Bayesian analysis of a spectral line (Brute-force approach)

The attached Python file contains experimental data showing a spectral line observed in a laboratory. Your task is to perform a Bayesian analysis on a grid to adjust the amplitude A of the spectral line and the background flux B. Assume that the spectral line has a Gaussian shape. For physical reasons, the value of A is constrained to be between 0 and 1000 a priori. The value of B is constrained to be between 0 and 3000 a priori. Use a uniform "prior" on A and B on a linear scale.

For uncertainties, you can assume Gaussian noise with standard deviation σ_k = sqrt(N_k). That is the photon noise in the limit of a large number of photons.

Exercise 1:

Run the attached code provided to you as is. The produced graphic shows the "data" you will be working with. Copy this figure into a Word document (or another note taking tool).

Exercise 2:

a) Test the function provided to you for the physics model of the spectral line. What is the vector of return values for A=200, B=1860? Copy the values into your document.
b) Code a function "prior" returning prior pdf p(A,B|I) for a given set of parameters (A, B). What is the return value for A=200, B=1860? Ideally, write the function in a vectorized form with numpy that can return the likelihood values with A and B values given on 2D grids.
c) Code a function "like" returning the likelihood function p(data|A,B,I) for a given set of parameters. What is the return value for A=200, B=1860? Ideally, write the function in a vectorized form with numpy that can return the likelihood values with A and B values given on 2D grids.
d) Code a function "like_times_prior" returning the products of the values of the likelihood and prior for a given set of parameters. That is the numerator of Bayes' Law. What is the return value for A=200, B=1860?

FYI, A=200 and B=1860 is not necessarily the best fit to the data. That is fine.

Exercise 3:

a) Add to the code to calculate the joint posterior probability density p(A,B|data,I) of A and B on a 300x300 grid spanning A and B. <u>Note:</u> Use np.meshgrid to generate a 2D grid for the values of A and B.

b) Make a graph of the posterior distribution in 2D using ax.pcolormesh(). Label the axes. Use a 300x300 grid over the entire parameter space. Label the axes. Include a color bar with the label.

c) Draw the same graph as in a) but use a 300x300 grid spanning only the non-zero probability region. Label the axes. Include a color bar with the label.

d) Show the point of maximum probability with a star symbol (marker="*") on the posterior of 2c). Include a color bar with the label.

e) Make a new graph identical to the one in Step 1 and show the best fit by superimposing the best fit model on the data. Write down the maximum probability values of A and B.

Exercise 4:

Calculate and give the value of Bayesian evidence (Z) on the basis of the grid in Questions 2a.

Exercise 5 (Optional):

Make a new graph identical to the one in Question 2c), but add to the graph the contours of the Bayesian "credible region" at 68%, 95% and 99.7% (think about how to find the level of these contours).

Exercise 6:

a) Calculate and give the mean values of A and B

b) Calculate and give the median values of A and B

c) Calculate and give the covariance matrix

Include the formulas you used.

Exercise 7:

a) Graph the marginalized "posterior" distribution of A and show the Bayesian 68% confidence interval on the graph.

b) Add to the graph in a) the conditional probability density distribution of A, $P(A|B=B^*)$ with B^* the value of B corresponding to the maximum probability (the "best-fit" for this uniform prior).

c) Explain why one distribution is wider than the other.