Telecom SudParis

Master of Science CCN

Evaluation of Network Algorithm Efficiency

December 9th, 2015

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Project Report

Formulation

The simulation area covers a square 1 km × 1 km. A mobile agent moves in this area according to the Random Way Point model with speed in [2 km/h, 4 km/h] and pause time [10 seconds, 1 minute]. This mobile agent is observed during three hours. Its initial position (x,y) is chosen uniformly in both the axes. Compute the mean speed of the agent in time with the granularity of one minute with precision 5% on confidence level 0.05. Draw a diagram of the mean speed of this agent in function of time.

Procedure

We started the project by implementing in ***Python*** the Random Way Point algorithm explained during the lectures:

Input: $d_u = d_u(x, y)$ coordinates of a destination of mobile u, v_u speed of mobile u, p_u pause length of mobile u

1 **forall the** *mobiles* u **do**

2 | Chose $d_u = d_u(x, y)$ uniformly in each direction, in $[x_{\min}, x_{\max}]$ and in $[y_{\min}, y_{\max}]$, respectively

3 | Chose v_u uniformly in $[v_{\min}, v_{\max}]$

4 | Move mobile u from its current position to d_u with speed v_u

5 | Chose p_u uniformly in $[p_{\min}, p_{\max}]$

6 | Wait during p_u

7 | go to 2

8 **end**

Image 1: Random Way Point algorithm obtained from the course slides.

As for the uniformly distributed variables; we used the Python library ***Numpy*** which already provides an implementation of randomly chosen numbers in an interval [x, y). Hence, the intervals proposed in the formulation of the project were not completely used (since the used function will never choose the upper bound of the interval).

The units where changed to meters (m) and minutes (min) to easily manage the granularity of the experiment. For each minute, we memorized the average speed of the mobile agent in a vector of 3 * 60 elements (3 hours of simulation). The following formula was used to compute the average speed:

$$avg\_speed = \frac{\sum_{i=1} speed_i * time_i}{1min}$$

Since the speed and the pause time are randomly picked, in a minute time-slot it is possible to have several different speeds; i.e. moving from the source point to the destination point with a speed of 3m/min during 30 seconds and then pause for 30 seconds. Replacing this in the formula we obtain:

$$avg\_speed = \frac{3\,m/min * 0.5\,\text{min} + 0\,m/min * 0.5min}{1min} = 1.5\,m/min$$

We ran the simulation N times and output each time the vector of per-minute-speeds to a file. The N was derived from the precision and confidence level as following:

1) First, we set a number N' = 30 as a first size to obtain a sample mean. This number was chosen based on the Central Limit Theorem, which states:

> *"Given certain conditions, the arithmetic mean of a sufficiently large number of iterates of independent random variables, each with a well-defined expected value and well-defined variance, will be approximately normally distributed, regardless of the underlying distribution." (Wikipedia, 2015)*

It is well known that usually N' ≥ 30 is sufficiently large for discrete samples.

2) After obtaining a first sample mean of N' vectors, we used the formula explained during the lectures to obtain the real number of N needed to correctly represent the confidence level (95%) and precision (5%):

$$N = \left(\frac{100 * Z_{1-\frac{\alpha}{2}} * \sigma}{r\bar{x}}\right)^2$$

We compute a vector of means from the N' vectors obtained (see *preliminary.txt* file included). With this vector, we applied the aforementioned formula on each element to obtain another vector of Ns, each $\mathrm{N_i}$ corresponding to the real sample size needed according to each mean $\overline{x'}_i$. Finally, we chose the greatest $\mathrm{N_i}$ and replaced it in the formula as well as the Zscore value 1.96 for a confidence level of 0.05.

Knowing the standard deviation of the uniform distribution in the velocity (m/min) is:

$$\sigma = \sqrt{\frac{(B-A)^2}{12}} = \sqrt{\frac{(66.6667 - 33.3333)^2}{12}} \cong 9.4868$$

We have then (see *preliminaty.txt*):

$$MAX(N_1, N_2, \dots, N_i) = MAX\left(\left[\frac{100 * (1.96) * (9.4868)}{(5) * \overline{x'}_i}\right]^2\right) \cong 91$$

With the resulting N vectors (see *simulation.txt*), we compute the confidence interval for each minute of the simulation according to the following formula:

$$\varepsilon = Z_{1-\frac{\alpha}{2}} * \frac{\sigma}{\sqrt{N}}$$

Where

$$\varepsilon = (1.96) * \frac{(9.4868)}{\sqrt{91}} \cong 1.9492$$

This ε represents then the half-width of the confidence interval used to estimate the mean speed per minute as following:

$$\mu_i = [\bar{x}_i - 1.9492\,,\ \ \bar{x}_i + 1.9492]$$

Results

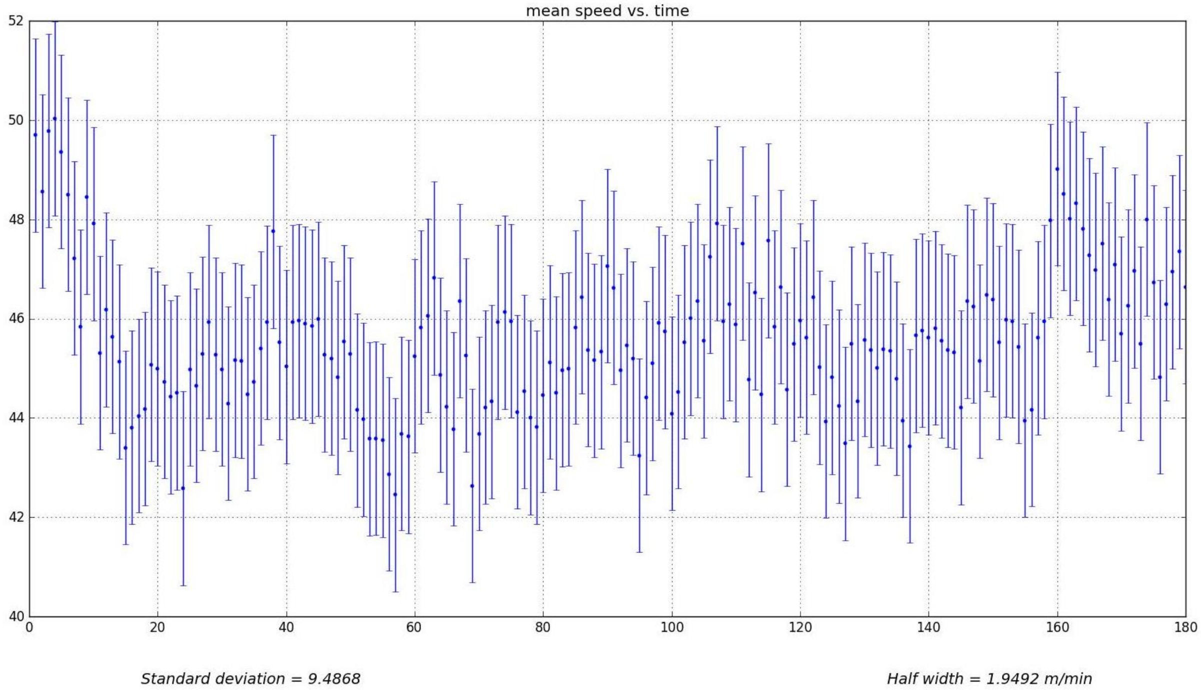

Figure 1: Mean speed (m/min) vs Time (min)

It is known that the uniform distribution represents equal probability to pick any of the values in the interval. This values will be independent between each other. Nevertheless, from one minute to the next one (or next ones) the speed of the agent is not completely independent.

Let's say, for example, that at the first minute, a speed of 35m/min is randomly and uniformly chosen. The destination point (x,y) is also picked uniformly so it could be anywhere in the 1 km x 1 km square. If the source point is really far from the destination point (i.e. 50m), the speed will be kept for more than a minute until the mobile agent reaches its destination. Thus, a fraction of the next minute of the simulation is already consumed by the movement and the speed the agent had. Therefore, we can see some dependency represented in the graph as a stabilization of the values in a middle point. We could assume then, that every time a considerable difference between two points is observed, a new value for the speed is chosen (the mobile agent starts another movement) at the very beginning of the minute. This dependency is also affected by the randomly chosen pause times of the agent and the independence between the N simulations done, which cause the irregularity of the graph.

This could also be observed in the following graph, the speed for just one simulation of 3 hours.

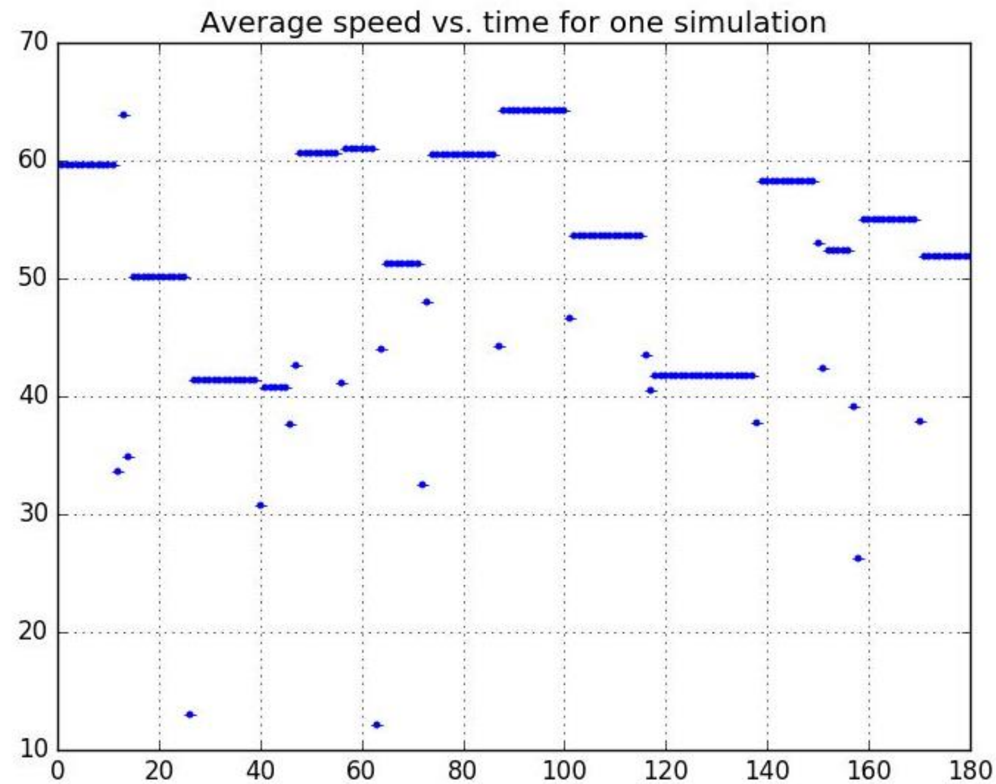

Figure 2: Average speed (m/min) vs time (min) for a single simulation of 3 hours

The single dots in the graph represent the minutes when the mobile agent stopped for some seconds, lowering the average speed. In the rest of the graph, it can be observed how a speed can be kept for up to 20 minutes, demonstrating a dependency between each timeslot.