x \wedge P_y \wedge A_{xy} \wedge \neg\exists z\ (B_z \wedge Q_z \wedge G_{xz}))$

      There exists an award-winning writer that has not written any best-seller.

Exercise I

Consider the following vocabulary:

- D_x: x is a doctor
- P_x: x is a patient
- R_x: x is rich (poor=not rich)
- E_x: x is an expert
- $T_{x,y}$: x takes care of y (x treats y, y is treated by x)

1. Formalize the following statements:

   - Each doctor takes care of at least one patient.
     $\forall x\ (D_x \rightarrow \exists y\ (P_y \wedge T_{xy}))$
   - There is at least one doctor that only takes care of rich patients.
     $\exists x\ (D_x \wedge \forall y\ (P_y \wedge T_{xy} \rightarrow R_y))$
   - There is at least one poor patient that is not treated by any doctor.
     $\exists x\ (P_x \wedge \neg R_x \wedge \neg\exists y\ (D_y \wedge T_{yx}))$
   - Rich patients are only treated by expert doctors.
     $\forall x \forall y\ (P_x \wedge R_x \wedge D_y \wedge T_{yx} \rightarrow E_y)$

2. Give a natural language description of the meaning of these formulas:

   - $\exists x\ (D_x \wedge R_x \wedge \forall y\ (P_y \wedge T_{xy} \rightarrow \neg R_y))$
     There exists at least one rich doctor that only takes care of poor patients.
   - $\neg\exists x\ (P_x \wedge R_x \wedge \forall y\ (D_y \rightarrow \neg T_{yx}))$
     There isn't any rich patient who is not treated by any doctor.
   - $\forall x\ (D_x \wedge E_x \rightarrow \forall y\ (P_y \rightarrow T_{xy}))$
     All expert doctors treat all patients.
   - $\forall x\ (D_x \rightarrow (P_x \rightarrow \neg\exists y\ (D_y \wedge T_{yx})))$
     Doctors who are patients are not treated by any doctor.