Lista 10

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Questão 1

$$EZ = \int_0^\infty \int_0^\infty \frac{y}{x} f_{X,Y}(x,y) dx dz$$
$$= ...$$

Questão 2

$X \sim X^2(n) = Gamma(n/2, 1/2)$

Seja $Y = \sqrt{X}$

$P(X > 0) = P(Y > 0) = 1$

Temos que

$$F_Y(y) = \int_0^y f_Y(u) du = \int_0^{y^2} f_X(x) dx = F_X(y^2)$$

assim

$$f_y(y) = f_X(y^2) 2y$$

Então

$$EY = \int_0^\infty f_X(y^2) 2y^2 dy = ...$$

ou

$$EY = \int_0^\infty \sqrt{x} f_X(x) dx = ...$$

Questão 3

$U_1 = U_2 \sim Exp(\lambda)$

$P(U_1 > 0) = P(U_2 > 0) = 1$

Para $Y = \max\{U_1, U_2\}$, $P(Y > 0) = 1$

$$F_Y(y) = P(\max\{U_1, U_2\} \le y) = P(U_1 \le y, U_1 > U_2) + P(U_2 \le y, U_2 > U_1) =$$
$$= P(U_1 \le y, U_2 < y) + P(U_2 \le y, U_1 < y) =$$
$$= P(U_1 \le y)P(U_2 \le y) + P(U_2 \le y)P(U_1 \le y) =$$
$$= 2P(U_1 \le y)P(U_2 \le y) =$$
$$= 2F_{U_1}(y)F_{U_2}(y)$$
$$= 2F_{U_1}(y)^2$$

assim $f_Y(y) = 4F_{U_1}(y)f_{U_1}(y)$

Então

$$f_Y(y) = 4e^{-\lambda y}\lambda \int_0^y e^{-\lambda u_1}\lambda du_1 = -4e^{-2\lambda y}$$

$$EY = \int_0^\infty -4ye^{-2\lambda y}dy$$

Questão 4

$$EX = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sen\theta f_\Theta(\theta)d\theta$$

Questão 5

a)

$$E|X| = \int_{-\infty}^\infty |x|f_X(x)dx$$

b)

$$EX^2 = \int_{-\infty}^\infty x^2 f_X(x)dx$$

Questão 6

$$EY = \int_{-\infty}^\infty y \int_{-\infty}^\infty f_{X,Y}(x,y)dxdy$$

$$EXY = \int_{-\infty}^\infty \int_{-\infty}^\infty xy f_{X,Y}(x,y)dxdy$$

$Cov(X,Y) = E(XY) - EYEX$

$\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$

Questão 7

$P(Y > 0) = P(X \in \mathbb{R}) = P(Z \in \mathbb{R}) = 1$

$$EXY = \int_0^\infty \int_{-\infty}^\infty xy f_X(x) f_Y(y) dx dy$$

Questão 8

$$
\begin{aligned}
\rho(X - \rho Y, Y) &= \frac{E((X - \rho Y)Y) - E(X - \rho Y)E(Y)}{\sigma_{X-\rho Y} \sigma_Y} = \\
&= \frac{E(XY - \rho YY)}{1 - \rho^2} = \\
&= \frac{E(XY) - \rho E(Y^2)}{1 - \rho^2} = \\
&= \frac{E(XY) - 1}{1 - \rho^2} = 0
\end{aligned}
$$

Questão 9

$$\int_b^a e^{tx} \frac{1}{a-b} dx = \frac{1}{a-b} \frac{e^{ta} - e^{tb}}{t}$$

Questão 10

a)

$$M_x(t) = \sum_{k=0}^n \binom{n}{k} e^{tk} p^k (1-p)^{n-k}$$

b)

$$M_x(t) = \sum_{k=0}^\infty e^{tk} \frac{e^\lambda \lambda^k}{k!}$$

Questão 11

a)

$$M_x(t) = \int_{-\infty}^\infty e^{tx} f_X(x) dx$$

b)

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} \frac{1}{2} e^{|x|} dx$$

Questão 12

Questão 13

a)

$$ES_n = n\mu \rightarrow E\bar{X} = \frac{ES_n}{n} = \mu$$

$$VarS_n = n\sigma^2 \rightarrow Var\bar{X} = Var(\frac{S_n}{n}) = \frac{1}{n^2} Var(S_n) = \frac{\sigma^2}{n}$$

b)

$$P(|\bar{X} - \mu| \leq \frac{\sigma}{10}) \geq 0.95$$
$$P(|\bar{X} - \mu| \geq \frac{\sigma}{10}) \leq 0.05$$
$$\frac{Var\bar{X}}{\frac{\sigma}{10}} \leq 0.05$$
$$\frac{\sigma^2 10}{n\sigma} \leq 0.05$$
$$\frac{\sigma 10}{0.05} \leq n$$
$$n \geq 200\sigma$$

Questão 14

a)

$$P(X > 85) \leq \frac{75}{85} = 0.88$$

b)

$$P(65 < X < 85) = P(65 - 75 < X - 75 < 85 - 75) =$$
$$= P(-10 < X - 75 < 10)$$
$$= P(|X - 75| < 10)$$
$$= 1 - P(|X - 75| \geq 10) \leq \frac{25}{100}$$
$$= P(|X - 75| \geq 10) \geq 1 - \frac{25}{100}$$
$$= P(|X - 75| \geq 10) \geq 0.75$$

Questão 15

$$P(|S_{100} - 100| \leq 5) = 2P(Z \leq \frac{105 - 97}{10\ 0.1}) - 1$$