

# Notation

This section provides a concise reference describing the notation used throughout this book. If you are unfamiliar with any of the corresponding mathematical concepts, we describe most of these ideas in chapters 2–4.

## Numbers and Arrays

$a$	A scalar (integer or real)
$\mathbf{a}$	A vector
$\mathbf{A}$	A matrix
$\mathbf{A}$	A tensor
$\mathbf{I}_n$	Identity matrix with $n$ rows and $n$ columns
$\mathbf{I}$	Identity matrix with dimensionality implied by context
$\mathbf{e}^{(i)}$	Standard basis vector $[0, \dots, 0, 1, 0, \dots, 0]$ with a 1 at position $i$
$\text{diag}(\mathbf{a})$	A square, diagonal matrix with diagonal entries given by $\mathbf{a}$
$a$	A scalar random variable
$\mathbf{a}$	A vector-valued random variable
$\mathbf{A}$	A matrix-valued random variable

## Sets and Graphs

$\mathbb{A}$	A set
$\mathbb{R}$	The set of real numbers
$\{0, 1\}$	The set containing 0 and 1
$\{0, 1, \dots, n\}$	The set of all integers between 0 and $n$
$[a, b]$	The real interval including $a$ and $b$
$(a, b]$	The real interval excluding $a$ but including $b$
$\mathbb{A} \setminus \mathbb{B}$	Set subtraction, i.e., the set containing the elements of $\mathbb{A}$ that are not in $\mathbb{B}$
$\mathcal{G}$	A graph
$Pa_{\mathcal{G}}(\mathbf{x}_i)$	The parents of $\mathbf{x}_i$ in $\mathcal{G}$

## Indexing

$a_i$	Element $i$ of vector $\mathbf{a}$ , with indexing starting at 1
$a_{-i}$	All elements of vector $\mathbf{a}$ except for element $i$
$A_{i,j}$	Element $i, j$ of matrix $\mathbf{A}$
$\mathbf{A}_{i,:}$	Row $i$ of matrix $\mathbf{A}$
$\mathbf{A}_{:,i}$	Column $i$ of matrix $\mathbf{A}$
$A_{i,j,k}$	Element $(i, j, k)$ of a 3-D tensor $\mathbf{A}$
$\mathbf{A}_{::,i}$	2-D slice of a 3-D tensor
$\mathbf{a}_i$	Element $i$ of the random vector $\mathbf{a}$

## Linear Algebra Operations

$\mathbf{A}^{\top}$	Transpose of matrix $\mathbf{A}$
$\mathbf{A}^{+}$	Moore-Penrose pseudoinverse of $\mathbf{A}$
$\mathbf{A} \odot \mathbf{B}$	Element-wise (Hadamard) product of $\mathbf{A}$ and $\mathbf{B}$
$\det(\mathbf{A})$	Determinant of $\mathbf{A}$

## Calculus

$\frac{dy}{dx}$	Derivative of $y$ with respect to $x$
$\frac{\partial y}{\partial x}$	Partial derivative of $y$ with respect to $x$
$\nabla_{\mathbf{x}} y$	Gradient of $y$ with respect to $\mathbf{x}$
$\nabla_{\mathbf{X}} y$	Matrix derivatives of $y$ with respect to $\mathbf{X}$
$\nabla_{\mathbf{X}} y$	Tensor containing derivatives of $y$ with respect to $\mathbf{X}$
$\frac{\partial f}{\partial \mathbf{x}}$	Jacobian matrix $\mathbf{J} \in \mathbb{R}^{m \times n}$ of $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$
$\nabla_{\mathbf{x}}^2 f(\mathbf{x})$ or $\mathbf{H}(f)(\mathbf{x})$	The Hessian matrix of $f$ at input point $\mathbf{x}$
$\int f(\mathbf{x}) d\mathbf{x}$	Definite integral over the entire domain of $\mathbf{x}$
$\int_{\mathbb{S}} f(\mathbf{x}) d\mathbf{x}$	Definite integral with respect to $\mathbf{x}$ over the set $\mathbb{S}$

## Probability and Information Theory

$a \perp b$	The random variables $a$ and $b$ are independent
$a \perp b \mid c$	They are conditionally independent given $c$
$P(a)$	A probability distribution over a discrete variable
$p(a)$	A probability distribution over a continuous variable, or over a variable whose type has not been specified
$a \sim P$	Random variable $a$ has distribution $P$
$\mathbb{E}_{\mathbf{x} \sim P}[f(\mathbf{x})]$ or $\mathbb{E}f(\mathbf{x})$	Expectation of $f(\mathbf{x})$ with respect to $P(\mathbf{x})$
$\text{Var}(f(\mathbf{x}))$	Variance of $f(\mathbf{x})$ under $P(\mathbf{x})$
$\text{Cov}(f(\mathbf{x}), g(\mathbf{x}))$	Covariance of $f(\mathbf{x})$ and $g(\mathbf{x})$ under $P(\mathbf{x})$
$H(\mathbf{x})$	Shannon entropy of the random variable $\mathbf{x}$
$D_{\text{KL}}(P \parallel Q)$	Kullback-Leibler divergence of $P$ and $Q$
$\mathcal{N}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$	Gaussian distribution over $\mathbf{x}$ with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$

## Functions

$f : \mathbb{A} \rightarrow \mathbb{B}$	The function $f$ with domain $\mathbb{A}$ and range $\mathbb{B}$
$f \circ g$	Composition of the functions $f$ and $g$
$f(\mathbf{x}; \boldsymbol{\theta})$	A function of $\mathbf{x}$ parametrized by $\boldsymbol{\theta}$ . (Sometimes we write $f(\mathbf{x})$ and omit the argument $\boldsymbol{\theta}$ to lighten notation)
$\log x$	Natural logarithm of $x$
$\sigma(x)$	Logistic sigmoid, $\frac{1}{1 + \exp(-x)}$
$\zeta(x)$	Softplus, $\log(1 + \exp(x))$
$\ \mathbf{x}\ _p$	$L^p$ norm of $\mathbf{x}$
$\ \mathbf{x}\ $	$L^2$ norm of $\mathbf{x}$
$x^+$	Positive part of $x$ , i.e., $\max(0, x)$
$\mathbf{1}_{\text{condition}}$	is 1 if the condition is true, 0 otherwise

Sometimes we use a function  $f$  whose argument is a scalar but apply it to a vector, matrix, or tensor:  $f(\mathbf{x})$ ,  $f(\mathbf{X})$ , or  $f(\mathbf{X})$ . This denotes the application of  $f$  to the array element-wise. For example, if  $\mathbf{C} = \sigma(\mathbf{X})$ , then  $C_{i,j,k} = \sigma(X_{i,j,k})$  for all valid values of  $i$ ,  $j$  and  $k$ .

## Datasets and Distributions

$p_{\text{data}}$	The data generating distribution
$\hat{p}_{\text{data}}$	The empirical distribution defined by the training set
$\mathbb{X}$	A set of training examples
$\mathbf{x}^{(i)}$	The $i$ -th example (input) from a dataset
$y^{(i)}$ or $\mathbf{y}^{(i)}$	The target associated with $\mathbf{x}^{(i)}$ for supervised learning
$\mathbf{X}$	The $m \times n$ matrix with input example $\mathbf{x}^{(i)}$ in row $\mathbf{X}_{i,:}$