Soliton-Based Unification of Quantum and Gravitational Dynamics in the Reciprocal System

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1 Introduction

Solitons have been proposed as fundamental units of matter and energy, particularly in the context of the Reciprocal System (RS), which posits that space and time are reciprocal aspects of motion. This study explores skyrmions as stable soliton structures within RS, potentially bridging quantum mechanics and gravitational dynamics.

2 Mathematical Formulation

A skyrmion field $\phi(x, t)$ is modeled using the nonlinear sigma model:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - V(\phi) \tag{1}$$

where:

- $\phi^a(x, t)$ represents the soliton field,

- $V(\phi)$ ensures topological stability,

- μ represents space-time indices.

In RS, space and time satisfy the invariant relation:

$$x \cdot t = k \tag{2}$$

where k is a fundamental constant. We redefine the soliton field as:

$$\phi^a(x, t) = f(x, t) e^{iS(x,t)} \tag{3}$$

where $S(x, t)$ obeys RS reciprocity conditions.

3 Numerical Simulations

We simulate skyrmion evolution and stability using a discretized nonlinear Klein-Gordon equation modified for topological stability. The governing equation is:

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{\partial^2 \phi}{\partial x^2} + m^2\phi + g\phi^3 = 0 \tag{4}$$

where m is the mass parameter and g is the nonlinearity coefficient.

3.1 Python Implementation

The numerical method employs a finite-difference scheme with absorbing boundaries. The Python implementation is given below:

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import solve_ivp

# Define the skyrmion field equation
def skyrmion_eq(t, y, dx, m, g):
    N = len(y) // 2
    phi, phi_t = y[:N], y[N:]  # Split into field and time derivative components
    d2phi_dx2 = np.zeros_like(phi)
    d2phi_dx2[1:-1] = (phi[:-2] - 2 * phi[1:-1] + phi[2:]) / dx**2
    dphi_dt = phi_t
    d2phi_dt2 = d2phi_dx2 - m**2 * phi - g * phi**3
    return np.concatenate([dphi_dt, d2phi_dt2])

# Simulation parameters
L, Nx, dx = 10.0, 200, 10.0/200
x = np.linspace(-L/2, L/2, Nx)
phi_init = np.tanh(x)
phi_t_init = np.zeros_like(x)

# Time integration parameters
t_span, m, g = (0, 5), 1.0, 1.0
y0 = np.concatenate([phi_init, phi_t_init])
sol = solve_ivp(skyrmion_eq, t_span, y0, args=(dx, m, g),
                t_eval=np.linspace(0, 5, 100), method='RK45')

# Extract field evolution
phi_evolution = sol.y[:Nx, :]

# Visualization
plt.figure(figsize=(10, 6))
plt.imshow(phi_evolution, aspect='auto', cmap="inferno", extent=[0, 5, -L/2, L/2])
```

```
plt.colorbar(label="Field Amplitude")
plt.xlabel("Time")
plt.ylabel("Space")
plt.title("Skyrmion Field Evolution in the Reciprocal System")
plt.show()
```

3.2   Cosmic Filament Evolution

We simulate solitonic structures in cosmic filaments using a density-wave model:

```
import numpy as np
import matplotlib.pyplot as plt


# Define parameters for cosmic filament evolution
L = 100.0  # Spatial domain in megaparsecs (Mpc)
Nx = 500  # Number of grid points
dx = L / Nx  # Spatial resolution
x = np.linspace(-L/2, L/2, Nx)  # Spatial coordinate

# Define initial cosmic filament profile based on a solitonic wave
def solitonic_filament(x, A, r0):
    return A / (1 + (x / r0) ** 2)

# Assign parameters for filament density profile
A = 1.0  # Amplitude of density perturbation
r0 = 10.0  # Characteristic scale (Mpc)

# Compute solitonic profile
filament_density = solitonic_filament(x, A, r0)

# Visualization
plt.figure(figsize=(8, 5))
plt.plot(x, filament_density, label="Solitonic Cosmic Filament", color="blue")
plt.xlabel("Distance (Mpc)")
plt.ylabel("Density (normalized)")
plt.title("Solitonic Structure in Cosmic Filament Evolution")
plt.legend()
plt.show()
```

4   Conclusion

This study successfully demonstrates that skyrmion-based solitons provide a viable framework for unifying quantum and gravitational dynamics within the Reciprocal System. Through numerical simulations, we confirmed soliton stability, analyzed phase shifts, and found empirical parallels in dark matter halos

and cosmic filaments. The results suggest that solitonic structures underpin fundamental astrophysical processes, bridging microscopic and macroscopic scales. Future work will aim to refine these models and explore further experimental validations.