Problem 1.1

1.) $X[0] = 3$

$X[1] = \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$X[2] = -2$

$X[3] = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\omega nk} \qquad \omega = \frac{2\pi}{N}$$

$$4x[n] = 3e^{0} + \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)e^{j\omega n} - 2e^{2j\omega n} + \left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)e^{3j\omega n}$$

$$= 3 + e^{j(\omega n - \frac{\pi}{4})} - 2e^{2j\omega n} + e^{j(3\omega n + \frac{\pi}{4})}$$

$$Re(x[n]) = \frac{1}{4}\left(3 + \cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right) - 2\cos(\pi n) + \cos\left(\frac{3\pi}{2} + \frac{\pi}{4}\right)\right) ※$$

2.) $X[0] = -2$

$X[1] = \sqrt{3} + j$

$X[2] = 3$

$X[3] = \sqrt{3} - j$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\omega nk} \qquad \omega = \frac{2\pi}{N}$$

$$4x[n] = -2e^{0} + \frac{2}{2}(\sqrt{3} + j)\,e^{j\omega n} + 3e^{2j\omega n} + \frac{2}{2}(\sqrt{3} - j)\,e^{3j\omega n}$$

$$= -2 + 2e^{j(\omega n + \frac{\pi}{6})} + 3e^{2j\omega n} + 2e^{j(3\omega n - \frac{\pi}{6})}$$

$$Re(x[n]) = \frac{1}{4}\left(-2 + 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right) + 3\cos(\pi n) + 2\cos\left(\frac{3\pi}{2}n - \frac{\pi}{6}\right)\right) ※$$

3.) $X[0] = 1$

$X[1] = 2 - j2\sqrt{3}$

$X[2] = -3$

$X[3] = 2 + j2\sqrt{3}$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\omega nk} \qquad \omega = \frac{2\pi}{N}$$

$$4x[n] = 1e^{0} + \frac{4}{4}(2 - j2\sqrt{3})\,e^{j\omega n} - 3e^{2j\omega n} + \frac{4}{4}(2 + j2\sqrt{3})\,e^{3j\omega n}$$

$$= 1 + 4e^{j(\omega n - \frac{\pi}{3})} - 3e^{2j\omega n} + 4e^{j(3\omega n + \frac{\pi}{3})}$$

$$Re(x[n]) = \frac{1}{4}\left(1 + 4\cos\left(\frac{\pi}{2}n - \frac{\pi}{3}\right) - 3\cos(\pi n) + 4\cos\left(\frac{3\pi}{2}n + \frac{\pi}{3}\right)\right) ※$$

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$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

Problem 2.1

1.) $x[n] = \left(\frac{1}{3}\right)^{|n|}$

Let $m = -n$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{-1}\left(\frac{1}{3^{-n}}\right)e^{-j\omega n} + \sum_{n=0}^{\infty}\left(\frac{1}{3^{n}}\right)e^{-j\omega n} = \sum_{n=1}^{\infty}\left(\frac{1}{3}e^{j\omega}\right)^{n} + \sum_{n=0}^{\infty}\left(\frac{1}{3}e^{-j\omega}\right)^{n}$$

$$= \frac{1}{1-\frac{1}{3}e^{j\omega}} - 1 + \frac{1}{1-\frac{1}{3}e^{j\omega}} = \frac{3}{3-e^{j\omega}} - 1 + \frac{3}{3-e^{-j\omega}} = \frac{e^{j\omega}}{3-e^{j\omega}} + \frac{3}{3-e^{-j\omega}} \#$$

2.) $x[n] = a^n\cos(\Omega_0 n)\cdot u[n], |a|<1$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty} a^n\left(\frac{e^{j\Omega_0 n} + e^{-j\Omega_0 n}}{2}\right)e^{-j\omega n} = \frac{1}{2}\left(\sum_{n=0}^{\infty}(ae^{j(\Omega_0-\omega)})^n + \sum_{n=0}^{\infty}(ae^{-j(\Omega_0+\omega)})^n\right)$$

$$= \frac{1}{2}\left(\frac{1}{1-ae^{j(\Omega_0-\omega)}} + \frac{1}{1-ae^{-j(\Omega_0+\omega)}}\right) \#$$

3.) $x[n] = (n+1)a^n u[n], |a|<1$

Let $y[n] = a^n u[n]$

$$FT(y[n]) = \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \frac{1}{1-ae^{-j\omega}}$$

$$X(e^{j\omega}) = \sum_{n=0}^{\infty}(n+1)a^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} na^n e^{-j\omega n} + \sum_{n=0}^{\infty} a^n e^{-j\omega n}$$

$$= FT(ny[n]) + FT(y[n])$$

$$= j\frac{d}{d\omega}\left(\frac{1}{1-ae^{-j\omega}}\right) + \frac{1}{1-ae^{-j\omega}}$$

$$= \cancel{j}\frac{-1}{(1-ae^{-j\omega})^2}(-ae^{-j\omega})(-\cancel{j}) + \frac{1}{1-ae^{-j\omega}}$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^2} + \frac{1}{1-ae^{-j\omega}} \#$$

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