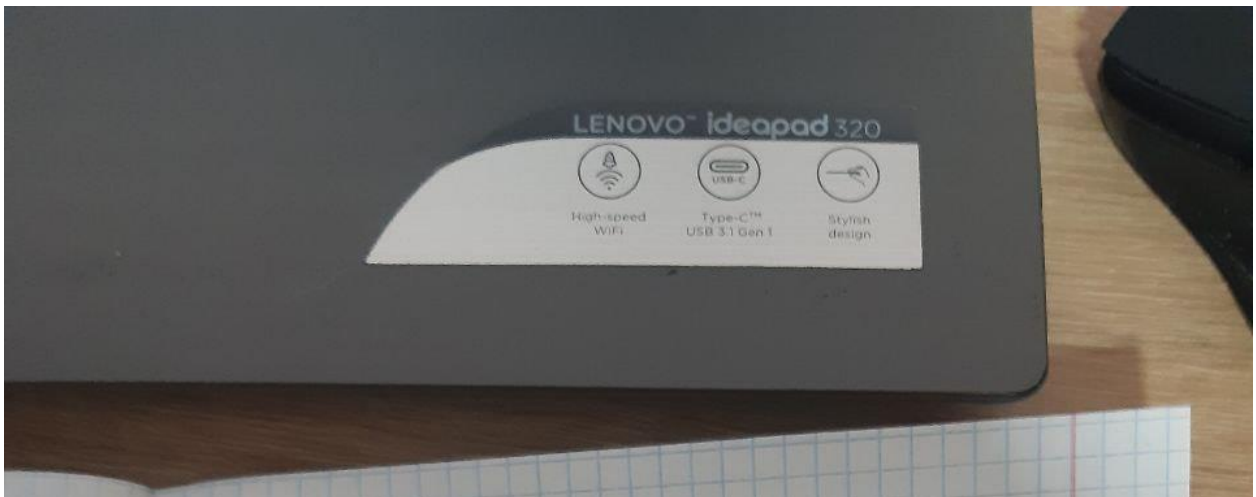


~~minim~~ Maxim

V0	–	–	–	10	8	6	4	0
V1	20	16	15	15	11	6	4	0
V2	23	21	15	15	11	6	4	0 1
V3	27							

c) Bellmann - Kalaba Minim zi maxim

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
x_1	0	6	8		12			+
x_2	ε	0	2	6				+
x_3			0	5	4		8	+
x_4				0	7	7		10
x_5					0	5	4	8
x_6						0		6
x_7							0	4
x_8								0
V_0	+	+	+	10	8	6	4	0
V_1	20	16	12	10	8	6	4	0
V_2	20	14	12	10	8	6	4	0

1 ~~- 8 - 7 - 8~~

1 - 3 - 5 - 8 V_3

1 - 5 - 8

$V_3 = V_2$ STOP

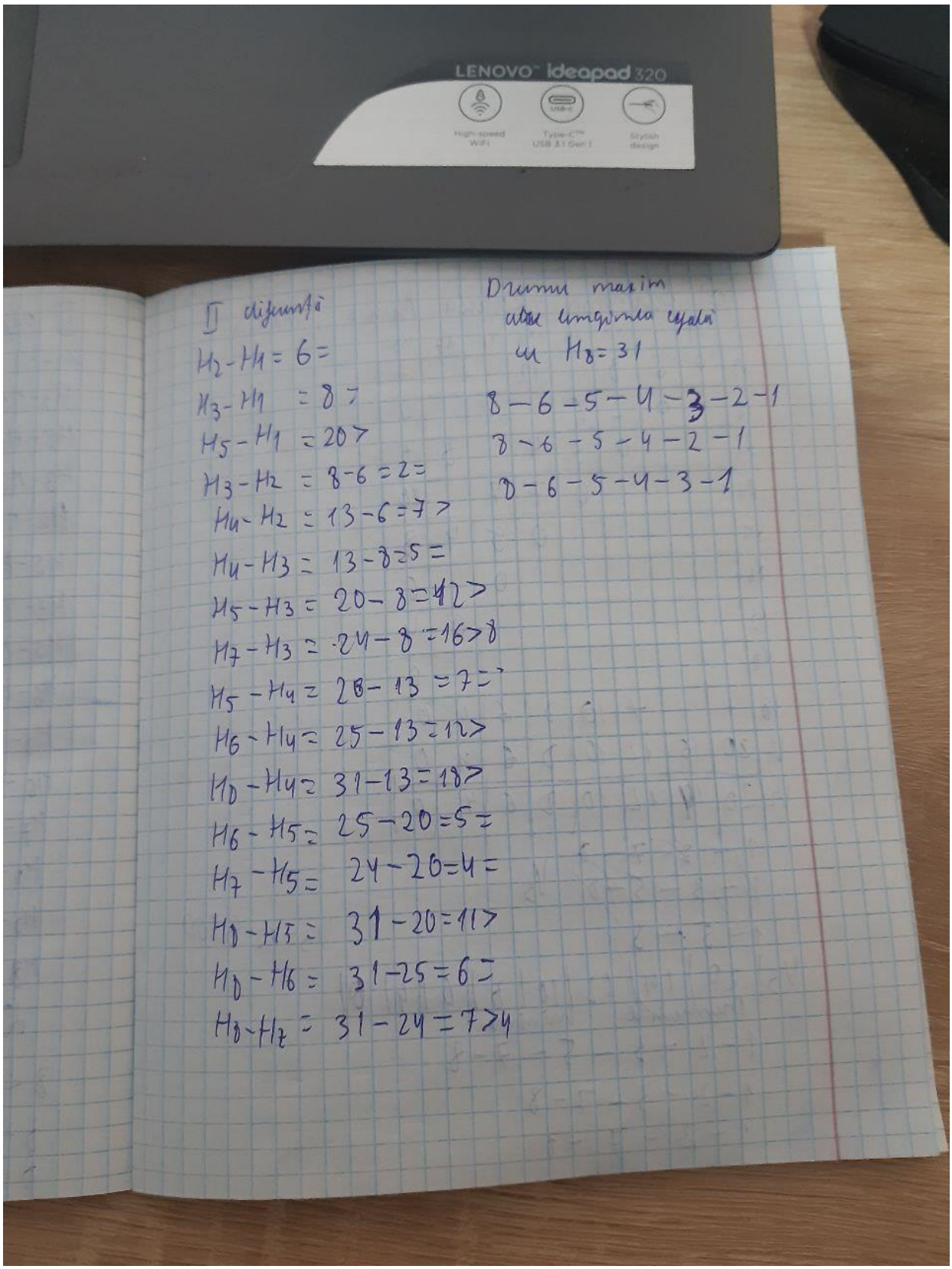
V_3 | 20 | 14 | 12 | 10 | 8 | 6 | 4 | 0 |

Drumurile minime

1 - 2 - 3 - 5 - 7 - 8

1 - 2 - 3 - 7 - 8

1 - 3 - 7 - 8



∏ divizibile

$H_2 - H_1 = 6 =$

$H_3 - H_1 = 8 =$

$H_5 - H_1 = 20 >$

$H_3 - H_2 = 8 - 6 = 2 =$

$H_4 - H_2 = 13 - 6 = 7 >$

$H_4 - H_3 = 13 - 8 = 5 =$

$H_5 - H_3 = 20 - 8 = 12 >$

$H_7 - H_3 = 24 - 8 = 16 > 8$

$H_5 - H_4 = 20 - 13 = 7 =$

$H_6 - H_4 = 25 - 13 = 12 >$

$H_8 - H_4 = 31 - 13 = 18 >$

$H_6 - H_5 = 25 - 20 = 5 =$

$H_7 - H_5 = 24 - 20 = 4 =$

$H_8 - H_5 = 31 - 20 = 11 >$

$H_8 - H_6 = 31 - 25 = 6 =$

$H_8 - H_7 = 31 - 24 = 7 > 4$

Drumul maxim atât ascendent cât și descendent cu $H_8 = 31$

8 - 6 - 5 - 4 - 3 - 2 - 1

8 - 6 - 5 - 4 - 2 - 1

8 - 6 - 5 - 4 - 3 - 1

8) Drum maxim Ford

$H_1 = 0$

$H_2 = -\infty \dots H_8 = -\infty$

I diferență

$H_2 - H_1 = -\infty < H_2 = 6$

$H_3 - H_1 = -\infty <; H_3 = 8$

$H_5 - H_1 = -\infty ; H_5 = 12$

$H_3 - H_2 = 8 - 6 = 2 =$

$H_4 - H_2 = -\infty ; H_4 = 12$

$H_4 - H_3 = 12 - 8 = 4 < 5 : H_4 = 13$

$H_5 - H_3 = 12 - 8 = 4 = 4$

$H_7 - H_3 = -\infty ; H_7 = 16$

$H_5 - H_4 = 12 - 13 < ; H_5 = 20$

$H_6 - H_4 = -\infty ; H_6 = 20$

$H_8 - H_4 = -\infty ; H_8 = 23$

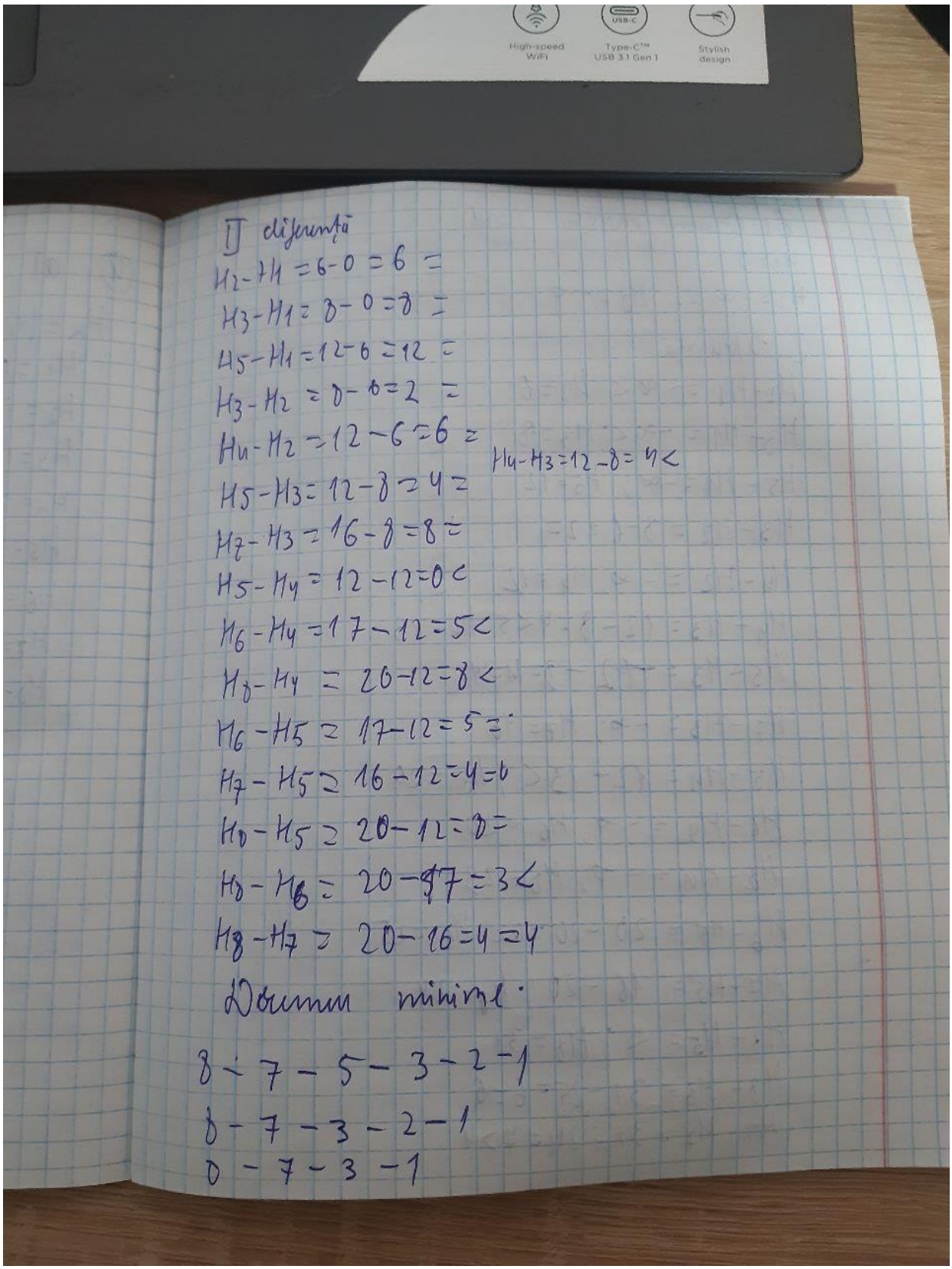
$H_6 - H_5 = 20 - 20 = 0 < H_6 = 25$

$H_7 - H_5 = 16 - 20 ; H_7 = 24$

$H_8 - H_5 = < ; H_8 = 31$

$H_8 - H_6 = 31 - 25 = 6 = 6$

$H_7 - H_7 = 31 - 24 = 7 > 4$



IJ diferență

$H_2 - H_1 = 6 - 0 = 6 =$

$H_3 - H_1 = 8 - 0 = 8 =$

$H_5 - H_1 = 12 - 6 = 12 =$

$H_3 - H_2 = 8 - 6 = 2 =$

$H_4 - H_2 = 12 - 6 = 6 =$

$H_5 - H_3 = 12 - 8 = 4 =$

$H_4 - H_3 = 12 - 8 = 4 <$

$H_7 - H_3 = 16 - 8 = 8 =$

$H_5 - H_4 = 12 - 12 = 0 <$

$H_6 - H_4 = 17 - 12 = 5 <$

$H_8 - H_4 = 20 - 12 = 8 <$

$H_6 - H_5 = 17 - 12 = 5 =$

$H_7 - H_5 = 16 - 12 = 4 = 6$

$H_8 - H_5 = 20 - 12 = 8 =$

$H_8 - H_6 = 20 - 17 = 3 <$

$H_8 - H_7 = 20 - 16 = 4 = 4$

Drumul minim:

8 - 7 - 5 - 3 - 2 - 1

8 - 7 - 3 - 2 - 1

0 - 7 - 3 - 1

I dijkstra

$H_2 - H_1 = inf - 0 = inf > P_{12}$; $H_2 = P_{12} + H_1$; $H_2 = 6$

$H_3 - H_1 = inf$; $H_3 = P_{13} + H_1$; $H_3 = 8$

$H_5 - H_1 = inf$; $H_5 = 12$

$H_3 - H_2 = 8 - 6 = 2 \Rightarrow 2$ Nu se modifică

$H_4 - H_2 = inf$; $H_4 = 6 + 6 = 12$

$H_4 - H_3 = 12 - 8 = <$ Nu se modifică

$H_5 - H_3 = inf$; $H_5 = 4 + 8 = 12$

$H_7 - H_3 = inf$; $H_7 = 8 + 8 = 16$

$H_5 - H_4 = 12 - 6 < 7$

$H_6 - H_4 = inf$; $H_6 = 7 + 12 = 19$

$H_8 - H_4 = inf$; $H_8 = 10 + 12 = 22$

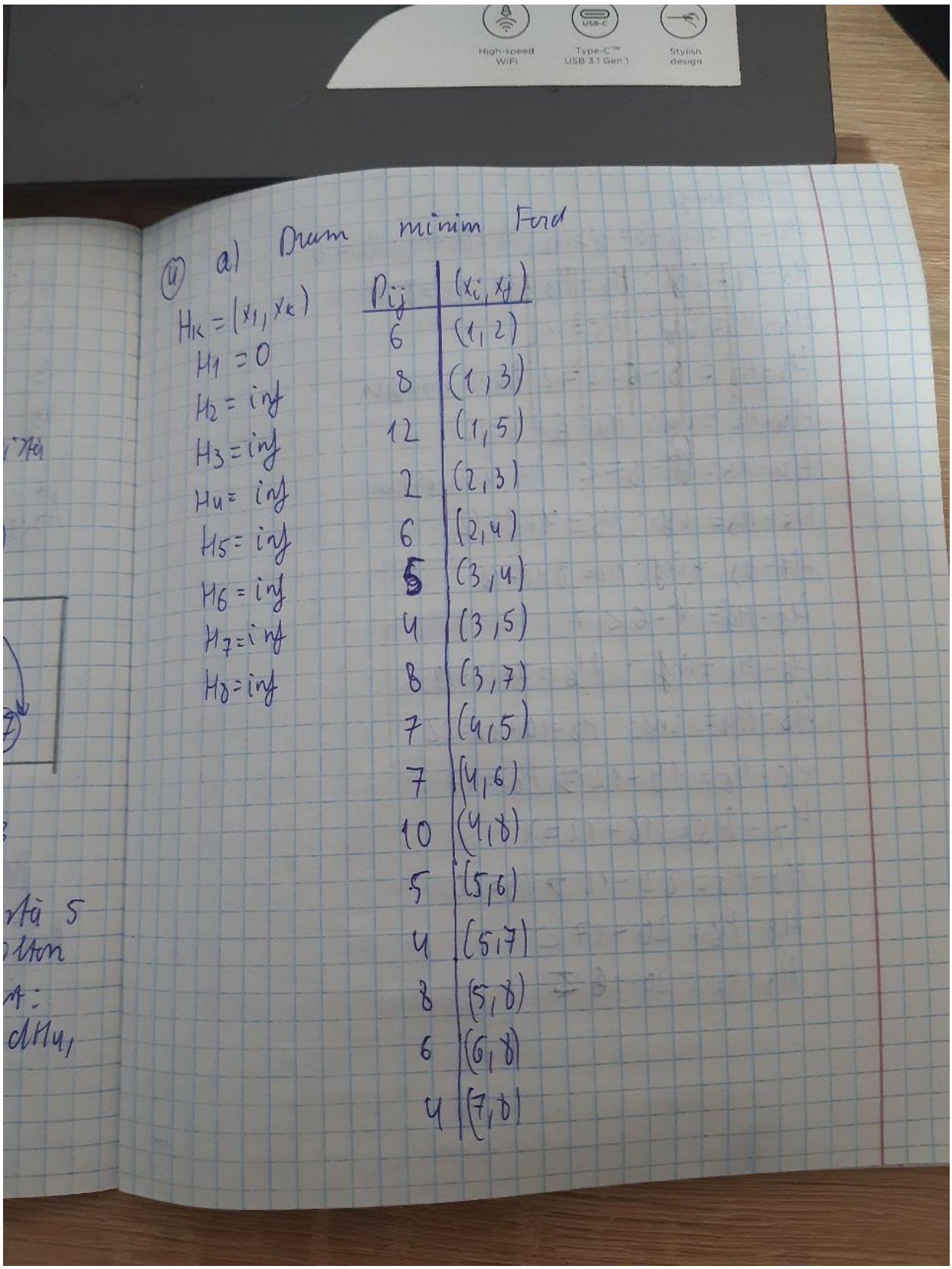
$H_6 - H_5 = 19 - 12 \Rightarrow H_6 = 17$

$H_7 - H_5 = 16 - 12 = 4$

$H_8 - H_5 = 22 - 12 \Rightarrow H_8 = 20$

$H_8 - H_6 = 20 - 17 <$

$H_8 - H_7 = 20 - 16 =$



(11) a) Drum minim Ford

$H_k = (x_1, x_k)$

$H_1 = 0$
$H_2 = inf$
$H_3 = inf$
$H_4 = inf$
$H_5 = inf$
$H_6 = inf$
$H_7 = inf$
$H_8 = inf$

P_{ij}	(x_i, x_j)
6	(1,2)
8	(1,3)
12	(1,5)
2	(2,3)
6	(2,4)
5	(3,4)
4	(3,5)
8	(3,7)
7	(4,5)
7	(4,6)
10	(4,8)
5	(5,6)
4	(5,7)
8	(5,8)
6	(6,8)
4	(7,8)

Matricea drumurilor:

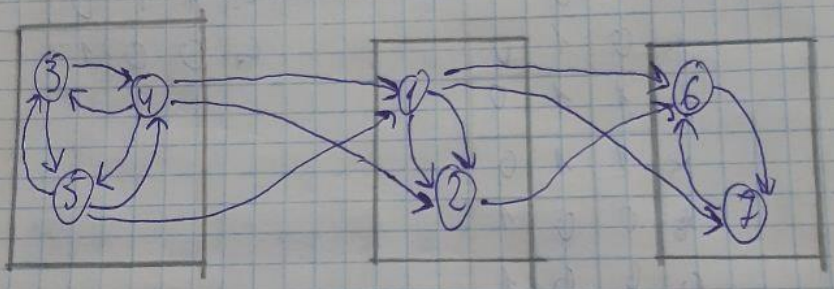
	c_1	c_2	c_3	$P(c_i)$
c_1	0	0	1	1
c_2	1	0	1	2
c_3	0	0	0	0

$\sum P(c_i) = 3$

$n = 3$

$n(n-1)/2 = 3 \Rightarrow$ există drum Hamiltonian

$\exists dH^* = (c_2, c_1, c_3)$



$c_2 \rightarrow c_1 \rightarrow c_3$

$dH_1 = (3, 4, 5, 1, 2, 6, 7)$

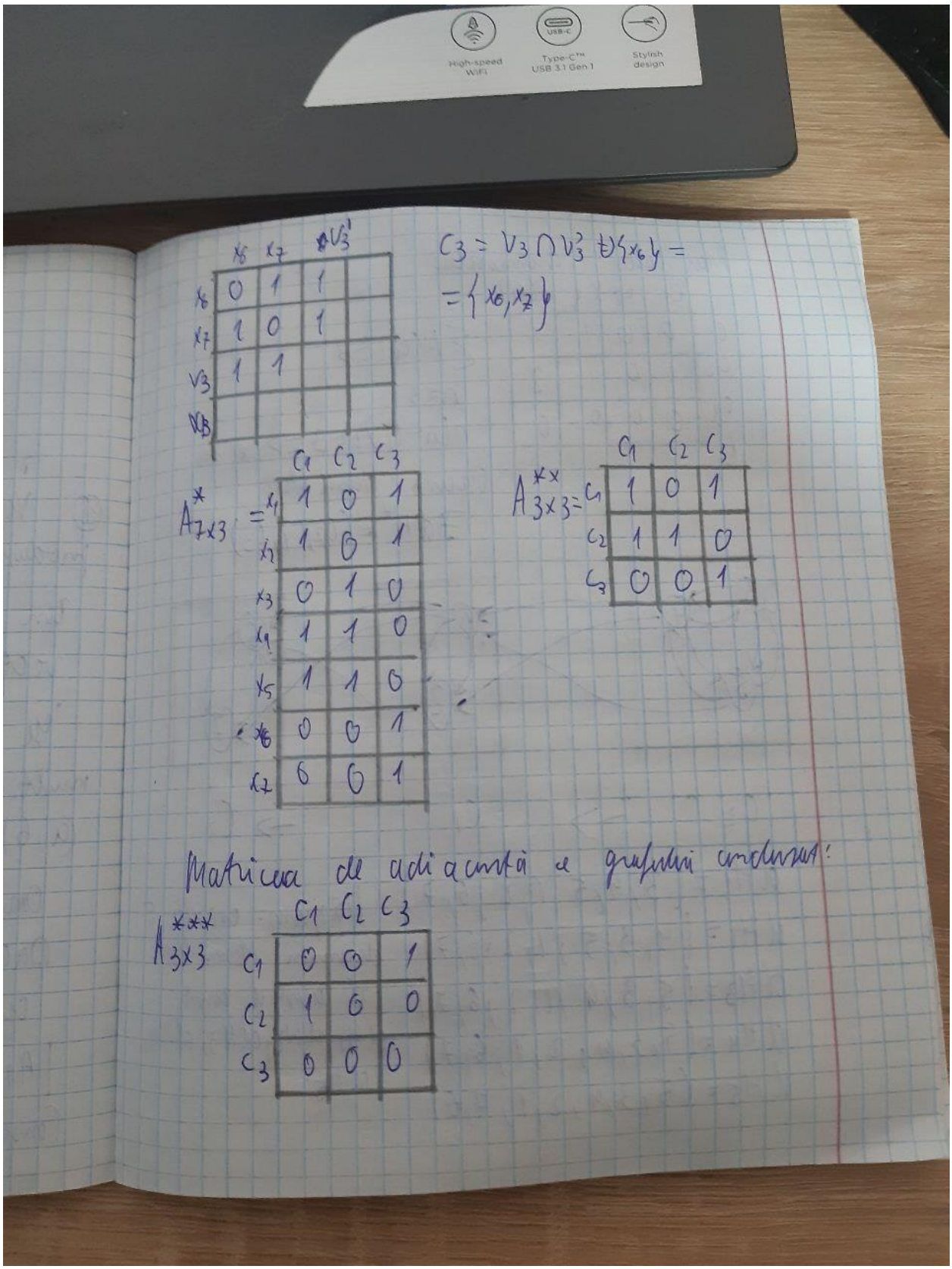
$dH_2 = (4, 3, 5, 1, 2, 6, 7)$

$dH_3 = (5, 3, 4, 1, 2, 6, 7)$

$dH_4 = (5, 3, 4, 2, 1, 6, 7)$

$dH_5 = (5, 3, 4, 2, 1, 7, 6)$

Răspuns: există 5 drumuri Hamiltonian și acestea sunt: $dH_1, dH_2, dH_3, dH_4, dH_5$.



	x_6	x_7	$*V_3'$	
x_6	0	1	1	
x_7	1	0	1	
V_3	1	1		
V_3'				

$C_3 = V_3 \cap V_3' \setminus \{x_6\} =$

$= \{x_6, x_7\}$

$A^{*}_{7\times3} =$

	C_1	C_2	C_3
x_1	1	0	1
x_2	1	0	1
x_3	0	1	0
x_4	1	1	0
x_5	1	1	0
x_6	0	0	1
x_7	0	0	1

$A^{**}_{3\times3} =$

	C_1	C_2	C_3
C_1	1	0	1
C_2	1	1	0
C_3	0	0	1

Matricea de adiacență a grafului condensat:

$A^{***}_{3\times3}$

	C_1	C_2	C_3
C_1	0	0	1
C_2	1	0	0
C_3	0	0	0

Busa Cătălin
TI-214

Evaluare No. 1 la
Matematică Discretă

① Vectori și produs cartezian. Cardinalul produsului cartezian

Un set ordonat de elemente se va numi vector sau cuplu.

Se numește produs cartezian a două mulțimi A și B mulțimea tuturor perechilor (a,b) pentru care $a \in A$, $b \in B$

Cardinalul produsului cartezian
Dacă $A_1, A_2, \dots, A_n$ sunt mulțimi finite cu $|A_1| = m_1$, $|A_2| = m_2$, ..., $|A_n| = m_n$, atunci
$|A_1 \times A_2 \times \dots \times A_n| = m_1 \cdot m_2 \cdot \dots \cdot m_n$
Consecință: $|A^n| = (A^n)$

1 Determinăm componentele tare conexe

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	V_1'
x_1	0	1	0	0	0	1	1	(1)
x_2	1	0	0	0	0	1	0	1
x_3	0	0	0	1	1	0	0	(1)
x_4	1	1	1	0	1	0	0	1
x_5	1	0	1	1	0	0	0	1
x_6	0	0	0	0	0	0	1	0
x_7	0	0	0	0	0	1	0	0
V_1	(1)	1	0	0	0	1	(1)	

$C_1 = V_1 \cap V_1' \cup \{x_1\} =$
$= \{x_1, x_2\}$

	x_3	x_4	x_5	x_6	x_7	V_2'
x_3	0	1	1	0	0	(1)
x_4	1	0	1	0	0	1
x_5	1	1	0	0	0	1
x_6	0	0	0	0	1	0
x_7	0	0	0	1	0	0
V_2	(1)	1	1	0	0	0
V_3						1

$C_2 = V_2 \cap V_2' \cup \{x_3\} =$
$= \{x_3, x_4, x_5\}$

③ Drumul Hamilton

~~x0 x1 x2 x6 x7~~

Lista de adiacență

1.	2_6_7_0
2	1_6_0
3	4_5_0
4	1_2_3_5_0
5	1_3_4_0
6	7_0
7	6_0

Aflăm componentele tare conexe:
