Chapter 2

Bond graphs modeling for simulation

Bond graph methodology uses energy as a *lingua franca* to describe interaction between parts of a system of interest in all physical domains. It allows describing the electrical, mechanical, hydraulic, pneumatic, thermal, thermodynamic, chemical and biological systems, including combinations thereof [10]. Since robotics is multidisciplinary by nature, the bond graph methodology suits well for complex robotic systems. The bond graph theory has been developed to create a process that would allow a computer to generate a mathematical description of a system of interest. In bond graph, all elements of a dynamic system are presented with unified components and their interrelationships are defined by energy transactions.

Figure 2.1: A power transfer form a system A to a system B

The bond graph methodology is a port-based modeling tool of dynamical systems with interconnection of any physical domains. Bond graph modeling requires the existence of two types of variables whose conjugation defines a *power*. These pair of variables, called the *power variables*, are associated to a power bond in order to represents an elementary interaction between two subsystems. Power variables whose conjugation is defined as a product are called generically the *effort* and the *flow*. The power variables were initially considered to be scalars, then this notion was extended to vectors. Multibonds are bonds with column vectors as power variables, they are equivalent to an array of simple bonds [11].

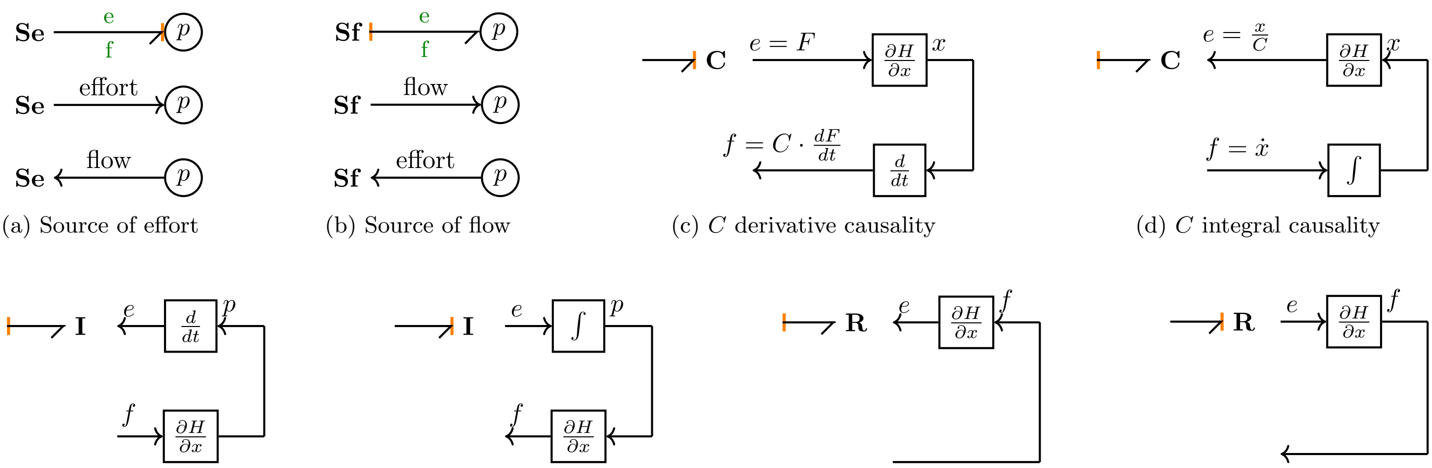
These two power variables describe bidirectional interaction of system's components, i.e., *effort* and *flow* headed in different directions. Therefore, one of them is an *input* while the second one is an *output* (causality). Integration of *power variables* gives *energy variables*, which represent the accumulated energy in an ideal energy storage elements. Fig. 2.1 shows a bond which is a half arrow that indicates power transfer from system A to a system B. An orange stroke, which is called *causal stroke*, indicates causality; here means that flow goes from A to B, while the effort goes in opposite direction. An orange stroke placed in opposite position would be mean an opposite directions for effort and flow. Tab. 2.1 shows the couples of power variables in different physical domains.

Energetic domain	**effort** e	**flow** f
Translation mechanics	Force F	Velocity v
Rotation mechanics	Torque τ	Angular velocity ω
Spatial mechanics	Wrench W	Twist T
Electrical	Voltage V	Current I
Hydraulic	Pressure P	Volume flow rate q

Table 2.1: Power variables in different physical domains

Tab. 2.2 shows bond graph's basic building blocks, which are elements of the ideal mathematical model.

Active elements supply power to a system. **Se** is an ideal source of constant effort; a typical example of that is a gravitational field acting on the matter. Fig. 2.2, (**a**) shows a power bond that indicates power transfer from source of effort **Se** to a port p. While one power variable is defined, the another one is determined by the connected load. An orange stroke indicates a causality; in this case effort cause flow.



(a) Source of effort (b) Source of flow (c) C derivative causality (d) C integral causality

(e) I derivative causality (f) I integral causality (g) R flow-in relation (h) R effort-in relation

Figure 2.2: One-port bond graphs building blocks

Fig. 2.2, (**b**) shows a power bond that indicates power transfer from source of flow **Sf** to a port p; an orange stroke indicates a causality, that in that case flow cause effort. **Sf** is an ideal source of flow, an example is an electric motor with a constant angular velocity. Power bond indicates that power transfer goes from a source to a load; thus, it is considered as a positive direction. In case of not constant effort or flow, we can use **MSe** or **MSf** for modulated sources of efforts and flows respectively; those are used in case if effort or flows are defined as a function of time.

Passive elements such as C and I accumulate energy, while R element dissipates energy, i.e., irreversibly transfer energy into heat. Integration of *power variables* gives *energy variables*, which represent the accumulated energy in an ideal energy storage elements C and I. Those elements accumulates and releases energy without losses.

"C" letter comes from *capacitor* or *compliance*, since in physical terms, C-element is used to model springs, flexible shafts, electrical capacitors, gravity tanks, pressure vessels, hydraulic accumulators, and electric accumulators.

There are two ways how can we model a C-element that differ in *causality*. In case of *derivative causality* (Fig. 2.2, **c**), an effort e goes to a C-element as an input and returns flow f as an output, which is proportional to the time derivation of effort. For a spring as an example we have

$$f = C \cdot \frac{de}{dt}, \rightarrow \ v = \frac{1}{k}\frac{dF}{dt},$$

where v is a velocity, compliance $C = \frac{1}{k}$ is inversely proportional to stiffness k, and F is a force generated in a spring. In case of *integral causality* (Fig. 2.2, **d**), an flow f goes to a C-element as an input and returns effort e as an output, which is proportional to the time integral of flow. For a spring as an example we have

$$e = \frac{1}{C}\int f dt, \rightarrow \ F = k\int v dt = kx.$$

A system is causal if and only if the signal that it produces is formed just through the use of present and past values from the signal that it receives. The derivative is not causal (i.e., it is considered to be uncausal) because it "looks at" the future of the signal, and looking at the future cannot be a causal operation. For computer simulation it is preferred to use only *integral causality.*

"I" letter comes from *inductance* or *inertia*, since in physical terms, I-element is used to model mass inertia in a translational or rotational movement, material housing, electrical inductance, flywheel, inertia of fluid in the hydraulics, coil, long pipe. In

One-port			Two and multi-ports		
Active elements	**Se**	Source of effort	**Conversion elements**	**TF**	Transformer
	Sf	Source of flow		**GY**	Gyrator
Passive elements	**C**	Flow storage element	**Interconnection elements**	**0**	All efforts are equal
	I	Effort storage element		**1**	All flows are equal
	R	Energy dissipation element			

Table 2.2: Bond-graphs building blocks

case of *derivative causality* (Fig. 2.2, **e**), an flow f goes to a I-element as an input and returns effort e as an output, which is proportional to the time derivation of flow. For a body with mass m as an example we have

$$e = I \cdot \frac{df}{dt}, \rightarrow\ F = m\frac{dv}{dt} = m\dot{v} = p,$$

where F is a inertia force applied to a body, v is a velocity, and p is a momentum. In case of *integral causality* (Fig. 2.2, **f**), an effort e goes to a I-element as an input and returns flow f as an output, which is proportional to the time integral of effort. For a body with mass m as an example we have

$$f = \frac{1}{I}\int edt, \rightarrow\ v = \frac{1}{m}\int Fdt.$$

"R" letter comes from *resistor*, energy consumer, dissipator of free energy, since in physical terms, R-element is used to model electrical resistors, dampers and friction in the mechanics, and other elements which convert the energy into heat. The causality means that we take one power variable as an independent variable and another one as dependent parameter. Performance variables are bound only on the static relationships, because of that causality can vote and vice versa.

In case of *flow-in relation* (Fig. 2.2, **g**), an flow f goes to a R-element as an input and returns effort e as an output. For a damper with coefficient b as an example we have

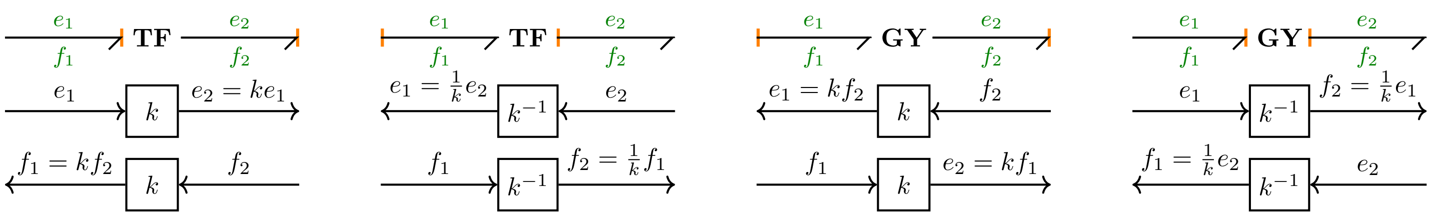
$$e = R \cdot f, \rightarrow\ F = b \cdot v,$$

where F is a damping force applied to a body and v is a velocity. In case of *effort-in relation* (Fig. 2.2, **h**), an effort e goes to a R-element as an input and returns flow f as an output. For Ohm's law as an example we have

$$f = \frac{1}{R} \cdot e, \rightarrow\ I = \frac{V}{R},$$

where I is a current, R is a resistor coefficient, and V stands for voltage.

Two-ports elements (Fig. 2.3) are the ones that can exchange energy system through two bonds. Two-ports elements preserve energy, meaning that the product of the effort e and the flow f at the outlet is equal to the product flow and effort of input.



(a) TF, inputs e_1 and f_2 (b) TF, inputs f_1 and e_2 (c) GY, inputs f_1 and f_2 (d) GY, inputs e_1 and e_2

Figure 2.3: Two-port bond graphs building blocks

Transformers TF and gyrators GY are needed to model power transmission. TF-element (Fig. 2.3, **a** and **b**) is used to model elements that scale power variables, such as lever or gearbox, or transform it, e.g., mechanical conversion of rotation into translation motion, or transformation of mechanical into hydraulic motion. GY-element (Fig. 2.3, **c** and **d**), is used to transform the performance of one physical nature to another, for an example, transition from electric to mechanical domains. Characteristics of TF and GY could be constant, e.g., with a coefficient k, or defined with a function of time as modulated transformer MTF or modulated gyrator MGY.

To construct a system from the mentioned basic building blocks, we need multi-port elements: 0 and 1 junctions.

Both of them have to follow two requirements:

- **Power continuity** which means that the junction can not dissipate or generate energy, so that the algebraic sum of the power of all the ports is zero
- **Port symmetry** implying that the constitutive relations of the ports are exchangeable. One can develop these relations to the effort or flow of any of the connected elements.

The 0-junction behaves analog to *Kirchhoff's junction rule* in the electrical domain, so the sum of all flow entering and leaving the junction is equal to zero and all the efforts have the same value. For a Fig. 2.4, **e** we have

$$\sum_i^n f_i = 0,\ e_1 = e_2 + e_3 + e_4.$$

(a) $0 - juntion$ (b) $1 - juntion$

Figure 2.4: Multi-port bond graphs building blocks

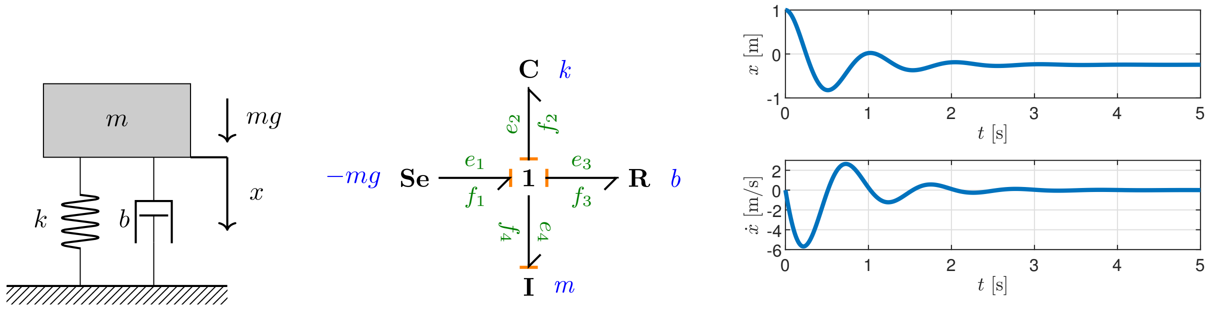
In the 1-junction on the other hand, the sum of all the efforts in the junction is zero and all the flows are equal, just like in Kirchhoff's loop rule for the electrical domain. For a Fig. 2.4, **f** we have

$$\sum_i^n e_i = 0,\ f_1 = f_2 + f_3 + f_4.$$

1D problem

Let us model the same mass-spring-damper system but using bond-graph methodology. For the convenience the schematic representation is shown (Fig. 2.5, **a**) next to a resulted bond graph (Fig. 2.5, **b**).

The system consists of three elements: a body with mass m, a spring with coefficient of stiffness k, and a damper with coefficient b. Here we want to model a conservative system, thus the only gravity force is applied to a body.



(a) Schematic representation (b) Bond graph (c) Simulation results

Figure 2.5: Schematic representation and block-diagram of mass-spring-damper system

Since the gravitational force is constant, we can use an ideal source of effort with coefficient equal to mg

$$\textbf{Se:}\ e_1 = -mg.$$

The applied force is distributed between the system's parts; to interconnect them we need to use 1-junction

$$\textbf{1-junction:}\quad \begin{aligned} f_1 &= f_2 = f_3 = f_4 = \dot{x} \\ e_1 &= e_2 + e_3 + e_4. \end{aligned}$$

To model a spring we use a C-element with integral causality

$$\textbf{C}:\ e_2 = \frac{1}{C}\int f dt = k\int \dot{x} dt = kx.$$

To model a damper we use a R-element with flow-in relation, since because of 1-junction the input flow is know

$$\textbf{R}:\ e_3 = b\dot{x}.$$

To model a mass we use a I-element with integral causality, however we don't know e_4 yet, but the one can be calculated from 1-junction equation for efforts

$$e_4 = e_1 - e_2 - e_3 = -mg - kx - b\dot{x} = F,$$

where $F = e_4$ is the total force applied to a body, which according to the second Newton's law equals to $e_4 = m\ddot{x}$. Fig. 2.5, **c** shows simulation results for a mass-spring-damper: position x and velocity $\dot{x}$.

Finally, we can verify that flow is a velocity indeed

$$\mathbf{I}\text{: } f_4 = \frac{1}{I}\int e_4 dt \ \rightarrow\ F = \frac{1}{m}\int m\ddot{x}dt = \dot{x} = v.$$

Classical block-diagram modeling procedure begins from creation of a mathematical model of the components of a system of interest and their interconnection, and only then a block-diagram can be built from the assembled model, which serves as a basis for simulation, e.g., in Simulink. The approach is straightforward for simple cases such that one-dimension systems, but it becomes tedious for complex systems.

An alternative way is to use bond graph methodology to build a bond graph of the whole system using basic building blocks, that can be used directly for simulation, e.g., in 20-sim. We have considered the basis of bond graphs and took a look at the simplest bond graph with one-dimensional scalar bonds. That approach is versatile and allows modeling any robotic systems as it done in [12, 13, 14]. However, it is not a convenient to model robotic systems that contains multiple components and interacts with environment. Because of that reason, chapter 4 focuses on screw theory and Lie groups. That section introduces power variable for spatial motion called twist and wrenches. Those power variable are vectors and forms multibonds, that makes modeling much easier.

Bibliography

[1] Stefano Stramigioli. *Modeling and IPC control of interactive mechanical systems—A coordinate-free approach.* 2001.

[2] Vincent Duindam and Stefano Stramigioli. *Modeling and control for efficient bipedal walking robots: A port-based approach*, volume 53. Springer, 2008.

[3] Vincent Duindam, Alessandro Macchelli, Stefano Stramigioli, and Herman Bruyninckx. *Modeling and control of complex physical systems: the port-Hamiltonian approach.* Springer Science & Business Media, 2009.

[4] Cristian Secchi, Stefano Stramigioli, and Cesare Fantuzzi. *Control of interactive robotic interfaces: A port-Hamiltonian approach*, volume 29. Springer Science & Business Media, 2007.

[5] Kevin M Lynch and Frank C Park. *Modern Robotics.* Cambridge University Press, 2017.

[6] Richard M Murray, Zexiang Li, S Shankar Sastry, and S Shankara Sastry. *A mathematical introduction to robotic manipulation.* CRC press, 1994.

[7] Antonio Tornambe and Claudio Melchiorri. *Modelling and Control of Mechanisms and Robots.* World Scientific, 1996.

[8] Jon M Selig. *Geometric fundamentals of robotics.* Springer Science & Business Media, 2004.

[9] Henry M Paynter. *Analysis and design of engineering systems.* MIT press, 1961.

[10] Darina Hroncová, Alexander Gmiterko, Peter Frankovskỳ, and Eva Dzurišová. Building elements of bond graphs. In *Applied Mechanics and Materials*, volume 816, pages 339–348. Trans Tech Publ, 2015.

[11] Catherine Bidard. Screw-vector bond graphs for multibody systems modelling. *SIMULATION SERIES*, 25:195–195, 1993.

[12] B Gola, J Kopec, J Rysiński, and S Zawislak. Bond graph model of a robot leg. In *Graph-Based Modelling in Engineering*, pages 69–80. Springer, 2017.

[13] Shengqi Jian, Cheng Yin, Luc Rolland, and Lesley James. Five bar planar manipulator simulation and analysis by bond graph. In *ASME International Mechanical Engineering Congress and Exposition*, volume 46476, page V04AT04A026. American Society of Mechanical Engineers, 2014.

[14] Zhaohong Wu, Matthew I Campbell, and Benito R Fernández. Bond graph based automated modeling for computer-aided design of dynamic systems. *Journal of Mechanical Design*, 130(4), 2008.