Problem Set 6

1 Synchrotron Cooling

1.1 Half-time for Energy Loss

Show that the time for an electron to lose half its energy is

```
using DrWatson
using Unitful: Mass, Length
include(srcdir("astr145/main.jl"))
```

```
escape_velocity (generic function with 1 method)
```

```
using Unitful: c, q, me, μ0, ϵ0

B_con = sqrt(4π / μ0)
q_factor = 1 / sqrt(4π * ϵ0)

t_half(B, γ0; m=me) = (3 * m^3 * c^5) / (2 * (q_factor * q)^4 * (B * B_con)^2
* γ0) |> upreferred
t_half(10u"μGauss", 1e6)
```

```
5.158665568940776e12 s
```

2 Compact Objects

White dwarfs, neutron stars, and stellar-mass black holes are all compact objects formed at the final stages of stellar evolution. Here, you will compare some of their fundamental properties. Assume typical white dwarfs have M ≈ M_sun and a radius comparable to Earth. For neutron stars, assume M ≈ 2 M_sun and a radius of 10 km.

2.1 Escape Velocities

Estimate the escape velocity at the surfaces of white dwarfs and neutron stars, and compare them to that of a black hole.

```julia
using UnitfulAstro: Msun, Rearth

M_white_dwarf = 1u"Msun"
M_neutron_star = 2u"Msun"
M_black_hole = 10u"Msun"

R_white_dwarf = Rearth
R_neutron_star = 10u"km"

v_es_white_dwarf = let M = M_white_dwarf, R = R_white_dwarf
    @show v_es(M, R)
end

v_es_neutron_star = let M = M_neutron_star, R = R_neutron_star
    @show v_es(M, R)
end

v_es_black_hole = let M = M_black_hole
    R = Rsch(M)
    @show v_es(M, R)
end

println("White dwarf to black hole: ", v_es_white_dwarf / v_es_black_hole)
println("Neutron star to black hole: ", v_es_neutron_star / v_es_black_hole)
```

```
v_es(M, R) = 6.450971939285288e6 m s⁻¹
v_es(M, R) = 2.3040177082652816e8 m s⁻¹
v_es(M, R) = 2.99792458e8 m s⁻¹
White dwarf to black hole: 0.021518126180763653
Neutron star to black hole: 0.7685375821780285
```

Neutron stars have the comparable escape velocity to black holes, while white dwarfs have a much lower escape velocity (1% of the black hole's).

2.2 Gravitational Acceleration

> Estimate the gravitational acceleration at the surface of a white dwarf and neutron star, and compare them with that at the Schwarzschild radius of a black hole with $M \approx 10\ M$. Would humans be able to survive this gravity?

```julia
g(M, R) = G * M / R^2 |> upreferred

g0 = 9.8u"m/s^2"

g_white_dwarf = let M = 1u"Msun", R = Rearth
```

```
    @show g(M, R)
end

g_neutron_star = let M = 2u"Msun", R = 10u"km"
    @show g(M, R)
end

g_black_hole = let M = 10u"Msun"
    R = Rsch(M)
    @show g(M, R)
end

NoUnits(minimum([g_white_dwarf, g_neutron_star, g_black_hole]) / g0)
```

```
g(M, R) = 3.262338232502328e6 m s⁻²
g(M, R) = 2.6542488e12 m s⁻²
g(M, R) = 1.5216374427790059e12 m s⁻²
```

```
332891.6563777886
```

Humans would not be able to survive the gravity of white dwarfs or neutron stars or black holes.

2.3 Gravitational Redshift

Recall that near a massive object, light experiences a gravitational redshift:

```
z(M, R) = sqrt(1 - 2 * G * M / (R * c^2))

@show z(M_white_dwarf, R_white_dwarf)
@show z(M_neutron_star, R_neutron_star)
@show z(M_black_hole, R_ISCO(M_black_hole))
```

```
z(M_white_dwarf, R_white_dwarf) = 0.999768458317059
z(M_neutron_star, R_neutron_star) = 0.6398046457942846
z(M_black_hole, R_ISCO(M_black_hole)) = 0.816496580927726
```

```
0.816496580927726
```

3 Ultra-High Energy Cosmic Rays

> Read this article about ultra-high energy cosmic rays: https://physicsworld.com/a/the-riddle-of-ultrahigh-energy-cosmic-rays/ Write a short reaction (a paragraph or so) to this video – you could comment on something you learned or particularly enjoyed or pose a question it raised to you, for example.

I was particularly intrigued by the challenges in identifying the sources of UHECRs due to their deflection by magnetic fields, which obscures their paths. This raises a compelling question: how can we develop methods or technologies to trace these particles back to their origins, thereby unveiling the astrophysical processes responsible for such extreme energies?

Bibliography