ON THE DIOPHANTINE EQUATION $2^s + p^k = m^2$ WITH A FERMAT PRIME p

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Abstract. Ramanujan [5] conjectured that the Diophantine equation

$$2^s - 7 = m^2$$

has five solutions in positive integers, namely $(s, m) = (3, 1), (4, 3), (5, 5), (7, 11)$ and $(15, 181)$. His conjecture was proved by Nagell [4]. The generalized Ramanujan-Nagell equation

$$2^s + D = m^2 \tag{1}$$

in positive integers s and m, where $D \neq 0$ is an integer parameter, was considered by several authors. See, for instance, Apéry [1], Hasse [3], Beukers [2]. In this talk, we consider equation (1) when the parameter D is of the form $D = p^k$, where k is a nonnegative integer and $p = 2^{2^\ell} + 1$ is a Fermat prime. Namely, our equation takes the form

$$2^s + p^k = m^2 \qquad \text{with a Fermat prime} \quad p = 2^{2^\ell} + 1. \tag{2}$$

In our talk, we find all the nonnegative integer solutions (m, p, k, s) of (2). This is a joint work with F. Luca.

References

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