Homework 4: Resolution in first order Logic

$\forall x\ P(f(x))$

$\forall x\ (P(x) \rightarrow Q(g(x)))$

$\therefore \neg \exists x\ \neg Q(f(g(x)))$

Domains of size 1

$D = \{o_1\}$ $\quad P_{o_1} <^{1}_{0} \quad Q_{o_1} <^{1}_{0} \quad f(o_1) = o_1 \quad g(o_1) = o_1$

so there are 2^2 possibilities

P_{o_1}

$\neg P_{o_1} \vee Q_{o_1}$

$\therefore\ Q_{o_1}$

P_{o_1}	Q_{o_1}	P_{o_1}	$\neg P_{o_1} \vee Q_{o_1}$	Q_{o_1}
0	0	0	1	0
0	1	0	1	1
1	0	1	0	0
1	1	1	1	1

As in the previous table there's no case in which the premises are both True and the conclusion false, there's no counterexample of size 1

Domains of size 2 $\quad D = \{o_1, o_2\}$

$P_{o_1} <^{1}_{0} \quad Q_{o_1} <^{1}_{0} \quad f(o_1) <^{o_1}_{o_2} \quad f(o_2) <^{o_1}_{o_2}$

$P_{o_2} <^{1}_{0} \quad Q_{o_2} <^{1}_{0} \quad g(o_1) <^{o_1}_{o_2} \quad g(o_2) <^{o_1}_{o_2}$

So there are 2^8 possibilities, which is 256

- Interpretation in which premises and conclusion are True $\quad P_{fo_1}$ and P_{fo_2} must be 1

$P_{fo_1} \wedge P_{fo_2}$

$(\neg P_{o_1} \vee Q_{go_1}) \wedge (\neg P_{o_2} \vee Q_{go_2})$

$\therefore\ Q_{fgo_1} \wedge Q_{fgo_2}$ $\quad Q_{fgo_1}$ and Q_{fgo_2} must be 1

So one possibility could be $f \equiv$ identity

$P_{o_1} = 1 \quad P_{o_2} = 1 \quad Q_{g_{o_1}} = 1 \quad Q_{g_{o_2}} = 1$

for every possible value of g_{o_1} and g_{o_2}.

For instance, if $g \equiv$ identity, a possible combination is $P_{o_1} = P_{o_2} = Q_{o_1} = Q_{o_2} = 1$

- Premises True but conclusion False

this means $Q_{f_{g_{o_1}}} = 0$ or $Q_{f_{g_{o_2}}} = 0$.

Again, let's f ~~be the identity, we have~~ ~~$P_{o_1} = P_{o_2} = 1$~~

$$\cancel{P_{o_1} \wedge P_{o_2}}$$

$$\cancel{(\neg P_{o_1} \vee Q_{g_{o_1}}) \wedge (\neg P_{o_2} \vee Q_{g_{o_2}})}$$

$$\cancel{Q_{g_{o_1}} \wedge Q_{g_{o_2}}}$$

$f(o_1) = o_1 \quad f(o_2) = o_1, \quad g(o_1) = o_2, \quad g(o_2) = o_1,$

$$P_{o_1} \wedge P_{o_1}$$

$$(\neg P_{o_1} \vee Q_{g_{o_1}}) \wedge (\neg P_{o_2} \vee Q_{g_{o_2}})$$

$$Q_{f_{g_{o_1}}} \wedge Q_{f_{g_{o_2}}}$$

~~$P_{o_1} = 1$~~

~~$Q_{g_{o_1}} = 1$~~

~~$Q_{o_2} = 1$~~

$Q_{o_1} = 0$

$$\overset{1}{P_{o_1}} \wedge \overset{1}{P_{o_1}} \Rightarrow 1$$

$$(\neg \overset{0}{P_{o_1}} \vee \overset{1}{Q_{o_2}}) \wedge (\neg \overset{1}{P_{o_2}} \vee \overset{0}{Q_{o_1}}) \Rightarrow 1 \qquad P_{o_1} = 1 \quad P_{o_2} = 0 \quad Q_{o_1} = 0 \quad Q_{o_2} = 1$$

$$\overset{0}{Q_{o_1}} \wedge \overset{0}{Q_{o_1}} \Rightarrow 0$$

Then,

$P_{o_1} = 1 \quad Q_{o_1} = 0 \quad f(o_1) = o_1, \quad g(o_1) = o_2$

$P_{o_2} = 0 \quad Q_{o_2} = 1 \quad f(o_2) = o_1, \quad g(o_2) = o_1$

is a counterexample, and hence, the argument is INVALID.

3. Using resolution

~~Prove $A, B, \neg C$ $C_1, \dots, C_k$ doesn't hold where $C_1, \dots C_k$ are the clauses for the premises and the rest for the conclusion.~~

$\forall x\, P(f(x))$

$\forall x\, (P(x) \rightarrow Q(g(x)))$

$\therefore \neg \exists x\, \neg Q(f(g(x)))$

a) Convert to SNF the negation of consequence because the premises are already.

$\neg(\neg \exists x\, \neg Q f g_x)$

- Apply double negation

$\exists x\, \neg Q f g x$

- Define Skolem constant a to remove the existential

$\neg Q f g a$

b) Extract the clauses:

$C_1: P f_{x_1}$

$C_2: \neg P_{x_2} \vee Q g_{x_2}$

$C_3: \neg Q f g a$

$P f_{x_1} \qquad \neg P_{x_2} \vee Q g_{x_2}$

$\{f x_1 / x_2\}$

$\boxed{Q g f x_1 \qquad \neg Q f g a}$ Not possible

we can't find a substitution

the other possibility:

$\boxed{\neg Q f g a \qquad \neg P x_2 \vee Q g x_2}$ Not possible as we can't find a substitution to x_2 so that $Q g x_1 = Q f g a$.

Therefore, the argument is invalid from theorems studied in Resolution lesson.