Modelling UDE & H∞ Controller for Maglev Application

Qing-Chang Zhong, *Fellow, IEEE*, and Anirudha Behera, *Student Member, IEEE*

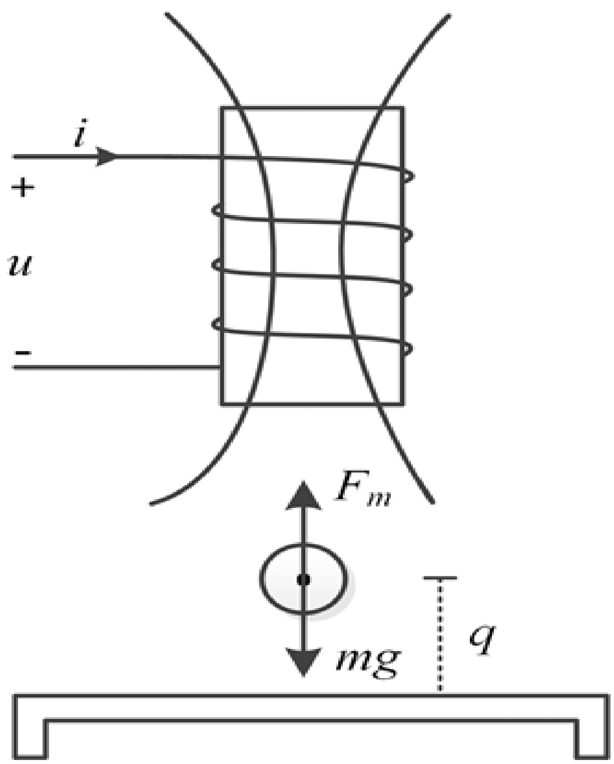
***Abstract*—In this paper, an uncertainty and disturbance estimator (UDE)-based robust controller and H∞ controller is proposed to address disturbance and noise problems. In this study, the sufficient and necessary sterilizability conditions of the UDE-based robust control and H∞ controller are investigated. According to the sterilizability conditions, a systematically design method is presented for the reference model based on the controllable canonical transformation and pole placement. This is then applied to a magnetic levitation system (Maglev) subject to model uncertainties and external disturbance as an example. Simulation results are presented to illustrate the effectiveness of the proposed control approach with respect to matched uncertainties.**

***Index Terms*—UDE controller, H∞ controller, Asymptotic controller, Disturbance, Maglev, Uncertainty, Stability**

I. Introduction

The automatic control systems have been widely used in modern industries, and many other fields. So far, we have an enormous spread of control methods, including UDE control, H∞ control, PID control, fuzzy control, adaptive control, sliding-mode cascaded control, multirate control, Quantitative feedback Control, asymptotic control, and active disturbance rejection control, etc. These control methods have their own characteristics and merits while the requirements on the designer's professional ability are also high**.** Out of all these control schemes we are going to use UDE and H∞ in this project for a Maglev. *The reason of using UDE-based control is, able to quickly estimate uncertainties and disturbances and hence is able to provide outstanding robust performance.* The UDE control algorithm is based on the assumption that a signal can be approximated and estimated using a filter with the right bandwidth and the reason behind H∞ is, it has advantages over classical control techniques where H∞ techniques are readily applicable to problems involving multivariable system with cross coupling between channels.

1) Our aim is to develop two control models using algorithm of UDE and H∞ for given maglev concept. We will perform different investigation of both the systems to analyze the stability of the model.



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II. PART-I

1) OPEN-LOOP MAGNETIC LEVITATION SYSTEM:

All practical magnetic levitation systems are inherently open-loop or unstable system, the relay on feedback control or closed-loop control system to produce desired levitation action.
Converting the given data into state space form,
where, $x_1 = q$, $x_2 = q^{\circ}$, $x_3 = i$

$$\begin{bmatrix} \mathring{x}_1 \\ \mathring{x}_2 \\ \mathring{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 3306.63 & 0 & -33.71 \\ 0 & 0 & -622.34 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 53.19 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad D = [\,0\,]$$

$$Assuming,\ p = 0, v = 0$$

$$So\ here,\ A = \begin{bmatrix} 0 & 1 & 0 \\ 3306.63 & 0 & -33.71 \\ 0 & 0 & -622.34 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 53.19 \end{bmatrix}$$

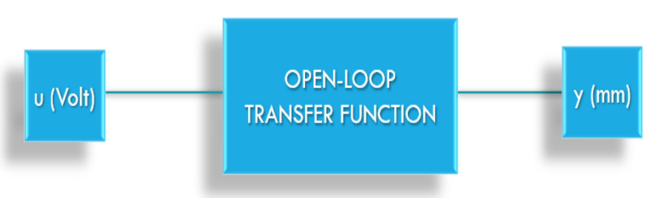
$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ D = [\,0\,]$$

$$Assuming,\ p = 0,\ v = 0$$

$$Open-loop\ transfer\ function\ = C[\,SI - A\,]^{-1}B + \triangle$$

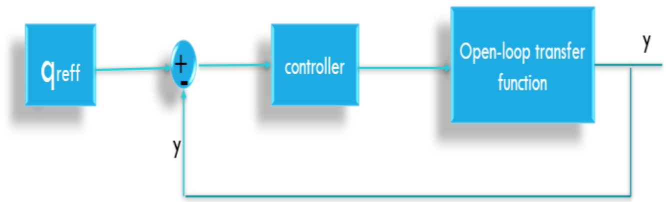
$$Open-loop\ transfer\ function\ = \frac{-1793}{S^3 + 622.3S^2 - 3307S - 2.058e06}$$

<u>Open-loop block diagram</u>

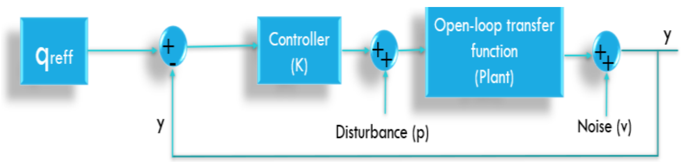


Note: It can be seen that the open-loop poles are,
$S_1 = 57.503$ ---------------(RHP)
$S_2 = -57.5033$
$S_3 = -622.3400$
Hence, open-loop system is statically unstable.

2) CLOSED-LOOP MAGNETIC LEVITATION SYSTEM:



[Closed-loop block diagram (p=0, v=0)]



[Closed-loop block diagram (with p and v)]

3) DESIGN OF UDE BASED ROBUST CONTROLLER:

Control framework:

consider the uncertain LTI system,

$$\mathring{x}(t) = (A + \triangle A)x(t) + Bu(t) + d(t) \text{ -------- } (1)$$

Uncertainty terms are denoted as Ud(t),

$$Ud(t) = \triangle A\, x(t) + d(t) \text{ --------- } (2)$$

$$from\ (1),\ Ud(t) = \mathring{x}(t) - Ax(t) - Bu(t) \text{ -------- } (3)$$

Ud(t) is estimated by an UDE law as follows,

$$\widehat{U}d(t) = [\mathring{x}(t) - Ax(t) - Bu(t)]^{*}\, gf(t) \text{ -------- } (4)$$

Where "*" = conventional convolution operator.
gf(t)= impulse response of a frequency filter with unity gain & zero phase shift covering the spectrum off 'Ud'.

Control Objective – To design robust control law such that system output could accurately track a reference signal 'r Є R' in presence of matched uncertainties (Δ A) and disturbances (d).
For two degree of freedom controller the reference model introduced as,

$$\mathring{x}_m(t) = A_m x_m(t) + B_m u_m(t) \text{ --------- } (5)$$

$$\mathring{e}(t) = (A_m + K)e(t) \text{ --------- } (6)$$

When 'K ∈ R\power{n*n}' is feedback gain such that e(t) reduces exponentially.

Putting (1) & (5) into (6),

$$Bu(t) = (A_m - A)x(t) + B_m u_m(t) - Ke(t) - Ud(t) \text{ --------- } (7)$$

Putting (4) into (7),

$$U(t) = B^{+}[(A_m - A)x(t) + B_m u_m(t) - Ke(t) - \widehat{U}d(t)] \text{ --------- } (8)$$

$$here,\ B^{+} = \left(B^T B\right)^{-1} B^T$$

$$U(t) = B^{+}\left\{ -Ax(t) + L^{-1}\left[\frac{1}{1-Gf(s)}\right] * [A_m x(t) + B_m u_m(t) - Ke(t)] - L^{-1}\left[\frac{sGf(s)}{1-Gf(s)}\right] x(t) \right\}$$

$$\text{----------} (9)$$

4) PROPOSED UDE MODEL:

The model matrix for controller:

$$A_m = \begin{bmatrix} 0 & 1 & 0 \\ 3306.63 & 0 & -33.71 \\ 849771 & 10483.9 & -1100 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 0 \\ -10000 \end{bmatrix}$$

$$C_m = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ D_m = \begin{bmatrix} 0 \end{bmatrix}$$

Above reference matrix is meant to track q_{ref}.

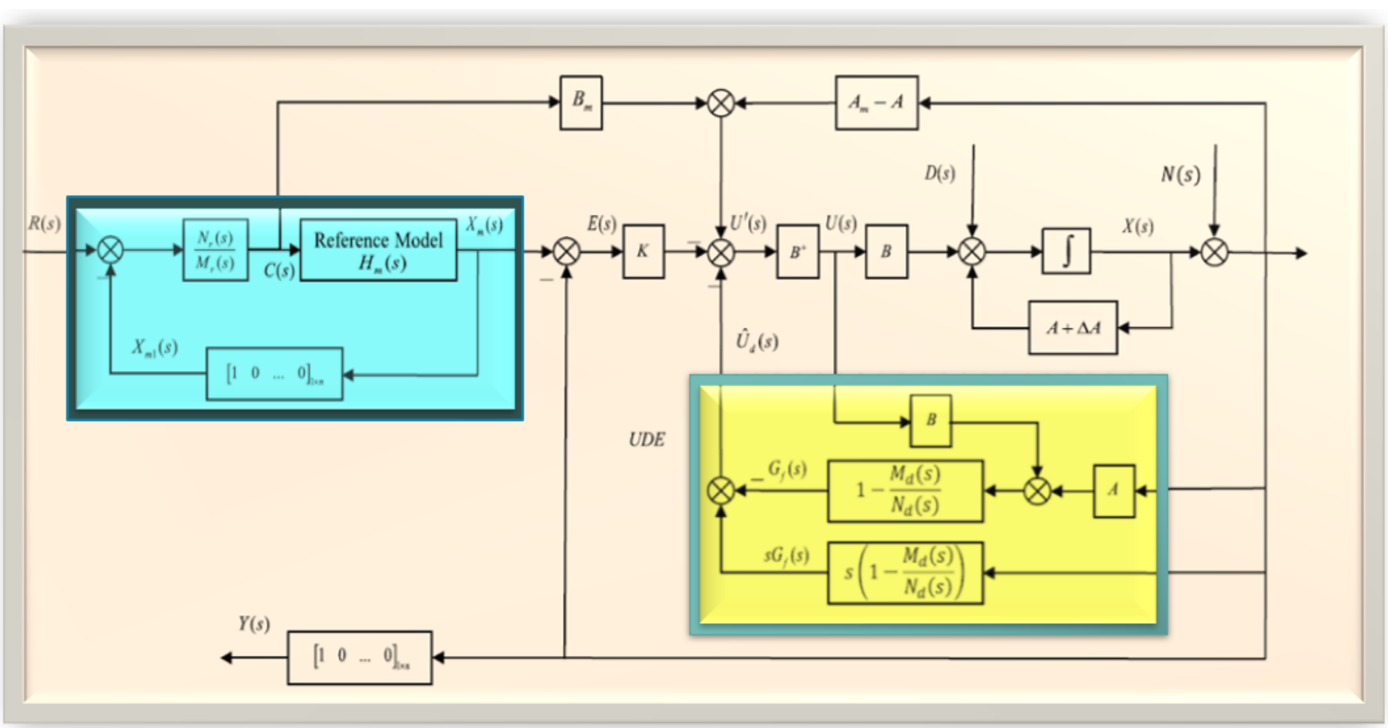
5) ACCORDING TO CONDITIONS WE HAVE SIX CASES FOR Q.3:

i.e. q_{reff} = 2mm, 4mm, 6mm } P=0, v=0

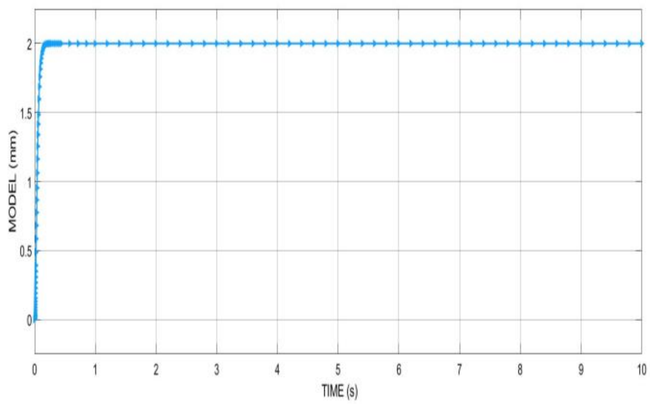
Then, q_{reff} = 2mm, 4mm, 6mm } P=50 at t=2s & v=0.1 at t=4s

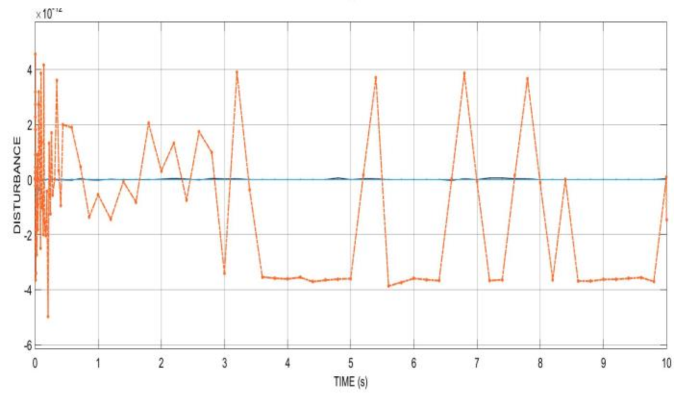
Note: Δ A= 0, for these six conditions.

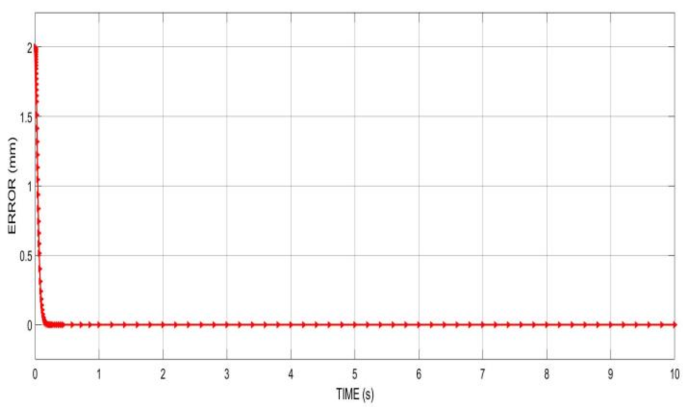
But we will mention here only two cases, one from each.

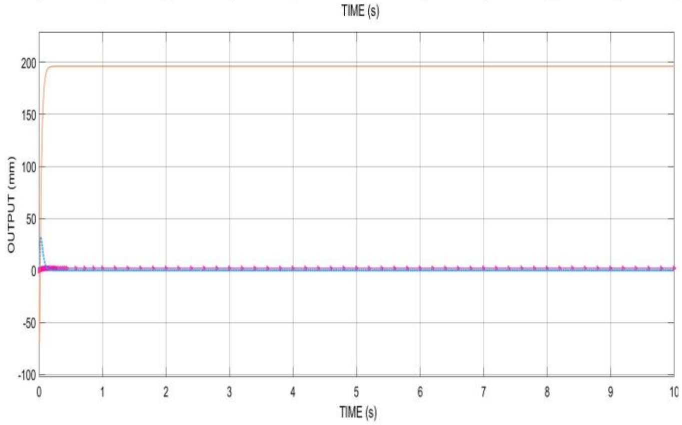


- Case:1-qreff=2mm, p=0, v=0

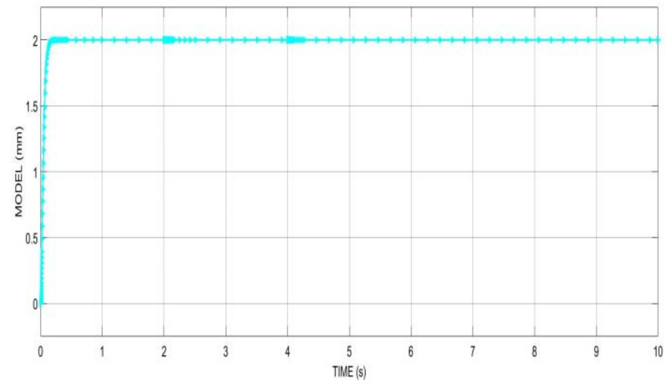


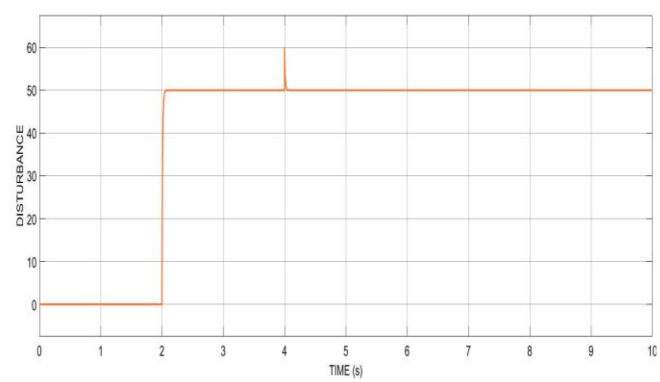


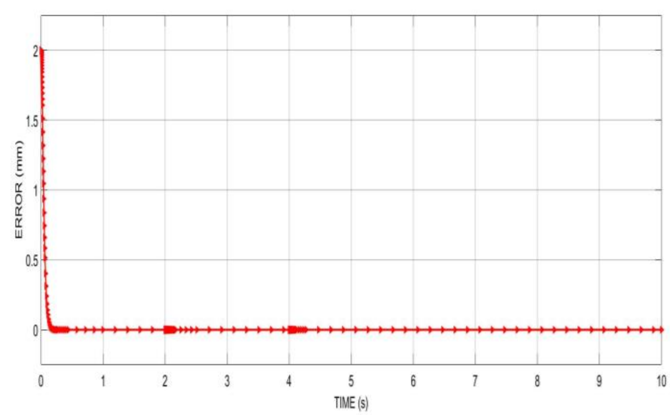


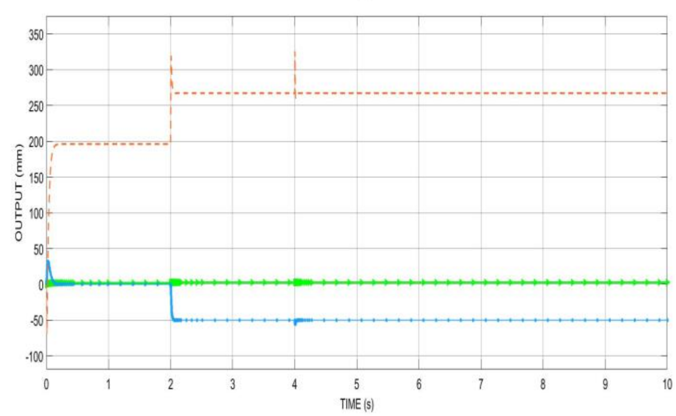


- Case:4-qreff=2mm, p=50 at t=2s, v=0.1 at t=4









6) ACCORDING TO CONDITIONS WE HAVE THREE CASES FOR Q.4:

q_{reff} = $4 + 2Sin(20\pi t)\ mm$ — P=0, v=0

q_{reff} = $4 + 2Sin(20\pi t)\ mm$ — P=50 at t=2s & v=0.1 at t=4s

q_{reff} = $4 + 2Sin(20\pi t)\ mm$ — P=50 at t=2s & v=0.1 at t=4s with 'Δ A'

Let,

$$\therefore\ \Delta A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

And respective Amplitude and Frequency is,

$$Amplitude : For\ (x) = A{*}Sin(Bx - C) + D$$

$$Here\ amplitude\ is\ |A|,$$

$$F(x) = 4 + 2Sin(20\pi t)$$

$$Therefore\ the\ Amplitude\ is\ = 2$$

$$Frequency :\ For,\ f(x) = Ag(Bx - C) + D$$

$$\therefore\ g(x)\ is\ basic\ trigonometric\ function$$

$$Frequency\ is\ \frac{B}{2\pi}$$

$$F(x) = 4 + 2Sin(20{*}pi{*}t)$$

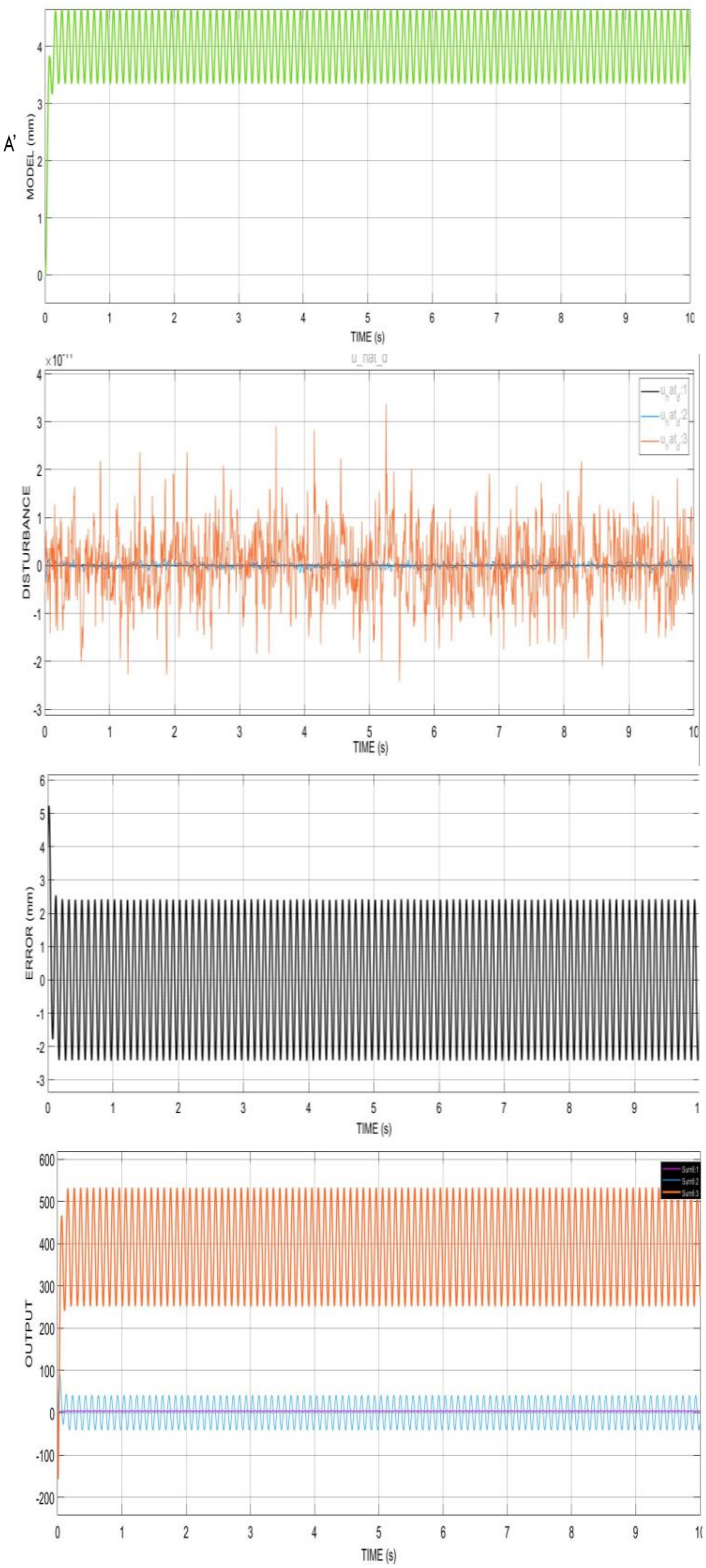
$$here,\ B = 20{*}\pi$$

$$g(Bx - C) = Sin(20{*}pi{*}t)$$

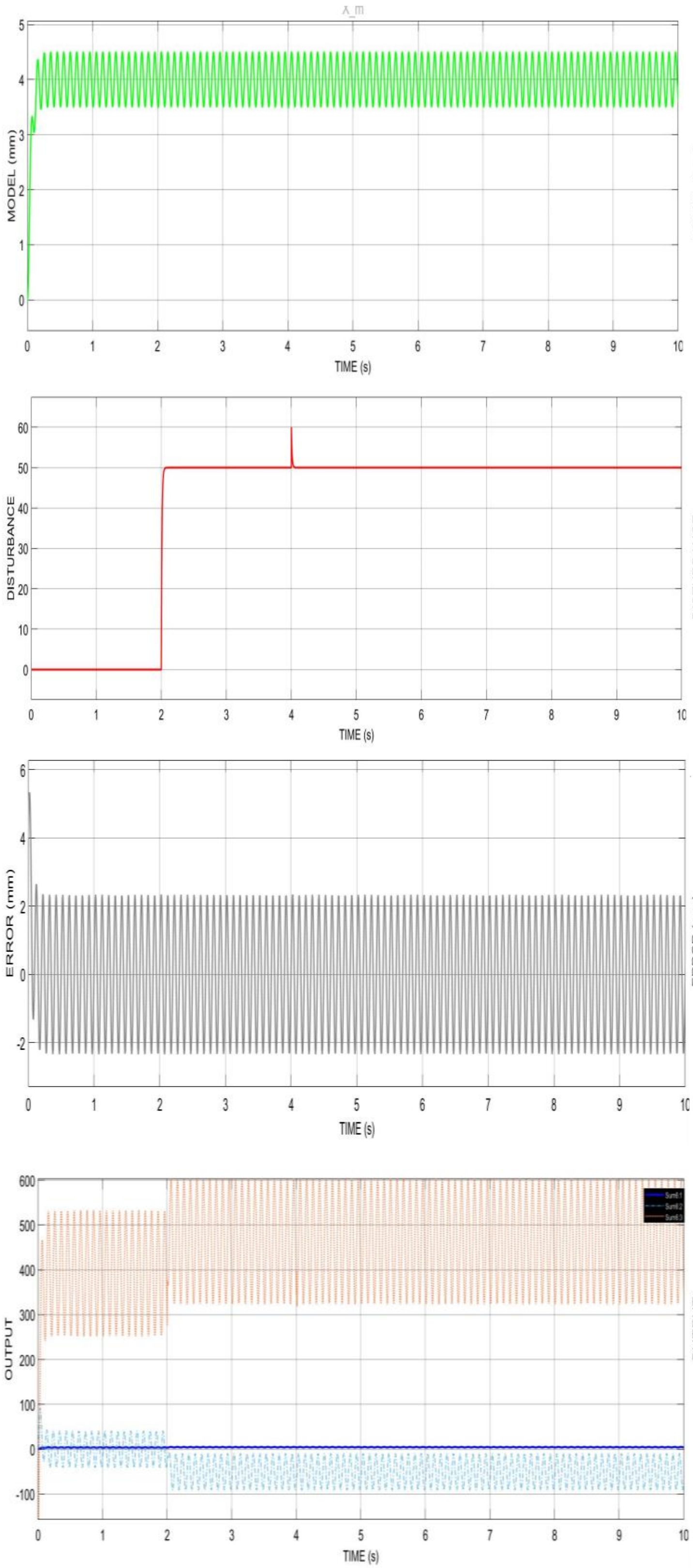
$$so,\ frequency = \frac{B}{2\pi} = \frac{20\pi}{2\pi} = 10$$

Here we will take three cases, one from each.

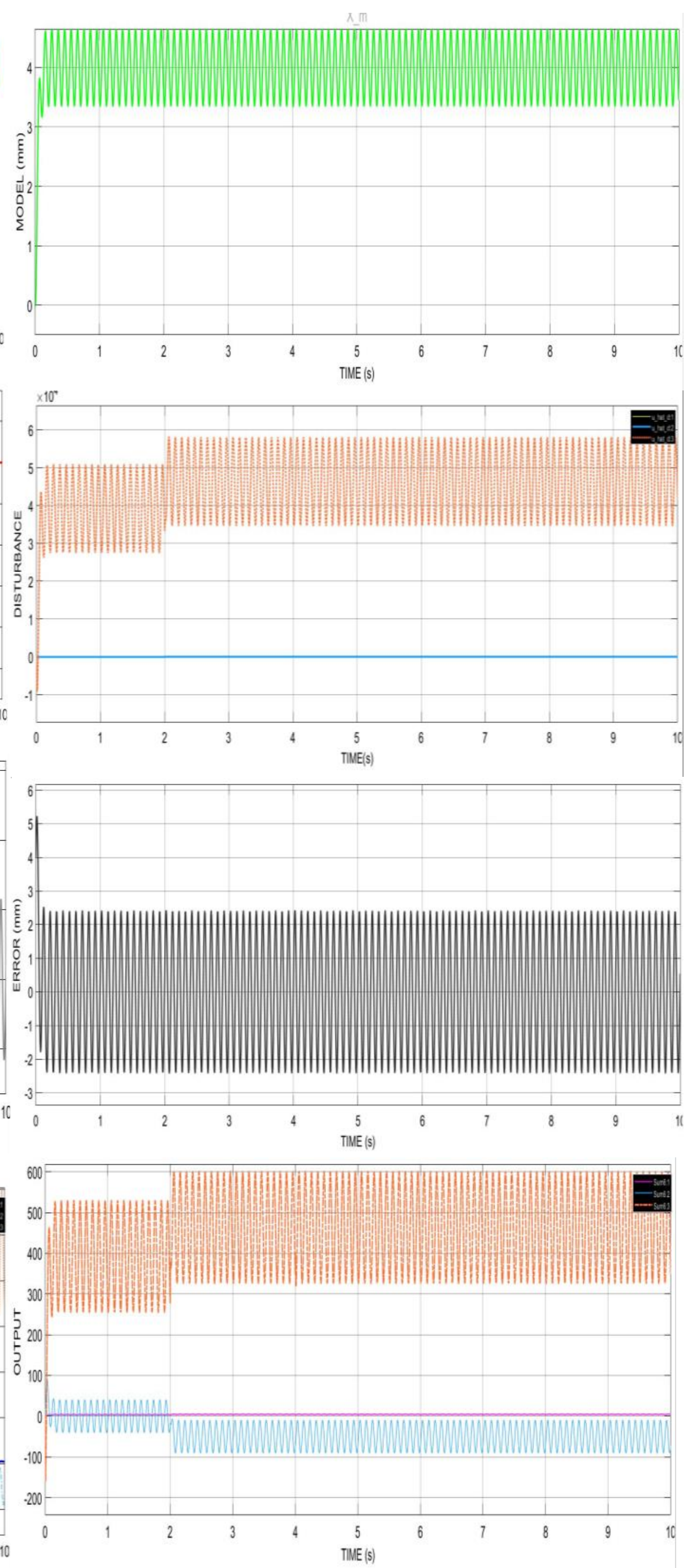
- Case:1-qreff=4+2Sin(20∏t) mm, p=0, v=0fr=10hz, A =2



- Case:2-qreff=4+2Sin(20∏t) mm, p=50 at t= 2, v=0.1 at t=4, fr=10hz, A =2



- Case:3-qreff=4+2Sin(20∏t) mm, p=50 at t= 2, v=0.1 at t=4, fr=10hz, A =2, with ΔA



III. PART-II

DESIGN OF H-∞ BASED ROBUST CONTROLLER:

Given condition,

$$Let,\ z = \begin{bmatrix} x \\ u \end{bmatrix},\ x = \begin{bmatrix} q & \frac{dq}{dt} & i \end{bmatrix}^T \ is\ a\ state\ vector$$

1) STATE SPACE MODEL

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 3306.63 & 0 & -33.71 \\ 0 & 0 & -622.34 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 53.19 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad D = [\,0\,]$$

$$Assuming,\ p = 0, v = 0$$

$$Open-loop\ transfer\ function\ = \frac{-1793}{S^3 + 622.3S^2 - 3307S - 2.058e06}$$

2) CLOSE LOOP H-∞ CONTROLLER DESIGN

$$Consider\ state\ space\ system,$$

$$\dot{x} = A_x + B_u$$

$$Let,\ \tilde{y} = r - y$$

$$where,\ r = reference$$

$$For\ input\ tracking\ we\ choose\ Z_\infty = e,\ w = r$$

$$where,\ Z_\infty = worst\ case\ error$$

H-infinity norm:

It is the max value of frequency norm or maximum amplification for any sinusoidal frequency.

The equation to be controlled is given as,

$$\dot{x} = A_x + B_u = A_x + B_1 w + B_2 u$$

$$Z_\infty = W - C_x = C_1 x + D_{11} w + D_{12} u$$

$$\tilde{y} = W - C_x = C2x + D_{21} w + D_{22} u$$

For the given plant,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 3306.63 & 0 & -33.71 \\ 0 & 0 & -622.34 \end{bmatrix},\ B_1 = [\,0\ \ 0\ \ 0\,],\ B_2 = \begin{bmatrix} 0 \\ 0 \\ 53.19 \end{bmatrix}$$

$$C_1 = C_2 = -C$$

$$D_{11} = [0], \quad D_{12} = [0]$$

$$D_{21} = [-1], \quad D_{22} = [0]$$

Here, D12 and D21 are key parameters.

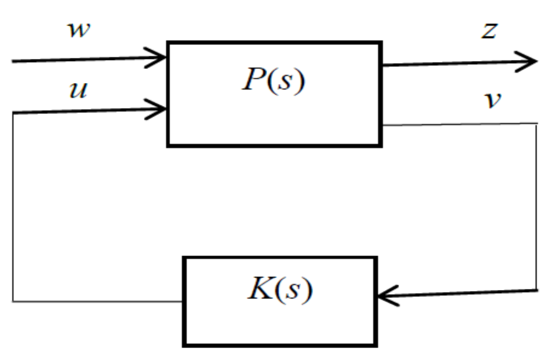
$$P(S) = \left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array}\right]$$

Now after applying H-infinity procedure we will get, Transfer function for controller,

$$K(S) = \frac{-2.561e15S^2 - 1.741e18S - 9.166e19}{S^3 + 1.719e05S^2 + 9.734e09S - 3.023e14}$$

$$r = 2.0076\ [H\infty\ norm\ of\ P_{ij}(S)]$$

<u>CLOSED-LOOP SYSTEM</u>



Where, K(S) = Transfer function of controller
r = qref
y = q
Z_∞ = e
This represents closed-loop system.

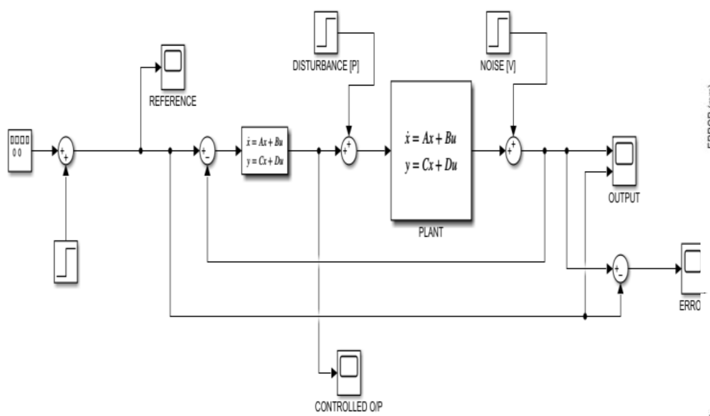
3) ACCORDING TO CONDITIONS WE HAVE SIX CASES FOR Q.3:

i.e. q_{reff} = 2mm, 4mm, 6mm } P=0, v=0

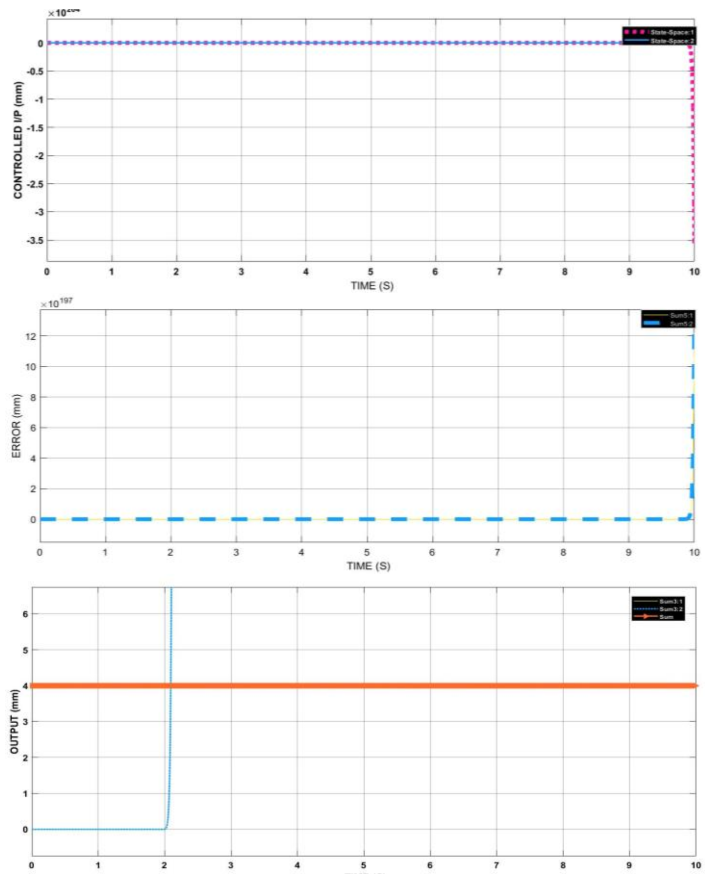
Then, q_{reff} = 2mm, 4mm, 6mm } P=50 at t=2s & v=0.1 at t=4s

Note: Δ A= 0, for these six conditions.

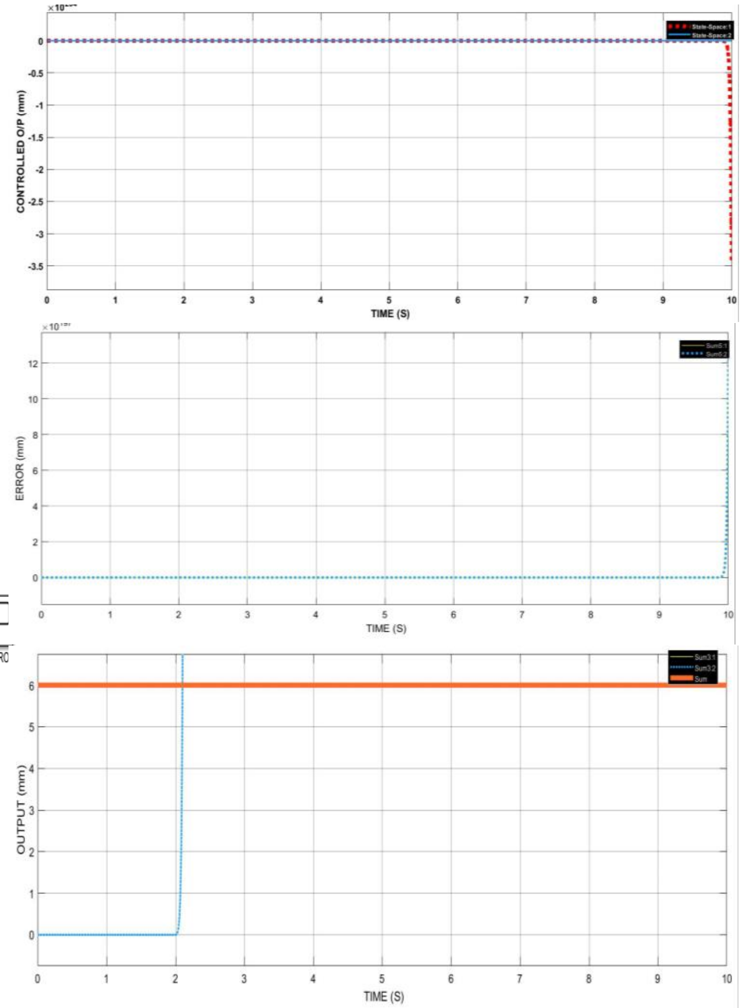
SIMULINK MODEL



- Case: -qref=4mm, p=50 at t=2s, v=0.1 at t=4



- Case: -qreff=6mm, p=50 at t=2s, v=0.1 at t=4

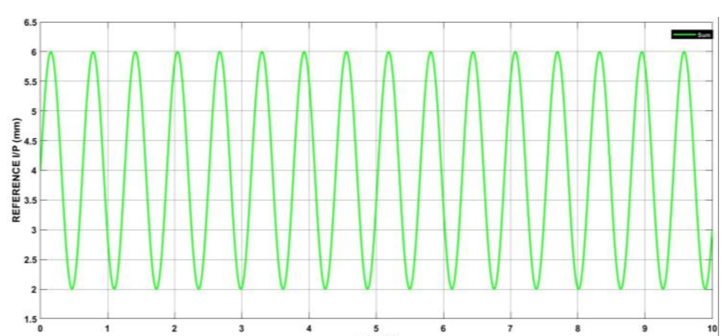


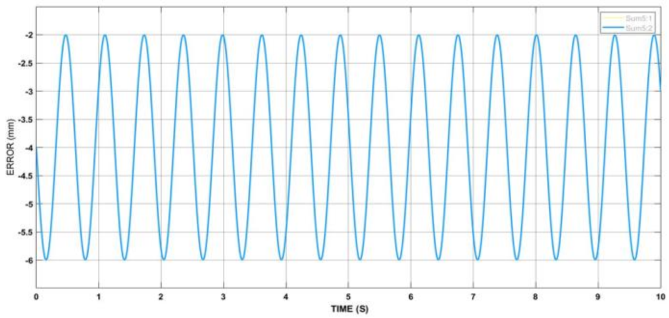
4) ACCORDING TO CONDITIONS WE HAVE TWO CASES FOR Q.4:

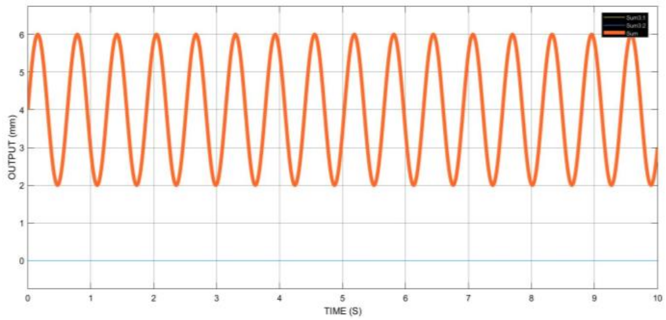
q_{reff} = $4 + 2Sin(20\pi t)\ mm$ } P=0, v=0

q_{reff} = $4 + 2Sin(20\pi t)\ mm$ } P=50 at t=2s & v=0.1 at t=4s

- Case:1-qreff=4+2Sin(20∏t) mm, p=0 v=0, fr=10hz, A =2







- Case:2-qreff=4+2Sin(20∏t) mm, p=50 at t= 2, v=0.1 at t=4, fr=10hz, A =2
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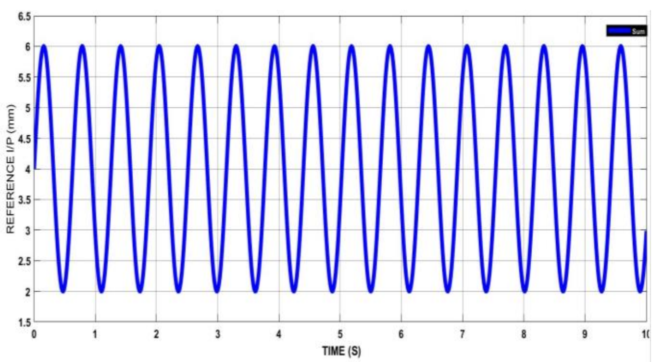

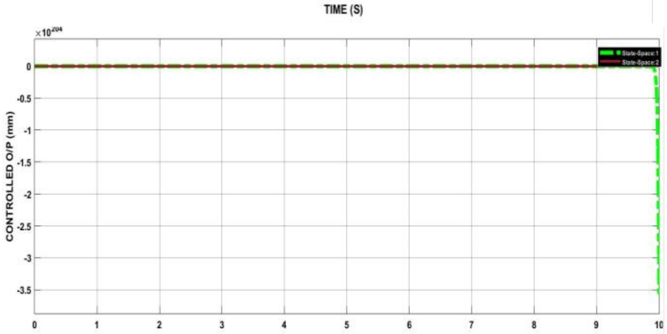

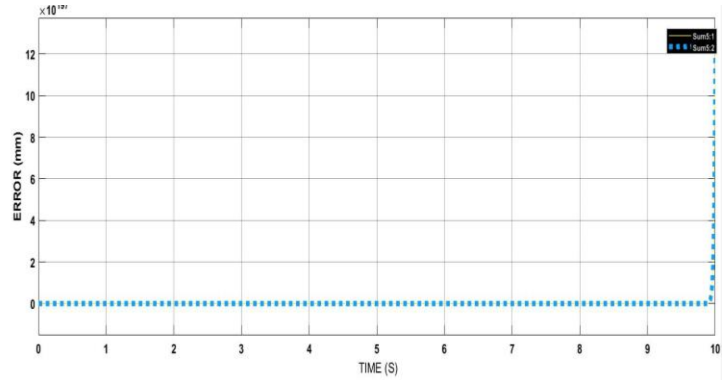

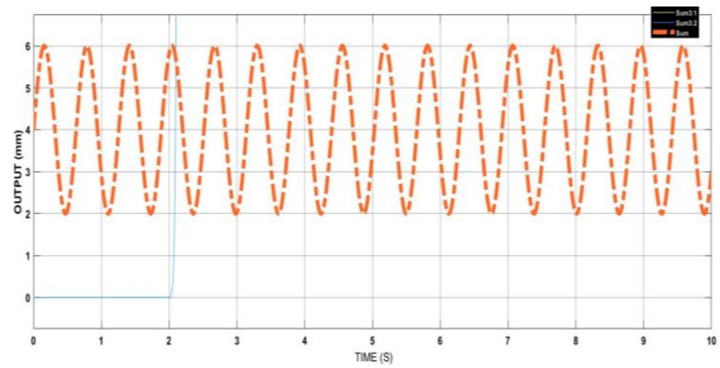

IV. CONCLUSION

In this project we investigated and designed UDE and H∞ controller for Maglev concept. Controller performance in presence of disturbance and noise is best in terms of disturbance rejection.

It tracks the desired performance in very short noise time.

It can be used to make stable an unstable plant.

Best part of UDE controller is it not only rejects disturbance; it also estimates the disturbance where H∞ controller modelling was simpler as compared to UDE but H∞ controller takes more time to obtain response during operation.

In this design, the controller parameters have been adjusted such that the system output tracks the output of the model reference. The proposed controller has achieved an asymptotic tracking of prescribed reference output with compensating the system parameters uncertainty. A variation of ±10% in system parameters has been considered.

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