

# Scaling Analysis of Soliton Behavior in the Fluxonic Klein-Gordon System

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## Abstract

This paper analyzes soliton scaling in the nonlinear Klein-Gordon system within a fluxonic framework, hypothesizing that mass ( $m$ ) and nonlinearity ( $g$ ) variations influence gravitational interactions, testable via Bose-Einstein Condensate (BEC) modulation akin to gravitational shielding experiments. Simulations quantify phase shifts and energy conservation across  $m$  and  $g$ , predicting measurable gravitational wave effects. These challenge General Relativity and quantum field theory, offering a deterministic gravitational model.

## 1 Introduction

Solitons in nonlinear systems offer insights into fundamental interactions (OCR Section 1). This study extends scaling analysis to a fluxonic context, aligning with OCRs shielding paradigm (Section 3), linking  $m$  and  $g$  to testable gravitational effects.

## 2 Hypothesis

Soliton properties scale with:

- **Mass ( $m$ ):** Affects stability and gravitational coupling.
- **Nonlinearity ( $g$ ):** Drives interaction strength, potentially measurable via wave attenuation (OCR Section 3).

Governed by:

$$\frac{\partial^2 \phi}{\partial t^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} + m^2 \phi + g \phi^3 = 8\pi G \rho, \quad (1)$$

where  $\phi(x, t)$  is the fluxonic field,  $c = 1$ ,  $m$  and  $g$  vary,  $\rho$  is mass density (negligible here).

## 3 Simulation Results and Observations

Simulations analyze soliton collisions across  $m$  and  $g$ :

### 3.1 Phase Shift Dependence

### 3.2 Energy Conservation

Energy increases with  $g$ , indicating stronger interactions.

$m$	$g$	Phase Shift (Soliton 1)	Phase Shift (Soliton 2)	Final Energy
0.5	0.500	0.00	2.81	20.01
0.5	0.875	-4.82	2.21	31.09
0.5	1.250	-5.82	8.94	42.02
0.5	1.625	-6.43	13.76	53.12
0.5	2.000	0.10	5.33	64.61

Table 1: Scaling Analysis Results

## 4 Simulation Code

Listing 1: Soliton Scaling Simulation

```

import numpy as np
import matplotlib.pyplot as plt

# Parameters
L = 20.0
Nx = 200
dx = L / Nx
dt = 0.01
Nt = 500
c = 1.0
G = 1.0
rho = np.zeros(Nx)
params = [(0.5, 0.5), (0.5, 0.875), (0.5, 1.25), (0.5, 1.625), (0.5, 2.0)]

# Grid
x = np.linspace(-L/2, L/2, Nx)
results = []

for m, g in params:
    # Initial conditions: two solitons
    phi_initial = np.tanh((x + 5) / np.sqrt(2)) + np.tanh((x - 5) / np.sqrt(2))
    phi = phi_initial.copy()
    phi_old = phi.copy()
    phi_new = np.zeros_like(phi)

    # Time evolution
    for n in range(Nt):
        d2phi_dx2 = (np.roll(phi, -1) - 2 * phi + np.roll(phi, 1)) / dx**2 # Periodic bou
        phi_new = 2 * phi - phi_old + dt**2 * (c**2 * d2phi_dx2 - m**2 * phi - g * phi**3 -
        phi_old, phi = phi, phi_new

    # Phase shift (simplified peak analysis)
    peak1 = x[np.argmax(phi[:Nx/2])]
    peak2 = x[np.argmax(phi[Nx/2:])] + Nx/2
    energy = np.sum(0.5 * ((phi - phi_old)/dt)**2 + 0.5 * (np.roll(phi, -1) - phi)/dx**2 +
    results.append((m, g, peak1 - (-5), peak2 - 5, energy))

# Plot for g = 2.0
plt.plot(x, phi_initial, label="Initial_State")
plt.plot(x, phi, label="Final_State_(m=0.5,g=2.0)")
plt.xlabel("x")

```

```
plt.ylabel(" (x,t)")
plt.title("Soliton_Collision_Simulation")
plt.legend()
plt.grid()
plt.show()
```

## 5 Experimental Proposal

Test via (OCR Section 3):

- **Setup:** BEC with solitonic excitations (OCR Section 3.2).
- **Source:** Rotating mass (OCR Section 3.1).
- **Measurement:** LIGO interferometers (OCR Section 3.3) for wave shifts.

## 6 Predicted Experimental Outcomes

Standard Prediction	Fluxonic Prediction
Unaltered gravitational waves	Attenuation with $g$ increase
No soliton-gravity link	Phase shift-induced wave effects
Fixed energy conservation	Energy scales with $g$

Table 2: Comparison of Predictions

## 7 Implications

If confirmed (OCR Section 5):

- Solitons influence gravity, challenging GR.
- Fluxonic framework unifies interactions.
- Engineering applications (OCR Section 5).

## 8 Future Directions

(OCR Section 6):

- Explore bound states via simulations.
- Test higher  $m, g$  values.
- Integrate with LIGO data analysis.