

CH2. Mathematical Building Blocks of Deep-learning

Il-Youp Kwak
Chung-Ang University



Mathematical Building Blocs of Deep-learning

A first look on neural network

Data representations for neural network

The gear of neural networks: tensor operations

The engine of neural networks: gradient-based optimization

Looking back at our first example

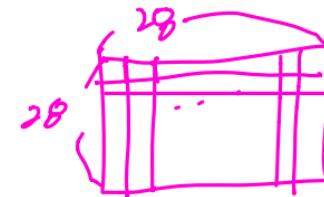


A first look on neural network



Figure 2.1 MNIST sample digits

Mnist data analysis example



Hand written digit (28,28 pixels) for 10 categories (0-9)

60,000 training set and 10,000 test set



Mnist data

Listing 2.1 Loading the MNIST dataset in Keras

```
from tensorflow.keras.datasets import mnist  
(train_images, train_labels), (test_images, test_labels) = mnist.load_data()
```

Try these codes from <https://colab.research.google.com/github/>

The screenshot shows a GitHub search interface. At the top, there are two tabs: "Examples" and "GitHub". Below the tabs is a search bar with the placeholder "Enter a GitHub URL or search by organization or user". To the right of the search bar is a checkbox labeled "Include private repos" and a magnifying glass icon. The search term "fchollet" is entered into the search bar. Underneath the search bar, there are two dropdown menus: "Repository:" with "fchollet/deep-learning-with-python-notebooks" selected and "Branch:" with "master" selected. Below these dropdowns is a "Path" input field. The main content area displays two repository entries. The first entry is "chapter02_mathematical-building-blocks.ipynb" with a GitHub icon, and the second is "chapter03_introduction-to-keras-and-tf.ipynb" also with a GitHub icon. Each entry has a magnifying glass icon and a copy icon to its right.



Welcome To Colaboratory

Share

Table of contents

- Getting started
 - Data science
 - Machine learning
 - More Resources

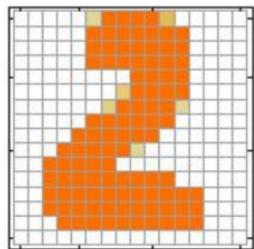
Featured examples

+ Section

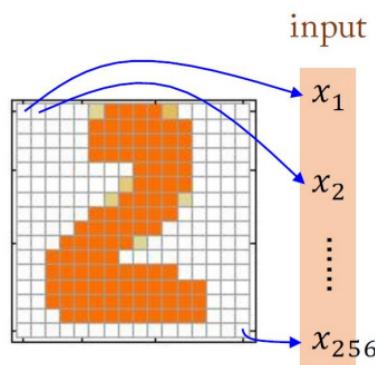
Recent		Google Drive	
 Filter notebooks			
Title	Last opened	First opened	
 Welcome To Colaboratory	12:29 PM	Sep 23, 2019	
 3.5-classifying-movie-reviews.ipynb	March 4	Mar 14, 2021	 
 2.1-a-first-look-at-a-neural-network.ipynb	March 4	Feb 27, 2021	 
 chapter02_mathematical-building-blocks.ipynb	March 3	January 24	 
 neurokit2.ipynb	February 28	February 28	 

Whether you're a **student**, a **data scientist** or an **AI researcher**, Colab can make your work easier. Watch [Introduction to Colab](#) to learn more, or just get started below!





“2”



output

\hat{y}_1 0.09 is “1”

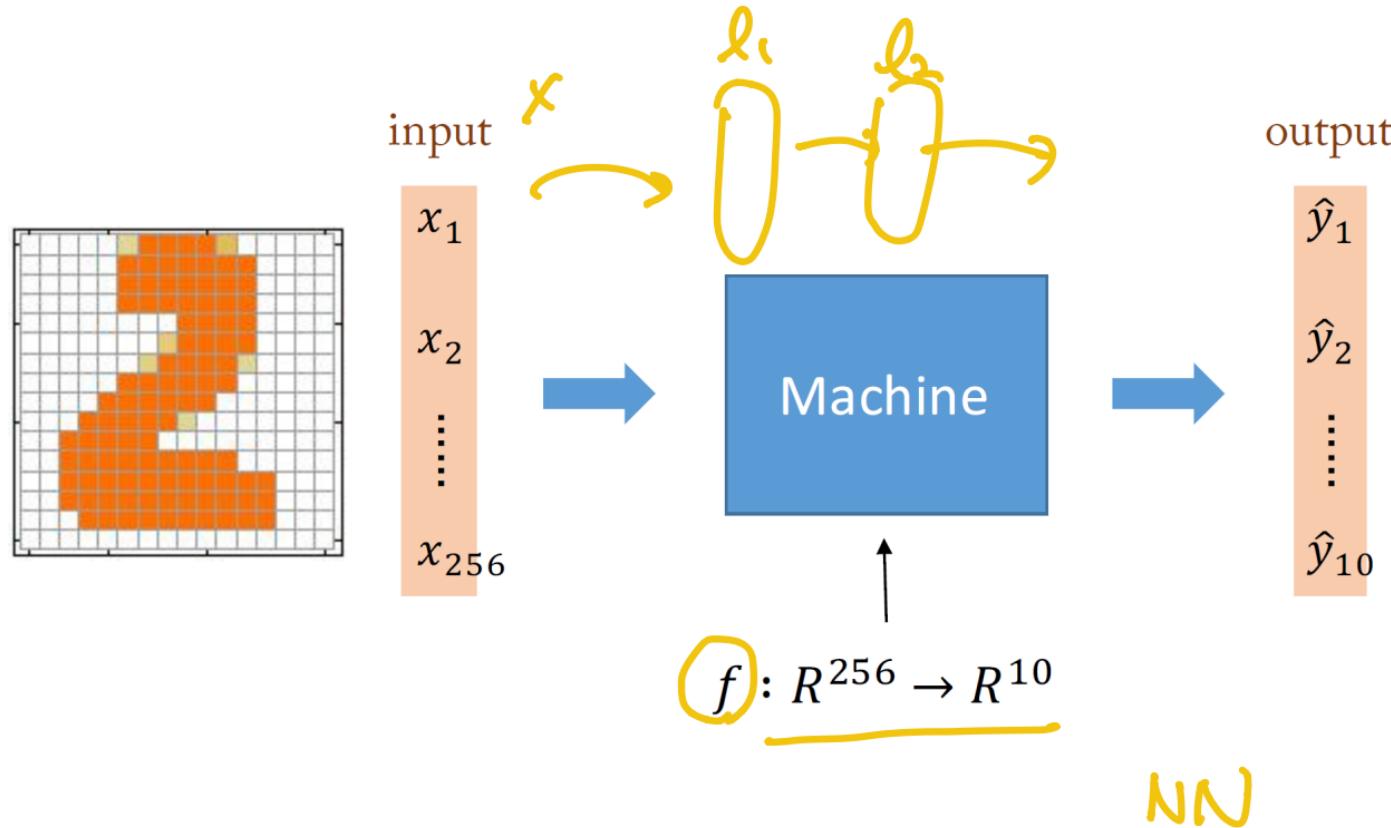
\hat{y}_2 0.71 is “2”

\hat{y}_{10} 0.03 is “0”

the image is “2”

$16 \times 16 = 256$
ink $\rightarrow 1$
no ink $\rightarrow 0$

Each dimension represents
the confidence of a digit.

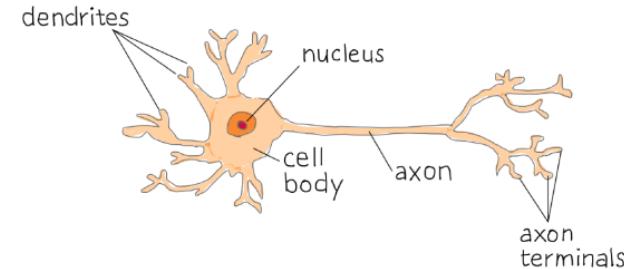
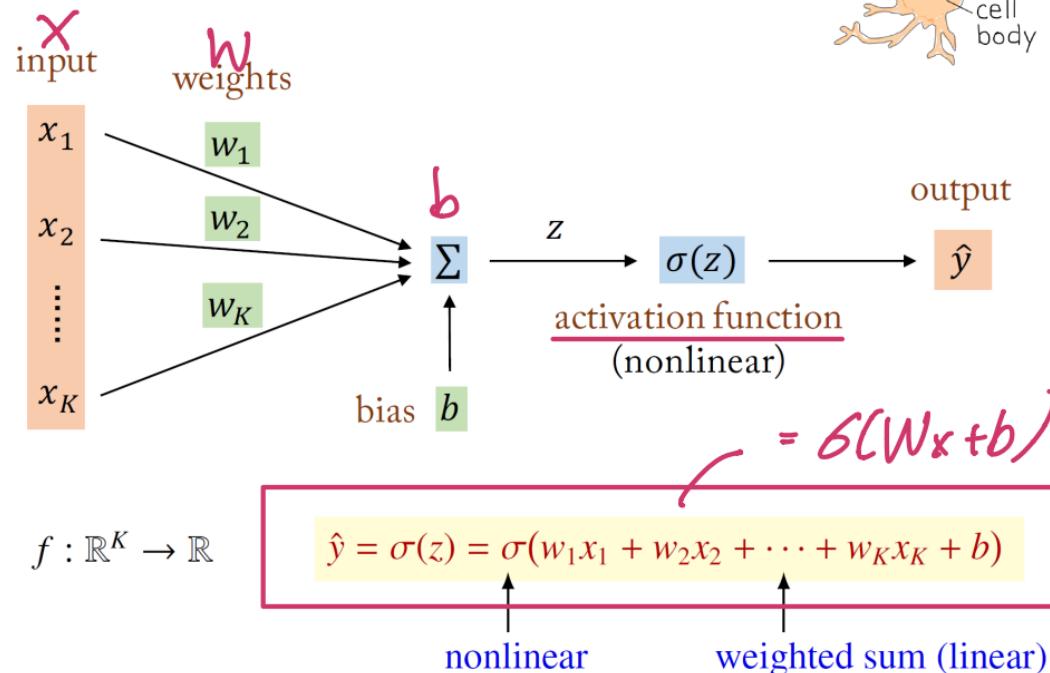


In Deep Learning, the function f is represented by a Neural Network.

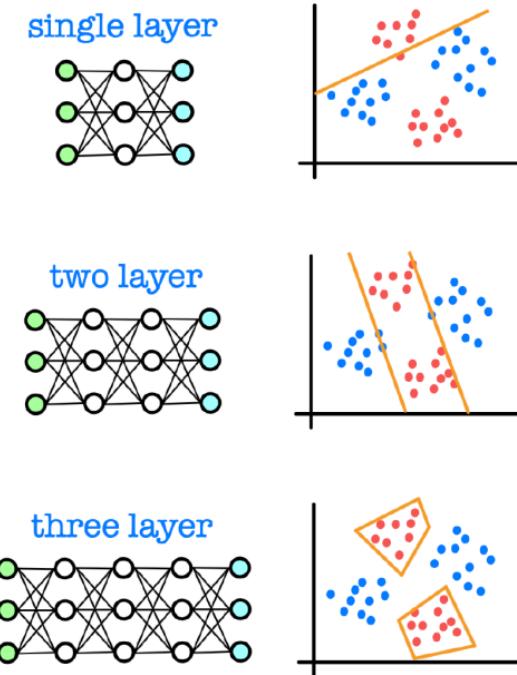
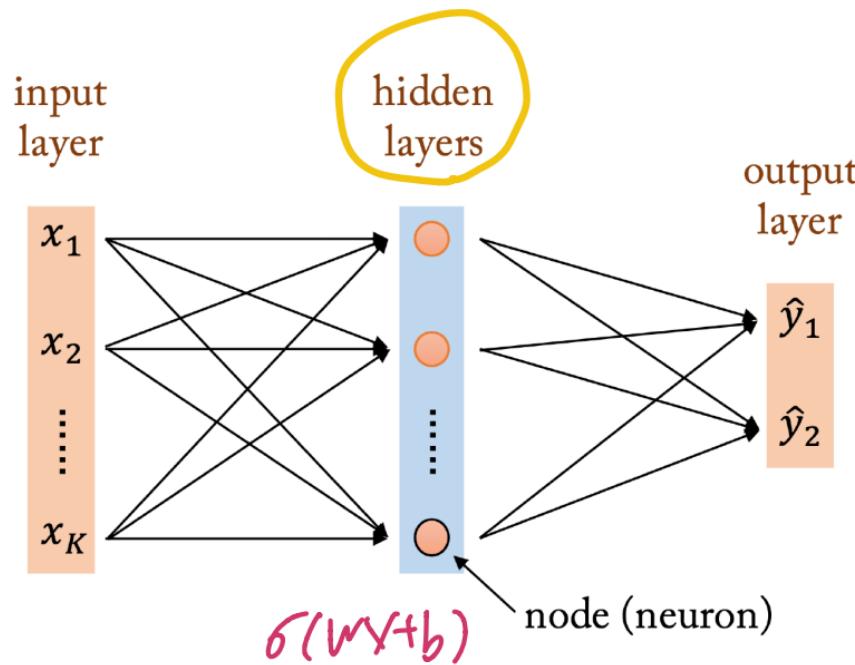
NN

Perceptron (neuron) [Frank Rosenblatt, 1957]

↳ One layer model



Multilayer Perceptron (MLP)

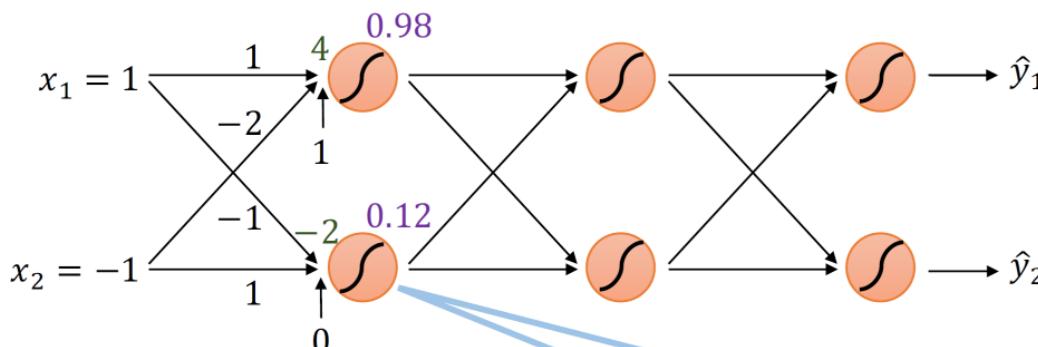


Forward pass computation

$$\sigma(Wx+b)$$

$$\sigma(w_1x_1 + w_2x_2 + \dots + w_Kx_K + b)$$

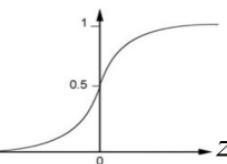
$$0.98 = \sigma(1 \cdot 1 + (-1) \cdot (-2) + 1)$$



$$\begin{aligned} & W \quad X \quad b \\ & \sigma \left(\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \\ & = \sigma \left(\begin{bmatrix} 4 \\ -2 \end{bmatrix} \right) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix} \end{aligned}$$

sigmoid function

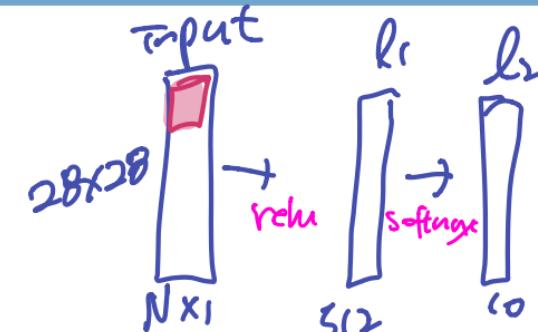
$$\sigma(z) = \frac{1}{1 + e^{-z}}$$



activation function

Listing 2.2 The network architecture

```
from tensorflow import keras
from tensorflow.keras import layers
model = keras.Sequential([
    layers.Dense(512, activation="relu"),
    layers.Dense(10, activation="softmax")
])
```



Core building block of NN is layers

Layers extract *representations*

$$\hat{Y} = \text{softmax}\left(\underbrace{W_2}_{10 \times 512} \underbrace{\text{relu}(W_1 x + b_1)}_{N \times 1} + \underbrace{b_2}_{10 \times 1}\right)$$

This step define our model $f(x; \text{weights})$

$$\begin{aligned} & \Rightarrow 512 \times 1 \\ & \Rightarrow 10 \times 1 \end{aligned}$$

parameters: w_1, w_2, b_1, b_2



The mathematical building blocks of neural networks

A first look at a neural network

- Data representations for neural networks
 - Scalars (rank-0 tensors)
 - Vectors (rank-1 tensors)
 - Matrices (rank-2 tensors)
 - Rank-3 and higher-rank tensors
 - Key attributes
 - Manipulating tensors in NumPy
 - The notion of data batches
 - Real-world examples of data tensors
 - Vector data
 - Timeseries data or sequence data
 - Image data
 - Video data
- The gears of neural networks: tensor

```
+ Code + Text | ⌂ Copy to Drive  
[8] train_images.shape  
(60000, 28, 28)  
[9] len(train_labels)  
60000  
▶ train_labels  
array([5, 0, 4, ..., 5, 6, 8], dtype=uint8)  
[11] test_images.shape  
(10000, 28, 28)  
[12] len(test_labels)  
10000  
▶ test_labels  
array([7, 2, 1, ..., 4, 5, 6], dtype=uint8)
```

The network architecture

Warning: you are connected to a GPU runtime, but not utilizing the GPU. [Change to a standard runtime](#)

85 completed at 12:36 PM

What's next?

$$f_{\theta}(x) = y$$

Loss function: How the network measure its performance

Optimizer: Mechanism through which the network will update itself based on the data it sees and its loss function

Metrics to monitor during training and testing: Accuracy

Listing 2.3 The compilation step

```
model.compile(optimizer="rmsprop",
              loss="sparse_categorical_crossentropy",
              metrics=["accuracy"])
```



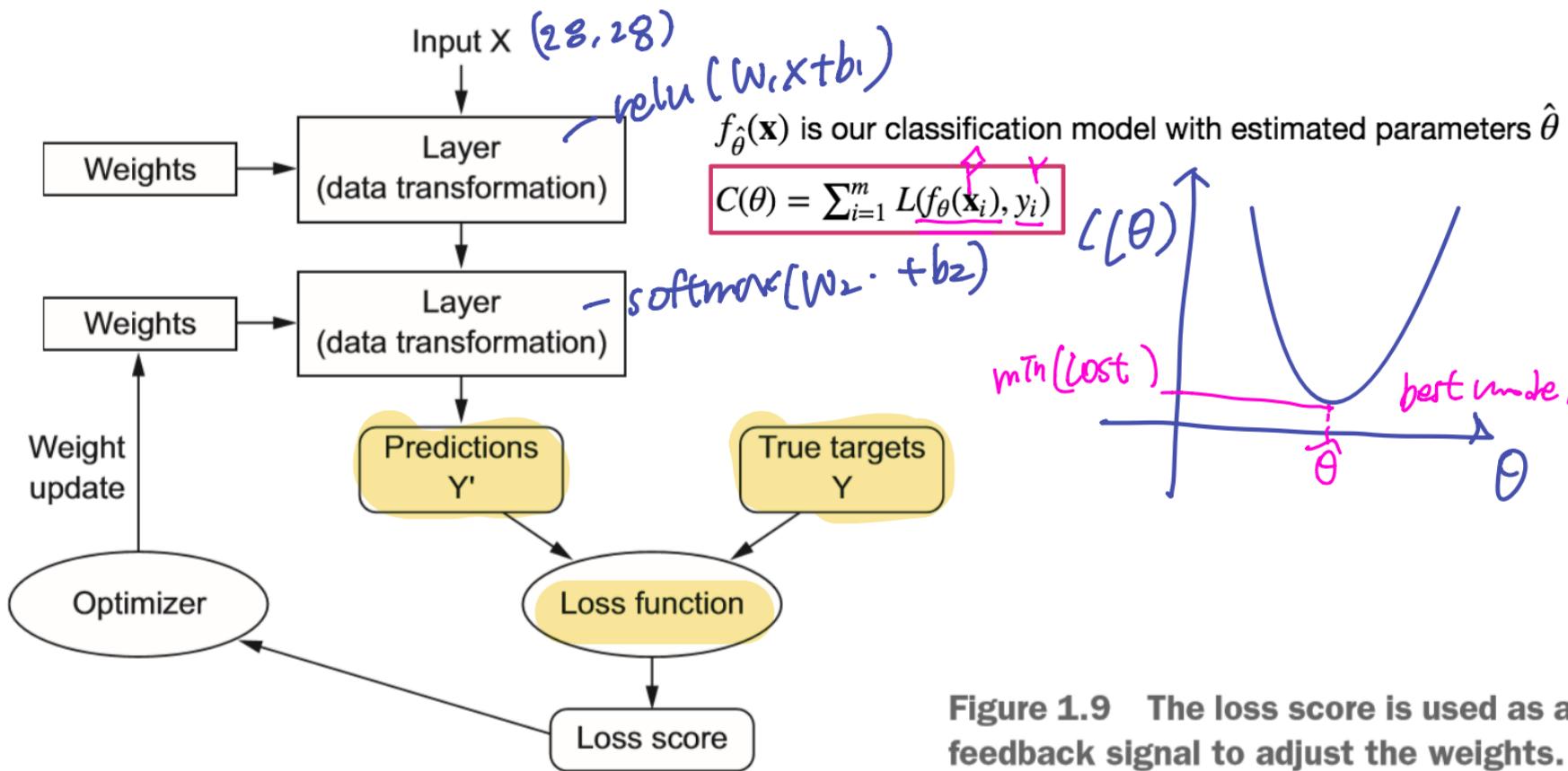


Figure 1.9 The loss score is used as a feedback signal to adjust the weights.



The mathematical building blocks of neural networks

A first look at a neural network

- Data representations for neural networks
 - Scalars (rank-0 tensors)
 - Vectors (rank-1 tensors)
 - Matrices (rank-2 tensors)
 - Rank-3 and higher-rank tensors
 - Key attributes
 - Manipulating tensors in NumPy
 - The notion of data batches
 - Real-world examples of data tensors
 - Vector data
 - Timeseries data or sequence data
 - Image data
 - Video data
- The gears of neural networks: tensor operations
 - Element-wise operations

```
+ Code + Text ⌂ Copy to Drive
```

[14] from tensorflow import keras
from tensorflow.keras import layers
model = keras.Sequential([
 layers.Dense(512, activation="relu"),
 layers.Dense(10, activation="softmax")])

The compilation step

```
model.compile(optimizer="rmsprop",
              loss="sparse_categorical_crossentropy",
              metrics=["accuracy"])
```

[

Preparing the image data

```
[ ] train_images = train_images.reshape((60000, 28 * 28))  
train_images = train_images.astype("float32") / 255  
test_images = test_images.reshape((10000, 28 * 28))  
test_images = test_images.astype("float32") / 255
```

"Fitting" the model

```
[ ] model.fit(train_images, train_labels, epochs=5, batch_size=128)
```

Rescale values from [0,255] to [0,1]

Listing 2.4 Preparing the image data

```
train_images = train_images.reshape((60000, 28 * 28))
train_images = train_images.astype("float32") / 255
test_images = test_images.reshape((10000, 28 * 28))
test_images = test_images.astype("float32") / 255
```

$\text{train} \rightarrow (60000, 28, 28)$
 $(60000, \frac{28 \times 28}{N})$

Fitting the model

Listing 2.5 “Fitting” the model

$\text{batch size} = 100 \rightarrow 60000 / 100 = 600 \text{ update}$ *weight*

```
>>> model.fit(train_images, train_labels, epochs=5, batch_size=128)
Epoch 1/5
60000/60000 [=====] - 5s - loss: 0.2524 - acc: 0.9273
Epoch 2/5
51328/60000 [=====>.....] - ETA: 1s - loss: 0.1035 - acc: 0.9692
```

600



Make Predictions

Listing 2.6 Using the model to make predictions

```
>>> test_digits = test_images[0:10]
>>> predictions = model.predict(test_digits)
>>> predictions[0] → 10개 이미지의 확률 예측값.
array([1.0726176e-10, 1.6918376e-10, 6.1314843e-08, 8.4106023e-06,
       2.9967067e-11, 3.0331331e-09, 8.3651971e-14, 9.9999106e-01,
       2.6657624e-08, 3.8127661e-07], dtype=float32)
```

test data는
) 10개의 예측값
10개 이미지의 확률 예측값.
9.9999106e-01,
↳ label=7이다!!

Evaluating the model

Listing 2.7 Evaluating the model on new data

```
>>> test_loss, test_acc = model.evaluate(test_images, test_labels)
>>> print(f"test_acc: {test_acc}")
test_acc: 0.9785
```

X *y*



The mathematical building blocks of neural networks

A first look at a neural network

- Data representations for neural networks
 - Scalars (rank-0 tensors)
 - Vectors (rank-1 tensors)
 - Matrices (rank-2 tensors)
 - Rank-3 and higher-rank tensors
 - Key attributes
 - Manipulating tensors in NumPy
 - The notion of data batches
 - Real-world examples of data tensors
 - Vector data
 - Timeseries data or sequence data
 - Image data
 - Video data
- The gears of neural networks: tensor operations
 - Element-wise operations

```
+ Code + Text | ⚡ Copy to Drive
```

▶ from tensorflow import keras
from tensorflow.keras import layers
model = keras.Sequential([
 layers.Dense(512, activation="relu"),
 layers.Dense(10, activation="softmax")
])

The compilation step

```
[22] model.compile(optimizer="rmsprop",
                      loss="sparse_categorical_crossentropy",
                      metrics=["accuracy"])
```

1

Preparing the image data

```
[ ] train_images = train_images.reshape((60000, 28 * 28))
train_images = train_images.astype("float32") / 255
test_images = test_images.reshape((10000, 28 * 28))
test_images = test_images.astype("float32") / 255
```

"Fitting" the model

```
[ ] model.fit(train_images, train_labels, epochs=5, batch_size=128)
```

[Paw]

$$40(920 = 512 \times 784 + 512)$$

$$5130 = 10 \times 512 + 10$$