2024 — 2025学年第一学期《拟阵基础》期末考试试卷

草 稿 区

任课教师: **专业:** **年级:** **学号:** **姓名:** **成绩:**

得 分

一、(20分) For a matroid M of rank $r+1$ on E and $k \leq r$ with rank function r, let r_k be a function on 2^E defined by

$$r_k(I) = \min\{r(I), k+1\}, \qquad \forall I \subseteq E.$$

Show that there exists some matroid $\mathrm{Trunc}_k(M)$ on E such that r_k is its rank function.

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二、(20分) Write down the matroid axioms for bases and independent sets. Show that if $B_1, B_2 \in \mathcal{B}$ are two bases, then $|B_1| = |B_2|$.

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三、(20分) Give a definition of a flat in terms of the rank function r of a matroid. Show that if F_1 and F_2 are flats then $F_1 \cap F_2$ is also a flat.

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四、(20分) Draw the Hasse diagram of the lattice of flats of the uniform matroid $U_{2,4}$, which is a matroid of rank 2 on the ground set $\{1,2,3,4\}$. Find an E-labeling of this lattice. (Recall that an E-labeling of a poset P is map from the edges of its Hasse diagram to the set of positive integers such that if $x < y$ then there exists a unique saturated chain with increasing labels.)

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五、(20分) Let L be a finite lattice with at least two elements, let $\mu(\cdot,\cdot)$ be its Möbius function, and let $\hat{0} \neq a \in L$. Show that

$$\sum_{b:a\vee b=\hat{1}} \mu(\hat{0},b) = 0,$$

and moreover, if a is atom then

$$\mu(\hat{0},\hat{1}) = -\sum_{\text{coatoms } b \text{ such that } b \not\geq a} \mu(\hat{0},b).$$