Quality Control of Risk Measures:

Backtesting Risk Models

"A Tale of Two Powers"*

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Outline

- Quality Control problem
- VaR backtesting
- Limitations of the Basel test
- QCRM hypothesis test
- Power of the test
- New rules for accepting/rejecting VaR models

The problem

• Regulators and risk managers have to decide a course of action; i.e., accept or reject a bank's model:

Model correct vs. Model incorrect

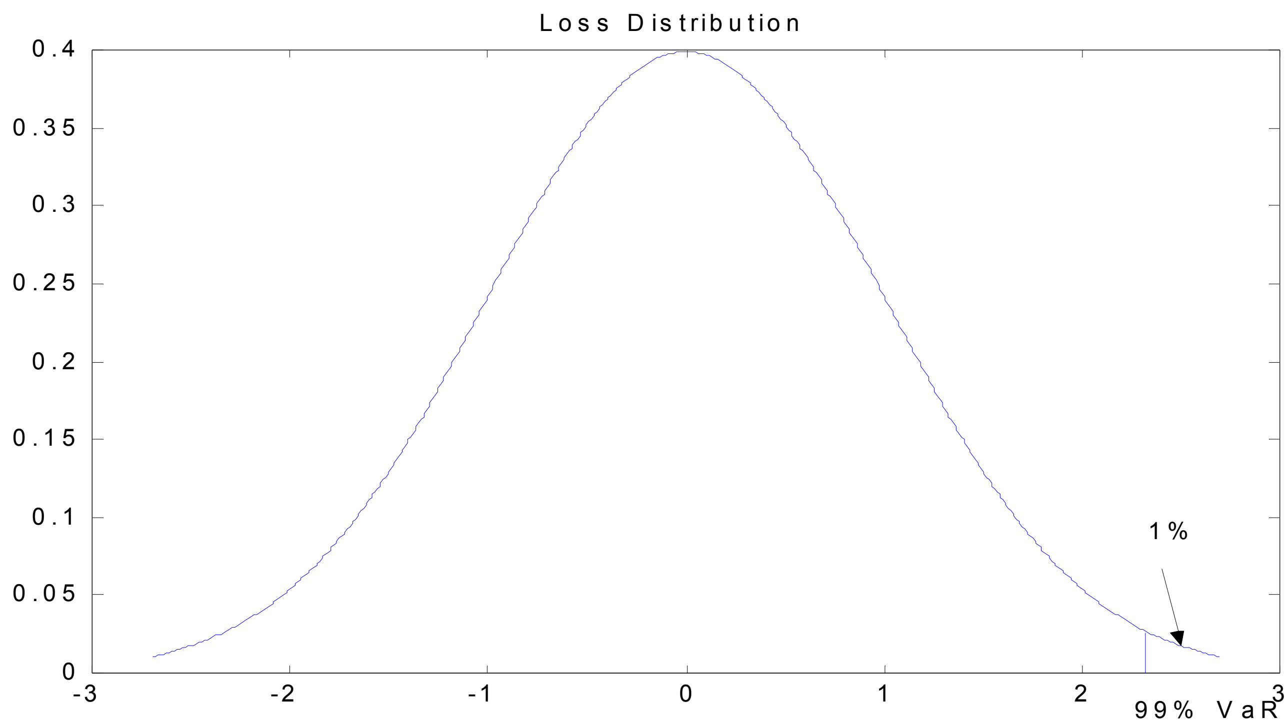
VaR backtesting

- A process by which financial institutions periodically compare daily profits and losses with VaR model-generated risk measures

- The goal is to evaluate the quality and accuracy of the bank's VaR risk model

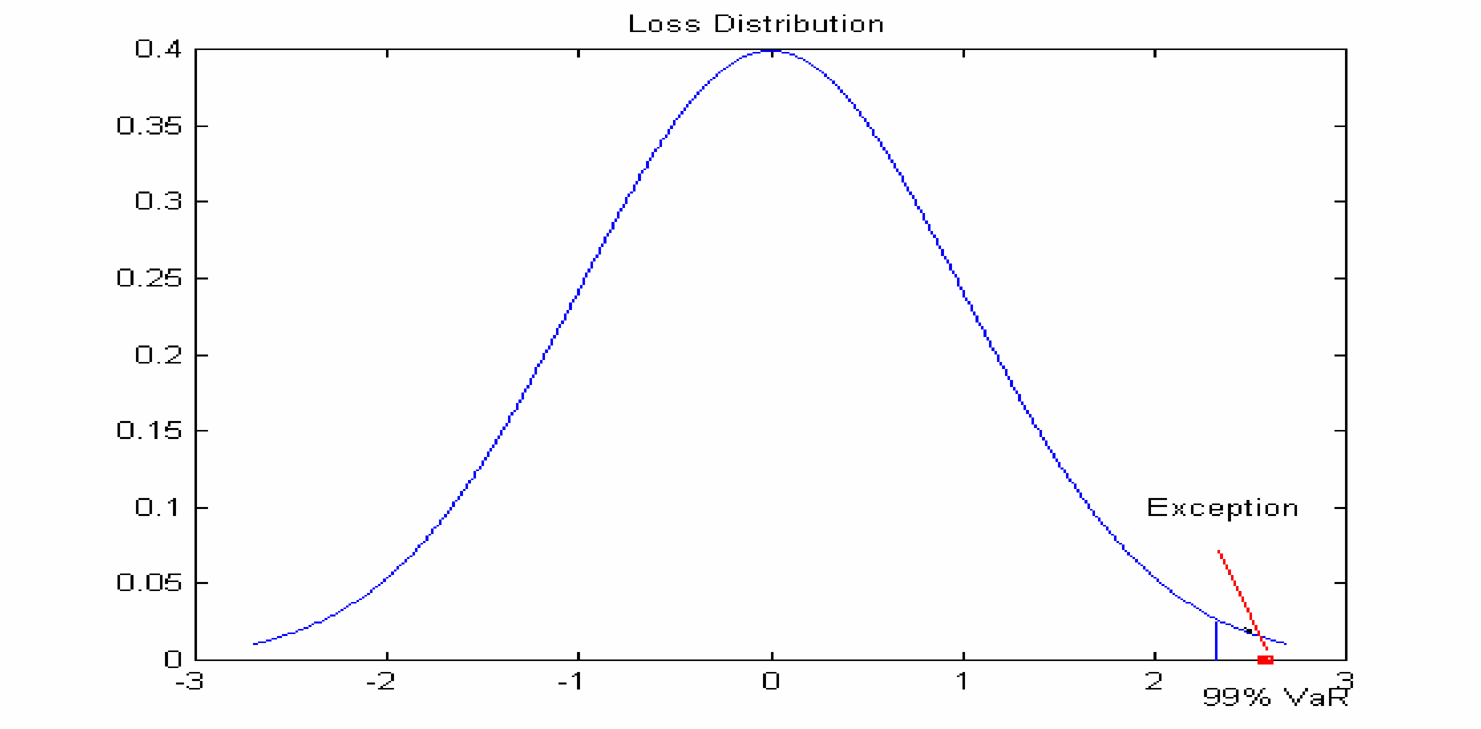
Value at Risk: refreshment

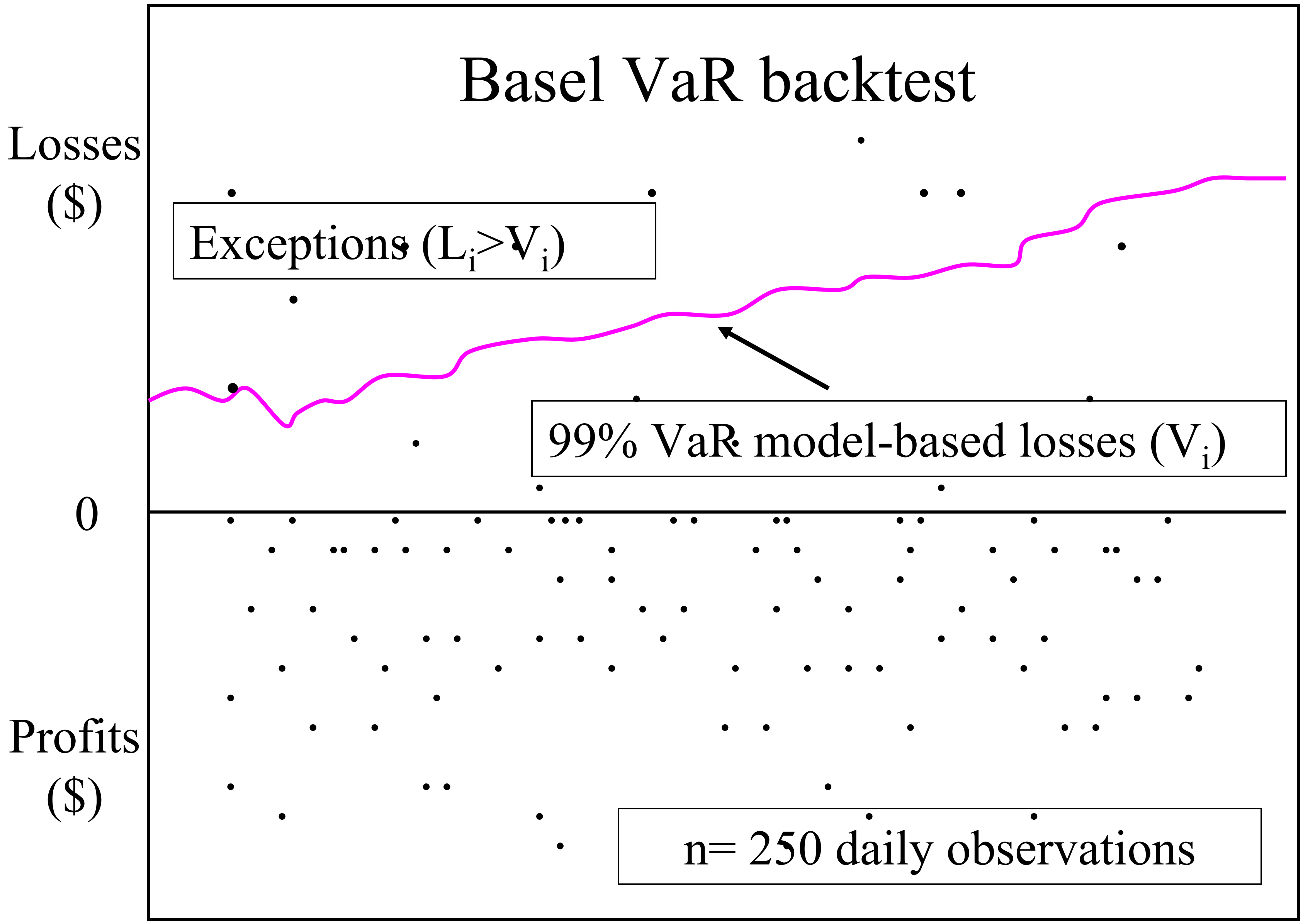
• The $(1-\alpha)\times 100\%$ Value at Risk is the percentile $(1-\alpha)$ of the distribution of the Portfolio losses



Exception (model failure)

• The event that the portfolio loss exceeds the corresponding VaR predicted for a trading day





Basel VaR backtest
Losses ($)
Exceptions ($L_i > V_i$)
99% VaR model-based losses (V_i)
0
Profits ($)
n= 250 daily observations

Notation

$V_{i-1}^{i}(\alpha)$: The $(1-\alpha)\times 100\%$ VaR estimate for trading day i using the information obtained until day i - 1,

L_i : Portfolio Loss observed on day i

• The indicator of the event of an exception on day i is given by

$$Y_i = 1_{\{L_i > V_{i-1}^{i}\}} = \begin{cases} 1 \text{ if } \mathrm{L_i} > V_{i-1}^{i} \\ 0 \text{ otherwise} \end{cases}$$

Assumptions

• We assume that the probabilities of observing an exception remain constant throughout time

$$P\left(Y_i = 1 \mid F_{i-1}\right) = p,$$

where F is the information available at time t

• Technical fact: if the indicators of exceptions have the same conditional probabilities then they are independent and so

$$\boxed{X = \sum_{i=1}^{n} Y_i \approx \text{Binomial } (\text{n, p})}$$

Basel accepting/rejecting regions

- **Green Zone** (0-4 exceptions): model is deemed accurate
- **Yellow Zone** (5-9 exceptions): Supervisor should encourage the bank to present additional information before taking action
- **Red Zone** (10+ exceptions): model is deemed inaccurate

Hypotheses

• Assume **p** is the true (unknown) probability of having an exception, risk managers test

$$\mathbf{H_0: p = p_0 = 0.01 \quad vs. \quad H_A: p > p_0 = 0.01}$$

• where $p_0 = 0.01$ (99% VaR) is the probability of an exception when the model is correct

Control Type I Error

- Basel VaR backtesting method seeks to control the probability of rejecting the VaR model when it is correct
    - Set the probability of rejecting the VaR model when it is correct to be as small as 0.0003 (0.03%)
    - Therefore, it controls the type I error at 0.03%
    - P(number of exceptions $\geq$ 10 when p = 0.01) = 0.0003

Basel on VaR Backtesting

"The Committee of course recognizes that tests of this type are limited in their power to distinguish an accurate model from an inaccurate model"[1]

(1) Basel Committee on Banking Supervision (Basel), page 5 of "Supervisory Framework for the use of "Back Testing" in conjunction with the internal models approach to Market Risk Capital requirements", January 1996

Change of hypotheses

- QCRM hypothesis testing problem:

H_0: VaR Model incorrect vs. H_A: VaR Model correct

- Accepting H_0 implies rejecting the model
- Rejecting H_0 implies accepting the model

New hypothesis test

- Assume **p** is the true probability of having one exception (unknown), QCRM tests:

$$\mathbf{H_0^Q: p > 0.01 \qquad vs. \qquad H_A^Q: p \le 0.01}$$

- This is the quality control problem

New acceptance and rejection regions

- **New Green zone = {0 to 5 exceptions}**: if p_0 is in the 95% one-sided confidence interval for p $[p_L(x,.05),1]$

- **New Yellow zone = {6 or 7 exceptions}**: if p_0 is in the 99% one-sided confidence interval for p $[p_L(x,.01),1]$ (and it is <u>not</u> in the 95% one-sided confidence interval)

- **New Red Zone = {8 or more exceptions}**: if p_0 is <u>not</u> the 99% one-sided confidence interval for p $[p_L(x,.01),1]$

Look at the power of the test!

• The power of the test is a function of the (unknown) parameter **p**, which is defined in terms of the rejection region R as

$$\boldsymbol{\beta}(\boldsymbol{p}) = \boldsymbol{P}_{\boldsymbol{p}}(\boldsymbol{X} \in \boldsymbol{R})$$

• This function contains all the information about the QCRM test

• We <u>redefine</u> the power of the test in terms of probability of accepting (rejecting) an incorrect (correct) model

Power: key comparison

Tests	P(rejecting the model|correct)	P(rejecting the model|incorrect)
Basel	0 – 0.0003*	P(X≥10|given p>0.01)
QCRM	0 – 0.004	P(X≥8|given p>0.01)

* Assume composite null hypothesis for Basel test with p≤0.01

Idea

• QCRM increases, with respect to the Basel test, the probability of rejecting an incorrect model

• QCRM's null hypothesis is then rejected when there is overwhelming evidence to accept the model $\Rightarrow$

• This lead to an statistically certification of the model

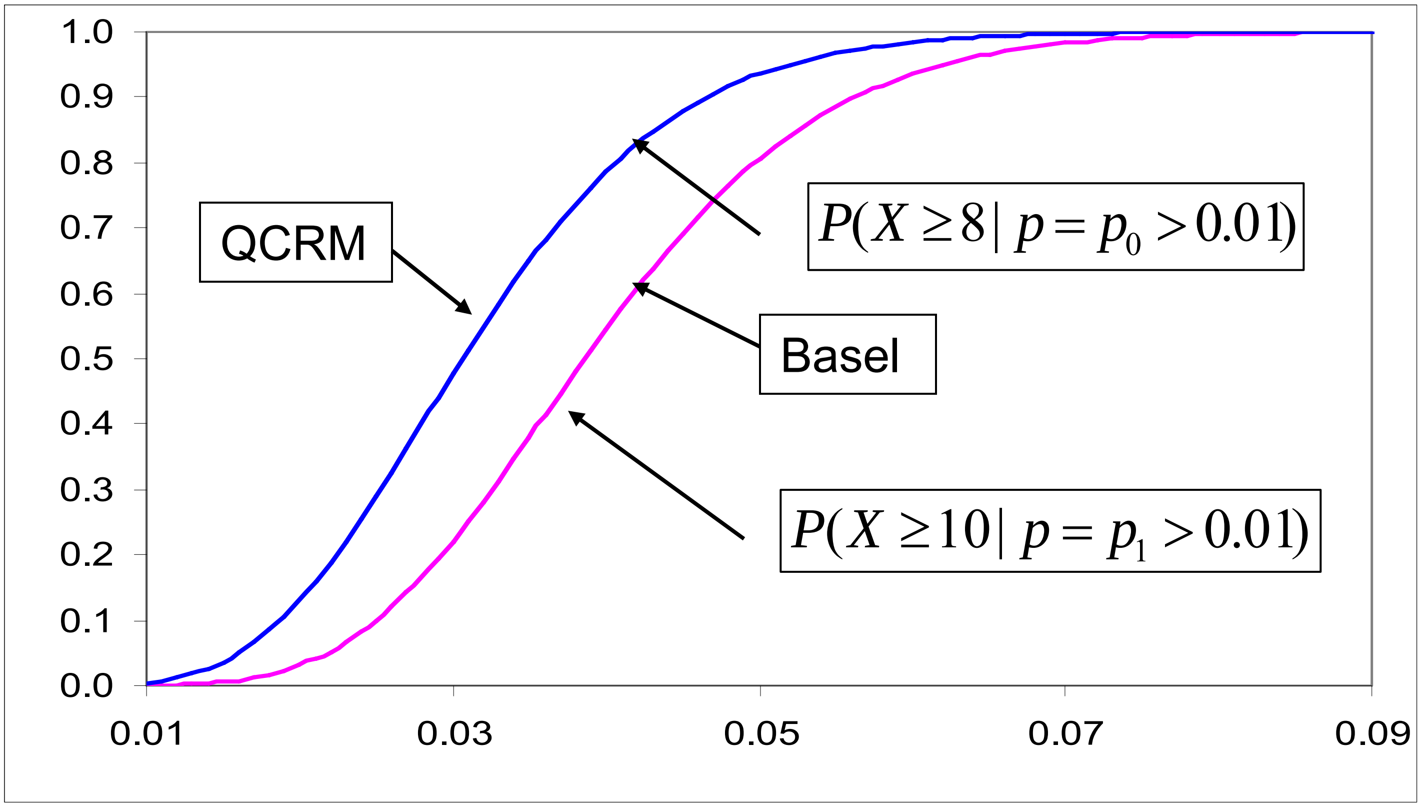
Probability of rejecting a correct model

- Basel: [0 – 0.0003] and QCRM [0 – 0.004]

- Suppose 10 model reviews per year. How many years, on average, are necessary for regulators to make a <u>wrong</u> assessment?…

Test	Max. Error	Model Reviews	Years	Years per Error
Basel	3	10,000	1,000	**333.3**
QCRM	4	1,000	100	**25**

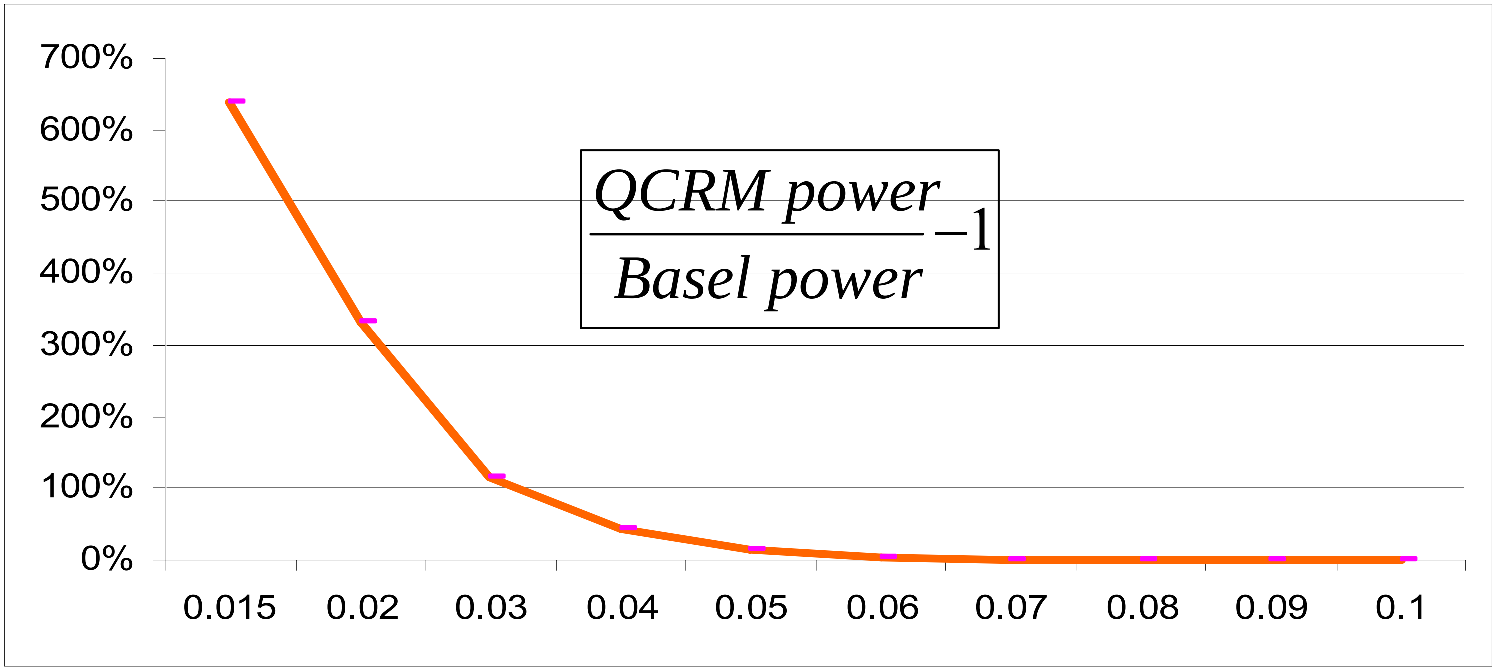
Probability of rejecting a wrong model



X-axis: different values of alternative hypotheses p

Power rate curve

• Percentage gains of QCRM over Basel in the probability of rejecting the wrong model



for different values of the alternative hypotheses p

Research in progress

- QCRM to test credit risk models for Basel II implementation

- The test can be applied to other areas within or outside finance

Summary

• We find that the Basel test is extremely conservative; i.e., it almost guarantees that regulators will not reject a correct model

•…but it may lead regulators to accept an incorrect model

• We propose a more balanced test that dramatically increases, with respect to Basel, the probability of rejecting a wrong model

• We propose new rules for accepting/rejecting a VaR model

• We can use QCRM to test the validity of credit risk models for Basel II implementation

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Preambulo: Riesgo de Mercado

- Que es el riesgo de mercado?
- Acuerdo de Basel
- Herramientas usadas
    - Binomial
    - Modelos de VaR (Value-at-Risk)
    - Teoria de pruebas de hipotesis

Que es el riesgo de mercado?

- Riesgo de perdidas en el portafolio del banco debido a cambios en los precios de los activos financieros
- Portafolio: conjunto de inversiones del banco en activos financieros
- Activos financieros incluye: acciones, bonos, prestamos, derivados, etc.
- Riesgo de credito: es el riesgo potencial de perdidas debido a la bancarrota de los deudores del banco

Acuerdo de Basel

- Basel es un organismo internacional dedicado a establecer normas para la “mejor practica” del manejo y control de los riesgos bancarios
- Basel establecio las normas para el uso de modelos internos (matematicos) de los bancos para la medicion y administracion del riesgo de mercado
- En 1996 establecio las reglas para la validacion de los modelos internos de los bancos, las que son utilizadas a nivel internacional

Herramientas usadas

- Binomial

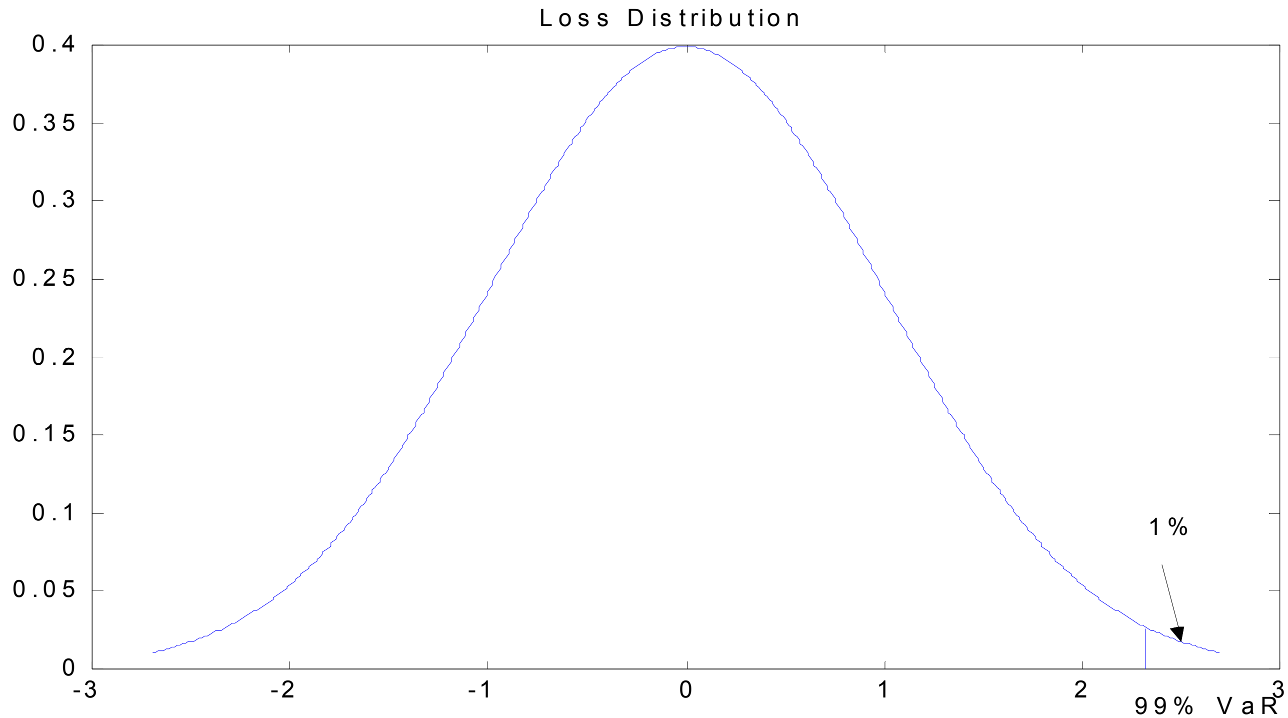
$$\boxed{\boldsymbol{P}(\boldsymbol{X} = \boldsymbol{k}) = \frac{\boldsymbol{n}!}{\boldsymbol{k}!(\boldsymbol{n}-\boldsymbol{k})!}\boldsymbol{p}^{\boldsymbol{k}}(1-\boldsymbol{p})^{\boldsymbol{n}-\boldsymbol{k}}}$$

- Ejemplo: cual es la probabilidad de obtener “cara”4 veces al tirar una moneda 10 veces?

$$\boldsymbol{P}(\boldsymbol{X} = 4) = \frac{10!}{4!(10-4)!}0.5^4(1-0.5)^{10-4} = 0.205078$$

Aplicacion: Valor a Riesgo (VaR)

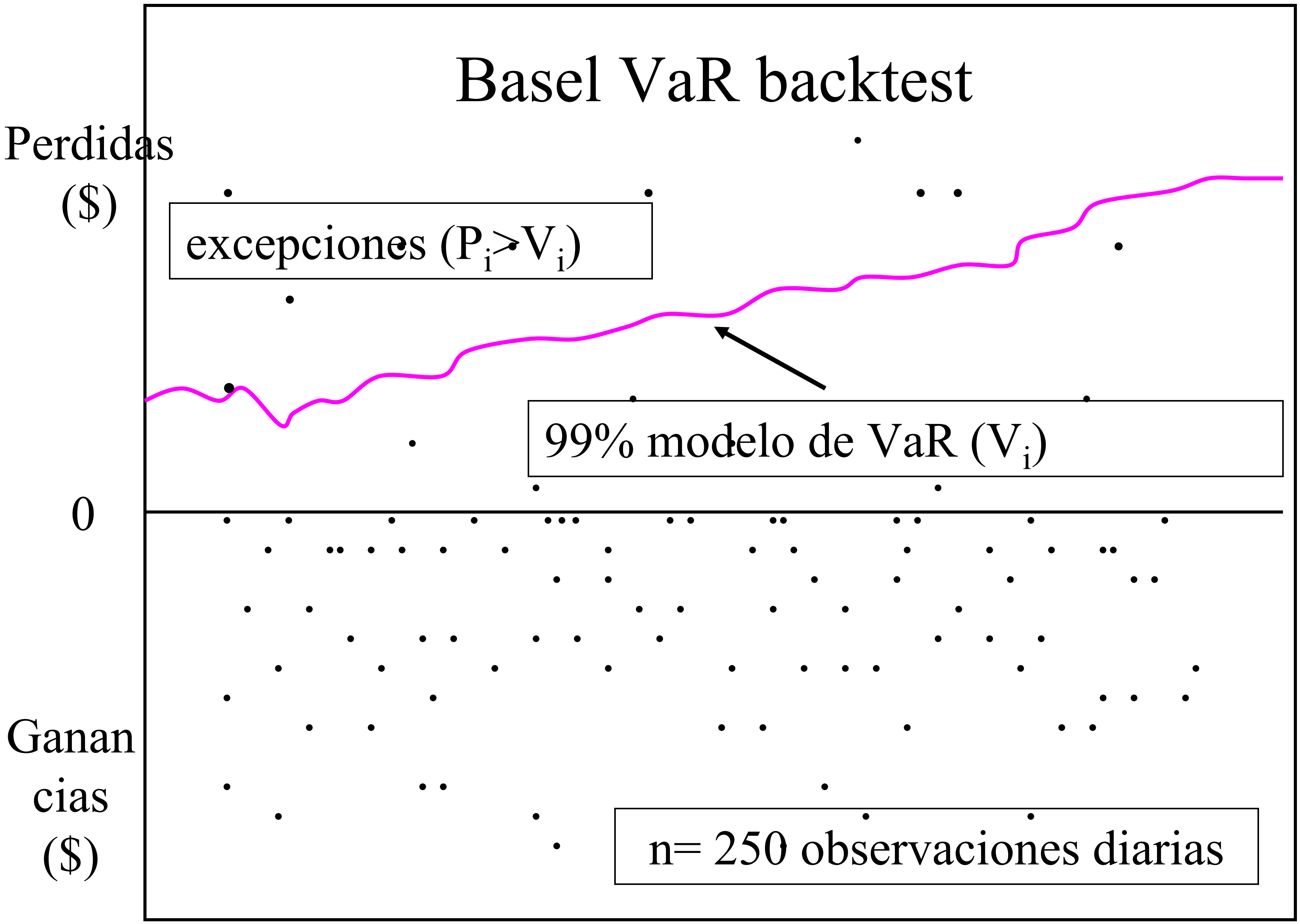
• El $(1-\alpha)\times 100\%$ VaR es el quantil $(1-\alpha)$ de la distribucion de las perdidas del portafolio del banco



VaR backtesting

- Es el proceso por el cual los bancos comparan periodicamente sus perdidas y ganancias diarias con los valores generados mediante el uso del model VaR

- El objetivo es el evaluar la calidad de las predicciones del modelo VaR


Basel VaR backtest
Perdidas
($)
excepciones ($P_i > V_i$)
99% modelo de VaR (V_i)
0
Ganan
cias
($)
n= 250 observaciones diarias

Hipotesis de la prueba de Basel

• Supongamos que **p** es la verdadera probabilidad de cometer un error (excepcion)

$$\mathbf{H_0: p = p_0 = 0.01 \quad vs. \quad H_A: p > p_0 = 0.01}$$

• donde $p_0 = 0.01$ (99% VaR) es la probabilidad de cometer un error cuando el model es correcto

• n es igual a 250 observaciones

• k el numero de ecepciones es mayor o igual a 10

• $P(X \geq 10) = 0.0003$