

## Milestone 1 - Mini Reviews

- ① (0) pretty bad
- ② (1) good plot
- ③ (0) not good
- ④ (1) pretty scenery

["pretty", "good", "bad", "plot", "not", "scenery"]

$$W = [0, 0, 0, 0, 0, 0], \text{ learning Rate} = 0.5$$

- ① (0) pretty bad

$$y_1 = 0 \longrightarrow \phi x = [1, 0, 1, 0, 0, 0]$$

$$h = 0 + 0 + 0 + 0 + 0 + 0 = 0$$

$$h = \sigma(0) = 1 / (1 + e^{-0}) = 0.5$$

$$w_1 = w_3 = 0 - 0.5 \times 0.5 \times 1 = -0.25$$

$$w_2 = w_4 = w_6 = 0 - 0.5 \times 0.5 \times 0 = 0$$

$$\longrightarrow W = [-0.25, 0, -0.25, 0, 0, 0] \times$$

- ② (1) good plot

$$y_2 = 1 \longrightarrow \phi x = [0, 1, 0, 1, 0, 0]$$

$$h = 0 + 0 + 0 + 0 + 0 + 0 = 0$$

$$h = \sigma(W \phi x) = 1 / (1 + e^{-0}) = 1/2 = 0.5$$

$$w_1 = w_3 = -0.25 - 0.5 \times (0.5 - 1) \times 0 = -0.25$$

$$w_2 = w_4 = 0 - 0.5 \times (0.5 - 1) \times 1 = 0.25$$

$$w_5 = w_6 = 0 - 0.5 \times (0.5 - 1) \times 0 = 0$$

$$\longrightarrow W = [-0.25, 0.25, -0.25, 0.25, 0, 0] \times$$

③ (0) not good

$$y_3 = 0 \longrightarrow \phi x = [0, 1, 0, 0, 1, 0]$$

$$h = 0.25$$

$$h = \sigma(w\phi x) = \frac{1}{1+e^{-0.25}}$$

$$w_1 = w_3 = -0.25 - 0 = -0.25$$

$$w_2 = 0.25 - 0.5 * (\frac{1}{1+e^{-0.25}}) * 1 \doteq -0.031088$$

$$w_4 = 0.25 - 0 = 0.25$$

$$w_5 = 0 - 0.5 * (\frac{1}{1+e^{-0.25}}) * 1 \doteq -0.281088$$

$$w_6 = 0 - 0 = 0$$

$$\longrightarrow W = [-0.25, -0.031088, 0.25, -0.281088, 0]_{*}$$

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④ (1) pretty scenery

$$y_4 = 1 \longrightarrow \phi x_4 = [1, 0, 0, 0, 0, 1]$$

$$h = -0.25$$

$$h = \sigma(-0.25) = \frac{1}{1+e^{0.25}}$$

$$w_1 = -0.25 - 0.5 * (\frac{1}{1+e^{0.25}} - 1) * 1 \doteq 0.031088$$

$$w_2 = -0.031088$$

$$w_3 = -0.25$$

$$w_4 = 0.25$$

$$w_5 = -0.281088$$

$$w_6 = 0 - 0.5 * (\frac{1}{1+e^{0.25}} - 1) * 1 \doteq 0.281088$$

$$\longrightarrow W = [0.031088, -0.031088, -0.25, 0.25, -0.281088, 0.281088]_{*}$$

## Milestone 2 - Derivatives

請試著推導  $(h-y)\Phi(x)$  是怎麼得到的？首先，請推導下列算式之答案

### Gradient Descent for Logistic Regression

$$\begin{cases} \text{Cost: } J = \frac{1}{m} \sum_{i=1}^M (-y_i \log h_i + (1-y_i) \log(1-h_i)) \\ \sigma(\text{Predict.val}) = h_i = \sigma(h_i) = \frac{1}{1+e^{-h_i}} = \frac{e^{h_i}}{e^{h_i}+1} \longrightarrow \frac{dJ}{d\theta} = \frac{dJ}{dh_i} \textcircled{2} * \frac{dh_i}{dh_i} \textcircled{2} * \frac{dh_i}{d\theta} \textcircled{3} \\ \text{Predict.val: } h_i = \theta x_i \end{cases}$$

\* chain rule

$$\begin{aligned} \textcircled{1} \quad \frac{dJ}{dh_i} &= -\frac{1}{m} \sum \left( \frac{d(-y_i \log h_i)}{dh_i} + \frac{d((1-y_i) \log(1-h_i))}{dh_i} \right) \\ &= -\frac{1}{m} \sum \left( y_i/h_i + (1-y_i) \cdot (-1)/(1-h_i) \right) \\ &= -\frac{1}{m} \sum \left( y_i/h_i - \frac{(1-y_i)}{(1-h_i)} \right) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \frac{dh_i}{dh_i} &= \frac{d\left(\frac{e^{h_i}}{e^{h_i}+1}\right)}{dh_i} \\ &= \frac{de^{h_i}}{dh_i} \left( \frac{e^{h_i}+1}{e^{h_i}} \right) - \frac{e^{h_i}}{e^{h_i}} \left( \frac{de^{h_i}}{dh_i} \right) / (e^{h_i}+1)^2 \\ &= e^{h_i}(e^{h_i}+1) - e^{h_i} \left( \frac{de^{h_i}}{dh_i} + \frac{de^{h_i}}{dh_i} \right) / (e^{h_i}+1)^2 \\ &= e^{h_i} / (e^{h_i}+1)^2 = \frac{e^{h_i}}{e^{h_i}+1} * \frac{1}{e^{h_i}+1} \\ &= h_i \cdot (1-h_i) \end{aligned}$$

$$\textcircled{3} \quad \frac{dh_i}{d\theta} = x_i$$

$$\begin{aligned} \longrightarrow \frac{dJ}{d\theta} &= \textcircled{1} * \textcircled{2} * \textcircled{3} \\ &= -\frac{1}{m} \sum \left( y_i/h_i - \frac{(1-y_i)}{(1-h_i)} \right) \cdot (h_i \cdot (1-h_i)) \cdot x_i \\ &= -\frac{1}{m} \sum \left( y_i(1-h_i) - (1-y_i)h_i \right) x_i \\ &= -\frac{1}{m} \sum (y_i - h_i) x_i = \frac{1}{m} \sum (h_i - y_i) x_i \end{aligned}$$