Assignment 6

Beeram Sandya cs21btech11006

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Outline

1. Question

2. Solution

Question

Papoulis ch7 Ex 7.27

An infinite sum is by definition a limit :

$$\sum_{k=1}^{\infty} x_k = \lim_{n\to\infty} y_n$$

$$y_n = \sum_{k=1}^{\infty} x_k$$

Show that if the random variables x_k are independent with zero mean and variance σ_k^2, then the sum exists in the MS sense if and only if

$$\sum_{k=1}^{\infty} \sigma_k^2 < \infty$$

Solution

Given mean of random variable is zero $=> E\{x_k\} = 0$

$$E\{x_k^2\} = \sigma_k^2$$

$$E\{(\Sigma x_k)^2\} = \Sigma E\{x_k^2\} = \Sigma \sigma_k^2$$

If $\sum_{k=1}^{\infty} \sigma_k^2 < \infty$then given $\epsilon > 0$ we can find n_o such that

$$\sum_{k=n+1}^{n+m} \sigma_k^2 < \epsilon$$

for any m and $n > n_o$ .Thus,

$$E\{(y_{m+n} - y_n)^2\} = E\{(\sum_{k=n+1}^{m+n} x_k)^2\} = \sum_{k=n+1}^{n+m} \sigma_k^2 < \epsilon$$

This shows that (from Cauchy), y_k converges in the MS sense.