APPENDIX 1: Comprehensive Governing Equation of Marketing Mixed Model

$$y_{i,t} = \sum_{k=1}^{m} m_k \widehat{M}_{i,t}^{k} + \sum_{k=1}^{n} e_k \widehat{E}_{i,t}^{k} + \sum_{k=1}^{l} d_k D_{i,t}^{k} + cC_{i,t} + \sum_{k=1}^{f} I^{k} + \epsilon_{i,t}$$

Where

$y_{i,t}$ is the dependent variable in the ith DMA and time t,
$\widehat{M}_{i,t}^{k}$ is the adstock variable for the kth media in the ith DMA at time t. It is defined as

$$\begin{cases} t < t_0 + p \ \ then \ \widehat{M}_{i,t}^{k} = missing \\ t = t_0 + p \ then \ \widehat{M}_{i,t}^{k} = (M_{i,t-p}^{k})^{\beta} \\ t > t_0 + p \ then \ \widehat{M}_{i,t}^{k} = (\alpha \ \widehat{M}_{i,t-1}^{k} + M_{i,t-p}^{k})^{\beta} \end{cases}$$

where

$M_{i,t}^{k}$ is the measured quantity (i.e. number of impressions) of the kth media in the ith DMA at time t, and α, β , and p are the decay rate, the saturation rate and the lag, respectively. The adstock variable is equal to the original non-transformed variable when α=0 , p=0 and β=1. t_0 is the initial time assumed to be equal to 1.

$\widehat{E}_{i,t}^{k}$ is the adstock variable for the kth econometric variable in the ith DMA at time t. It is defined as

$$\begin{cases} t < t_0 + p \ \ then \ \widehat{E}_{i,t}^{k} = missing \\ t = t_0 + p \ then \ \widehat{E}_{i,t}^{k} = (E_{i,t-p}^{k})^{\beta} \\ t > t_0 + p \ then \ \widehat{E}_{i,t}^{k} = (\alpha \ \widehat{E}_{i,t-1}^{k} + E_{i,t-p}^{k})^{\beta} \end{cases}$$

APPENDIX 1 (Continued): Comprehensive Governing Equation of Marketing Mixed Model

where

$E_{i,t}^{k}$ is the measured quantity (i.e. unemployment rate, inflation) of the kth econometric variable in the ith DMA at time t, and α and p are the decay rate, and the lag, respectively, t_0 is the initial time assumed to be equal to 1.

$D_{i,t}^{k}$ is the kth dummy variable capturing the kth seasonality factor (i.e. weekly and monthly effect) in the ith DMA at time t. It is defined as $D_{i,t}^{k} = \begin{cases} 1 & if\ k = t \\ 0 & else \end{cases}$,

$C_{i,t}$ is a measurement of capacity (i.e. number of active hotels room, number of active cruise boats) in the ith DMA at time t. This variable accounts for structural break in the underlying phenomenon affecting the dependent variable,

$\sum_{k=1}^{f} I^{k}$ is the sum of all fixed and random effect intercepts,

$m_k, e_k, d_k, and\ c$ are the regression coefficients, and

$\epsilon_{i,t}$ is the error at the ith DMA and time t.