Razor Financial Principals

Version 3.2

RISK TECHNOLOGIES

Contents

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Chapter 1

Introduction

1.1 RAZOR Financial Principles Document

This document examines the fundamental financial concepts RAZOR utilizes in determining financial risk. RAZOR is an enterprise wide fully integrated credit and market risk management system.

1.2 Credit Risk

Credit risk is the risk of a counterparty not fully meeting their financial obligations. In attempting to manage this risk the probability, magnitude, and possible offsetting effects must be estimated.

Traditionally, credit exposure arose from lending activities, and was measured as simply the book value of all outstanding obligations from a counterparty. Although book value is a fairly poor measure of credit exposure — the market value of an obligation often diverges significantly from its book value — it does have the advantage of being a fairly simple and consistent measure that can provide a reasonable sense of the credit exposure to a counterparty. Unfortunately the notion of book value starts to break down when examining pre-settlement risk on many derivative instruments. Par swaps and bonds have a market value of zero when they are first booked. Many other derivatives have a credit exposure far higher than their book value. A more accurate, probabilistic measure for potential credit exposure is required. The moment a transaction is committed, its market value or exposure changes as time progresses and market rates change. If this market value is in profit then there exists a credit risk exposure, as the unrealised profit will be lost upon the party defaulting. The problem is that only the current value is known, and what exposure the trade will obtain during its future life.

RAZOR solves this problem by taking the current market rates and predicting what they may be tomorrow, and measuring the exposure of the trade under these new conditions. These predictions are produced statistically to simulate the way market rates move in the market place. Historical data is examined to determine the behaviour of each rate and how that rate interacts with all the other rates. This information is used as parameters in the statistical simulation of future market rates called market scenarios.

This process is repeated for the next day and so on until the trade expires. The results are market values of the trade, for all the days in the duration of the trade, which is called an exposure profile. The problem with this approach is that a single statistical prediction of future market rate is unlikely to be correct so this whole process is repeated many times, with the statistically generated future markets being different each time. This produces a distribution of possible values for each future date. Applying a confidence level to this distribution will select a single value for each future date and therefore produce an exposure profile. To save time only a sample, defined by the user, of future exposures are calculated.

Exposure profiles for each trade are calculated, and then combined together in portfolios, taking into account netting agreements, collateralisation agreements and guarantees. Path-dependant trades are transitioned appropriately as price barrier levels are encountered, or payments are rolled off. In this way, RAZOR is able to accurately determine the sensitivities of the portfolio to different rate and price movements, and determine the spectrum of possible losses given specific rate movement scenarios.

RAZOR allows the risk manager to quantify and analyse credit exposures based on the rules set by the business. Each trade is mapped into various portfolios depending on the characteristics of the trade, the counterparty hierarchy, or the internal business unit managing the deal.

RAZOR allows the risk manager to set and manage credit limits and also to track the utilization of the credit limits. Different types of limits thresholds may be set and alerts generated if threshold levels are breached.

Settlement risk is the risk that when an exchange of assets is supposed to happen upon settlement of a trade, one of the counterparties fails to meet their obligations. Settlement risk usually creates very large credit exposures which last for brief periods of time. RAZOR can calculate the risk exposures arising from settlement risk and spread the risk across days depending on the characteristics of the trade.

1.3 Market Risk

Market risk is the risk of a decrease in value of a portfolio of investments, trades, and positions due to changes in the market factors which determine the market value of the portfolio.

The standard market risk factors or drivers which influence market risk are:

Equity risk, or the risk of a decrease in value due to changes in stock prices

Interest rate risk, or the risk of a decrease in value due to changes in interest rates

Currency risk, or the risk of a decrease in value due to changes in foreign exchange rates

Commodity risk, or the risk of a decrease in value due to changes in commodity prices (i.e. grains, metals, etc.)

Spread risk, or the risk of a decrease in value due to changes in spreads between interest rates

Volatility risk, or the risk of a decrease in value due to changes in implied volatilities, as used in derivative/option revaluations

Market risk is typically measured using a Value at Risk methodology. Market risk can also be contrasted with Specific risk, which measures the risk of a decrease in ones investment due to a change in a specific industry or sector, as opposed to a market-wide move.

Razor provides support for the modelling of all market risk factors for both Historical VaR, and Monte Carlo VaR. Historical VaR for a particular time horizon is derived from a distribution of portfolio values generated from scenarios created by applying historical changes to the current market rates. Monte Carlo VaR for a particular time horizon is derived from a distribution of portfolio values generated from scenarios created by modelling the evolution of current market rates using the stochastic processes driving the market risk factors as implied by the historical data.

Further market risk analytic measures to understand fully the dynamics of market risk include Partial Risk Analysis, Market Risk Factor Sensitivity Analysis, Scenario Analysis, Stress Testing, and Back Testing. These are all supported by RAZOR.

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Chapter 2

Monte Carlo Simulation

2.1 Introduction

RAZOR employs a Monte Carlo simulation approach to predict credit exposure distributions at predefined future dates (credit node). Exposure is determined by sampling results from the exposure distributions based upon the required confidence interval.

The essence of the Monte Carlo process is the stochastic simulation of market rates. To produce the future market scenarios, RAZOR uses the form of parametric simulations that assumes the normality or log normality of future distributions of the market rates. The future values of the market rates representing financial markets snapshots are simulated according to a specified stochastic model.

Currently lognormal mean reverting, normal mean reverting and lognormal are supported.

The stochastic models may be specifically associated with each individual rate class, or sub class of rate classes, or simply defaulted globally to the one model which is applied to all rate classes. In other words the user may override the default global simulation model with a specific simulation model for individual asset types or rate classes.

Currently all market rates which must be simulated, must not change state moving from one day to the next. In other words the term to maturity must be relative and not an explicit date such as would be the case for specific security or futures for example. The historical time series of data used to generate the stochastic parameters for a specific rate must also be constant in state throughout the time series.

The statistical parameters of these models (e.g. volatilities, level and speed of mean reversions, correlations, etc.) are either specified explicitly or derived from historical time series.

The statistical parameters of simulation are pre-calculated using these past prices of some historical sampling period. To produce realistic future scenarios outcomes these statistical parameters can be reviewed and adjusted for all or selected set of market rates.

The same price history can be also used for generation of meaningful correlation matrices that reflects interdependency among market rates and should be used to achieve the co-integration of simulated scenarios. It is assumed that the correlations of market rates historical price changes are stable.

Thus, the length of the historical sampling periods is important for both the generation of parametric coefficients of stochastic simulations and for building of non-degenerated correlation matrices. Ideally the number of price records of the historical time series should be at least the same as the number of simulated market rates. However, even this condition can not guarantee positive semi-definite feature of correlation matrix when some of the price series may be very close to linear dependency. In these situations some fast methods of matrix decomposition, as, for example, Cholesky decomposition, fail to perform. RAZOR uses the standard iterative QL algorithm for derivation of the principal components from the correlation matrix.

2.2 Specification of the Monte Carlo Simulation Process

Results from the Monte Carlo simulation procedure critically depend on the models used to describe the relevant market price process. Currently there are three models implemented in RAZOR.

Lognormal Model

This model assumes that the evolution of market rates is governed by:

$$d\ln(R^t) = (a - \frac{\sigma^2}{2})dt + \sigma dW_t$$

where dW_t is the Wiener process.

The model results in a lognormal distribution of the underlying rates.

Normal Mean-Reverting model

Market rates are simulated according to the following formula:

$$dR^t = (\theta - aR^t)dt + \sigma dW_t$$

The use of this model will produce normally distributed samples of possible simulated market rates outcomes.

Lognormal Mean-Reverting model

The process of market rate simulations is described by:

$$d(\ln R^t) = (\theta - a\ln(R^t))dt + \sigma dW_t$$

The distribution of the resulting simulated market rates will be lognormal.

The last two models have the attraction that they capture the stylised fact that some market rates tend to revert to their mean. The use of these models makes consistent the dispersion of the simulated values with one's expectation of "likely" values over a given time horizon: in particular rates should not be allowed to become negative, or to assume "implausibly" large values.

2.2.1 Historical Statistical Simulation Parameters

The Monte Carlo Simulations of market rates according to the selected model of simulation requires the generation of the appropriate stochastic differential equation parameters.

In RAZOR the statistical parameters are calculated from the historical observations of corresponding market rates.

2.2.2 Lognormal model

There are two parameters to be estimated: lognormal drift a and volatility σ.

In the calculation of these values it is assumed the historical observation period is continuous, that is there are no missing observations for the period, with there being 252 observations per year.

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Let's assume that we have one year's history of observations of market rate R.

$$R = r_{-251}, r_{-250}, r_{-249}, \ldots, r_{-1}, r_0$$

Define:

$$\Delta_1 = \ln r_{-250} - \ln r_{-251}$$
$$\Delta_2 = \ln r_{-249} - \ln r_{-250}$$
.
.
.
$$\Delta_{251} = \ln r_0 - \ln r_{-1}$$

mean:

$$\overline{\Delta} = \frac{1}{N-1}\sum_{i=1}^{N-1}\Delta_i$$

variance:

$$\overline{\sigma}^2 = \frac{1}{N-2}\sum_{i=1}^{N-1}(\Delta_i - \overline{\Delta})^2$$

Because we consider daily perturbations, on business days only, introduce:

$\delta = \dfrac{1}{M}$ Where M = 252

The statistical parameters can be derived from the given data as:

$\sigma = \sqrt{\overline{\sigma}^2}\sqrt{M} = \dfrac{\sqrt{\overline{\sigma}^2}}{\sqrt{\delta}}$ - annualised standard deviation

$a = \dfrac{\overline{\Delta}}{\delta} + \dfrac{\sigma^2}{2}$ - drift

2.2.3 Normal Mean-Reverting model

Parameters to be estimated for this model are the drift parameter θ, speed of mean reversion a and volatility σ.

As for the Lognormal model it is assumed the historical observation period is continuous, with 252 observations per year.

Historically for these R:

$$R = r_{-251}, r_{-250}, r_{-249,} r_{-248}, \ldots, r_{-1}, r_0$$

Calculate:

$$\Delta_1 = r_{-250} - r_{-251}$$
$$\Delta_2 = r_{-249} - r_{-250}$$
.
.
.
$$\Delta_{251} = r_0 - r_{-1}$$

mean:

$$\overline{\Delta} = \frac{1}{N-1}\sum_{i=1}^{N-1}\Delta_i$$

variance:

$$\overline{\sigma}^2 = \frac{1}{N-2}\sum_{i=1}^{N-1}(\Delta_i - \overline{\Delta})^2$$

Let's consider:

$$\tilde{r}_1 = r_{-251}, \tilde{r}_2 = r_{-250}, \ldots, \tilde{r}_{251} = r_{-1}, \tilde{r}_{252} = r_0$$

Parameters of the model can be estimated using the Method of Maximum Likelihood which maximises the maximum likelihood estimator as a function of the probability that the selected set of fitted parameters is "most likely" to be correct.

The likelihood function in this case is:

$$L(\theta, a, \sigma, \Delta_1 \ldots \Delta_{251}, r_1 \ldots r_{251}) = f(\Delta_1, r_1) f(\Delta_2, r_2) \ldots f(\Delta_{251}, r_{251})$$

where

$$f(\Delta_i, r_i) = \frac{1}{\sqrt{2\pi\delta\sigma}} e^{-\frac{(\Delta_i - (\theta - ar_i)\delta)^2}{2\sigma^2\delta}}$$

The maximum of this likelihood estimator coincides with the maximum of its logarithm, so after maximising function

$$F = \ln L = \sum_{i=1}^{251} \ln \frac{1}{\sqrt{2\pi\delta\sigma}} - \sum_{i=1}^{251} \frac{(\Delta_i - (\theta - ar_i)\delta)^2}{2\sigma^2\delta}$$

the desired parameters can be calculated using expressions:

$$\theta = \frac{\sum_{i=1}^{N-1}\Delta_i \sum_{i=1}^{N-1}\tilde{r}_i^2 - \sum_{i=1}^{N-1}\Delta_i\tilde{r}_i \sum_{i=1}^{N-1}\tilde{r}_i}{\delta\left((N-1)\sum_{i=1}^{N-1}\tilde{r}_i^2 - \left(\sum_{i=1}^{N-1}\tilde{r}_i\right)^2\right)}$$

$$a = \frac{\sum_{i=1}^{N-1}\Delta_i \sum_{i=1}^{N-1}\tilde{r}_i - (N-1)\sum_{i=1}^{N-1}\Delta_i\tilde{r}_i}{\delta\left((N-1)\sum_{i=1}^{N-1}\tilde{r}_i^2 - \left(\sum_{i=1}^{N-1}\tilde{r}_i\right)^2\right)}$$

and

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$$\sigma = \sqrt{\frac{1}{(N-1)\delta}\sum_{i=1}^{N-1}(\Delta_i - (\theta - a\tilde{r}_i)\delta)^2}$$

Proof:

For better readability, we replace all $\tilde{r}_i$ by r_i.

The log-likelihood function is given by:

$$F = \ln L = \sum_{i=1}^{251} \ln \frac{1}{\sqrt{2\pi\delta\sigma}} - \sum_{i=1}^{251} \frac{(\Delta_i - (\theta - ar_i)\delta)^2}{2\sigma^2\delta}.$$

To find the maximum likelihood estimators:

$$\frac{\partial F}{\partial \theta} = \sum \frac{(\Delta_i - (\theta - ar_i)\delta)}{\sigma^2}$$
$$\frac{\partial F}{\partial a} = \sum \frac{(\Delta_i - (\theta - ar_i)\delta)r_i}{\sigma^2}$$

Set $\frac{\partial F}{\partial \theta}$ and $\frac{\partial F}{\partial a}$ to 0, we obtain:

$$\frac{\partial F}{\partial \theta} = \sum \frac{(\Delta_i - (\theta - ar_i)\delta)}{\sigma^2} = 0 \quad \ldots \quad (1)$$
$$\frac{\partial F}{\partial a} = \sum \frac{(\Delta_i - (\theta - ar_i)\delta)r_i}{\sigma^2} = 0 \quad \ldots \quad (2)$$

To estimate θ, from (1), we obtain:

$$a = \frac{(N-1)\theta\delta - \sum \Delta_i}{\delta \sum r_i}$$ sub in (2),

$$\theta = \frac{\sum \Delta_i \sum r_i^2 - \sum \Delta_i r_i \sum r_i}{\delta\left((N-1)\sum r_i^2 - \left(\sum r_i\right)^2\right)}.$$

To estimate a, from (1), we obtain:

$$\theta = \frac{\sum \Delta_i + a\delta \sum r_i}{(N-1)\delta}$$ sub in (2),

$$a = \frac{\sum \Delta_i \sum r_i - (N-1)\sum \Delta_i r_i}{\delta\left((N-1)\sum r_i^2 - \left(\sum r_i\right)^2\right)}.$$

Volatility can be obtained similarly by differentiating with respect to volatility.

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2.2.4 Lognormal Mean-Reverting model

The set of parameters for the lognormal mean reverting model of simulation can be obtained using the same procedure and expressions as for normal mean reverting model with the exception that the input vector of historical market rate observations:

$$R = r_{-251}, r_{-250}, r_{-249}, \ldots, r_{-1}, r_0$$

should be replaced by logs of the corresponding values:

$$R = \ln r_{-251}, \ln r_{-250}, \ln r_{-249}, \ldots, \ln r_{-1}, \ln r_0$$

2.3 Random Number Generation

The Monte Carlo method relies on a random draw from a standardised (i.e. $N(0,1)$ normal distribution based on the generation of uniformly distributed pseudo random numbers.

RAZOR uses the Box-Muller algorithm for generating random deviates with a Gaussian distribution and *ran2* procedure of long period random number generation for creating a uniform random deviate.

2.4 Correlation Matrix and Eigenvalues Analysis

In RAZOR the future values of the market rates are simulated according to the chosen stochastic model with statistical parameters derived from historical time series.

The simulation procedure should take into account the historical interdependency amongst selected market rates. This is achieved using the correlation information of historical shifts around assumed parametric drifts across the whole set of rates.

To build the correlation matrix, define new time series of normal perturbations:

2.4.1 Lognormal Model

$$Z_1 = \frac{\Delta_1 - (a - \frac{\sigma^2}{2})\delta}{\sigma\sqrt{\delta}},$$

$$Z_2 = \frac{\Delta_2 - (a - \frac{\sigma^2}{2})\delta}{\sigma\sqrt{\delta}},$$

$$\vdots$$

$$Z_{251} = \frac{\Delta_{251} - (a - \frac{\sigma^2}{2})\delta}{\sigma\sqrt{\delta}}$$

2.4.2 Mean-Reverting models

$$Z_1 = \frac{\Delta_1 - a(\frac{\theta}{a} - \tilde{r}_1)\delta}{\sigma\sqrt{\delta}},$$
$$Z_2 = \frac{\Delta_2 - a(\frac{\theta}{a} - \tilde{r}_2)\delta}{\sigma\sqrt{\delta}},$$
$$\vdots$$
$$Z_{251} = \frac{\Delta_{251} - a(\frac{\theta}{a} - \tilde{r}_{251})\delta}{\sigma\sqrt{\delta}}$$

2.4.3 Correlation Matrix

The correlation matrix $V = \{\upsilon_{k,l}\}$ is defined for K market rates, where $k = 1,\ldots,K$, and $l = 1,\ldots,K$, and where for each market rate the time series is represented by $\{Z_i\}, i = 1,\ldots,251$.

The correlation co-efficient $\upsilon_{k,l}$ between two market rates k and l for the time series $Z^k = \{Z_i^k\}$ and $Z^l = \{Z_i^l\}$ is defined by

$$\upsilon_{k,l} = \frac{\operatorname{cov}\left(Z^k, Z^l\right)}{\sigma_{Z^k}\sigma_{Z^l}},$$ where

$\operatorname{cov}\left(Z^k, Z^l\right)$ is the covariance defined by

$$\operatorname{cov}\left(Z^k, Z^l\right) = \frac{1}{N-2}\sum_{i=1}^{N-1}\left(Z_i^k - \overline{Z^k}\right)\left(Z_i^l - \overline{Z^l}\right),$$ and where

σ_{Z^k} is the standard deviation defined by

$$\sigma_{Z^k} = \sqrt{\frac{1}{N-2}\sum_{i=1}^{N-1}\left(Z_i^k - \overline{Z^k}\right)^2}$$, and where $\overline{Z^k}$ is the mean of Z^k for k'th market rate.

In Razor the correlation matrix is calculated directly from

$$\upsilon_{k,l} = \frac{\operatorname{cov}\left(Z^k, Z^l\right)}{\sigma_{Z^k}\sigma_{Z^l}},$$ ie

$$\upsilon_{k,l} = \frac{\sum_{i=1}^{N-1}\left(Z_i^k - \overline{Z^k}\right)\left(Z_i^l - \overline{Z^l}\right)}{\sqrt{\sum_{i=1}^{N-1}\left(Z_i^k - \overline{Z^k}\right)^2}\sqrt{\sum_{i=1}^{N-1}\left(Z_i^l - \overline{Z^l}\right)^2}}$$

Note that since by definition as the sample size increase, the sample series $\{Z_i\}$ will approach a N(0,1) distribution, with mean $\overline{Z}$ of 0, and standard

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deviation σ_Z of 1, and so with increasing sample size the correlation will approach

$$\upsilon_{k,l} \rightarrow \frac{1}{N-2}\sum_{i=1}^{N-1} Z_i^k Z_i^l$$

This correlation matrix

$$V = \{\upsilon_{k,l}\}$$

should be used for the transformation of the generated random vector of independent normal components to the random vector of correlated normal components.

2.4.4 Eigenvalues and Eigenvectors

Let $G = [g_1, \dots, g_k]$ be a vector of independent Gaussian deviates. In order to generate the vector of correlated Gaussian deviates with the given correlation features, this vector G should be multiplied by the square root of the given correlation matrix V.

There are various methods of finding a square root of a matrix, although not all of them are equally robust. The numerical difficulty is that the correlation matrix will be of a significant dimension and very possibly not well defined and singular. Some fast methods like Cholesky decomposition are not always able to perform these calculations, as a precondition is that the input correlation matrix is positive semi-definite.

To find the square root of the correlation matrix V Razor uses principal value decomposition in the eigenvectors space with the consequent representation of it as:

$$V = B\Lambda B^T$$

where B is the matrix of eigenvectors:

$$B = \begin{pmatrix} \beta_{1,1} & \beta_{2,1} & \cdots & \beta_{K,1} \\ \beta_{1,2} & \beta_{2,2} & \cdots & \beta_{K,2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,K} & \beta_{2,K} & \cdots & \beta_{K,K} \end{pmatrix}$$

(each column is an eigenvector) and $\sqrt{\Lambda}$ is the diagonal matrix where $\sqrt{\lambda_i}$ are the elements of the main diagonal, and $\lambda = [\lambda_1, \dots, \lambda_k]$ is the vector of eigenvalues.

Householder transformation and QL algorithm are used to perform this decomposition. This approach has proven to be practically more robust.

Ref:

“Numerical Recipes in C”, Second Edition.

Householder transformation , p470-p475.

QL algorithm, p478-p481.

We know define:

$$\beta_j = \left[\beta_{j,1}, \beta_{j,2}, \ldots, \beta_{j,K}\right]^T$$

= the eigenvector corresponding to the j^{th} eigenvalue λ_j .

It can be shown $V = B\Lambda B^T$ and another way of writing V is:

$$V = \sum_{j=1}^{K} \lambda_j \boldsymbol{\beta}_j \boldsymbol{\beta}_j^T .$$

2.4.5 Principal Component Analysis

For simulation purposes, RAZOR uses only dominant components of the obtained eigenvalues, i.e. we approximate $V = \sum_{j=1}^{K} \lambda_j \boldsymbol{\beta}_j \boldsymbol{\beta}_j^T$ as:

$V \approx \sum_{j=K-n}^{K} \lambda_j \boldsymbol{\beta}_j \boldsymbol{\beta}_j^T$ where $\lambda = [\lambda_1, \ldots, \lambda_K]$ are sorted in ascending order here. To determine the 'dominant' eigenvalues (i.e. to find n), we calculate the ratio of convergence or level of representation of the square root matrix calculated using selected components to the square root matrix derived from all components. The ratio Rd_n is defined as:

$Rd_n = \dfrac{\sum_{k=K-n}^{K} \lambda_k}{\sum_{k=1}^{K} \lambda_k}$, for $n = 0, \ldots, K-1$, and note that $\lambda = [\lambda_1, \ldots, \lambda_K]$ are sorted in ascending order.

It appears that for the correlation noise matrices prepared on two years observation history of approximately 1000 market rates, only a few principal components are required (very often less than 20) in order to represent 95% (this percentage is configurable) of the correlation matrix transformational behaviour. In other words, there will be a significant reduction in the dimensionality of the problem of the correlation noise building if only principal components are chosen to represent the required level of correlation significance.

Example:

If we have three asset classes and we obtained the covariance matrix:

$$V = \begin{bmatrix} 1 & 0.5 & 0.9 \\ 0.5 & 1 & 0.3 \\ 0.9 & 0.3 & 1 \end{bmatrix}.$$

After decomposition,

$$V$$

$$= B\Lambda B^T$$

$$= \begin{bmatrix} 0.73 & 0.17 & 0.66 \\ -0.18 & -0.88 & 0.44 \\ -0.65 & 0.44 & 0.61 \end{bmatrix} \begin{bmatrix} 0.07 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 2.17 \end{bmatrix} \begin{bmatrix} 0.73 & -0.18 & -0.65 \\ 0.17 & -0.88 & 0.44 \\ 0.66 & 0.44 & 0.61 \end{bmatrix}.$$

Recall $Rd_n = \dfrac{\sum_{k=K-n}^{K} \lambda_k}{\sum_{k=1}^{K} \lambda_k}$.

If we set the confidence interval to be 95%, then we can find that $n = 2$ where

$$Rd_2$$

$$= \frac{2.17 + 0.75}{2.17 + 0.75 + 0.07}$$

$$= 97.66\%.$$

Then

$$V \approx \sum_{j=K-n}^{K} \lambda_j \boldsymbol{\beta}_j \boldsymbol{\beta}_j^T$$

$$= 2.17 \cdot \begin{bmatrix} 0.66 \\ 0.44 \\ 0.61 \end{bmatrix} \begin{bmatrix} 0.66 & 0.44 & 0.61 \end{bmatrix} + 0.75 \cdot \begin{bmatrix} 0.17 \\ -0.88 \\ 0.44 \end{bmatrix} \begin{bmatrix} 0.17 & -0.88 & 0.44 \end{bmatrix}$$

$= \begin{bmatrix} 0.97 & 0.52 & 0.93 \\ 0.52 & 1.00 & 0.29 \\ 0.93 & 0.29 & 0.95 \end{bmatrix}$ which is a very good approximation for V.

It is computationally important to include only the dominant eigenvalues. For example in daily simulations, if we have 1000 asset classes but only 20 dominant eigenvalues, then instead of looping through all 1000 eigenvalues and eigenvectors, we only need to loop through 20 eigenvalues and eigenvectors.

It is also possible to filter the eigenvalues using a fixed threshold instead of a confidence interval.

2.4.6 Daily Simulations

The formulae of discrete daily simulations could be presented as:

For the lognormal simulation model:

$$R_i^1 = R_i^0 \exp(a_i\delta - \tfrac{\sigma_i^2}{2}\delta + \sigma_i\sqrt{\delta}(\beta_{1,i}\sqrt{\lambda_1}\varpi_1^1 + \beta_{2,i}\sqrt{\lambda_2}\varpi_2^1 + \ldots + \beta_{19,i}\sqrt{\lambda_{19}}\varpi_{19}^1))$$

$$R_{i+1}^1 = R_{i+1}^0 \exp(a_{i+1}\delta - \tfrac{\sigma_{i+1}^2}{2}\delta + \sigma_{i+1}\sqrt{\delta}(\beta_{1,i+1}\sqrt{\lambda_1}\varpi_1^1 + \beta_{2,i+1}\sqrt{\lambda_2}\varpi_2^1 + \ldots + \beta_{19,i+1}\sqrt{\lambda_{19}}\varpi_{19}^1))$$

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and for time t and k market rate:

$$R_k^t = R_k^{t-1}\exp(a_k\delta - \tfrac{\sigma_k^2}{2}\delta + \sigma_k\sqrt{\delta}(\beta_{1,k}\sqrt{\lambda_1}\varpi_1^t + \beta_{2,k}\sqrt{\lambda_2}\varpi_2^t + \ldots + \beta_{19,k}\sqrt{\lambda_{19}}\varpi_{19}^t))$$

For mean reverting models:

$$R_j^1 = R_j^0 + (\theta_j - a_j R_j^0)\delta + \sigma_j\sqrt{\delta}(\beta_{1,j}\sqrt{\lambda_1}\varpi_1^1 + \beta_{2,j}\sqrt{\lambda_2}\varpi_2^1 + \ldots + \beta_{19,j}\sqrt{\lambda_{19}}\varpi_{19}^1)$$

$$R_{j+1}^1 = R_{j+1}^0(\theta_{j+1} - a_{j+1}R_{j+1}^0)\delta + \sigma_{j+1}\sqrt{\delta}(\beta_{1,j+1}\sqrt{\lambda_1}\varpi_1^1 + \beta_{2,j+1}\sqrt{\lambda_2}\varpi_2^1 + \ldots + \beta_{19,j+1}\sqrt{\lambda_{19}}\varpi_{19}^1)$$

and for time t and k market rate:

$$R_k^t = R_k^{t-1} + (\theta_k - a_k R_k^{t-1})\delta + \sigma_k\sqrt{\delta}(\beta_{1,k}\sqrt{\lambda_1}\varpi_1^t + \beta_{2,k}\sqrt{\lambda_2}\varpi_2^t + \ldots + \beta_{19,k}\sqrt{\lambda_{19}}\varpi_{19}^t)$$

2.5 Generation of the Monte Carlo Simulation

The Monte Carlo simulation can be pre-generated and stored for use in later risk analysis jobs. This provides additional flexibility is scheduling operational procedures, and allows multiple jobs to utilise the same simulation reducing required processing resources.

The Razor simulation produces a daily simulated value for each rate defined within the market. When the future simulated tenor corresponds to a credit node the ratio of the simulated rate to the most recent observed market rate is stored to a memory mapped file. For the purposes of simulation business days are included, assuming 365 days per year. For example a node at 1 week would require daily rates be generated for 7 days with the 7th rate begin recorded for that node. This process is repeated so that the ratio file contains a ratio for each market rate, path, and credit node.

Within the simulation the ratios for the required path, credit node, and market rates are multiplied with the current base market values to obtain the perturbed market rate values. Whilst the number of paths and credit nodes are defined within the system configuration, the market rates required are determined by performing a rate dependency analysis for the trades to be priced for that scenario.

The ratio file can be overridden by defining a market perturbation process within a user definable scenario adapter, to perturb the market during the risk processing.

2.6 Equity Simulation

Note:
Simulation method for base equities coincides with the method to simulate log-normal rates in Razor Financial Principle section 2.2.2.

2.6.1 Introduction

This section discusses on the generation of future equity prices using Monte-Carlo simulation. Equities are categorised into two fundamental types: base equity and secondary equity. Base equities are broad-market

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indices like S&P 500 and secondary equities are those equities that are not broad enough to represent any particular market but whose movement is correlated closely with a particular base equity.

2.6.2 Model Specification

Base Equities

Base equities are broad-market indices. Common base equities are like S&P500, DJIA, etc. Base equities represent a particular section of the market thus it is linked to the whole market movement closely. If each of the base equities moves closely with the overall market, it is apparent that all base equities should be correlated to each other. We are now to construct the base equity model mathematically.

Define:

$S_i(t)$ = price of the i-th base equity at time t.

μ_i = drift coefficient of the i-th base equity.

σ_i = volatility of the i-th base equity.

β = Principal decomposition of the covariance matrix.

β_{ij} = element of matrix β.

$W_j(t)$ = Brownian motion for base equity j at time t.

N = the set of base indices $\{1,2,\ldots,n\}$.

The stochastic differential equation (SDE) representing the base equity is:

$$dS_i(t) = \mu_i S_i dt + S_i \sum_{j=1}^{n} \beta_{ij} dW_j(t).$$

Note that if there is only one base index, then $\beta_{11} = \sigma_1$.

Apply Ito's Lemma to solve the SDE and obtain:

$$S_i(t) = S_i(0)\exp\left[\left(\mu_i - \frac{1}{2}\sigma_i^2\right)dt + \sum_{j=1}^{n} \beta_{ij} dW_j\right].$$

Secondary Equities

Secondary equities are equities that are not broad enough to represent any particular market but whose movement is linked closely with a particular base equity. The secondary equities have two major sources of risks, i.e. the risk associated with the overall market and the risk associated with the linked base equity. We are now to construct the secondary equity model mathematically.

Define:

$S_k^i(t)$ = price of secondary equity k linked to base equity i at time t.

μ_k^i = drift coefficient of the secondary equity k linked to base equity i.

β_k^i = coefficient relating secondary equity k to base equity i.
β_k = coefficient specific to secondary index k.
$W_k^i(t)$ = Brownian motion for secondary equity k linked base equity i at time t.

The stochastic differential equation (SDE) for secondary equity k linked to base equity i is:

$$\frac{dS_k^i(t)}{S_k^i(t)} = \mu_k^i dt + \beta_k^i\left[\frac{dS_i(t)}{S_i(t)} - \mu_i dt\right] + \beta_k dW_k^i(t)$$
$$= \mu_k^i dt + \beta_k^i\left[\sum_{j=1}^{n}\beta_{ij} dW_j(t)\right] + \beta_k dW_k^i(t)$$

Solve the SDE to obtain:

$$S_k^i(t) = S_k^i(0)\left[\frac{S_i(t)}{S_i(0)}\right]^{\beta_k^i} \exp\left[\left(\mu_k^i - \frac{1}{2}\sigma_k^{i2}\right)dt - \beta_k^i\left(\mu_i - \frac{1}{2}\sigma_i^2\right)dt + \beta_k dW_k^i(t)\right]$$

where
$\sigma_k^{i2} = \beta_k^{i2}\sigma_i^2 + \beta_k^2$.

2.6.3 Simulation Algorithm

Base Equity

Step 1: To obtain μ_i and σ_i
Obtain historical prices say 252 days for base index i and calculate the historical daily log returns:

$$\Delta_i(1) = \log S_i(-250) - \log S_i(-251)$$
$$\Delta_i(2) = \log S_i(-249) - \log S_i(-250)$$
$$\vdots$$
$$\Delta_i(251) = \log S_i(0) - \log S_i(-1)$$

Then we calculate

$$\Delta_i = \frac{M}{N-1}\sum_{j=1}^{N}\Delta_i(j)$$
$$\sigma_i^2 = \frac{M}{N-1}\sum_{j=1}^{N}\left(\Delta_i(j) - \overline{\Delta}_i\right)^2$$

where N is the number of log returns (251 number of returns in this case) and M is the day convention per year (e.g. 365 if convention is 365 days per year).

$$\mu_i = \Delta_i + \frac{\sigma_i^2}{2}$$
$$\sigma_i = \sqrt{\sigma_i^2}$$

Step 2: To obtain μ_i and σ_i for all base indices
Follow step 1 to calculate μ_i and σ_i for each of the base index $i = 1, 2, ..., n$.

Step 3: To obtain covariance matrix for base indices.
Use the historical prices for each of the index i to generate the covariance matrix V for all base indices.

Step 4: To obtain β_{ij}
β_{ij} is defined as the element of matrix β and β is defined as the decomposition of covariance matrix V, i.e. $V = \beta^T \beta$. We can obtain β from V using principal decomposition algorithm.

Step 5: To generate random vectors from standard normal distribution.
For each base equity i, we need to generate $\phi_{t_l}^i$ from N(0,1) for each of the future simulating time point t_l. Thus if we want to simulate stock path at the future time point $t_1, t_2, ..., t_n$, we need to generate a random vector $\phi^i = \left(\phi_{t_1}^i, \phi_{t_2}^i, ..., \phi_{t_n}^i\right)$. We do the same for each of the base equity $i = 1, 2, ..., n$.

Step 6: To simulate the base equity price path
We can simulate the future base equity price path using the formula:

$$S_i(t_{m+1}) = S_i(t_m)\exp\left[\left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t + \sum_{j=1}^{n} \beta_{ij}\phi_{t_m}^j \sqrt{\Delta t}\right].$$

Secondary Equity
Step 1: To obtain μ_k^i, β_k^i and β_k
We obtain μ_k^i, β_k^i and β_k using linear regression. Refer to the next section for the details of using linear regression to obtain the secondary equity parameters.

Step 2: To generate random vector from standard normal distribution
For secondary equity i, we need to generate $\zeta_{t_l}^i$ from N(0,1) for each of the future simulating time point t_l. Thus if we want to simulate stock path at the future time point $t_1, t_2, ..., t_n$, we need to generate a random vector $\zeta^i = \left(\zeta_{t_1}^i, \zeta_{t_2}^i, ..., \zeta_{t_n}^i\right)$.

Step 3: To simulate the secondary equity price path

We can simulate the future secondary equity price path using the formula:

$$S_k^i(t) = S_k^i(t_m)\left[\frac{S_i(t_{m+1})}{S_i(t_m)}\right]^{\beta_k^i} \times$$
$$\exp\left[\left(\mu_k^i - \frac{1}{2}\beta_k^{i2}\sigma_k^{i2} - \frac{1}{2}\beta_k^2\right)dt - \beta_k^i\left(\mu_i - \frac{1}{2}\sigma_i^2\right)dt + \beta_k \zeta_{t_m}^{ik}\sqrt{\Delta t}\right]$$

Note that $\frac{S_i(t_{m+1})}{S_i(t_m)}$ can be obtained from the already simulated base equity prices.

2.6.4 Parameter Estimation for Secondary Equities

Model Explanation

The equation governing the price of the secondary equity index is:

$$\frac{\Delta S_k^i(t_m)}{S_k^i(t_m)} = \mu_k^i \Delta t + \beta_k^i\left[\sum_{j=1}^{i}\beta_{ij}\phi_{t_m}^j\sqrt{\Delta t}\right] + \beta_k \gamma_{t_m}^k\sqrt{\Delta t}\ .$$

The left-hand-side component
$S_k^i(t_m)$ is the price of the k-th secondary equity index linked to the i-th base equity at time t_m. Historical prices of the secondary equity index are used in linear regression, thus $S_k^i(t_m)$ is known and observable in linear regression.

$\Delta S_k^i(t_m)$ is defined as $S_k^i(t_{m+1}) - S_k^i(t_m)$. It is the change of secondary equity index price in one day (if we use daily data). Since all the historical prices of the secondary equity index are observable, the changes in the prices of the secondary equity index are also observable.

Thus the left-hand-side of the equation is completely observable and known. The left-hand-side $\frac{\Delta S_k^i(t_m)}{S_k^i(t_m)}$ is actually the historical daily return of the secondary index. We denote the left-hand-side constant as Y.

For example, if we have historical secondary index prices for six days, then we will have five historical daily returns and we will have five values for Y.

The $\mu_k^i \Delta t$ component

Δt is the change in time. If the historical data used in linear regression are daily data, then $\Delta t = \frac{1}{365}$ (if the day convention is 365 days per year). Since we know the frequency of the historical data, thus Δt is known too.

μ_k^i is the deterministic drift for secondary index k and it is an unknown constant we want to find using linear regression.

Thus only μ_k^i is unknown in the $\mu_k^i \Delta t$ component.

The $\mu_k^i \Delta t$ consists of two known constants and one unknown constant, thus $\mu_k^i \Delta t$ is still a constant. We denote $\mu_k^i \Delta t$ as α_0.

The $\beta_k^i \left[\sum_{j=1}^{i} \beta_{ij} \phi_{t_m}^j \sqrt{\Delta t} \right]$ component

β_k^i is the correlation coefficient relating secondary index k to diffusions in the i-th base index. It is an unknown constant we want to estimate using linear regression and we denote it α_1.

β_{ij} is the principal decomposition of the covariance matrix and it was determined when estimating the base index. Thus at the time when we want to estimate the secondary equity index, β_{ij} is a known value already.

Δt is a known value as we mentioned before.

$\phi_{t_m}^j$ is a standard normal variable $N(0,1)$. However, it has already been generated when we estimated the base index. Thus $\phi_{t_m}^j$ is already known when we try to estimate the secondary equity index.

The term $\left[\sum_{j=1}^{i} \beta_{ij} \phi_{t_m}^j \sqrt{\Delta t} \right]$ is a constant and observable from the base index.
We denote $\left[\sum_{j=1}^{i} \beta_{ij} \phi_{t_m}^j \sqrt{\Delta t} \right]$ as X_1. Note that X_1 is a constant because it is observable from the process when we estimated the base index. When doing linear regression, if we have five historical returns, i.e. five Y, then we need to get the corresponding five values of X_1 from the base index.
X_1 can be recovered from the historical prices of the base index by the following formula:

$X_1 = \ln\left(\frac{S_i(t_{m+1})}{S_i(t_m)}\right) - \left(\mu_i - \frac{1}{2}\sigma_i^2\right)\Delta t$.

Note the summation $\sum_{j=1}^{i} \beta_{ij}\phi_{t_m}^j \sqrt{\Delta t}$ over j is based on the number of base indexes. If there are 10 base indexes, we need to sum the j from 1 to 10. Then the term becomes $\sum_{j}^{10} \beta_{ij}\phi_{t_m}^j \sqrt{\Delta t}$. However, we do not need to calculate the summation explicitly for regression purposes because they are already included in the X_1 term. We do need to calculate the summation term later for simulation.

The $\beta_k \gamma_{t_m}^k \sqrt{\Delta t}$ component

β_k is a coefficient unique to secondary index k and it is an unknown constant we want to determine using linear regression. We denote it as α_2.

Δt is a known constant as we mentioned previously.

$\gamma_{t_m}^k$ is standard normal variable $N(0,1)$. It is random and different every time we generate it. Thus it is not a constant.

In the term $\gamma_{t_m}^k \sqrt{\Delta t}$, only $\gamma_{t_m}^k$ is not a constant, we denote $\gamma_{t_m}^k \sqrt{\Delta t}$ as X_2. Note that X_2 is not a constant because $\gamma_{t_m}^k$ is not a constant. When doing linear regression, if we have five historical returns, i.e. five Y, then we need to generate five $\gamma_{t_m}^k$ values from the standard normal distribution randomly and we will obtain five X_2.

To summarise, the equation for the secondary equity index can be re-written as:

$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2$.

We can determine α_0, α_1 and α_2 using linear regression.

Note that $\alpha_0 = \mu_k^i \Delta t$ and Δt is a known constant. Thus once we obtain α_0 from regression, we can calculate μ_k^i as $\frac{\alpha_0}{\Delta t}$. Also $\alpha_1 = \beta_k^i$ and $\alpha_2 = \beta_k$, thus if we know a_1 and α_2, we automatically know β_k^i and β_k.

Linear Regression

To carry out linear regression, we write the linear regression equation as:

$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \varepsilon$, where $\varepsilon \sim N(0,1)$.

Our aim is to minimise the sum of the square term ε.

Example of linear regression

We have the equation for the secondary equity index:

$$\frac{\Delta S_k^i(t_m)}{S_k^i(t_m)} = \mu_k^i \Delta t + \beta_k^i \left[\sum_{j=1}^{i} \beta_{ij} \phi_{t_m}^j \sqrt{\Delta t} \right] + \beta_k \gamma_{t_m}^k \sqrt{\Delta t} .$$

We assume in this example that all the known constants are equal to 1 and we have yearly historical secondary index data, thus both β_{ij} and Δt are all equal to 1. We also assume that there is only one base index.

Thus the equation becomes:

$$\frac{\Delta S_k^i(t_m)}{S_k^i(t_m)} = \mu_k^i + \beta_k^i \phi_{t_m}^j + \beta_k \gamma_{t_m}^k .$$

Assume the annual historical prices for secondary equity index in the past six years are:

Year	Price
2001	10
2002	20
2003	30
2004	40
2005	50
2006	60

Then the vector of $Y = \frac{\Delta S_k^i(t_m)}{S_k^i(t_m)}$ becomes:

$$Y = \begin{bmatrix} \frac{20-10}{10} \\ \frac{30-20}{20} \\ \frac{40-30}{30} \\ \frac{50-40}{40} \\ \frac{60-50}{50} \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0.33 \\ 0.25 \\ 0.2 \end{bmatrix} .$$

$\alpha_0 = \mu_k^i$ is constant.

$\alpha_1 = \beta_k^i$ is a constant and $X_1 = \phi_{t_m}^j$. There are five values for Y, thus we need to have five values for X_1. The five values of X_1 should have already been generated in the base index thus they are observable and can be recovered by the formula we mentioned previously. For example, if the base index price for 2001 and 2002 are 30 and 40 respectively and μ_i and σ_i from base index estimation are 0.01 and 0.05, then the first element of X_1 is $\ln\left(\frac{40}{30}\right)-\left(1\cdot 0.01-\frac{1}{2}\cdot 1\cdot 0.05^2\right)\cdot 1 = 0.28$. If the five values of X_1 in the base index are 0.28, 0.16, 0.81, 0.53 and 0.65 then the vector of X_1 becomes:

$$X_1 = \begin{bmatrix} 0.28 \\ 0.16 \\ 0.81 \\ 0.53 \\ 0.65 \end{bmatrix}.$$

$\alpha_2 = \beta_k$ is a constant and $X_2 = \gamma_{t_m}^k$, $\gamma_{t_m}^k \sim N(0,1)$. Again, we need to generate five values for X_2. We generate five values randomly from the standard normal distribution. If the five random numbers we generated are 0.1,0.9,0.35,0.65 and 0.95 then the vector of X_2 becomes:

$$X_2 = \begin{bmatrix} 0.1 \\ 0.9 \\ 0.35 \\ 0.65 \\ 0.95 \end{bmatrix}.$$

The regression equation is thus:

$$\begin{bmatrix} 1 \\ 0.5 \\ 0.33 \\ 0.25 \\ 0.2 \end{bmatrix} = \alpha_0 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \alpha_1 \begin{bmatrix} 0.28 \\ 0.16 \\ 0.81 \\ 0.53 \\ 0.65 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0.1 \\ 0.9 \\ 0.35 \\ 0.65 \\ 0.95 \end{bmatrix} + \varepsilon$$

$$\varepsilon = \begin{bmatrix} 1 \\ 0.5 \\ 0.33 \\ 0.25 \\ 0.2 \end{bmatrix} - \alpha_0 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \alpha_1 \begin{bmatrix} 0.3 \\ 0.1 \\ 0.8 \\ 0.5 \\ 0.6 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0.1 \\ 0.9 \\ 0.35 \\ 0.65 \\ 0.95 \end{bmatrix}$$

To find α_0, α_1 and α_2 that minimise $\varepsilon^2 = \varepsilon^T \varepsilon$, we can use the linear least square algorithm.

Chapter 3

Credit Exposure Calculation

3.1 Pre-Settlement Risk

Unlike settlement risk, which entails exposure equal to a counterparty's gross obligation, pre-settlement risk entails exposure equal to a counterparty's net obligation on that contract. For example, suppose an institution enters into a forward contract to exchange $1M Australian dollars for $0.6M US dollars in three months. Settlement risk exposes the institution to a possible loss of $1M AUD. Pre-settlement risk exposes the institution to just the difference in market value between the US and Australian dollar payments.

3.1.1 Calculation of Credit Exposure

The credit exposure is calculated by valuing each trade according to the current market rates in the simulation. Each trade's exposure is then integrated into the portfolios that that trade impacts.

3.1.2 Treatment of Credit Nodes

Credit nodes are the points in time on which exposures are calculated. On each credit node, the market rates are rolled forward to that particular day, and the various yield curves, exchange rates and volatility surfaces are generated.

3.1.3 Economic Offsetting

Individual trades in a portfolio can be quite risky in isolation, but their addition in a portfolio can serve to reduce the aggregate risk if the correlation of losses between these credits and the rest of the portfolio is less than fully correlated.

3.1.4 Netting Agreements

The netting of cash flows or obligations is a means of reducing the credit exposure to a counterparty:

Payment netting reduces settlement risk and makes the payment processing more efficient: If counterparties are to exchange multiple cash flows during a given day, they can agree to net those cash flows to one payment per currency.

Closeout netting reduces pre-settlement risk: If counterparties have multiple offsetting obligations to one another — for example, multiple interest rate swaps or foreign exchange forward contracts — they can agree to net those obligations. In the event that a counterparty defaults, or some other termination event occurs, the outstanding contracts are all terminated. They are marked to market and settled with a net payment. This technique eliminates "cherry picking" whereby a defaulting counterparty fails to make payment on its obligations, but is legally entitled to collect on the obligations owed to it.

With a bilateral netting agreement, two counterparties agree to net with one another. They sign a master agreement specifying the types of netting to be performed as well as the existing and future contracts, which will be affected.

3.1.5 Collateral Agreements

A counterparty will enter into a collateral agreement in order to enhance their credit quality. A collateral agreement obligates the counterparty to margins in cash with the dealer to cover costs in the event of default.

3.2 Credit Exposure Measures

3.2.1 Introduction

In July 2005, the Basel Committee on Banking Supervision (BCBS), as established by the Bank of International Settlements (BIS) published a major amendment to the Basel 2 framework in its application to a bank's Trading Book. This amendment describes changes relating to Credit Exposure of the Trading Book, treatment of Guarantees, and changes to the regulatory specific risk calculation.

3.2.2 Key Measures

Within the Basel regulatory framework there are various models for the calculation of exposure to Counterparty Credit Risk (CCR). This Exposure at Default (EAD) can be measured using:

1. Current Exposure Method (CEM) – MtM+Addon
2. Standardised Method (SM) –Under SM, the exposure amount represents the product of:
   (i) the larger of the net current market value or a "supervisory EPE", times
   (ii) a scaling factor, termed beta.
3. Internal Model Method (IMM) – Potential Future Exposure (PFE), Expected Exposure (EE), Expected Positive Exposure (EPE) – all derived from the distribution of counterparty exposures as generated from the stochastic simulation process as described elsewhere in the Financial Principles.

In support for the Internal Model the following additional measures are provided in Razor to support the aforementioned Trading Book Amendments:

1. Expected Exposure (EE)
   EE is the probability-weighted average exposure estimated to exist on a future date.
2. Expected Positive Exposure (EPE)
   EPE is the time-weighted average of individual expected exposures estimated for a given forecasting horizon:
   $$EPE = \sum_{k=1}^{\min(1year, maturity)} EE_{t_k} \times \Delta t_k \ .$$
3. Effective Expected Exposure (EEE)

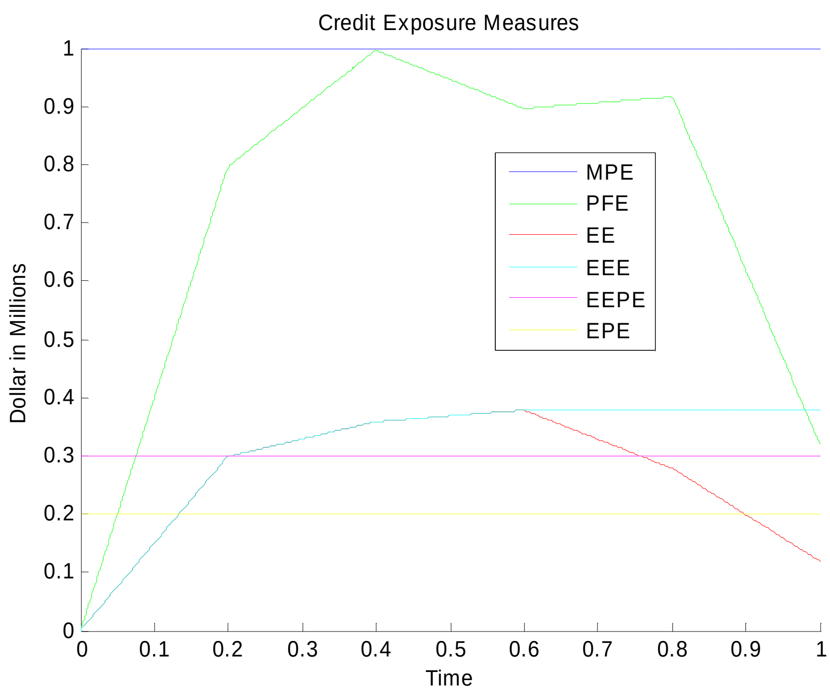
EEE is defined recursively as $EEE_{t_k} = \max\left(EEE_{t_{k-1}}, EE_{t_k}\right)$ where exposure is measured at future dates $t_1, t_2, t_3, \ldots$, and EEE_{t_0} equals current exposure.

4. Effective Expected Positive Exposure (EEPE)
   EEPE is the average EEE during during the first year of future exposure. If all contracts in the netting set mature before one year, EPE is the average of EE until all contracts in he netting set mature. EEPE is computed as a weighted average of EEE:

$$EEPE = \sum_{k=1}^{\min(1year, maturity)} EEE_{t_k} \times \Delta t_k \ .$$

The effective EPE estimate corresponds with the exposure that can be calculated in the current economic climate. However in order to capture a deteriorating economic climate the final number must be scaled by a regulatory defined factor called “Alpha”, this value is currently set at 1.4.

The following diagram illustrates different credit exposure measures:



Note that MPE is the maximum peak exposure and PFE is the maximum exposure estimated to occur on a future date at a high level of statistical confidence.

Reference

Basel Committee on Banking Supervision (July 2005), 'The Application of Basel 2 to Trading Activities and the Treatment of Double Default Effects', Bank for International Settlements.

3.3 Settlement Risk

Settlement Risk is the exposure a bank faces to the loss of the full principal value of a deal where the counterparty defaults in the payment of their part of an exchange.

Settlement Risk is primarily incurred by foreign exchange transactions and cross-currency swaps, where the bank pays away their side of the deal and does not receive the counterparty's matching payment.

In order to recognise the settlement risk inherent in a portfolio of deals, the system must perform the following steps:

Identify the transactions/cashflows within the portfolio that generate settlement risk

Identify any Settlement Netting agreements that cover transactions within the portfolio, condense the netting agreements to a reduced set of net cashflows.

For each exchange, determine the duration of the settlement exposure

Combine the transactions to form a settlement exposure profile

For each date, check the peak exposure from the profile to determine the utilisation

Check the utilisation against limits to determine breaches.

3.3.1 Transaction/Cashflow Identification

Only those transactions that generate settlement risk against a portfolio should be processed.

All cashflows that represent a matched exchange generate settlement risk, eg FX, Cross Currency Swaps and Bond transactions.

Some transactions, e.g. swaps, can generate multiple settlement risks on multiple dates.

An 'exchange' occurs, where both parties make payments to each other, where these payments are intended, but are not guaranteed, to be simultaneous.

3.3.2 Exposure Duration

It is the nature of settlement risk, that exposure is derived from the mismatch between the point at which the bank is irrevocably committed to making a payment, and the point at which the bank can recognise that the counterparty's payment has been received.

Deals/Cashflows pass through the following states as part of the settlement process:

Revocable - The bank's payment instruction for the sold currency either has not been issued or may be unilaterally cancelled without the consent of the bank's counterparty or any other intermediary. The bank faces no current settlement exposure for this trade. During this period the trade represents a pre-settlement exposure.

Irrevocable - The bank's payment instruction for the sold currency can no longer be cancelled unilaterally either because it has been finally processed by the relevant payments system or because some other factor (e.g. internal procedures, correspondent banking arrangements, local payments system rules, laws) makes cancellation dependent upon the consent of the counterparty or another intermediary; the final receipt of the bought currency is not yet due. In this case, the bought amount is clearly at risk.

Uncertain - The bank's payment instruction for the sold currency can no longer be cancelled unilaterally; receipt of the bought currency is due, but the bank does not yet know whether it has received these funds with finality. In normal circumstances, the bank expects to have received the funds on time. However, since it is possible that the bought currency was not received when due (e.g. owing to an error or to a technical or financial failure of the counterparty or some other intermediary), the bought amount might, in fact, still be at risk.

Failed - The bank has established that it did not receive the bought currency from its counterparty. In this case the bought amount is overdue and remains clearly at risk. Failed trades can be converted to a cash exposure and should appear effectively as a Loan in the Pre-Settlement exposure to the Counterparty.

Settled - The bank knows that it has received the bought currency with finality. From a settlement risk perspective the trade is considered settled and the bought amount is no longer at risk.

The key states for settlement risk measurement purposes are Irrevocable and Uncertain.

3.3.3 Settlement Exposure Calculation

Settlement Date Exposure

The system calculates the settlement risk by determining the cashflows that fall due on each forward date.

In the event that the exposures are being calculated on a gross basis, or if the trades are not covered by a netting agreement, the settlement exposure is calculated as the sum of the receipts that fall due on that date.

For example the following trades are being processed and all settle on the same date:

Trade REF	Direction	Currency	AUD
T1	Pay	NZD	85
	Recv	AUD	95
T2	Pay	USD	100
	Recv	NZD	90
T3	Pay	AUD	50
	Recv	USD	45

These trades generate the following exposures:

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Trade REF	Currency	AUD
T1	AUD	95
T2	NZD	90
T3	USD	45

These trades will arise at a gross settlement exposure of 230.

3.3.4 Settlement Netting

Where netting is involved, the system must divide the cashflows into netting pools organised by agreement (multiple netting agreements may be in operation, for example and country settlement limit) and by currency.

Within each pool the payments and receipts are offset together to arise at net payment or receipt for a counterparty/currency combination. The system sums all net receipts to arise at the settlement exposure for the date in question.

For example if the trades encountered in Table 1 above are covered by a netting agreement, the processing will be as follows:

The cashflows are divided into currency pools:

Currency	Cashflows		Position
AUD	Pay 50	Recv 95	Recv 45
USD	Pay 100	Recv 45	Pay 55
NZD	Pay 85	Recv 90	Recv 5

The net settlement date exposure for these trades will be 50 (AUD 45 + NZD 5).

The system supports the processing of settlement exposure with and without settlement netting enabled for different portfolio types.

3.3.5 Spread Risk

The concept of spread risk means that the system is enabled to take into account the Irrevocable and Uncertain periods defined in the section “Exposure Duration” above.

The system supports the processing of settlement exposures with or without spread risk for different portfolio types.

Currently the spread risk is specified by currency, in terms of *Receipt Identification*; the amount of time needed to identify that the receipt of the counterparty’s payment has indeed occurred, and *Cancellation Deadline*; the amount of time before settlement occurs needed to cancel the payment to the counterparty.

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Chapter 4
Credit Risk Measurement

Credit risk arises from the changes in credit quality of a counterparty to a trade. It is made up of default risk and downgrade credit spread risk. Default risk is measuring the risk of failure of a counterparty meeting its financial obligations, and what level of recovery is likely in the event of default. Downgrade risk is the risk that a counterparty's credit rating falls and therefore the value of the positions with that counterparty may also fall.

The traditional approaches to the management of credit risk by setting limits to measures such as counterparty credit exposure are described in Chapter 3. Additional to the measurement of counterparty credit exposure, which measure current and potential future exposure assuming default, it is useful to have more sophisticated measures of credit risk which adequately model credit losses.

A credit risk model calculates credit losses by deriving at each credit or time node a distribution of potential credit losses. These models not only consider the changes in the market risk factors, but also credit rating migration and rates of recovery if default occurs for a counterparty or an issuer.

Factors which more commonly determine portfolio credit risk include,

At a trade level include but are not limited to such factors as:

Current and future potential exposure as defined in 3.1.1, and Recovery rates, which are a measure of the amount which may still be recovered from a defaulting counterparty.

2. At a counterparty level the factors are:

Credit migration probabilities, that is the probability of a counterparty credit rating transitioning to a lower credit rating and the subsequent change in credit exposure to that counterparty,

Default probabilities, that is the probability a counterparty will default, and the subsequent loss incurred by the counterparty failing to meet the obligations associated with the trade.

The default probability may be embedded in the credit migration probabilities by specifying a probability to transition to the default rating. Alternatively default risk can be separated from credit risk by only specifying probabilities to maintain the current rating or transition to default.

Recovery rates, which are a measure of the amount which may still be recovered from the defaulting counterparty,

At the counterparty level the aggregation of exposures will take into consideration offsetting, netting agreements and collateral as described in 3.1.3, 3.1.4, 3.1.5,

3. At a portfolio level such factors as joint credit migration probability distribution, which recognize the level of co-movement between credit events occurring for different counterparties.

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4.1 Modelling Credit Transition Losses

In addition to simulating the market rates changes the credit rating of counterparties are also allowed to vary stochastically, independent of the market simulation.

The Credit Rating Transition process evolves according to the probability distribution specified via a Credit Rating Transition Matrix (CRTM). The CRTM specifies probabilities of transitioning from an initial credit rating to other ratings defined in the rating scheme. This distribution can vary over time by defining multiple matrices for differing tenors.

The Credit Spread Risk can be modelled by letting the pricing model be dependent on the simulated credit rating.

Similarly by analysing distribution of the exposure for default in the events of a transition to the default rating the Default Risk can be determined.

4.1.1 Credit Rating Transition Matrices

The transition matrices may include any type of rating (i.e. provided by external ratings models used by S&P, Moodys, Fitch, or internal ratings models) and should support any level of rating granularity desired by the user. The only constraint is that each credit rating in the matrix must be set up as a separately simulated pricing curve, and for a given matrix the same ratings must be used across all transition tenors.

The credit rating transition probabilities are derived from the analysis of historical data and subsequent derivation into an external and/or internal ratings model.

The migrations are specified as the % chance for an issuer of a given rating to migrate to each of the other potential ratings within the matrix. Hence, this matrix specifies the probabilities p_{ij} of transition from rating state i to rating state j (including default) over a fixed time horizon τ.

Example transition matrix (annual):

	Transitioned Rating								
		AAA	AA	A	BBB	BB	B	CCC	D
Initial Rating	AAA	92.97%	6.43%	0.48%	0.08%	0.04%	0.00%	0.00%	0.00%
	AA	2.60%	91.81%	4.78%	0.60%	0.06%	0.12%	0.03%	0.00%
	A	0.70%	2.27%	91.68%	4.49%	0.56%	0.25%	0.01%	0.04%
	BBB	0.07%	0.20%	5.60%	87.87%	4.83%	1.02%	0.17%	0.24%
	BB	0.00%	0.10%	0.61%	7.79%	81.48%	7.90%	1.11%	1.01%
	B	0.00%	0.10%	0.28%	0.46%	6.95%	82.80%	3.96%	5.45%
	CCC	0.00%	0.00%	0.37%	0.75%	2.43%	12.32%	60.44%	23.69%

Therefore the % chance that a AAA rated issuer will stay AAA in a year's time is 92.97%, while the chance that it will migrate to AA within a year is 6.43%.

This matrix can be simplified by just specifying the probabilities of default from each credit rating which simulates default events but ignores transitioning between other rating values.

Transition Matrix Tenor

During simulation, the tenor between credit nodes will not match the frequency of the rating transition matrix; i.e. if an annual credit transition matrix is used but the credit time nodes are weekly, using the annual matrix would drastically overstate the transitioning.

Any given transition matrix may be specified at a number of different tenors. For example for a US Industrials transition matrix, one matrix may be specified with tenor 1W, another for 1M, another for 6M etc. During simulation of credit transitions, the time between the current time node and the next simulated node will determine the tenor used. When the time between credit nodes falls between two transition matrices, the transition percentages will be interpolated between the two matrices to determine the appropriate transition values.

Linear interpolation is performed on each CRTM element. Hence, to interpolate between CRTM at tenors t_1 and t_2 to tenor t, each element in the interpolated matrix is given by:

$$r = \frac{t - t_1}{t_2 - t_1}(r_2 - r_1) + r_1$$

Where:

r_1 element in $CRTM_1$

t_1 tenor of $CRTM_1$

r_2 corresponding element in $CRTM_2$

t_2 tenor of $CRTM_2$

r corresponding element in interpolated CRTM

t tenor of interpolated CRTM

If only a single CRTM is defined then the migration process is assumed to be time independent and no interpolation is performed.

The Process of Transition Probability Interpolation

The process of interpolation in some more detail, can be described in the context of simulation of credit rating migration for each credit party. Each credit rating migration probability matrix of a specific term represents the probability of migrating from the start credit rating to the end credit rating over that term, so if there are two matrices one for 1 week, and one for 1 month, these represent the probability of migrating over a 1 week and a 1 month period with respect to the current point - irrespective if this current valuation time node is today or at some future credit node point.

In the transition/migration to the current credit node from the previous credit node the term for this period is calculated, and then the probabilities for this calculated term is linearly interpolated from the set of provided term based probabilities.

1. If there is only one provided matrix then the probabilities are constant with respect to term.

2. Similarly if the calculated term required is shorter in duration compared with the first sorted matrix, or longer in duration compared with the last sorted matrix, then again the probabilities are flat with respect to term, and the closest (respectively the first and last) matrix will be used.

3. If the duration of the term being transitioned from one credit node to the next lies between two termed based credit rating migration probability matrices then it is a straight line time based linear interpolation to derive the probabilities to use.

So for example if a 1 month and 3 month credit rating migration probability matrix are provided, and all the credit nodes are 2 months apart then the probabilities used for each migration of 2 months duration in the simulation will be those same linearly interpolated (half way) between the 1 month and 3 month matrices.

As another example if using the same matrices as above and just 3 credit nodes of 1 month apart, then the probabilities for each credit rating migration of 1 month duration in the future will be those from the 1 month credit rating migration probability matrix.

As another example if using the same matrices as above and just 2 credit nodes, at 1 month and 4 months, then the transition to the 1 month credit node will use the 1 month probabilities as supplied, and the transitioning from 1 month node to 4 month credit node will use the 3 month probabilities as supplied.

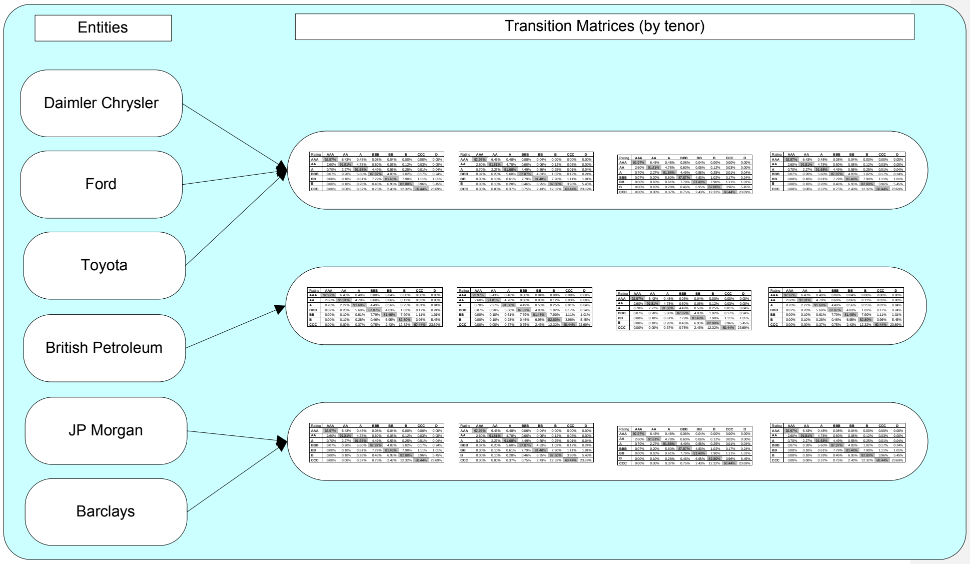
Naturally as transitioning along a path the credit party credit rating is migrating according to the probabilities and the simulation going from one credit node to the next along the path.

Entity – Transition Rating Mapping

Adding a separate mapping to indicate the rating used to drive the migration enables gives greater control over the migration; they don’t necessarily need to use the actual S&P etc rating of a given issuer.

Entity – Transition Matrix Mapping

The ability to map an entity to a particular transition matrix allows the simulation to capture different transition volatilities across industry, country, or other attributes. For example it is possible to use one transition matrix for US software companies, and another for US Utilities. Adding a transition matrix field to the entity definition allows for this mapping and provides complete flexibility to map any issuer to any matrix.



Hence in summary, CRTM are defined by tenor and interpolated to the current time node. Multiple CRTM structures can be defined relating to different segments of the market.

Configuration determines to which CRTM structure a trade is assigned. Typically this would be determined by any combination of trade attributes such as industry, location, and so on.

4.1.2 Simulation of Credit Rating Changes

The CRTM defines the credit transition probabilities. This section describes how they are used within the credit simulation. For each credit party within a simulation scenario a correlated uniform random number is drawn. This random number is then used to index into the CRTM to determine the transitioned credit rating, using the row of the CRTM for the current credit rating of that counterparty. For instance using the previous CRTM example with a current rating of A the probabilities of transition are:

New Rating	Probability	Cumulative Probability
AAA	0.70%	0.70%
AA	2.27%	2.97%
A	91.68%	94.65%
BBB	4.49%	99.14%
BB	0.56%	99.70%
B	0.25%	99.95%
CCC	0.01%	99.96%
D	0.04%	100.00%

Hence a simulation value of 0 - 0.7% (ie the correlated uniform random number) would indicate a new rating of AAA, 0.7% - 2.97% would transition to AA, and so on to 99..96% - 100% for a default event. This perturbed credit rating is passed to the trade pricing routines for all trades with that counterparty in the scenario. Note that as in this example it is typical that the

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most likely outcome is the transitioned credit rating will be the same as the current rating.

This perturbed rating is used as the initial rating for the next time node in the simulation. At the beginning of each path the credit rating is reset to its initial condition. In this manor a credit rating can transition up or down during the simulation. Transitioning to the default rating causes a default event to be raised and no further transitioning can occur for the remainder of the path.

4.1.3 Credit Transition Co-Movement

The process described thus far fails to take into account co-movement of rating transition between counterparties.

This is required to accommodate the observation that credit rating transitions and defaults are not independent events, as confirmed from statistical analysis of historical observations. Many simple examples can illustrate this point, for example a severe drought is likely to have a similar financial impact on similar counterparties from the agricultural industry.

In razor this co-dependence is captured by a correlation matrix of the counterparties, to produce correlated uniform random numbers to index into the CRTM.

As part of the simulation of credit migration, the correlation matrix would be decomposed using the same eigenvalue algorithm as is currently utilized for market drivers (including performing Principal Component Analysis on the correlation matrix) and then multiplied against a uniform random number distribution to obtain correlated random numbers. This is similar to the current implementation for simulation of market rates, with the exception being that when simulating credit migration it is not necessary to transform the uniform distribution to a N(0,1) distribution using a cumulative distribution function.

Result of Matrix Decomposition	Uniform Random Numbers		Correlated random numbers
$\begin{vmatrix} b_{11} & 0 & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{vmatrix}$	$\begin{vmatrix} z_1 \\ z_2 \\ z_3 \end{vmatrix}$	=	$\begin{matrix} b_{11} * z_1 \\ b_{21} * z_1 + b_{22} * z_2 \\ b_{31} * z_1 + b_{32} * z_2 + b_{33} * z_3 \end{matrix}$

This yields a set of correlated random numbers on the interval (0,1) for each credit entity, and for each scenario in a path.

4.1.4 Generation of Credit Rating Changes

As described in section 2.5 for market simulation the credit rating simulation can be pre-generated, separately from the market. For a Credit Rating Simulation a uniform random method is required with the correlation matrix defined for all counterparties being simulated. This will result in a memory mapped file containing a random number for each counterparty and scenario to determine the perturbed credit rating.

As for a market simulation this pre-generated file can be overridden to simulate transitions during the pricing phase.

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Note: Within RAZOR it is possible to set up multiple credit loss models running in parallel where each model is using data from a different external or internal ratings model, and hence it is possible to compare measures such as expected loss, and economic capital as derived from each model.

The Credit Rating simulation has been described it remains to detail the calculation of Credit Losses.

4.1.5 Credit Spread Risk Loss

To measure the credit spread risk the credit rating can be used to map to the discount curve with the corresponding credit spread. The client can configure any combination of credit rating, and other attributes such as industry, country to map to any discount curve defined within the market. These discount curves can in turn be defined as spread curves on top of risk free curve to reflect the credit spread premium. The resulting simulated pricing of a trade will now reflect the adjusted credit spread for the counterparty rating. In general all pricing models will have access to the simulated credit rating and can therefore make use of it in pricing.

4.1.6 Default Risk Loss

In the case of a default event Razor can be configured to calculate a Loss Given Default (LGD). This value is calculated by pricing the trade at default, called the Exposure at Default (EAD) using the simulated market for the corresponding scenario, and a recovery rate (RR) applied to determine the loss suffered.

LGD = EAD * (1 – RR)

The recovery rate in Razor is currently defined as a parameter at the trade level.

From these results a distribution of losses is built up for each simulation path, and various statistical analyses can be performed on this including Expected Loss, and Capital. The aggregation of these losses will be examined in relation to Credit Value-at-Risk and Economic Capital requirements.

4.2 Credit Value-at-Risk

Credit VaR is the simulated losses to a portfolio due to both market movements and changes to the credit quality of underlying entity. The principles of credit rating migration and default have been described in Chapter 4, and the general market risk VaR methodology is described in 5.3.

To capture the market, credit spread and default risk a Credit VaR simulation must stochastically vary market rates and credit ratings. The credit simulation must then impact the credit spread used in pricing and generate LGD in the case of default events. Any credit mitigation effects such as netting/collateral agreements or right to break agreements are taken into account in aggregating exposure in a portfolio.

Then similar to market risk VaR, Credit VaR measures the worst loss that can be expected over the specified time horizon with the degree of certainty defined by the confidence level. It is the extension of VaR to also take account of the effects of credit risk on the portfolio value, effects such as a change in credit rating causing a revaluation of credit sensitive instruments in the portfolio such as corporate and semi-government bonds, interest rate swaps, and credit derivatives. It is normally performed using similar time horizons as used for VaR.

4.3 Economic Capital

Economic capital is an estimate of capital needed by a financial organization to manage their own risk and allow the allocation of the cost of maintaining regulatory capital to different organizational units. Economic capital differs from "Regulatory Capital". Economic capital is an internal allocation against risk while "Regulatory Capital" is mandated by financial regulators.

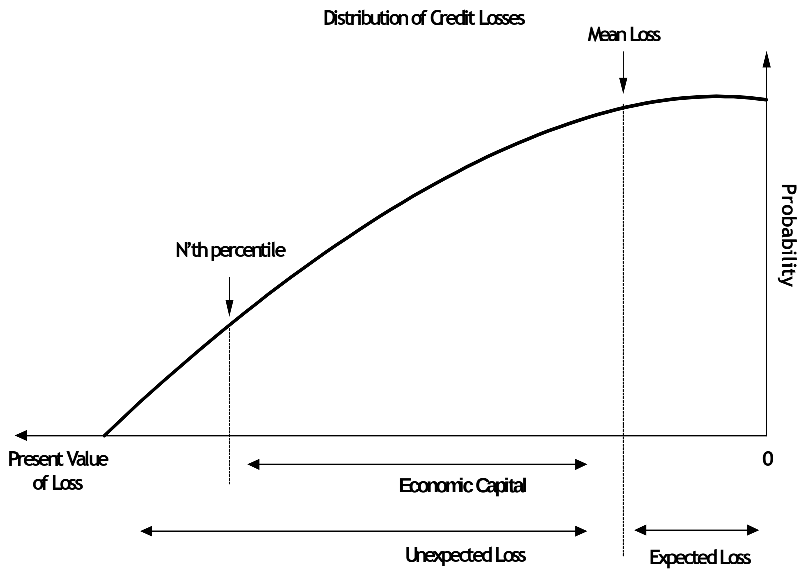
Economic Capital is calculated from the modelling of credit losses as generally described in, but is typically over a 1 year time horizon, but may involve any number of interim credit nodes.

For Economic capital only losses due to default are considered not credit spread risk. Hence, the CRTM need only specify the probability of default, and exclude other rating transition probabilities, although this is not required. Market rates are still simulated. Moreover, trade exposure is only computed when there is a default event, with pricing disabled in non-default scenarios and do not contribute to the exposure.

The resulting distribution of default exposures are aggregated taking into account any credit mitigation effects such as netting/collateral agreements or right to break agreements.

The aggregation of results in Razor is derived from this distribution of credit losses, and is equal to the maximum unexpected loss greater than the expected (mean) loss to a specified confidence level.

This can be represented by the following diagrammatic distribution of losses



Currently RAZOR does not support a distribution of credit losses for a particular node which have been present valued back to the current date. The present value of the simulated credit losses back to the start date can be supported by a simple method with a small enhancement in RAZOR. This simplistic approach will use rates from the base (or current) market. Alternatively a more sophisticated model could be introduced which would derive the discount factor to use for present value from the simulated rates along each path, and this discount factor is used with the specific loss generated along this path.

The distribution of credit losses as used for calculation of economic capital is derived from those losses where the credit entity has transitioned to default, and the credit exposure aggregated for that portfolio is positive. In other words an actual loss has been simulated. The value for that actual loss is calculated from the positive credit exposure less any amount recovered (as defined by the recovery rate). In all other instances the credit loss is set to zero. In RAZOR the recovery rates are assumed to be static, and may be set at any level, down to the individual trade level. As a future extension to RAZOR the recovery rates will be stochastically modelled.

4.3.1 Present Value of Losses

The credit loss may be present valued at the credit node where loss occurs, by discounting to the current value date from the date of the credit node where the loss is incurred. This is a configurable setting which may be turned on or off. The curve to use for discounting is specified as part of the configuration.

```
<economicCapital>
  <presentValue />
  <presentValueCurve>RF</presentValueCurve>
</economicCapital>
```

4.3.2 Aggregation of Losses to Final Credit Node

The credit losses may be aggregated along the simulation path to the final credit node of the simulation. This is a configurable setting which may be turned on or off.

```
<economicCapital>
  <aggregate />
</economicCapital>
```

4.3.3 Default Treatment

There are various other configurable settings for default treatment using the following xml under the simulatedRiskConfig (or distributedRiskConfig).

```
<defaultTreatment>
  <valueAtDefault>false</valueAtDefault>
  <valueAfterDefault>false</valueAfterDefault>

<pushValueAtDefaultForward>true</pushValueAtDefaultForward>
```

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</defaultTreatment>

1. valueAtDefault will value on the first node after the default event and then assume 0 value for the remainder of the path.
2. valueAfterDefault will value on all nodes after the default event.
3. pushValueAtDefaultForward will value on the first node after the default event and then use that same value for the remainder of the path.

All options default to false.

Chapter 5

Market Risk Measurement

Market risk is the risk of a decrease in value of a portfolio of investments, trades, and positions due to changes in the value of the market risk factors which determine the market value of the portfolio.

Measuring this risk provides financial institutions with the knowledge to properly manage the risk of portfolio losses, and the understanding to the composition of this risk, and enables the setting aside of regulatory capital to protect them against these potential portfolio losses.

There are various methods for measuring this risk, including the more traditional measures of sensitivity such as interest rate sensitivity such as PV01, basis point sensitivity, duration, convexity, and option based sensitivity measures (the 'Greeks') such as delta, gamma, rho, vega, theta.

Market risk is also typically measured using a Value at Risk methodology. Razor provides support for both the Historical, and the Monte Carlo VaR methodologies

5.1 Value-at-Risk (VaR)

VaR, under the assumption of prevailing normal market conditions, is a probabilistic bound of market losses. It is the worst-case loss expected over a defined period of time, t (the holding period) within the defined level of certainty, or probability defined by the confidence interval c. In other words larger losses are still possible, but with a lower probability than the confidence interval.

There are various non-simulation based statistical VaR methodologies, including RiskMetrics, and Parametric VaR which use variance-covariance data, with delta, and extension to gamma approximations.

The various simulation based VaR methodologies are scenario based methods, whereby the revaluation is performed for and using each generated market scenario, and the resultant values are compared to the current value. All these methods derive the statistical analysis from a distribution of changes in value, created from the difference between each of the simulated calculated values and the current value.

Each simulated calculated value is calculated from a revaluation using a specific simulated market scenario for that value. The simulation generates a distribution of market scenarios, by generating a new set of market risk factors for each of the scenarios.

Portfolio VaR is the same conceptually as that for an individual trade. The portfolio VaR is derived from the current portfolio value, and a distribution of portfolio values, where each portfolio value is the sum of the values for each trade valued with the specific generated scenario.

The simulation methodologies differ in how they generate the distribution of scenarios used.

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More formally, the VaR for a particular holding period may be derived from a probability density function $f_\tau(w)$ for the continuous random variable W of the change in portfolio value with values w, for that holding period τ.

The probability p of a change in portfolio value w being lower than a level V is $\int_{-\infty}^{V} f_\tau(w)dw$.

$$p = P(W < V) = \int_{-\infty}^{V} f_\tau(w)dw.$$

Therefore the probability of a change in portfolio value not being less than this value V is $1-p$. Hence $1-p$ represents the confidence level c.

$$c = 1 - p = P(W > V) = \int_{V}^{\infty} f_\tau(w)dw$$

In other words, with confidence level c the worst change in portfolio value time horizon τ is V. Hence V is the value for VaR, and it is reported as a positive number.

So for example, over a time horizon of 1 day, if 5% of the changes in portfolio value are less than -10,000 (ie more negative than -10,000), then 95% of the changes in portfolio value are greater than or equal to -10,000 (ie more positive than -10,000). In other words if 5% of the portfolio losses are greater than a loss of 10,000, then 95% must be less than or equal to 10,000. Hence to a confidence level of 95% the largest likely loss will be 10,000, and this is the reported VaR.

5.2 Historical Simulation

Historical simulation is a scenario-based approach to measuring market risk, where the distribution of values is derived from revaluations using the historic simulated scenarios S_t^τ.

Using the notation, where

S is a scenario, which contains market risk values F_i,

$F_i(t)$ is the value of a market risk factor i observed at time t,

S_0 is the current market scenario,

$F_i(0)$ are the current market risk values $F_i(0)$ in scenario S_0,

then scenario S_t^τ is defined by the application to the current market scenario S_0, containing the current market risk values $F_i(0)$, the historic changes in value for these market risk factors from time t to time $t+\tau$, as obtained from the historic time series of market risk factor values.

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Hence for S_t^τ, each market risk factor value $F_i^\tau(t)$ is derived as $F_i^\tau(t) = F_i(0) + \Delta^\tau F_i(t)$, where $\Delta^\tau F_i(t) = F_i(t+\tau) - F_i(t)$ is the change in value of a market risk factor i observed from time t to time $t+\tau$.

This change $\Delta^\tau F_i(t)$ can be set for each class of risk factors to be absolute ($F_i(t+\tau) - F_i(t)$, as above), relative ($F_i(t+\tau)/F_i(t)$), or zero (for example as use in partial risk as described below).

Historical simulation has the advantage of using actual real historic changes in market risk factors to generate the distribution of values, and hence there are no assumptions with respect to the distribution of risk factor values, the volatility/correlations are as they actually occurred, and takes account of outliers, and fat tails. Some of the disadvantages to be aware of include that trends reflected in the historical data do not necessarily reflect current market conditions, and also since it is possible for outliers to distort the VaR, careful analysis of the outliers may be required.

An adequate VaR number depends on the availability of an adequate set of historic data. It is recommended that at least 1 year of historic data is used. The regulatory framework of the Basle Internal Models Approach requires Historic VaR to be calculated for 1 day and 10 day holding periods with a 99% confidence level. The 10 day VaR may be calculated as an approximation from the 1 day VaR ($VaR_{10day} = \sqrt{10} * VaR_{1day}$), but ideally should be directly calculated using the same historic simulation methodology as the 1 day VaR.

5.2.1 Filtered Historical VaR Simulation

Razor provides an additional volatility weighting method, called the filtered HSVaR method, to adjust each scenario return value based on its history. The procedures are as follow:

For a set of returns r, first calculate the set of volatilities σ^2 then the set of weightings w. The new modified return r* is r multiplied by w.

$$\sigma_t^2 = \alpha\sigma_{t-1}^2 + (1-\alpha)r_{t-1}^2$$

$$w = \frac{\sigma_t + \sigma_N}{2\sigma_t}$$

r_t is the return for scenario t

σ_t^2 is the volatility for scenario t

w_t is the weighting for scenario t

r_t^* is the modified return for scenario t

α is the constant decay factor

σ_N is the last in the sequence t = 1..N

The decay factor constant can be defined by the user, and if not specified, Razor will assume a value of 0.97.

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5.3 Monte Carlo Simulation

Monte Carlo simulation is a scenario-based approach to measuring market risk, where the distribution of values is derived from revaluations using the Monte Carlo simulated scenarios S^{τ}.

Monte Carlo simulation uses the generalized Monte Carlo simulation framework as described in detail in Chapter 2, to generate the set of simulated changes in the risk factors which are then used to create the scenarios S_j^{τ}, for time horizon τ.

The market risk factors are stochastically modelled forward in time τ according to a set of correlated diffusion processes. The returns for each market risk factor are assumed to have a specific probability distribution, such as normal, or lognormal, and evolve using a stochastic process defined by volatility and with or without mean reversion. From the set of market risk factors and the probability distributions, it is necessary to estimate or define the stochastic parameters for each of the risk factors and the correlations between all of the risk factors. This can be by various methods, such as deriving from a historical set of values for the market risk factors or defining externally by any other mechanism, or from some combination of these methods.

From this input the Monte Carlo simulation process generates m simulated relative changes or returns $S\Delta_j^{\tau}F_i$, in the n market risk factors F_i, over the time horizon τ, by correlated random sampling.

Hence, in a similar fashion as for Historic VaR, for scenario S_j^{τ}, each market risk factor value $F_i^{\tau}(j)$ is defined as the value of the i'th risk factor in the j'th scenario for time horizon τ, and is derived as $F_i^{\tau}(j) = F_i(0) * S\Delta_j^{\tau}F_i$.

Advantages of the Monte Carlo simulation methodology include the ability to apply a theoretical distribution framework, and have a very large number of distributions without the need for historical data of the same size. Disadvantages include the high level of computational processing, and that outliers are not included, and the assumed distributions do not adequately describe the real world.

5.4 Analysis of VaR

To assist in understanding the important factors which influence the value of the overall portfolio VaR (as calculated using either Historical or Monte Carlo VaR), it is useful to analyse the composition of the portfolio VaR, by measuring the contributions made by market risk factors, either single, multiple collections, or general classes, and also by measuring the contributions made by single trade, set of trades, or sub-portfolio.

The general classes of risk factors include equity stock prices, interest rates, foreign exchange rates, commodity prices/rates, credit spreads, and implied market volatilities. It is possible to define any collection of risk factors and/or set of trades and hence to measure the contribution made by this collection to the portfolio VaR

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5.4.1 Partial Risk

Partial risk measures the effect on portfolio VaR due to the volatility of the individual defined set of market risk factors, where this set is composed of either a single, a collection, or a class of market risk factors.

The portfolio VaR due to the volatility of the specific set of risk factors is calculated by performing a VaR calculation where the volatility of all other risk factors is set to zero. This will then measure the VaR due to that specific set of risk factors.

A typical use of this would be to measure the VaR due to each individual market risk class, and within the foreign exchange rate risk class measure the VaR due to the individual common cross rates. The sum of these partial portfolio VaR values can the be compared to the overall portfolio VaR to measure the portfolio effect due to the correlation in movements between all of the individual risk factors.

5.4.2 Incremental VaR

Incremental VaR measures the incremental effect on portfolio VaR due to a change in the portfolio composition through for example the introduction of a new set of trades, or due to the inclusion of the effects of the volatility of a single risk factor or a class of risk factors. It is calculated from the difference in the overall portfolio VaR before and after the change.

So for example, the incremental VaR for a portfolio due to the introduction of a set of trades is equal to the portfolio VaR after the set of trades is included in the portfolio less the portfolio VaR before the trades were added.

As another example, the incremental VaR for a portfolio due to the volatility of the interest rate risk factors is equal to the overall portfolio VaR less the portfolio VaR calculated by a partial risk calculation as described above which includes all risk factors except the interest rate risk factors. This is the same effect as setting the volatility for the interest rate factors to zero.

5.4.3 Marginal VaR

Marginal VaR is similar to incremental VaR, but measures the changes in portfolio VaR due to more subtle changes in risk factors and portfolio composition, rather than a straight out inclusion/exclusion. Examples of such changes would be the change in portfolio VaR due to small change in positions within the portfolio, small changes in the current value of individual risk factors, and changes in volatility of a risk factor.

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Chapter 6
Market Rates, Prices and Curves

The various pricing routines in RAZOR make use of both observed and implied security prices and theoretical structures that describe the characteristics of the financial market at a point in time. This information is encapsulated by an object in RAZOR called a Market.

6.1 The Market - Rates and Prices

The market object contains a set of rates and prices for various assets, yield curves, forward curves, and volatility surfaces implied from those rates and prices. The market object is arranged into a set of term structures, where each term structure is defined for a specific asset as a collection of related rate classes by term (relative term or discrete date), and where each rate class is a single observable element of market data. For a more complete description of these definitions and the relationship between each of these components please refer to the Razor technical documentation.

Rates and prices in RAZOR are uniquely identified in a market by a *rate class identifier*. Each rate class is differentiated by the elements that make that type of rate unique — for example, a one month BAB rate class identifies that the rate is based on a one month BAB future. The *rate identifier* by contrast, identifies the particular instance of the rate.

6.2 Interest Rate Curves

6.2.1 Interest Rates and Security Prices

Zero Rate

Zero rate is simply the rate of return on a zero-coupon bond or any non-coupon bearing instruments, e.g. cash deposits, bills or any single cashflow.

$Z_{t\to t_1}$ = the annualised zero rate at time t_1 .

$d_{t\to t_1} = t_1 - t$ = number of days from t to t_1.

D = number of days in a year (depends on the day convention used, see reference)

$P(t,t_1)$ = the price of a zero-coupon bond at time t maturing at time t_1.

F = the face value of the zero-coupon bond.

The price of a zero-coupon bond can be determined as:

$$P(t,t_1) = F\left(\frac{1}{1+Z_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}}\right)$$

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If the zero-coupon bond price is observed, the zero rate can be uniquely determined as:

$$Z_{t\to t_1} = \left(\frac{F}{P(t,t_1)} - 1\right)\cdot\frac{D}{d_{t\to t_1}}$$

When $F = 1$, the price of the zero-coupon bond $P(t,t_1)$ is called the discount factor at time t_1 and we denote it as $D(t,t_1)$, i.e.

$$D(t,t_1) = \frac{1}{1 + Z_{t\to t_1}\dfrac{d_{t\to t_1}}{D}}$$ and equivalently

$$Z_{t\to t_1} = \left(\frac{1}{D(t,t_1)} - 1\right)\cdot\frac{D}{d_{t\to t_1}}$$

Forward Rate

Forward rate is the interest rate implied by current zero rates for a specified future time period.

Define:

$f(t,t_1,t_2)$

$=$the forward rate

$=$the implied annualised interest rate for future time period t_1 to t_2 at current time t.

The forward rate must mathematically satisfy the following equality:

$$\left(1 + Z_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}\right)\left(1 + f(t,t_1,t_2)\cdot\frac{d_{t_1\to t_2}}{D}\right) = \left(1 + Z_{t\to t_2}\cdot\frac{d_{t\to t_2}}{D}\right)$$

The reason for the above equality to hold is explained by the arbitrage-free theory:

If LHS > RHS

Short \$1 of a t_2-year zero-coupon bond.

Use the shorted \$1 to invest in a t_1-year zero-coupon bond and lock in a forward interest rate to invest for the period t_1 to t_2.

At the end of time t_2, we will have the invested amount equal to the LHS and we need to return the shorted amount with interest equal to the RHS. Since LHS > RHS, an arbitrage profit of LHS – RHS is earned.

If LHS < RHS

Short \$1 of a t_1-year zero-coupon bond and lock in a forward interest rate to pay for the period t_1 to t_2.

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Use the shorted $1 to invest in a t_2-year zero-coupon bond.

At the end of time t_2, we will have the invested amount equal to the RHS and we need to return the shorted amount with interest equal to the LHS. Since LHS < RHS, an arbitrage profit of RHS – LHS is earned.

In an efficient market, investors will exploit the arbitrage opportunities. This will force any arbitrage opportunity to be eliminated as soon as they arise and fail to exist in the market. This means that the above mathematical equality has to hold, i.e. LHS = RHS, in an efficient and arbitrage-free market.

The above mathematical equality can also be written as:

$$D(t,t_2) = D(t,t_1)\left(1 + f(t,t_1,t_2)\cdot\frac{d_{t_1\to t_2}}{D}\right)^{-1}$$

Thus if the zero rates (or the price of the zero-coupon bonds) are observed, then the forward rate can be uniquely determined as:

$$\begin{aligned} f(t,t_1,t_2) &= \left[\left(1+Z_{t\to t_2}\cdot\frac{d_{t\to t_2}}{D}\right)/\left(1+Z_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}\right)-1\right]\cdot\frac{D}{d_{t_1\to t_2}} \\ &= \left(\frac{D(t,t_1)}{D(t,t_2)}-1\right)\cdot\frac{D}{d_{t_1\to t_2}} \end{aligned}$$

Swap Rate

Swap rate is the fixed rate in an interest rate swap that causes the swap to have a value of zero.

It is important to note that on the floating side of the swap, the present value of all cash flows must equal to the principal amount. Thus for the interest rate swap to have a value of zero, the present value of all the cash flows on the fixed side of the swap must also equal to the principal amount. It implies that the fixed rate on the swap is equivalent to the yield-to-maturity of a bond that equates the present value of all cashflows of the bond to the face value of the bond (i.e. the par yield). As a result, it is convenient to think the swap rate as a par yield rate.

Bond Yield-to-Maturity Rate and Bond Price (Currently Under Development)

Bill Discount Rate (Currently Under Development)

6.2.2 Market specific interest rate curves

Market specific interest rate curves include Swap, Treasury, Corporate, Minor Market (CP, BA, CD, Prime, Tbill, Fedfund, etc)), BMA, OIS, cross currency curves, 1M Libor, 3M Libor Curves.

Specific interest rate curves in the market are built from collections of specific interest rate market rates and security prices.

Currently Razor supports the configuration of any specific curves as long as these curves are composed of rates/prices which Razor's bootstrapping supports. This includes cash deposit, cash rates, swap rates, and futures points. Currently security (bond) yield to maturity rates and security (bond) prices are not supported in Razor, and there is no support for convexity adjustments. Inflation Curves are currently not supported.

Specific market curves which contain rates/prices which are not directly supported by Razor's current bootstrapping function can still be supported by a conversion of these points to points in the form above. This will involve some integration layer / ETL processing.

6.2.3 Spread Yield Curve

This curve is defined by the standard bootstrapping as described in detail above for simple yield curves, where each of the input points on this curve is first derived from the corresponding reference curve input point plus the spread point. The dates for the resultant set of input points for the spread yield curve matches the dates for the reference curve input points.

For each input rate $r_{t_i}^r$, at time t_i, on the reference yield curve r, the resultant input rate $r_{t_i}^c$ at time t_i, on the spread yield curve c is defined by

$r_{t_i}^c = r_{t_i}^r + s_{t_i}$, where s_{t_i}, is the spread rate at time t_i

If no spread rate s_{t_i} exists at time t_i, then the spread rate to use is derived by linear interpolation between the previous and next spread rates. If there is no previous spread rate, then linear interpolate between the next spread rate and a spread rate of zero at start of curve, time t. If there is no next spread rate, then $r_{t_i}^c = r_{t_i}^r$.

6.2.4 Composite Yield Curve

This curve is defined as a function of two bootstrapped yield curves. The discount factor $D_c(t,t_1)$, for a specific date t_1 on the composite curve c is calculated from the corresponding respective discount factors from each of the two bootstrapped yield curves, where each of these discount factors is converted into a zero rate, the zero rates are summed, and the result $Z_{t\to t_1}^c$ is then converted back to a discount factor. Hence

$$D_c(t,t_1) = \frac{1}{1 + Z_{t\to t_1}^c \cdot \frac{d_{t\to t_1}}{D_c}}$$

$$Z_{t\to t_1}^c = Z_{t\to t_1}^1 + Z_{t\to t_1}^2$$

$$Z_{t\to t_1}^1 = \left(\frac{1}{D_1(t,t_1)} - 1\right) \cdot \frac{D_c}{d_{t\to t_1}}$$, the zero rate at time t_1 from first yield curve

$$Z_{t\to t_1}^2 = \left(\frac{1}{D_2(t,t_1)} - 1\right) \cdot \frac{D_c}{d_{t\to t_1}}$$, the zero rate at time t_1 from second yield curve

D_c, the number of days in year, as defined on composite curve

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$d_{t\to t_1}$, number of days from time t to time t_1

t, current time

t_1, time for which discount factor required from composite curve

6.2.5 Adjustment Factors Applied to Yield Curves

The yield curve framework in Razor supports an interface which allows for customization of the bootstrapped curve, to adjust the curve by an additive factor and/or a multiplicative factor.

The additive factor is applied before the multiplicative factor. The default additive factor is 0.0, and the default multiplicative factor is 1.0. The factors however are only applied if explicitly supplied.

The adjusted discount factor is calculated from the unadjusted discount factor $D(t,t_1)$, as provided from the bootstrapped curve as follows.

$$Z_{t\to t_1} = \left(\frac{1}{D(t,t_1)} - 1\right) \cdot \frac{D}{d_{t\to t_1}}$$, unadjusted zero rate

$$Z^A_{t\to t_1} = \left(Z_{t\to t_1} + A\right) * M$$, adjusted zero rate

$$D_A(t,t_1) = \frac{1}{1 + Z^A_{t\to t_1} \cdot \frac{d_{t\to t_1}}{D_c}}$$, adjusted discount factor

6.2.6 Negative Forward Rates

To eliminate the creation of simulated yield curves which may contain negative forward rates and hence cause certain trades to fail to price during a simulation, Razor applies the following method to remove the existence of negative forward rate in all yield curves when they occur under simulation.

Post the bootstrapping of all simulated yield curves or scenarios, the pricing server validates each yield curve (including the base scenario) by iterating through the ordered set of discount factors, checking each discount factor against the previous good discount factor from the term structure, and if it is greater then removing it. From the resultant yield curve the normal interpolation and extrapolation will then operate on the remaining discount factors

Since the base scenario is representative of the current market yield curve, if validation detects negative forward rates then the yield curve will be rejected rather than be modified and any trade which uses this curve for valuation will fail to find the curve and hence be rejected as failed to price.

There is a detailed log of all analysis and actions for any yield curve (base or simulated) that contains negative forward rates (ie increasing discount factors).

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6.3 Yield Curve Construction

By definition, yield curve is the curve that depicts the relationship between zero rates and their maturities. Thus the objective of the yield curve construction routines is to determine the values of zero coupon bond rates for any maturity forward in time given the observed market prices of interest-based securities.

The price of market securities embodies several considerations for a market participant. When an interest rate security is exchanged the buyer of the security is giving up an amount of equity in order to gain a certain return. In general, the longer the cash is tied up in the security, the greater the return the investor will require from the security.

Term structure analysis examines market securities to imply a curve that exhibits an equilibrium time value of money. The process of determining the attributes of this curve from market securities is known as “bootstrapping”, as the process uses the value of near-dated securities to help determine the theoretical value of further dated zero coupon bonds.

6.3.1 Bootstrapping

In RAZOR, the bootstrapping process is driven by a *bootstrapping configuration*. The bootstrapping configuration determines what assets to use to generate the yield curve. Assets are referenced by an *asset class*, an identifier that represents the type of security used at a particular point on the yield curve - for example; a one month bank accepted bill rate.

Different types of securities have different cash flow profiles and different rate calculation methods. These differences need to be taken into account when determining the theoretical value of a zero coupon bond at the specific maturity.

6.3.2 Bootstrapping the Yield Curve Using Different Securities

Cash Deposits

Cash deposits are short-term non-coupon-bearing instruments issued to raise short term capital.

The method to determine the price of a bill is the same as the zero-coupon bond. We simply discount the face value by the corresponding zero rate, i.e.

$$P(t,t_1) = F\left(\frac{1}{1+Z_{t\to t_1}\cdot\frac{d_{t\to d_1}}{D}}\right)$$

It is very straightforward to bootstrap the zero rates if we are given the prices of the bills. The zero rates can be found using the formula

$$Z_{t \to t_1} = \frac{(F - P(t,t_1))D}{P(t,t_1)d_{t \to t_1}}.$$

Example:

If we have the price for a 30-day bill with face value 100 equal to 99 (assuming 360 days in a year), the annualised 30-day zero rate is:

$$Z_{t \to t_1} = \frac{360(100-99)}{30 \times 99} = 12.12\%.$$

While it is very simple to determine zero rates using bills, it should be realised that bills have very short-maturities. To construct a yield curve with maturities up to say 40 years, it is obvious that we do not have bills with equivalent time-to-maturities.

Bill Futures

A bill futures contract is a futures contract on bills. It obliges the buyer/seller of the futures contract to buy/sell the underlying bill at a predetermined delivery price in the specified future time.

Define:

$B(t,t_1,t_2)=$ the futures price of a bill futures contract that begins at time t_1 and matures at time t_2

$B(t,t_1,t_2)$ is quoted in a way such that $\frac{100-B(t,t_1,t_2)}{100}$ is the annualised interest rate applied to the period t_1 to t_2 at the current time t.

Thus it is intuitively clear that $\frac{100-B(t,t_1,t_2)}{100}$ is just the annualised forward rate applied to the period t_1 to t_2 from the perspective of current time t. Mathematically, we can write:

$$f(t,t_1,t_2) = \frac{100-B(t,t_1,t_2)}{100}$$

Thus if we are given the bill futures price, we can uniquely determine the forward rate and then bootstrap the zero rate for t_2 from this forward rate and the zero rate we bootstrapped or given at time t_1.

Example:

Assume there are 360 days in a year and given the 180-day discount factor is 0.9730. We have a 90-day bill futures contract that begins in 180 days and the

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rate of the bill futures contract is 95. We want to determine the 270-day zero rate, i.e. we want to calculate $Z_{t\to\frac{270}{360}}$.

Answer:

We are given $B\left(t,\frac{180}{360},\frac{270}{360}\right)=95$. We can use the forward formula

$$f(t,t_1,t_2)=\frac{100-B(t,t_1,t_2)}{100}$$

to calculate the annualised forward rate implied for the period from 180 days to 270 days, i.e.

$$f\left(t,\frac{180}{360},\frac{270}{360}\right)=\frac{100-95}{100}=0.05$$

We have calculated $f\left(t,\frac{180}{360},\frac{270}{360}\right)$ and we are given the 180-day discount factor $D\left(t,\frac{180}{360}\right)=0.9730$, we can use the formula

$$D(t,t_1)\cdot\left(1+f(t,t_1,t_2)\cdot\frac{d_{t_1\to t_2}}{D}\right)^{-1}=D(t,t_2)$$

to work out the 270-day discount factor $D\left(t,\frac{270}{360}\right)$ as:

$$D\left(t,\frac{270}{360}\right)$$

$$=D\left(t,\frac{180}{360}\right)\cdot\left(1+f\left(t,\frac{180}{360},\frac{270}{360}\right)\cdot\frac{d_{\frac{180}{360}\to\frac{270}{360}}}{360}\right)^{-1}$$

$$=0.9730\times\left(1+0.05\cdot\frac{270-180}{360}\right)^{-1}$$

$$=0.9610$$

The 270-day zero rate can be obtained directly from the formula

$$Z_{t\to t_1}=\left(\frac{1}{D(t,t_1)}-1\right)\cdot\frac{D}{d_{t\to t_1}}$$ so that

$$Z_{t\to\frac{270}{360}}$$

$$=\left(\frac{1}{D\left(t,\frac{270}{360}\right)}-1\right)\cdot\frac{360}{d_{t\to\frac{270}{360}}}$$

$$= \left(\frac{1}{0.9610} - 1 \right) \cdot \frac{360}{270}$$

$$= 5.41\%$$

Swaps

A swap contract is a contract that exchanges a fixed rate of interest on a certain notional principal for a floating rate of interest on the same notional principal.

The swap rate is the fixed rate in a swap that causes the swap to have a value of zero. As we have discussed in section 6.1.2, the swap rate can be thought as a par yield (i.e. the yield-to-maturity that makes the present value of all cash flows of a bond equal to the face value of the bond).

The construction of a yield curve usually requires a very long maturity term. Bills and bill futures generally do not have the equivalent maturities. However, swaps can have very long maturity term (e.g. 40 years). It is thus very important to find a method to construct the yield curve using swaps.

Define:

$r_s(c, N) =$ the annualised swap rate of a swap contract that exchanges

interest payment c times a year with maturity term N

$F =$ the principal amount under the swap

By the definition of a swap rate, the following mathematical equality must be satisfied:

$$F = F\frac{r_s(c,N)}{c}D\left(t,\frac{1}{c}\right) + F\frac{r_s(c,N)}{c}D\left(t,\frac{2}{c}\right) + \ldots + F\frac{r_s(c,N)}{c}D\left(t,\frac{cN}{c}\right) + FD\left(t,\frac{cN}{c}\right)$$

The above can be simplified to

$$D(t,N) = \frac{1 - \frac{r_s(c,N)}{c}\sum_{i=1}^{cN-1} D\left(t,\frac{i}{c}\right)}{1 + \frac{r_s(c,N)}{c}}$$

Using the above formula, we can bootstrap the yield curve based on the swap contract.

Example:

Assume there are 30 days in a month and 360 days in a year. We have a 1 year swap contract with the annualised swap rate 6% that exchanges interest payment quarterly. We are given the 3-month, 6-month and 9-month discount factors equal to 0.9851, 0.9698 and 0.9557 respectively. We need to bootstrap the 1 year zero rate using the information given above.

Answer:

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From the question, we are given

$$r_s(4,1) = 0.06$$

$$D\left(t,\frac{90}{360}\right) = D\left(t,\frac{1}{4}\right) = 0.9851$$

$$D\left(t,\frac{180}{360}\right) = D\left(t,\frac{2}{4}\right) = 0.9698$$

$$D\left(t,\frac{270}{360}\right) = D\left(t,\frac{3}{4}\right) = 0.9557$$

Using the formula

$$D(t,N) = \frac{1 - \dfrac{r_s(c,N)}{c}\displaystyle\sum_{i=1}^{cN-1} D\left(t,\frac{i}{c}\right)}{1 + \dfrac{r_s(c,N)}{c}}$$

$$\begin{aligned}
D(t,1) &= \frac{1 - \dfrac{r_s(4,1)}{4}\displaystyle\sum_{i=1}^{3} D(t,i)}{1 + \dfrac{r_s(4,1)}{4}} \\
&= \frac{1 - \dfrac{r_s(4,1)}{4}\left(D(t,1) + D(t,2) + D(t,3)\right)}{1 + \dfrac{r_s(4,1)}{4}} \\
&= \frac{1 - \dfrac{0.06}{4}(0.9851 + 0.9698 + 0.9557)}{1 + \dfrac{0.06}{4}} \\
&= 0.9422
\end{aligned}$$

Using the zero rate formula

$$Z_{t\to t_1} = \left(\frac{1}{D(t,t_1)} - 1\right)\cdot\frac{D}{d_{t\to t_1}}$$

The 1 year zero rate is:

$$\begin{aligned}
Z_{t\to 1} &= \left(\frac{1}{D(t,1)} - 1\right)\cdot\frac{360}{d_{t\to 1}} \\
&= \left(\frac{1}{0.9422} - 1\right)\cdot\frac{360}{360} \\
&= 6.13\%
\end{aligned}$$

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Effective Swap Rate

As described above, when bootstrapping the next zero rate or discount factor at t_k from a swap rate at t_k of frequency c, then the solution requires the $(t_k c - 1)$ number of zero rates or discount factors for times $t_{\frac{1}{c}}$ to $t_{k-\frac{1}{c}}$. If all of these required discount factors are readily available as they have all been previously directly bootstrapped, or may be derived by interpolation (support different types of interpolation) as they fall before the term of the last zero rate bootstrapped t_i, then deriving at t_k is straight forward.

For the case where the term of the last known zero bootstrapped t_i does not cover the terms for all required zero rates, then we need to derive in order these interim zero rates first.

Assume t_j is the term of the next interim zero rate which is required to be bootstrapped but for which there is no direct swap rate of frequency c available, t_i is the term for the most recently bootstrapped zero rate, $t_{j-1} = t_j - 1/c$, $t_{j-1} \le t_i$, and $t_i < t_j < t_k$.

To bootstrap the zero rate at t_j, we firstly need to derive a swap rate at this term. This is achieved by interpolating between the next available swap rate at t_k, and an effective swap rate of the same frequency derived at t_i, or is this t_{j-1}.

Once we find the effective swap rate at time t_i, with the given swap rate at time t_k, we linearly interpolate the swap rate at time t_j. After obtaining the swap rate at time t_j, the discount factor and zero rate at time t_j can be evaluated as we have done in the previous section.

Define Effective Swap Rate

An effective swap rate is a swap rate that is implied by the discount factors

An effective swap rate of maturity ti, and frequency c, can be derived from an existing bootstrapped yield curve, or known series of zero rates or discount factors for terms t1 to tn, spanning ti.

It is very simple to calculate the effective swap rate:

Recall:

$$D(t,N) = \frac{1 - \frac{r_s(c,N)}{c}\sum_{i=1}^{cN-1} D\left(t, \frac{i}{c}\right)}{1 + \frac{r_s(c,N)}{c}}$$

Rearrange the formula and we can calculate the effective swap rate as:

$$r_s(c,N) = \frac{c(1 - D(t,N))}{\sum_{i=1}^{cN} D\left(t, \frac{i}{c}\right)}$$

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Example:

Assume there are 30 days in a month and 360 days in a year. We are given the 3-month, 6-month and 9-month and 12-month discount factors equal to 0.9851, 0.9698, 0.9557 and 0.9422 respectively. We want to find the effective swap rate for a 1-year swap contract with quarterly exchanges of interest payments.

Answer:

We are given

$$D\left(t,\frac{90}{360}\right)=D\left(t,\frac{1}{4}\right)=0.9851$$

$$D\left(t,\frac{180}{360}\right)=D\left(t,\frac{2}{4}\right)=0.9698$$

$$D\left(t,\frac{270}{360}\right)=D\left(t,\frac{3}{4}\right)=0.9557$$

$$D\left(t,\frac{360}{360}\right)=D(t,1)=0.9422$$

Using the effective swap rate formula

$$r_s(c,N)=\frac{c(1-D(t,N))}{\sum_{i=1}^{cN}D\left(t,\frac{i}{c}\right)}$$

$$\begin{aligned}
r_s(4,1)&=\frac{4(1-D(t,1))}{\sum_{i}^{4}D\left(t,\frac{i}{4}\right)}\\
&=\frac{4(1-D(t,1))}{D\left(t,\frac{1}{4}\right)+D\left(t,\frac{2}{4}\right)+D\left(t,\frac{3}{4}\right)+D(t,1)}\\
&=\frac{4(1-0.9422)}{0.9851+0.9698+0.9557+0.9422}\\
&=6\%
\end{aligned}$$

Thus the 1-year effective swap rate is 6%.

The effective swap rate formula is very useful to bootstrap zero rates. Consider the following situation:

We want to find the 1.25-year zero rate. We have a 3-year swap with quarterly interest exchange. We also have the 90-days, 180-days, 270-days and 1-year discount factors obtained from other short term instruments.

From the formula we had before, to find the 1.25-year discount factor, we will need a swap contract with maturity at 1.25 years. However, we only have a 3-year swap contract. We even cannot interpolate the 1.25-year swap rate because we do not have any other swap contract besides the 3-year swap. The method on effective swap rate is now useful for us to calculate an effective 1-year swap rate with the corresponding discount factors. We then are able to interpolate the 1.25-year swap rate and apply our usual formula to find the 1.25-year discount factor. Once we know the discount factor, the zero rate is obtained easily.

Example:

Assume there are 30 days in a month and 360 days in a year. We have a 3-year swap contract with the annualised swap rate 5.82% that exchanges interest payments quarterly. We are given the 3-month, 6-month, 9-month and 12-month discount factors equal to 0.9851, 0.9698, 0.9557 and 0.9422 respectively. We need to bootstrap the 1.25-year zero rate using the information given.

Answer:

We can use the discount factors to calculate the 1-year effective swap rate and then interpolate the 1.25-year swap rate with the 1-year effective swap rate and the 3-year swap rate.

Given:

$$D\left(t,\frac{90}{360}\right)=D\left(t,\frac{1}{4}\right)=0.9851$$

$$D\left(t,\frac{180}{360}\right)=D\left(t,\frac{2}{4}\right)=0.9698$$

$$D\left(t,\frac{270}{360}\right)=D\left(t,\frac{3}{4}\right)=0.9557$$

$$D\left(t,\frac{360}{360}\right)=D(t,1)=0.9422$$

$$r_s(4,3)=0.0582$$

The 1-year effective swap rate can be obtained from the formula

$$r_s(c,N)=\frac{c(1-D(t,N))}{\sum_{i=1}^{cN} D\left(t,\frac{i}{c}\right)}$$

and we have calculated in the previous example that the 1-year effective swap rate $r_s(4,1)$ is 6%.

Using linear interpolation (for details on linear interpolation, please refer to the next section):

$$\begin{aligned} r_s(4,1.25) &= r_s(4,1)+\frac{r_s(4,3)-r_s(4,1)}{3-1}(1.25-1) \\ &= 0.06+\frac{0.0582-0.06}{3-1}(1.25-1) \\ &= 5.98\% \end{aligned}$$

With the interpolated 1.25-year swap rate and the corresponding discount factors, we can find the 1.25 discount factor using the formula:

$$D(t,N)=\frac{1-\frac{r_s(c,N)}{c}\sum_{i=1}^{cN-1}D\left(t,\frac{i}{c}\right)}{1+\frac{r_s(c,N)}{c}}$$

We can now find $D(t,1.25)$ as:

$$\begin{aligned}D(t,1.25)&=\frac{1-\frac{r_s(4,1.25)}{4}\sum_{i=1}^{4}D\left(t,\frac{i}{4}\right)}{1+\frac{r_s(4,N)}{4}}\\&=\frac{1-\frac{r_s(4,1.25))}{4}\left(D\left(t,\frac{1}{4}\right)+D\left(t,\frac{2}{4}\right)+D\left(t,\frac{3}{4}\right)+D(t,1)\right)}{1+\frac{r_s(4,1.25)}{4}}\\&=\frac{1-\frac{0.0598}{4}(0.9851+0.9698+0.9557+0.9422)}{1+\frac{0.0598}{4}}\\&=0.9285\end{aligned}$$

It is straightforward to find the 1.25-year zero rate $Z_{t\to 1.25}$:

$$Z_{t\to t_1}=\left(\frac{1}{D(t,t_1)}-1\right)\cdot\frac{D}{d_{t\to t_1}}$$

$$\begin{aligned}Z_{t\to 1.25}&=\left(\frac{1}{D(t,1.25)}-1\right)\cdot\frac{360}{d_{t\to 1.25}}\\&=\left(\frac{1}{0.9285}-1\right)\cdot\frac{360}{360(1.25)}\\&=6.16\%\end{aligned}$$

6.3.3 Interpolation Methods

From previous sections, we can see that our rates are all given or calculated at discrete points in time. It is often possible that we require rate that lies between two discrete points. Thus we need some methods to interpolate the required rate using rates on the discrete points.

Linear Interpolation

This is the simplest way to interpolate the required rate. If the rate at time t_i is r_i and the rate at time t_j is r_j and we want to work out the rate at a time between t_i and t_j, i.e. to calculate r_α at time t_α for some $\alpha\in(i,j)$. Linear interpolation assumes there is a linear relationship between r_i and r_j during

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the time from t_i to t_j. Thus we can calculate any rate r_α at time t_α using linear interpolation formula:

$$r_\alpha = r_i + \frac{r_j - r_i}{t_j - t_i}(t_\alpha - t_i).$$

Rate here is a very general definition. It can be spot rate, discount rate, log-rate, etc.

We will now discuss four options that are based on linear interpolation.

Linear Spot

This is the situation where we have got two spot rates $Z_{t\to t_a}$ and $Z_{t\to t_b}$ at time t_a and t_b respectively. We need to find the spot $Z_{t\to t_i}$ at time t_i for some $i \in (a,b)$ using linear interpolation.

Example:

The 3-month spot rate is 6.01% and the 6-month spot rate is 6.11%. Calculate the 5-month spot rate using the concept of linear spot.

Answer:

$$Z_{t\to\frac{3}{12}} = 0.0601$$

$$Z_{t\to\frac{6}{12}} = 0.0611$$

Using the linear interpolation formula:

$$\begin{aligned} Z_{t\to\frac{5}{12}} &= Z_{t\to\frac{3}{12}} + \frac{Z_{t\to\frac{6}{12}} - Z_{t\to\frac{3}{12}}}{\frac{6}{12}-\frac{3}{12}}\left(t_{\frac{5}{12}} - t_{\frac{3}{12}}\right) \\ &= 0.0601 + \frac{0.0611-0.0601}{\frac{6}{12}-\frac{3}{12}}\left(\frac{5}{12}-\frac{3}{12}\right) \\ &= 6.08\% \end{aligned}$$

Linear Discount Factor

This is similar to linear spot except linear discount factor is to linearly interpolate discount factors rather than spot rate.

Example:

The 3-month discount factor is 0.9852 and the 6-month discount factor is 0.9704. Calculate the 5-month discount factor using linear discount factor.

Answer:

$$D\left(t,\frac{3}{12}\right) = 0.9852$$

$$D\left(t,\frac{6}{12}\right)=0.9704$$

Using linear interpolation formula:

$$D\left(t,\frac{5}{12}\right)$$

$$=D\left(t,\frac{3}{12}\right)+\frac{D\left(t,\frac{6}{12}\right)-D\left(t,\frac{3}{12}\right)}{\frac{6}{12}-\frac{3}{12}}\cdot\left(\frac{5}{12}-\frac{3}{12}\right)$$

$$=0.9852+\frac{0.9704-0.9852}{\frac{6}{12}-\frac{3}{12}}\cdot\left(\frac{5}{12}-\frac{3}{12}\right)$$

$$=0.9753$$

Log Linear Spot

This is similar to linear spot. However, instead of linearly interpolate spot rate, we will linearly interpolate the log of the spot rate and then obtain the spot rate by calculating the exponential of the linearly interpolated log-spot rate.

Example:

Calculate the 5-month spot rate from the information in the example of linear spot using log linear spot method.

Answer:

$$Z_{t\to\frac{3}{12}}=0.0601$$

$$Z_{t\to\frac{6}{12}}=0.0611$$

Use log linear spot with the linear interpolation formula:

$$\log\left(Z_{t\to\frac{5}{12}}\right)$$

$$=\log\left(Z_{t\to\frac{3}{12}}\right)+\frac{\log\left(Z_{t\to\frac{6}{12}}\right)-\log\left(Z_{t\to\frac{3}{12}}\right)}{\frac{6}{12}-\frac{3}{12}}\cdot\left(\frac{5}{12}-\frac{3}{12}\right)$$

$$=\log(0.0601)+\frac{\log(0.0611)-\log(0.0601)}{\frac{6}{12}-\frac{3}{12}}\cdot\left(\frac{5}{12}-\frac{3}{12}\right)$$

$$=-2.80074$$

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$$Z_{t\to\frac{5}{12}}$$

$$= \exp\left(\log\left(Z_{t\to\frac{5}{12}}\right)\right)$$

$$= \exp(-2.80074)$$

$$= 6.08\%$$

Log Linear Discount Factor

This is similar to log linear discount factor except that we need to linearly interpolate the log-discount factor instead of the discount factor. We can obtain the discount factor by calculating the exponential of the linearly interpolated log discount factor.

Example:

Assuming the information in the example of linear discount factor, calculate the 5-month discount factor using log linear discount factor.

Answer:

$$D\left(t,\frac{3}{12}\right) = 0.9852$$

$$D\left(t,\frac{6}{12}\right) = 0.9704$$

Use log linear discount factor with the linear interpolation formula:

$$\log\left(D\left(t,\frac{5}{12}\right)\right)$$

$$= \log\left(D\left(t,\frac{3}{12}\right)\right) + \frac{\log\left(D\left(t,\frac{6}{12}\right)\right) - \log\left(D\left(t,\frac{3}{12}\right)\right)}{\frac{6}{12}-\frac{3}{12}} \cdot \left(\frac{5}{12}-\frac{3}{12}\right)$$

$$= \log(0.9852) + \frac{\log(0.9704) - \log(0.9852)}{\frac{6}{12}-\frac{3}{12}} \cdot \left(\frac{5}{12}-\frac{3}{12}\right)$$

$$= -0.025$$

$$D\left(t,\frac{5}{12}\right)$$

$$= \exp\left(\log\left(D\left(t,\frac{5}{12}\right)\right)\right)$$

$$= \exp(-0.025)$$

$$= 0.9753$$

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Exponential Discount Factor Interpolation

Introduction
Exponential interpolation is one of the methods to interpolate discount factors. This section explains the methodology of exponential interpolation. We also show that exponentially interpolating discount factors is the same as linearly interpolating continuous compounding rates.

Notation

t_i	=	time point i.
r_i	=	continuous compounding rate at t_i.
df_i	=	discount factor at t_i.

Implementation
If we have got discount factors at t_1 and t_2, i.e. df_1 and df_2 respectively, then we can interpolate the discount factor df_i at time $t_1 < t_i < t_2$ as:

$$df_i = df_1^{\frac{t_i}{t_1}\times\frac{t_2-t_i}{t_2-t_1}} \times df_2^{\frac{t_i}{t_2}\times\frac{t_i-t_1}{t_2-t_1}}.$$

Example
If the discount factor at year 1 is 0.9900 and the discount factor at year 2 is 0.9800 then the discount factor at year 1.5 can be interpolated as:

$$df = 0.9900^{\frac{1.5}{1}\times\frac{2-1.5}{2-1}} \times 0.9800^{\frac{1.5}{2}\times\frac{1.5-1}{2-1}} \approx 0.9850.$$

Razor supports this directly and also via linearly interpolating continuous compounding rate.
We now show that to exponentially interpolate discount factors is the same as linearly interpolate continuous compounding rate.
We have the formulas:

$$df_i = e^{-r_i t_i}.$$

$$r_i = -\frac{1}{t_i}\ln df_i.$$

We substitute $df_i = df_1^{\frac{t_i}{t_1}\times\frac{t_2-t_i}{t_2-t_1}} \times df_2^{\frac{t_i}{t_2}\times\frac{t_i-t_1}{t_2-t_1}}$ into $r_i = -\frac{1}{t_i}\ln df_i$, then we get

$$\begin{aligned} r_i &= -\frac{1}{t_i}\ln df_i \\ &= -\frac{1}{t_i}\ln\left(df_1^{\frac{t_i}{t_1}\times\frac{t_2-t_i}{t_2-t_1}} \times df_2^{\frac{t_i}{t_2}\times\frac{t_i-t_1}{t_2-t_1}} \right) \end{aligned}$$

$$=-\frac{1}{t_i}\left[\ln\left(df_1^{\frac{t_i}{t_1}\times\frac{t_2-t_i}{t_2-t_1}}\right)+\ln\left(df_2^{\frac{t_i}{t_2}\times\frac{t_i-t_1}{t_2-t_1}}\right)\right]$$

$$=-\frac{1}{t_i}\left[\ln\left(e^{-r_1\times t_i\times\frac{t_2-t_i}{t_2-t_1}}\right)+\ln\left(e^{-r_2\times t_i\times\frac{t_i-t_1}{t_2-t_1}}\right)\right]$$

$$=-\frac{1}{t_i}\left[-r_1\times t_i\times\frac{t_2-t_i}{t_2-t_1}-r_2\times t_i\times\frac{t_i-t_1}{t_2-t_1}\right]$$

$$=r_1\times\frac{t_2-t_i}{t_2-t_1}+r_2\times\frac{t_i-t_1}{t_2-t_1}$$

$$=\frac{r_1t_2-r_1t_i+r_2t_i-r_2t_1}{t_2-t_1}$$

$$=\frac{r_1t_2-r_1t_i+r_1t_1-r_1t_1+r_2t_i-r_2t_1}{t_2-t_1}$$

$$=\frac{r_1(t_2-t_1)t_2+(r_2-r_1)(t_i-t_1)}{t_2-t_1}$$

$$=r_1+\frac{(r_2-r_1)}{t_2-t_1}(t_i-t_1).$$

Cubic Spline Interpolation

Given a piecewise continuous function that is twice differentiable $y=f(x), x\in$ we can interpolate a point $\hat{x}$ by defining a set of piecewise smooth continuous polynomials of degree 3 on intervals of $f(x)$ or in tabular form $y_i=f(x_0\ldots x_n)$ where an interval is defined as $[x_i,x_{i+1}]$ $i=[0,n-1]$. It is advantageous to use sets of lower order polynomials to estimate $f(x)$ to avoid oscillation errors on the edges of $f(x)$ that occur when using higher order polynomials. This is known as Runge's phenomenon.

A spline polynomial segment over the interval $[x_i,x_{i+1}]$ is of the form:

$$S_i(x)=a_i(\Delta x)^3+b_i(\Delta x)^2+c_i(\Delta x)+d_i \text{ for } \Delta x\in[x_i,x_{i+1}] \quad (0.1)$$

Therefore, the spline polynomial $S(x)$ is the set of $S_i(x)$ segments $i=[0,n-1]$. Since there are 4 coefficients a,b,c,d and n segments, there are 4n parameters that are needed to fully define $S(x)$. Since it is required that the spline function be piecewise continuous, the following condition must hold:

$$S_i(x_i)=y_i=S_{i+1}(x_{i+1}) \quad (0.2)$$

In order to ensure smoothness, the first and second derivatives of the spline segments must also be continuous:

$$S_i'(x_i) = S_{i+1}'(x_{i+1}), \quad S_i''(x_i) = S_{i+1}''(x_{i+1}) \tag{0.3}$$

Here we will consider the "Natural Spline" with boundary conditions $S_0''(x_0) = 0$ and $S_n''(x_n) = 0$ this will result in a linear interpolation off the edges.

We start by rewriting Eq. (0.1) where we substitute $S_i(x) = y_i$ as:

$$\begin{aligned} &\Delta x = x - x_i \\ &y_i = d_i \text{ since } \Delta x = x_i - x_i = 0 \end{aligned} \tag{0.4}$$

$$y_{i+1} = a_i \Delta x^3 + b_i \Delta x^2 + c_i \Delta x + d_i \tag{0.5}$$

Since we require the first and second derivatives to be smooth where the spline segments join we differentiate Eq. (0.4) twice:

$$y' = 3a_i \Delta x^2 + 2b_i \Delta x + c_i \tag{0.6}$$

$$y'' = 6a_i \Delta x + 2b_i \tag{0.7}$$

We determine the parameters a_i, b_i, c_i, d_i by rewriting the equations in terms of the of the second derivatives starting from Eq. (0.7):

$$\begin{aligned} S_i'' &= 6a_i \Delta x_i + 2b_i \\ &= 2b_i \\ S_{i+1}'' &= 6a_i \Delta x_i + 2b_i \end{aligned}$$

We can now define coefficients a_i, b_i as:

$$b_i = \frac{S_i''}{2}, \, a_i = \frac{\Delta S''}{6\Delta x} \tag{0.8}$$

Since we now have a_i, b_i, d_i (from Eq. (0.4)) we can substitute these into Eq. (0.5) in order to solve for c_i:

$$\begin{aligned} y_{i+1} &= \frac{\Delta S''}{6\Delta x} \Delta x^3 + \frac{S_i''}{2} \Delta x^2 + c_i \Delta x + y_i \\ c_i &= \frac{\Delta y}{\Delta x} - \frac{\Delta x (2S_i'' + S_{i+1}'')}{6} \end{aligned}$$

Based on Eq (0.3) we must ensure the first derivative at each i^{th} interval is that same such that Eq. (0.6) can be rewritten with $x = x_i$ as:

$$y' = 3a_i \Delta x^2 + 2b_i \Delta x + c_i = c_i \text{ since } \Delta x = (x_i - x_i)$$

We also need to look at the previous interval $\Delta x_{-1} = (x_i - x_{i-1})$ which from Eq. (0.6) becomes:

$$y' = 3a_{i-1} \Delta x_{-1}{}^2 + 2b_{i-1} \Delta x_{-1} + c_{i-1}$$

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Setting these two equations for interval i and $i-1$ equal to each other and substituting for a_i, b_i, c_i, d_i in terms of

$$y_i' = \frac{\Delta y}{\Delta x} - \frac{\Delta x\left(2S_i'' + S_{i+1}''\right)}{6}$$
$$= 3\left(\frac{\Delta S_{-1}''}{6\Delta x_{-1}}\right)\Delta x_{-1}{}^2 + 2\left(\frac{S_{i-1}''}{2}\right) + \frac{\Delta y_{-1}}{\Delta x_{-1}} - \frac{\Delta x_{-1}\left(2S_{i-1}'' + S_i''\right)}{6}$$

This equation is now simplified as:

$$\Delta x_{-1} S_{i-1}'' + (2\Delta x_{-1} + \Delta x) + \Delta x S_{i+1}'' = 6\left(\frac{\Delta y}{\Delta x} - \frac{\Delta y_{-1}}{\Delta x_{-1}}\right)$$

And finally substituting $\Delta x = h, S'' = \mathrm{S}$ and writing in a vector form:

$$h_{i-1}\mathrm{S}_{i-1} + (2h_{i-1} + h_i) + h_i \mathrm{S}_{i+1} = 6\left(\frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}}\right) \qquad (0.9)$$

Next apply the interval $i = [1, n]$ to Eq. (0.9) to generate a system of equations in matrix form:

$$\begin{bmatrix} h_0 2(h_0 + h_1) h_1 & & & \\ & h_1 2(h_1 + h_2) h_2 & & \\ & & \ddots & \\ & & & h_{n-3} 2(h_{n-3} + h_{n-2}) h_{n-2} \end{bmatrix} \begin{bmatrix} \mathrm{S}_1 \\ \mathrm{S}_2 \\ \vdots \\ \mathrm{S}_{n-2} \end{bmatrix} = 6 \begin{bmatrix} \frac{y_2 - y_1}{h_1} - \frac{y_1 - y_0}{h_0} \\ \frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1} \\ \vdots \\ \frac{y_{n-1} - y_{n-2}}{h_{n-2}} - \frac{y_{n-2} - y_{n-3}}{h_{n-3}} \end{bmatrix}$$

(0.10)

We do not solve for either S_0 or S_{n-1} since these end-point second derivatives are bound at 0 (natural spline). Since this is a symmetric, tridiagonal matrix we use a sparse matrix algorithm for reduction and back-substitution to solve for S_i. From Eq. (0.8) the values for a_i, b_i, c_i, d_i are:

$$a_i = \frac{\mathrm{S}_{i+1} - \mathrm{S}_i}{6h_i}, b_i = \frac{\mathrm{S}_i}{2}, c_i = \frac{y_{i+1} - y_i}{h_i} - \frac{2h_i\left(\mathrm{S}_i + \mathrm{S}_{i+1}\right)}{6}, d_i = y_i \qquad (0.11)$$

Example (Interpolate point .4 years)

N=5	Time X (in years)	Discount Factor Y
0	0	1
1	.25	0.99635611
2	.5	0.99111989
3	.75	0.98683149
4	1	0.970428

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$$\begin{bmatrix} 1 & .25 & 0 \\ .25 & 1 & .25 \\ 0 & .25 & 1 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} -0.0382 \\ 0.0227 \\ -0.2908 \end{bmatrix}$$

$$A^{-1}b = S$$

$$\begin{bmatrix} 1.0714 & -0.2857 & 0.0714 \\ -0.2857 & 1.1429 & -0.2857 \\ 0.0714 & -0.2857 & 1.0714 \end{bmatrix} \begin{bmatrix} -0.0382 \\ 0.0227 \\ -0.2908 \end{bmatrix} = \begin{bmatrix} -0.0682 \\ 0.1200 \\ -0.3208 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 \\ -0.0682 \\ 0.1200 \\ -0.3208 \\ 0 \end{bmatrix}$$

From equations (0.11) we derive the coefficients for the cubic spline polynomial

$$a = \begin{bmatrix} -0.0455 \\ 0.1255 \\ -0.2938 \\ 0.2138 \end{bmatrix}, b = \begin{bmatrix} 0 \\ -0.0341 \\ 0.0600 \\ -0.1604 \end{bmatrix}, c = \begin{bmatrix} 0.0033 \\ -0.0078 \\ 0.0139 \\ 0.0158 \end{bmatrix}, d = \begin{bmatrix} 1 \\ 0.9964 \\ 0.9911 \\ 0.9868 \end{bmatrix}$$

Since the point we are interpolating $\hat{x} = .4$ is on the second interval the spline polynomial from Eq. (0.1):

$$\begin{aligned} h &= \hat{x} - x_1 \\ \hat{y} &= a_1 h^3 + b_1 h^2 + c_1 h + d \\ &= 0.9948 \end{aligned}$$

Extrapolation Methods

Suppose we have N number of rates in our data sample.

Define:

$t^{(k)}$ = the time of the k^{th} rate in our sample such that $t^{(1)} < t^{(2)} < t^{(3)} < ... < t^{(N)}$

$r^{(k)}$ = the rate at time $t^{(k)}$

We have demonstrated in the previous section that we can find any rate r_j at time t_j for any $t_j \in \left(t^{(1)}, t^{(N)}\right)$. For example, for some i such that $t^{(i)} < t_j < t^{(i+1)}$, we can use the linear interpolation formula

$$r_j = r^{(i)} + \frac{r^{(i+1)} - r^{(i)}}{t^{(i+1)} - t^{(i)}} \cdot \left(t_j - t^{(i)}\right)$$

to find r_j.

However, sometimes we need to find r_j for $t_j < t^{(1)}$ or $t_j > t^{(N)}$ and it is particularly true at the beginning or the end of the yield curve where we do not have two rates to interpolate the rate in between. Under this situation, we will need extrapolation techniques to help find the rates. Again as in interpolation, r_j here has a very general definition, it can be zero rates, discount factors, etc.

Last Zero

This is a very simple and straightforward extrapolation method to extrapolate zero rates $Z_{t \to t_j}$ for some $t_j < t^{(1)}$ or $t_j > t^{(N)}$. This method is saying that we use the boundary zero rate as an approximation for any zero rate beyond the boundary rate. Thus, using last zero method, $Z_{t \to t_j} = r^{(1)}$ for any $t_j < t^{(1)}$ and $Z_{t \to t_j} = r^{(N)}$ for any $t_j > t^{(N)}$.

Example:

The 3-month zero rate $Z_{t \to \frac{3}{12}}$ is 6.01% and the 40-year zero rate $Z_{t \to 40}$ is 5.75%. Calculate the 1-month zero rate and 41-year zero rate using last zero method.

Answer:

$$r^{(1)} = Z_{t \to \frac{3}{12}} = 6.01\%$$

$$t^{(1)} = \frac{3}{12}$$

$$r^{(N)} = Z_{t \to 40} = 5.75\%$$

$$t^{(N)} = 40$$

Since $t_{\frac{1}{12}} = \frac{1}{12} < t^{(1)}$, using last zero method, we then obtain:

$$Z_{t \to \frac{1}{12}}$$
$$= r^{(1)}$$
$$= 6.01\%$$

Also $t_{41} = 41 > t^{(N)}$, using last zero method, we then obtain:

$$Z_{t \to 41}$$
$$= r^{(N)}$$
$$= 5.75\%$$

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Linear Zero

It is another method to extrapolate the zero rates $Z_{t\to t_j}$ for some $t_j < t^{(1)}$ and $t_j > t^{(N)}$. This method works as follows:

For $t_j < t^{(1)}$, we assume $Z_{t\to t_j}$ lies on the straight line produced by the two points $\left(t^{(1)}, r^{(1)}\right)$ and $\left(t^{(2)}, r^{(2)}\right)$. Mathematically, $Z_{t\to t_j}$ can be written as:

$$Z_{t\to t_j} = r^{(1)} + \frac{r^{(2)} - r^{(1)}}{t^{(2)} - t^{(1)}} \cdot \left(t_j - t^{(1)}\right)$$

For $t_j > t^{(N)}$, we assume $Z_{t\to t_j}$ lies on the straight line produced by the two points $\left(t^{(N-1)}, r^{(N-1)}\right)$ and $\left(t^{(N)}, r^{(N)}\right)$. Mathematically, $Z_{t\to t_j}$ can be written as:

$$Z_{t\to t_j} = r^{(N-1)} + \frac{r^{(N)} - r^{(N-1)}}{t^{(N)} - t^{(N-1)}} \cdot \left(t_j - t^{(N-1)}\right)$$

Example:

Given the 3-month zero rate is 6.01% and 6-month zero rate is 6.11%. Calculate the 1-month zero rate using linear zero.

Answer:

$$r^{(1)} = 0.0601$$

$$t^{(1)} = \frac{3}{12}$$

$$r^{(2)} = 0.0611$$

$$t^{(2)} = \frac{6}{12}$$

Since $t_j = \frac{1}{12} < t^{(1)}$, we use linear zero formula to extrapolate $Z_{t\to\frac{1}{12}}$:

$$Z_{t\to\frac{1}{12}}$$

$$= 0.0601 + \frac{0.0611 - 0.0601}{\frac{6}{12} - \frac{3}{12}} \cdot \left(\frac{1}{12} - \frac{3}{12}\right)$$

$$= 5.94\%$$

Example:

Given the 39.5-year zero rate is 5.75% and 40-year zero rate is 5.35%. Calculate the 41-year zero rate using linear zero.

Answer:

$$r^{(N-1)} = 0.0575$$

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$t^{(N-1)} = 39.5$

$r^{(N)} = 0.0535$

$t^{(N)} = 40$

Since $t_j = 41 > t^{(N)}$, we use linear zero formula to extrapolate $Z_{t \to 41}$:

$$Z_{t \to 41}$$

$$= 0.0575 + \frac{0.0535 - 0.0575}{40 - 39.5} \cdot (41 - 39.5)$$

$$= 4.55\%$$

Linear Discount Factor

This is exactly the same as linear zero except that we are extrapolating discount factors instead of zero rates. If we are given the discount rate $r^{(1)}, r^{(2)}, r^{(N-1)}$ and $r^{(N)}$

For $t_j < t^{(1)}$

$$D(t, t_j) = r^{(1)} + \frac{r^{(2)} - r^{(1)}}{t^{(2)} - t^{(1)}} \cdot (t_j - t^{(1)})$$ and

for $t_j > t^{(N)}$

$$D(t, t_j) = r^{(N-1)} + \frac{r^{(N)} - r^{(N-1)}}{t^{(N)} - t^{(N-1)}} \cdot (t_j - t^{(N-1)})$$

Example:

Given the 3-month discount factor is 0.9850 and the 6-month discount factor is 0.9698, calculate the 1-month discount factor using linear discount factor method.

Answer:

$r^{(1)} = 0.9850$

$t^{(1)} = \frac{3}{12}$

$r^{(2)} = 0.9698$

$t^{(2)} = \frac{6}{12}$

Since $t_j = \frac{1}{12} < t^{(1)}$, we use linear discount factor formula to extrapolate $D\left(t, \frac{1}{12}\right)$:

$$D\left(t, \frac{1}{12}\right)$$

$$= 0.9850 + \frac{0.9698 - 0.9850}{\frac{6}{12} - \frac{3}{12}} \cdot \left(\frac{1}{12} - \frac{3}{12}\right)$$

$$= 0.9951$$

Example:

Given the 39.5-year discount factor is 0.1062 and the 40-year discount factor is 0.1032, calculate the 41-year discount factor using linear discount factor method.

Answer:

$$r^{(N-1)} = 0.1062$$

$$t^{(N-1)} = 39.5$$

$$r^{(N)} = 0.1032$$

$$t^{(N)} = 40$$

Since $t_j = 41 > t^{(N)}$, we use linear discount factor formula to extrapolate $D(t,41)$:

$$D(t,41)$$

$$= 0.1062 + \frac{0.1032 - 0.1062}{40 - 39.5} \cdot (41 - 39.5)$$

$$= 0.09720$$

Linear Zero-Zero

Linear zero-zero is an extrapolation method to extrapolate zero rate for some $t_j < t^{(1)}$. This method assumes the zero rate r_0 at time 0 (t_0) is 0 and all the zero rates $Z_{t \leftarrow t_j}$ for some time $t_0 < t_j < t^{(1)}$ will lie on the straight line of the two points (t_0, r_0) and $(t^{(1)}, r^{(1)})$. Note that $(t_0, r_0) = (0,0)$ by assumption.

This allows us to calculate the zero rates $Z_{t \to t_j}$ using linear zero-zero extrapolation formula:

$$Z_{t \to t_j} = \frac{r^{(1)}}{t^{(1)}} \times t_j$$

Example:

The 3-month zero rate is 6.01%. Find the 1-month zero rate using linear zero-zero method.

Answer:

$$r^{(1)} = 0.0601$$

$$t^{(1)} = \frac{3}{12}$$

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Since $t_{\frac{1}{12}} = \frac{1}{12} < t^{(1)}$, we need to extrapolate the data to find $Z_{t \to \frac{1}{12}}$. Using linear-zero-zero formula:

$$Z_{t \to \frac{1}{12}}$$

$$= \frac{0.0601}{\frac{3}{12}} \times \frac{1}{12}$$

$$= 2.00\%$$

Linear Discount Factor-One (Currently Under Development)

6.3.4 Zero Treatment

Zero treatment can be specified at the curve and/or asset level. At the curve level the zero mode is an input to the interpolation and extrapolation in the curve building process.

At the asset level the cash deposit allows rate inputs to be specified as zeros. The modes available are:

Simple Zero

Compounded Zero

Continously-Compounded Zero

Cash Deposit

These are similar to the input zero methods at the curve level which are explained below. Hence a set of cash deposit assets with a zero method of continuously-compounded selected, is in affect a zero curve with no bootstrapping required.

When the interpolation and extrapolation method selected requires the computation of the zero rate (i.e. linear zero, log linear zero, last zero or linear zero zero), Razor users can further specify the definition of the zero rate in a number of ways. We currently support the following modes: simple, compounding, continuously-compounding, and cash-deposit. Default setting is cash-deposit if user specification is omitted.

Using similar denotations as above, that is,

$d_{t \to t_1} = t_1 - t =$ number of days from t to t_1

$D =$ number of days in a year

$P(t, t_1) =$ the price of a zero-coupon bond at time t maturing at time t_1

$F =$ the face value of the zero-coupon bond,

$D(t, t_1) =$ the discount factor at time t_1

we provide the following different definitions of the zero rate:

Simple Zero

Let us denote the annualised simple zero by $Z^S_{t\to t_1}$. The relationship between the future and present values is given by

$$P(t,t_1) = F\left(\frac{1}{1+Z^S_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}}\right) = F\cdot D(t,t_1)$$

and the conversions between $D(t,t_1)$ and $Z_{t\to t_1}$ can be stated as:

$$D(t,t_1) = \left(1+Z^S_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}\right)^{-1} \text{ and } Z^S_{t\to t_1} = \left(\frac{1}{D(t,t_1)}-1\right)\frac{D}{d_{t\to t_1}}$$

Compounded Zero

For compounded zero, users can specify different compounding frequencies (number of times compounding is performed in a year). For example, annual compounding is represented by setting the frequency to “1” (once a year), semi-annual to “2”, quarter to “4”, etc. We provide the following derivations for different compounding frequency cases.

Annual Compounding

Denoting the annualised compounded zero rate by $Z^C_{t\to t_1}$, The relationship between the present and future values is given by

$$P(t,t_1) = F\left(1+Z^C_{t\to t_1}\right)^{-\frac{d_{t\to t_1}}{D}} = F\cdot D(t,t_1)$$

and the conversions between the discount factor and the zero rate are

$$D(t,t_1) = \left(1+Z^C_{t\to t_1}\right)^{-\frac{d_{t\to t_1}}{D}} \text{ and } Z^C_{t\to t_1} = D(t,t_1)^{-\frac{D}{d_{t\to t_1}}} - 1$$

Semi-Annual Compounding

The relationship between the present and future values is given by

$$P(t,t_1) = F\left(1+\frac{Z^C_{t\to t_1}}{2}\right)^{-\frac{2d_{t\to t_1}}{D}} = F\cdot D(t,t_1)$$

and the conversions between the discount factor and the zero rate are

$$D(t,t_1)=\left(1+\frac{Z^C_{t\to t_1}}{2}\right)^{\frac{2d_{t\to t_1}}{D}} \quad \text{and} \quad Z^C_{t\to t_1}=2\left(D(t,t_1)^{-\frac{D}{2d_{t\to t_1}}}-1\right)$$

The Generalised Case

Generalising from above, the relationship between the present and future values for compounding frequency K is given by

$$P(t,t_1)=F\left(1+\frac{Z^C_{t\to t_1}}{K}\right)^{-\frac{Kd_{t\to t_1}}{D}}=F\cdot D(t,t_1)$$

and the conversions between the discount factor and the zero rate are

$$D(t,t_1)=\left(1+\frac{Z^C_{t\to t_1}}{K}\right)^{\frac{Kd_{t\to t_1}}{D}} \quad \text{and} \quad Z^C_{t\to t_1}=K\left(D(t,t_1)^{-\frac{D}{Kd_{t\to t_1}}}-1\right)$$

Continuously-Compounded Zero

For continuous-compounding, we have the following relationships

$$P(t,t_1)=F\exp\left(-Z^e_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}\right)=F\cdot D(t,t_1)$$

$$D(t,t_1)=\exp\left(-Z^e_{t\to t_1}\cdot\frac{d_{t\to t_1}}{D}\right) \text{and } Z^e_{t\to t_1}=-\frac{D}{d_{t\to t_1}}\log(D(t,t_1))$$

where $Z^e_{t\to t_1}$ represents the continuously-compounded zero rate.

Cash-Deposit

When $\frac{d_{t\to t_1}}{D}\le 1$, it is treated as a simple zero, and when $\frac{d_{t\to t_1}}{D}>1$, it is treated as an annually-compounded zero.

6.3.5 Summary of Bootstrapping Formulas

Number	Description	Formula
1	Zero rate from price of non-coupon bearing instrument	$Z_{t\to t_1}=\left(\frac{F}{P(t,t_1)}-1\right)\cdot\frac{D}{d_{t\to t_1}}$

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2	Discount factor from zero rate	$D(t,t_1) = \dfrac{1}{1 + Z_{t\to t_1} \dfrac{d_{t\to t_1}}{D}}$
3	Zero rate from discount factor	$Z_{t\to t_1} = \left(\dfrac{1}{D(t,t_1)} - 1\right) \cdot \dfrac{D}{d_{t\to t_1}}$
4	Discount factor from forward rate and previous corresponding discount factor	$D(t,t_2) = D(t,t_1)\left(1 + f(t,t_1,t_2) \cdot \dfrac{d_{t_1\to t_2}}{D}\right)^{-1}$
5	Forward rate from discount factors	$f(t,t_1,t_2) = \left(\dfrac{D(t,t_1)}{D(t,t_2)} - 1\right) \cdot \dfrac{D}{d_{t_1\to t_2}}$
6	Discount factor from swap rate and previous discount factors	$D(t,N) = \dfrac{1 - \dfrac{r_s(c,N)}{c} \sum_{i=1}^{cN-1} D\left(t, \dfrac{i}{c}\right)}{1 + \dfrac{r_s(c,N)}{c}}$
7	Effective swap rate from discount factors	$r_s(c,N) = \dfrac{c(1 - D(t,N))}{\sum_{i=1}^{cN} D\left(t, \dfrac{i}{c}\right)}$
8	Linear Interpolation	$r_\alpha = r_i + \dfrac{r_j - r_i}{t_j - t_i}(t_\alpha - t_i)$

6.3.6 Demonstration of Bootstrapping

Sample Curve

	Contracts Available	
Contract Number	Contract Type	Contract Maturity
1	Cash Deposit	1-month
2	Cash Deposit	3-month
3	Bill Futures starting in 3 months	3-month
4	Bill Futures starting in 6 months	3-month

5	Swap with quarterly payments	1-year
6	Swap with quarterly payments	3-year
7	Swap with **semi-annual** payments	5-year
8	Swap with **semi-annual** payments	10-year

Bootstrapping Method

Time	Contract Used	Method to obtain zero rates
1-month	Contract (1)	Formula (1)
3-month	Contract (2)	Formula (1)
6-month	Contract (3)	Formula (4) then (3)
9-month	Contract (4)	Formula (4) then (3)
1-year	Contract (5)	Formula (6) then (3)
1.25-year	Contract (6)	Linear interpolate the 1-year swap rate and the 3-year swap rate with formula (8) to find the 1.25-year swap rate. Use formula (6) to bootstrap the 1.25-year discount factor and convert to 1.25-year zero rate by formula (3).
3-year	Contract (6)	Formula (6) then (3)
3.5-year	Contract (7)	Use formula (7) to find the 3-year effective swap rate with **semi-annual** payments. Linear interpolate the 5-year swap rate and 3-year effective swap rate with formula (8) to find the 3.5-year swap rate. Use formula (6) to bootstrap the 3.5-year discount factor and convert to 3.5-year zero rate by formula (3).

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5-year	Contract (7)	Formula (6) then (3)
5.5-year	Contract (8)	Linear Interpolate the 5-year swap rate and the 10-year swap rate with formula (8) to find 5.5-year swap rate. Use formula (6) to bootstrap the 5.5-year discount factor and convert to 5.5-year zero rate by formula (3).
10-year	Contract (8)	Formula (6) then (3)

Up to the moment, we assume each coupon payment period is equal. In reality, payment periods may not be equal for each period *i* (due to day adjustments and day count convention). Razor also caters for this case. A new formula is used for unequal coupon payment period. For equal coupon payment periods, the formula is:

$$D(t,N) = \frac{1 - \frac{r_s(c,N)}{c}\sum_{i=1}^{cN-1} D\left(t, \frac{i}{c}\right)}{1 + \frac{r_s(c,N)}{c}}$$

where c is the number of payments per year, and

$r_s(c,N)$ is the annualised swap rate.

To cater for unequal coupon payment period, the formula becomes:

$$D(t,N) = \left(1 - \sum_{i=1}^{N-1} Coupon_i \cdot D(t,i)\right) / \left(1 + Coupon_N\right)$$

where $Coupon_i = r_s \cdot DCF_i$, and

DCF_i is the year count fraction for period *i* based on day count convention of the swap.

Similarly, instead of defining effective swap rate (implied by discount factors) using same assumption in the following formula:

$$r_s(c,N) = \frac{c(1 - D(t,N))}{\sum_{i=1}^{cN} D\left(t, \frac{i}{c}\right)}$$

the year count fraction for period *i* should also be used:

$$Coupon_N = \frac{1 - D(t,N)}{\sum_{i=1}^{N} D(t,i) \cdot DCF_i}.$$

In addition, when linearly interpolating between two swap rates, the year count fractions (instead of number of calendar days) should be used:

$$r_\alpha = r_i + \frac{r_j - r_i}{DCF_j - DCF_i}\left(DCF_\alpha - DCF_i\right)$$

where DCF_i is the year count fraction from t_0 to t_i

6.4 CDS Spread Curves

For details of the support for CDS spread curves, please refer to section 16.1.5 .

6.5 FX Market Data

Razor supports a term structure of FX spot and FX forward rates.

6.6 Equity Market Data

Razor supports Equity/Stock spot prices and Equity Index spot, additionally with a term structure of either continuously compounding forecast dividend yield, or discrete dividend payments.

6.7 Commodity Forward Curves

Razor supports Commodity Forward curves. So for example ('NGO Spot', ‘NGO 1M', ‘NGO 3M', ‘NGO 6M', ‘NGO 1Y', ‘NGO 2Y', ‘NGO 3Y’)

6.8 Volatility Market Data

The basic building block for implied volatilities in Razor is the termstructure of implied volatilities for different option expiry dates. A volatility value for a specified expiry date can be obtained from this curve. If the requested date does not exist as a volatility data point on this term structure, then the volatility is derived.

If the expiry date for the required volatility is between two existing expiry dates on the volatility curve then the required volatility is derived by linear interpolation. Otherwise if the expiry date is before the start date, or after the end date then straight line extrapolation is used.

A set of these term structures will represent the volatility surface for a specific asset.

6.8.1 Interest Rate Volatility

For interest rate volatilities, Razor supports an implied volatility surface, by option expiry date, and underlying term to maturity. In other words the volatility surface is represented as a set of volatility term structures, one for each underlying term to maturity.

The volatilities are implied from the market option prices. Currently to support volatilities for different strikes or deltas (degree of moneyness) requires the user to define a volatility surface for each strike or delta, and then a specific surface with the closest strike or delta will be associated with the trade. Hence currently there is no interpolation across the strike dimension.

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If the underlying term to maturity for which a volatility is required does not match a known one on the surface, then linear interpolation is employed between volatilities found from term structures for known underlying terms to maturity, which span the underlying term to maturity of the option for which an implied volatility is required. Otherwise straight line extrapolation is used beyond either edge of the surface.

The interest rate volatility surface as described above is currently implemented and used by the swaption and bond option pricing routines.

All other interest rate option products (including caps and floors) currently use the volatility surface defined by the option expiry date and delta, as described below. Currently only the volatility for the ATM delta is used.

6.8.2 Equity Volatility

For equity volatilities, Razor supports a term structure of implied equity volatilities by option expiry date, and strike. Alternatively the volatility data may be provided by option expiry date and delta. In other words the volatility surface is represented as a set of volatility term structures, one for each strike or delta.

If the strike or delta does not match, then linear interpolation is used between volatilities for known strikes or deltas which span the strike or delta for which an implied volatility is required. Otherwise straight line extrapolation is used beyond either edge of the surface.

Note that even though volatilities for different deltas may be provided, currently for this method only volatility for the ‘at-the-money’ delta is used by the pricing / revaluation.

At present, local volatility modelling is not currently supported in Razor.

Currently only the vanilla equity option and vanilla index option (European and American) use volatilities for specific expiry and strike, on the basis that volatility data is provided in this form. Otherwise only a single ATM volatility can be used as provided specifically for that trade or product by configuration.

6.8.3 FX Volatility

Treatment of FX volatilities is as above for Equity volatility.

Parametric stochastic volatility (Heston), construction from risk reversal and broker strangle spreads for 10% and 25% deltas and ATM volatility are not currently supported.

Currently within each of the fx option models, only a single ATM volatility can be used as provided specific for that trade or product by configuration.

6.8.4 Commodity Volatility

Treatment of Commodity volatilities is as above for Equity volatility.

Currently only the vanilla commodity option uses volatilities for specific expiry and strike, if volatility data is provided in this form. Otherwise only a single ATM volatility can be used as provided specific for that trade or product by configuration.

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6.9 Calendars

There are various named calendars in RAZOR. Calendars contain date events, which define events such as holidays. There should be a calendar defined for every currency code, as these are used in the date roll calculations.

Chapter 7
RAZOR'S Product Support

7.1 Design Objectives in RAZOR 'S Product Support

RAZOR's product support is designed with a number of goals in mind:

Completeness of representation: Financial products differ greatly in characteristics even within the same product class. Any representation of the product should strive to embody as much information about the characteristics of the financial product as possible.

Flexibility: Innovation is rife within the financial industry. RAZOR's product support must be flexible enough to handle new products as they come along.

Accessibility: RAZOR's representation of the product should, as much as is possible, be based around an open standard, to make collaboration with other systems easier and to minimise vendor-specific risk. The pricing strategy of the products should be able to incorporate user models easily.

7.2 Product Representation

FPML (Financial products Markup Language) is the industry-standard protocol for complex financial products. It is based on XML (Extensible Markup Language), the standard meta-language for describing data shared between applications. All categories of privately negotiated derivatives will eventually be incorporated into the standard. Version 1.0 of FPML covers interest rate swaps and Forward Rate Agreements (FRA's). Version 2.0 extends the interest rate product coverage to the most common option products, including caps, floors, swaptions, and cancellable and extendible swaps. Version 3.0 covers different asset classes, this version includes the interest rate work of version 2.0 and additionally covers FX and Equity Derivatives.

The standard, which is freely licensed, is intended to automate the flow of information across the entire derivatives partner and client network, independent of the underlying software or hardware infrastructure supporting the activities related to these transactions.

RAZOR uses FPML version 3.0 Markup when that product is covered by the standard. Further information on FPML can be found on the FPML website.

7.2.1 The FinMark Schema

FINMARK is Razor's representation of an organisation's deals. It encapsulates FPML, but adds additional elements in order to better support the risk management process.

Schema Diagram

FinMark Schema			
Name: Type	**Occurs**	**Size**	**Description**
scheme string	1..1	20	An identifier that allows us to run multiple trade sets in the one database.
deal Deal	0..n		The deal
ccyPair CurrencyPair	0..n		
party Party	0..n		
whatIfDealScenarios WhatIfDealScenarios	0..1		

Where a deal is represented as:

Deal Schema			
Name: Type	**Occurs**	**Size**	**Description**
sysId int	1..1		
dealHeaderDealId string	1..1	20	The unique identifier for the deal
dealHeaderDealType string	1..1	20	This is used to identify the type of structure for structured deals - e.g. CONDOR
dealHeaderDealDate date	1..1		The date the deal was traded
dealHeaderStatus string	1..1	20	The current status of the deal
trade Trade	0..n		

And the Trade is:

Trade Schema			
Name: Type	**Occurs**	**Size**	**Description**
sysId int	1..1		
tradeHeaderTradeId string	1..1	30	This identifier uniquely identifies the trade
tradeHeaderTradeDate date	1..1		The date that this particular trade was done

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Trade Schema			
Name: Type	**Occurs**	**Size**	**Description**
tradeHeaderTradeType string	1..1	20	This code identifies the trade type and is used to map to the appropriate pricing model
tradeHeaderCreditLine string	0..1	20	This identifier provides the capability to link trades with specific lines of credit
tradeHeaderDealer string	0..1	30	Identifies the dealer who did the trade
tradeHeaderCounterparty string	0..1	20	Identifies the counterparty of the trade
tradeHeaderInternalUnit string	1..1	20	Identifies the internal unit who is responsible for the trade
tradeHeaderBuySell string	1..1	4	Indicates whether the internal unit is long or short the trade
tradeHeaderStatus string	1..1	20	Indicates the current status of this particular trade
guarantees Guarantees	0..1		Guarantees provide information about any guarantors in the deal
collateralAgreementId string	0..1	20	ID of the collateral agreement if the trade has been lodged against
rightToBreakSchedule RightToBreakSchedule	0..1		Defines a schedule for allowing the bank to break the contract extensions
extensions TradeExtensions	0..1		Extensions allow clients to extend the trade with additional data
productGenericInterestRate GenericInterestRate	1..1		A product that can representing generic interest rate payments or cashflows
productCashPayment CashPayment	1..1		A product that can represent single cash payments
productCreditLine CreditLine	1..1		A product that represents a line of credit
productLetterOfCredit LetterOfCredit	1..1		A product that represents a letter of credit
productFxleg fpmlFXLeg	1..1		A product that represents an FX deal
productFxSimpleOption fpmlFXOptionLeg	1..1		A product that represents an FX Vanilla Option deal
productFxBarrierOption fpmlFXBarrierOption	1..1		A product that represents an FX Barrier Option deal
productFxDigitalOption fpmlFXDigitalOption	1..1		A product that represents an FX Digital Option deal

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Trade Schema			
Name: Type	Occurs	Size	Description
productFxAverageRateOption fpmlFXAverageRateOption	1..1		A product that represents an FX Average Rate deal
productFxOption FXOption	1..1		A product that represents an FX vanilla option or a vanilla option with single or double barriers
productSwap fpmlSwap	1..1		A product that represents an interest rate swap
productFra fpmlFRA	1..1		A product that represents an interest rate FRA
productEquityOption fpmlEquityOption	1..1		A product that represents an equity option
productCapFloor fpmlCapFloor	1..1		A product that represents an interest rate cap or floor
productSwaption fpmlSwaption	1..1		A product that represents an interest rate
productBondOption BondOption	1..1		A product that represents a bond option
productEquity Equity	1..1		product that represents a held or forward equity
productCreditDefaultSwap fpmlCreditDefaultSwap	1..1		A product that represents an credit default swap
productBond fpmlBond	1..1		A product that represents a bond
otherPartyPayment OtherPartyPayment	0..n		Any additional payments made to the counterparty

7.3 Adding New Products

In the back-end products are mapped to pricing models via mapping rules specified in XML. Products are also mapped to input screens on the front- end using mapping rules.

Adding pricing support for new products into RAZOR involves adding a DLL into the *Master/bin* directory and also the *Slave/bin* directory on the master computer and all its slave computers. This DLL prices the product and also contains functions to extract information from the product that the RAZOR engine needs in order to incorporate the product in the risk management process.

The XML representation of the product can be incorporated in one of two ways:

The easiest way to represent a new product is to choose a product schema that's similar to the one being represented. Any information not contained in the existing schema can be added using the product extension schema.

The other way to add a new product representation to RAZOR is to create an XML Schema for the product, and then to generate the necessary support files to link into RAZOR using the *metatype framework*.

7.4 Pricing Contexts

When evaluating the impact a financial product has on credit risk, there may be several different exposures depending on the *context* you are looking at the product at. As an example; a bond can have two exposures - the exposure to the counterparty who issued the bond, and an exposure to the seller of the bond. When determining the exposure of the bond the particular pricing context must be taken into account.

7.5 Transitioning

7.5.1 Market Transitioning

The pricing adapter for a trade may need to transition certain rates between different node dates. An example for when this needs to occur is on swaps. If a rate set occurs between two node dates, the swap pricing model linear interpolates between the rate that occurred on the last node date and the rate that occurs on the current node date to determine the implied rate occurring at the rate set date.

Obviously, this means that the pricing model has to have the information available for the relevant rates on previous node dates. The pricing adapter achieves this through the use of the *market transition cache* parameter on the pricing request structure. The pricing adapter is responsible for allocating an area of memory that it can use as a cache, in order to keep any historical market information around that it needs to fulfil pricing requests.

Historical Data processing between time nodes

Any processing on the current time node which requires historical data for market data events which have occurred in the past on dates between the current time node and the previous is derived by linear interpolation between the cached observed values saved from the last time node and current values at the current time node.

The processing at the current time node will be a function of cached historical market data, linearly interpolated missing historical market data, and the current market data.

So for example for an average rate option, the market transition cache will contain the set of rate fixings which have already occurred, or a current average representative of these fixings. At each time node this cache will be updated if there is a fixing on the current time node, and also with any fixings that should have occurred since the last time node. These fixings will be derived by linear interpolation between current observations and those observations from the last time node.

This processing is specific and customized for each product supported so to fully support the transitioning of the product forward through time along a path of the simulation process. This also supports the situation during simulation where the expiry date falls between two time nodes, and hence there is the need to determine whether the option was exercised at expiry.

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Rate Resets between time nodes

The process for supporting rate resets as transitioning through time along a path in the simulation process is as follows. The full set of floating flow cashflows are initialized from the current market on the valuation date and then cached, so that it may be updated during the transition along a path. For each forward time node transitioning along a path, the full set of floating cashflows are reprocessed and updated where needed, to effect the transitioning along the path.

For each floating flow cashflow c_i we have the fixing date t_{f_i}, the start date t_{s_i}, and the end date t_{e_i}.

The initial set of floating flow cashflows.

The floating flow cashflows are initialized from today's market (ie time 0, t_0) and as has been already been fixed with the trade. All previous fixings and possibly also todays should already be set and marked as fixed.

a. If today is a fixing date and the floating rate is not already set then it is observed and set from today's market, and marked as fixed. It is set to the forward simple rate from the start date t_{s_i} to the end date t_{e_i}.

b. All other fixings which occur in the future will be forward forecast from today's market, with each flows observation date set to today's date. They are not marked as fixed. The rate is set to the forward simple rate from the start date t_{s_i} to the end date t_{e_i}.

c. For any previous fixings which have not been observed as yet, then it is set to an observation from today's market, and marked as fixed. It is set to the forward simple rate from the start date $t_0 + \left(t_{s_i} - t_{f_i}\right)$ to the end date $t_0 + \left(t_{e_i} - t_{f_i}\right)$. This represents the best estimate that can be made from today as to what the fixing should have been.

The set of floating flow cashflows at a forward time node

The market scenario is simulated and built with respect to the time node date t_j. All floating rate flows which are already fixed are skipped. Those floating flow rates which are not already fixed are processed as outlined in the cases above (except t_0 is replaced with t_j), but with one additional case to be handled.

For any previous fixings which have been previously observed as forecast in the future, but for which the fixing date now lays between last time node and the current time node, then the rate will be calculated as follows. It is the rate which is linearly interpolated between the last observed rate, and the forecast observed rate from the current time node – ie the forward simple rate from the start date $t_j + \left(t_{s_i} - t_{f_i}\right)$ to the end date $t_j + \left(t_{e_i} - t_{f_i}\right)$. The observed date is set to the fixing date t_{f_i}, and the cashflow is market as fixed.

7.5.2 Trade Transitioning

Path-dependent trades may need to transition into different kinds of products if, for example, barrier levels are breached. The pricing adapter may make use of a *trade transition cache* in order to store the state of trades during a path.

Chapter 8
Common XML Structures

fpmlInterestRateStream Schema			
Name: Type	**Occurs**	**Size**	**Description**
payerPartyReference fpmlPartyReference	1..1		The identifier of the paying party
receiverPartyReference fpmlPartyReference	1..1		The identifier of the receiving party
calculationPeriodDates fpmlCalculationPeriodDates	1..1		Indicates the schedule that the floating rate calculations occurs
paymentDates fpmlPaymentDates	1..1		Indicates the schedule that the date payments occur
resetDates fpmlResetDates	1..1		This structure indicates when floating rate resets occur
calculationPeriodAmount fpmlCalculationPeriodAmount	1..1		This structure indicates how the amounts to be paid is determined
stubCalculationPeriodAmount fpmlStubCalculationPeriodAmount	0..1		This structure indicates how the stub period amounts to be paid are determined
principalExchanges fpmlPrincipalExchanges	0..1		Determines if and when the exchange
cashflows fpmlCashflows	0..1		This type gives us the fixed cash flows represented by the product

The fpmlCalculationPeriodDates structure allows us to generate the schedule for working out when floating rate calculations are made.

fpmlCalculationPeriodDates Schema			
Name: Type	**Occurs**	**Size**	**Description**
effectiveDate fpmlUnadjustedDate	1..1		
terminationDate fpmlUnadjustedDate	1..1		
calculationPeriodDatesAdjustments fpmlDateAdjustments	1..1		
calculationPeriodFrequency fpmlCalculationPeriodFrequency	0..1		
id string	1..1	50	

The fpmlPaymentDates structure allows us to generate the dates payments occur on.

fpmlPaymentDates Schema			
Name: Type	Occurs	Size	Description
calculationPeriodDatesReference fpmlRef	0..1		
paymentFrequency fpmlTenor	1..1		
payRelativeTo fpmlDateRelativeTo	0..1		
paymentDatesAdjustments fpmlDateAdjustments	0..1		

The fpmlResetDates indicate when floating rate resets are done.

fpmlResetDates Schema			
Name: Type	Occurs	Size	Description
calculationPeriodDatesReference fpmlRef	1..1		
resetRelativeTo fpmlDateRelativeTo	1..1		
fixingDates fpmlRelativeDateOffset	1..1		
resetFrequency fpmlTenor	1..1		
resetDatesAdjustments fpmlDateAdjustments	1..1		

The fpmlCalculationPeriodAmount element determines how the amounts to be paid are calculated.

fpmlCalculationPeriodAmount Schema			
Name: Type	Occurs	Size	Description
calculation fpmlCalculation	1..1		
knownAmountSchedule fpmlAmountSchedule	1..1		

The fpmlStubCalculationPeriodAmount structure determines how the amounts paid on stub periods are calculated.

fpmlStubCalculationPeriodAmount Schema			
Name: Type	Occurs	Size	Description
calculationPeriodDatesReference fpmlRef	1..1		
initialStub fpmlStub	0..1		

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fpmlStubCalculationPeriodAmount Schema			
Name: Type	**Occurs**	**Size**	**Description**
finalStub fpmlStub	0..1		

The fpmlCashflows type gives us the fixed cash flows represented by the product.

fpmlCashflows Schema			
Name: Type	**Occurs**	**Size**	**Description**
cashflowsMatchParameters boolean	0..1		Indicates whether the cashflows could be regenerated from the parametric information without any loss of information
principalExchange fpmlPrincipalExchange	0..n		The initial, intermediate and final principal exchange amounts
paymentCalculationPeriod fpmlPaymentCalculationPeriod	0..n		The adjusted payment date and associated calculation period parameters required to calculate the payment amount

The fpmlPrincipalExchanges structure determines if and when the exchange of principal is done. Note that any exchange of principal could have settlement risk implications.

Generic Interest Rate Example

```
<genericInterestRate>
 <interestRateStream>
    <payerPartyReference href="#ITE"/>
    <receiverPartyReference href="#BAN"/>
    <cashflows>
       <paymentCalculationPeriod>
          <adjustedPaymentDate>2002-09-30</adjustedPaymentDate>
          <fixedPaymentAmount>
             <currency>AUD</currency>
             <amount>1000000.000000</amount>
          </fixedPaymentAmount>
       </paymentCalculationPeriod>
    </cashflows>
 </interestRateStream>
 <interestRateStream>
    <payerPartyReference href="#BAN"/>
    <receiverPartyReference href="#ITE"/>
    <cashflows>
       <paymentCalculationPeriod>
          <adjustedPaymentDate>2002-09-30</adjustedPaymentDate>
          <calculationPeriod>
             <adjustedStartDate>2002-09-30</adjustedStartDate>
             <adjustedEndDate>2002-12-30</adjustedEndDate>
             <notionalAmount>
```

```
                <currency>AUD</currency>
                <amount>1000000.000000</amount>
            </notionalAmount>
            <floatingRateDefinition/>
        </calculationPeriod>
    </paymentCalculationPeriod>
    <paymentCalculationPeriod>
        <adjustedPaymentDate>2002-12-30</adjustedPaymentDate>
        <calculationPeriod>
            <adjustedStartDate>2002-12-30</adjustedStartDate>
            <adjustedEndDate>2003-03-30</adjustedEndDate>
            <notionalAmount>
                <currency>AUD</currency>
                <amount>1000000.000000</amount>
            </notionalAmount>
            <floatingRateDefinition/>
        </calculationPeriod>
    </paymentCalculationPeriod>
    <paymentCalculationPeriod>
        <adjustedPaymentDate>2003-03-30</adjustedPaymentDate>
        <calculationPeriod>
            <adjustedStartDate>2003-03-30</adjustedStartDate>
            <adjustedEndDate>2003-06-30</adjustedEndDate>
            <notionalAmount>
                <currency>AUD</currency>
                <amount>1000000.000000</amount>
            </notionalAmount>
            <floatingRateDefinition/>
        </calculationPeriod>
    </paymentCalculationPeriod>
    <paymentCalculationPeriod>
        <adjustedPaymentDate>2003-06-30</adjustedPaymentDate>
        <calculationPeriod>
            <adjustedStartDate>2003-06-30</adjustedStartDate>
            <adjustedEndDate>2003-09-30</adjustedEndDate>
            <notionalAmount>
                <currency>AUD</currency>
                <amount>1000000.000000</amount>
            </notionalAmount>
            <floatingRateDefinition/>
        </calculationPeriod>
    </paymentCalculationPeriod>
    <paymentCalculationPeriod>
        <adjustedPaymentDate>2003-09-30</adjustedPaymentDate>
        <fixedPaymentAmount>
            <currency>AUD</currency>
            <amount>1000000.000000</amount>
        </fixedPaymentAmount>
    </paymentCalculationPeriod>
  </cashflows>
 </interestRateStream>
</genericInterestRate>
```

Chapter 9
FX and FX Derivatives

9.1 FX Forwards

9.1.1 Description of Instrument

Spot and Forward FX transactions involve the exchange of cash in one currency for cash of another currency. With a spot FX transaction the money is exchanged on the "spot" date, which is determined by the market convention for the two currencies being exchanged. Normally the convention is that the spot date is two business, or "good", days forward from today, and must be business days in each currency's home country.

An FX Swap is a combination of two FX Forwards; one being the reverse of the other, but being forward in time. This is represented in RAZOR as being a deal containing the two FX Forward products.

9.1.2 XML Representation

fpmlFXLeg Schema			
Name: Type	**Occurs**	**Size**	**Description**
productType string	1..1	50	Indicates the type of product
exchangedCurrency1 fpmlCurrencyFlow	1..1		This is the first of the two currency flows that define a single leg of a standard foreign exchange transaction.
exchangedCurrency2 fpmlCurrencyFlow	1..1		This is the second of the two currency flows that define a single leg of a standard foreign exchange transaction.
valueDate date	1..1		The date on which both currencies traded will settle.
exchangeRate fpmlFXRate	1..1		The rate of exchange between the two currencies.
nonDeliverableForward fpmlFXCashSettlement	0..1		Used to describe a particular type of FX forward transaction that is settled in a single currency.

FX Forward Example

```
<fxleg>
  <exchangedCurrency1>
     <payerPartyReference href = "ITE"/>
     <receiverPartyReference href = "BAN"/>
     <paymentAmount>
        <currency>AUD</currency>
        <amount>10000000</amount>
```

```
        </paymentAmount>
    </exchangedCurrency1>
    <exchangedCurrency2>
        <payerPartyReference href = "BAN"/>
        <receiverPartyReference href = "ITE"/>
        <paymentAmount>
            <currency>USD</currency>
            <amount>5600000</amount>
        </paymentAmount>
    </exchangedCurrency2>
    <valueDate>2003-12-21</valueDate>
    <exchangeRate>
        <quotedCurrencyPair>
            <currency1>AUD</currency1>
            <currency2>USD</currency2>
            <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
        </quotedCurrencyPair>
        <rate>0.5600</rate>
    </exchangeRate>
</fxleg>
```


9.1.3 Pricing

9.1.4 Currency Calculations

Exchange rates are the rate at which you exchange one unit of one currency for another currency. For example, an exchange rate of AUD/USD 0.5406 means that we are exchanging Australian dollars for US dollars and that we are getting 0.5406 cents of one currency for one unit of the other currency. The question becomes - which currency are we getting 0.5406 cents for 1 dollar of the other currency?

The answer is that this depends on the *quote convention*. The quote convention describes the market convention for which currency is the numerator in the fraction $\frac{0.5406}{1}$, and which is the denominator. The numerator and denominator currency is specified for each unique *currency pair*.

Interest rate parity describes the relationships of interest rates in different currencies with the behaviour of exchange rates. This mechanism com- pares investing in the risk-free rate locally with exchanging the money into a foreign currency and investing offshore. The investor should be able to achieve the same rate of return by exchanging money to the foreign exchange rate and investing offshore and then exchanging the money back into local terms, as he is to investing locally at the risk-free rate or an arbitrage opportunity will exist.

FX spot and forward deals are agreements to exchange amounts denominated in differing currencies at an agreed exchange rate. FX physicals can be priced in two ways:

The value of the deal can be seen as the difference in the agreed exchange rate and the forward exchange rate when the deal is to be settled.

The value of the deal can be seen as the difference in the discounted value of the forward cash flows, converted to a common currency. RAZOR uses this approach to value FX Physical trades.

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Let

C_{ccy1} = the cash flow denominated in currency $_{ccy1}$

C_{ccy2} = the cash flow denominated in currency $_{ccy2}$

df_{ccy1} = the $ccy1$ discount factor from spot to settlement

df_{ccy2} = the $ccy2$ discount factor from spot to settlement

S_{ccy1}^{val} = the $ccy1 \rightarrow val$ exchange rate

S_{ccy2}^{val} = the $ccy2 \rightarrow val$ exchange rate

Then

$$V = C_{ccy1} df_{ccy1} S_{ccy1}^{val} + C_{ccy2} df_{ccy2} S_{ccy2}^{val}$$

9.2 FX Vanilla Options

9.2.1 Description of Instrument

FX Vanilla Options give the purchaser the right to buy (vanilla calls) or sell (vanilla put) one of the currency in exchange for the other currency at the *strike price*. Note that a call on one currency is the same as a put on the other currency.

9.2.2 XML Representation

FX Vanilla Options are specified in the FPML version 3 standards.

XML Schema

fpmlFXOptionLeg Schema			
Name: Type	**Occurs**	**Size**	**Description**
productType string	1..1	50	Indicates the type of product
buyerPartyReference fpmlPartyReference	1..1		
sellerPartyReference fpmlPartyReference	1..1		
expiryDateTimeExpiryDate date	1..1		
expiryDateTimeExpiryTime time	1..1		
expiryDateTimeCutName string	0..1		
exerciseStyle string	1..1		The manner in which the option can be exercised

fpmlFXOptionLeg Schema			
Name: Type	**Occurs**	**Size**	**Description**
fxOptionPremium fpmlFXOptionPremium	0..n		Premium amount or premium installment amount for an option.
valueDate date	1..1		The date on which both currencies traded will settle
cashSettlementTerms fpmlFXCashSettlement	0..1		This optional element is only used if an option has been specified at execution time to be settled into a single cash payment. This would be used for a non-deliverable option
putCurrencyAmountCurrency string	1..1		The currency in which an amount is denominated
putCurrencyAmountAmount decimal	1..1		The monetary quantity in currency units
callCurrencyAmountCurrency string	1..1		The currency in which an amount is denominated
callCurrencyAmountAmount decimal	1..1		The monetary quantity in currency units
fxStrikePriceRate decimal	1..1		
fxStrikePriceStrikeQuoteBasis string	1..1		The method by which the strike rate is quoted
quotedAs fpmlQuotedAs	1..1		Describes how the option was quoted

FX European Option Example

```
<fxSimpleOption>
  <productType>Foreign Exchange</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
     <expiryDate>2004-05-31</expiryDate>
     <expiryTime>1200</expiryTime>
  </expiryDateTime>
  <exerciseStyle>European</exerciseStyle>
  <fxOptionPremium>
     <payerPartyReference href="ITE"/>
     <receiverPartyReference href="BAN"/>
     <premiumAmount>
        <currency>AUD</currency>
        <amount>1000.00</amount>
     </premiumAmount>
     <premiumSettlementDate>2003-06-02</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2004-06-02</valueDate>
  <putCurrencyAmount>
```

```
    <currency>USD</currency>
    <amount>550000.000000</amount>
  </putCurrencyAmount>
  <callCurrencyAmount>
    <currency>AUD</currency>
    <amount>1000000.000000</amount>
  </callCurrencyAmount>
  <fxStrikePrice>
    <rate>0.550000</rate>
    <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
  </fxStrikePrice>
</fxSimpleOption>
```


9.2.3 Pricing

When valuing currency options we will call one currency involved in the option contract the *foreign currency*. This currency is the *denominator* in the quote convention for the currency pair. The other currency, the *numerator* in the currency pair's quote convention, we will call the *domestic currency*. Our valuation will then be pricing in terms of the domestic currency. We are going to assume that the exchange rate follows a geometric Brownian motion process.

Let

S = the exchange rate

r = the annualised domestic rate, continuously compounded

r_f = the annualised foreign interest rate, continuously compounded

σ = the exchange rate volatility

X = the strike rate

T_x = the time from today to expiry, annualised

T_d = the time from spot to delivery, annualised

$F_0 = S_0 e^{(r-r_f)T_d}$

N is the cumulative normal distribution function.

$$d_1 = \frac{\ln\left(\frac{F_0}{X}\right) + \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}} \qquad d_2 = \frac{\ln\left(\frac{F_0}{X}\right) - \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

The European call price is given by

$$c = e^{-rT_d}(F_0 N(d_1) - XN(d_2))$$

The European put price is given by

$$p = e^{-rT_d}(XN(-d_2) - F_0 N(-d_1))$$

Please refer to section 17.1.4 - generalised Black-Scholes option pricing formula for further detail and the Greeks.

9.3 FX Single Barrier Options

9.3.1 Description of Instrument

Barrier options are a family of options that cause an underlying product to come into existence or cease to exist when predetermined trigger levels are reached. There are two major types of single barrier options - Knock Ins and Knock Outs. Knock In options cause an underlying vanilla option to be created when the trigger level is breached and Knock out options cause the underlying vanilla option to cease to exist upon a breach of the barrier. The barrier can be any traded variable and may or may not be directly related to the underlying of the original option.

9.3.2 XML Representation

XML Schema

fpmlFXBarrierOption Schema			
Name: Type	Occurs	Size	Description
productType string	1..1	50	Indicates the type of product
buyerPartyReference fpmlPartyReference	1..1		
sellerPartyReference fpmlPartyReference	1..1		
expiryDateTimeExpiryDate date	1..1		
expiryDateTimeExpiryTime time	1..1		
expiryDateTimeCutName string	0..1		
exerciseStyle string	1..1		The manner in which the option can be exercised
fxOptionPremium fpmlFXOptionPremium	0..n		Premium amount or premium instalment amount for an option
valueDate date	1..1		The date on which both currencies traded will settle
cashSettlementTerms fpmlFXCashSettlement	0..1		This optional element is only used if an option has been specified at execution time to be settled into a single cash payment. This would be used for a non-deliverable option.
putCurrencyAmountCurrency string	1..1		The currency in which an amount is denominated
putCurrencyAmountAmount decimal	1..1		The monetary quantity in currency units
callCurrencyAmountCurrency	1..1		The currency in which an amount is

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fpmlFXBarrierOption Schema			
Name: Type	**Occurs**	**Size**	**Description**
string			denominated
callCurrencyAmountAmount decimal	1..1		The monetary quantity in currency units
fxStrikePriceRate decimal	1..1		
fxStrikePriceStrikeQuoteBasis string	1..1		The method by which the strike rate is quoted
quotedAs fpmlQuotedAs	0..1		Describes how the option was quoted
spotRate decimal	0..1		An optional element used for FX forwards and certain types of FX OTC options. For deals consummated in the FX Forwards Market, this represents the current market rate for a particular currency pair. For barrier and digital/binary options, it can be useful to include the spot rate at the time the option was executed to make it easier to know whether the option needs to move "up" or "down" to be triggered.
fxBarrier fpmlFXBarrier	0..n		Information about a barrier rate in a Barrier Option - specifying the exact criteria for a trigger event to occur.
triggerPayout fpmlFXOptionPayout	0..1		The amount of currency which becomes payable if and when a trigger event occurs

fpmlFXBarrier Schema			
Name: Type	**Occurs**	**Size**	**Description**
fxBarrierType string	0..1		This specifies whether the option becomes effective ("knock-in") or is annulled ("knock- out") when the respective trigger event occurs.
quotedCurrencyPairCurrency1 string	1..1		The first currency specified when a pair of currencies is to be evaluated
quotedCurrencyPairCurrency2 string	1..1		The second currency specified when a pair of currencies is to be evaluated
quotedCurrencyPairQuoteBasis string	1..1		The method by which the exchange rate is quoted
triggerRate decimal	1..1		The market rate is observed relative to the trigger rate, and if it is found to be on the predefined side of (above or

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fpmlFXBarrier Schema			
Name: Type	Occurs	Size	Description
			below) the trigger rate, a trigger event is deemed to have occurred
informationSource fpmlInformationSource	0..n		
observationStartDate date	0..1		The start of the period over which observations are made to determine whether a trigger has occurred
observationEndDate date	0..1		The end of the period over which observations are made to determine whether a trigger event has occurred

FX Single Barrier Option Example

```
<fxBarrierOption>
  <productType>FX Single Barrier</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
    <expiryDate>2004-05-31</expiryDate>
    <expiryTime>1200</expiryTime>
  </expiryDateTime>
  <exerciseStyle>European</exerciseStyle>
  <fxOptionPremium>
    <payerPartyReference href="ITE"/>
    <receiverPartyReference href="BAN"/>
    <premiumAmount>
      <currency>AUD</currency>
      <amount>1000.000000</amount>
    </premiumAmount>
    <premiumSettlementDate>2003-06-02</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2004-06-02</valueDate>
  <putCurrencyAmount>
    <currency>USD</currency>
    <amount>550000.000000</amount>
  </putCurrencyAmount>
  <callCurrencyAmount>
    <currency>AUD</currency>
    <amount>1000000.000000</amount>
  </callCurrencyAmount>
  <fxStrikePrice>
    <rate>0.550000</rate>
    <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
  </fxStrikePrice>
  <spotRate>0.585000</spotRate>
  <fxBarrier>
    <fxBarrierType>KNOCKIN</fxBarrierType>
    <quotedCurrencyPair>
      <currency1>AUD</currency1>
```

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<currency2>**USD**</currency2>
<quoteBasis>**CURRENCY2PERCURRENCY1**</quoteBasis>
</quotedCurrencyPair>
<triggerRate>**0.650000**</triggerRate>
<informationSource>
<rateSource>**Reuters**</rateSource>
<rateSourcePage>**AUD**=</rateSourcePage>
</informationSource>
</fxBarrier>
</fxBarrierOption>

9.3.3 Pricing

The equations below are derived from Reiner and Rubinstein [1991a] equations for pricing standard barrier options. The implementation in RAZOR identifies two different time periods; the time from the spot date to the delivery date of the underlying product, which is applied to the exchange and strike rate components in the pricing model, and the time from the pricing date to option expiry, which is applied to the volatility components.

Let

$$x_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \qquad x_2 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(H^2/SX)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \qquad y_2 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad \mu = \frac{b-\sigma^2/2}{\sigma^2} \qquad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$$

$$A = \phi S e^{(b-r)T} N(\phi x_1) - \phi X e^{-rT} N(\phi x_1 - \phi\sigma\sqrt{T})$$

$$B = \phi S e^{(b-r)T} N(\phi x_2) - \phi X e^{-rT} N(\phi x_2 - \phi\sigma\sqrt{T})$$

$$C = \phi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_1) - \phi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T})$$

$$D = \phi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_2) - \phi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$E = K e^{-rT} (N(\eta x_2 - \eta\sigma\sqrt{T}) - (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}))$$

$$F = K((\frac{H}{S})^{\mu+\lambda} N(\eta\zeta) + (\frac{H}{S})^{\mu-\lambda} N(\eta\zeta - 2\eta\lambda\sigma\sqrt{T}))$$

$$c_{di}(X > H) = C + E, \eta = 1, \phi = 1$$

$$c_{di}(X < H) = A - B + D + E, \eta = 1, \phi = 1$$

$$c_{ui}(X > H) = A + E, \eta = -1, \phi = 1$$

$$c_{ui}(X < H) = B - C + D + E, \eta = -1, \phi = 1$$

$$p_{di}(X > H) = B - C + D + E, \eta = 1, \phi = -1$$

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$$p_{di}(X<H)=A+E,\eta=1,\phi=-1$$

$$p_{ui}(X>H)=A-B+D+E,\eta=-1,\phi=-1$$

$$p_{ui}(X<H)=C+E,\eta=-1,\phi=-1$$

$$c_{do}(X>H)=A-C+F,\eta=1,\phi=1$$

$$c_{do}(X<H)=B-D+F,\eta=1,\phi=1$$

$$c_{uo}(X>H)=F,\eta=-1,\phi=1$$

$$c_{uo}(X<H)=A-B+C-D+F,\eta=-1,\phi=1$$

$$p_{do}(X>H)=A-B+C-D+F,\eta=1,\phi=-1$$

$$p_{do}(X<H)=F,\eta=1,\phi=-1$$

$$p_{uo}(X>H)=B-D+F,\eta=-1,\phi=-1$$

$$p_{uo}(X<H)=A-C+F,\eta=-1,\phi=-1$$

Rebates

An optional rebate may be paid out the barrier is hit (for a knock-out option) or not hitting the barrier (for a knock-in option).
Rebates in Razor are cash rebates factored into the intrinsic value of the option as either a digital no-touch (knock-in) or a digital (binary) one-touch (knock-out). Rebates are implemented for knock-ins that do not knock-in at expiry as a digital no-touch paid in units of CCY2. Rebates for options that knock out prior to expiry are implemented as a digital one-touch paid in units of CCY2.

Knock-In Rebate at Expiry

If the option has not been knocked-in during its lifetime a rebate can be specified which is paid out at expiration. The implementation of the rebate in this model is equivalent to a ***single barrier digital no-touch*** option.

Rebate of K units of CCY2

$$r=Ke^{-rT}\left[N\left(\eta x_2-\eta\sigma\sqrt{T}\right)-\left(\frac{H}{S}\right)^{2}N\left(\eta y_2-\eta\sigma\sqrt{T}\right)\right]$$

Rebate of K units of CCY1

$$r=K\cdot Se^{-rT}\left[N\left(\eta x_1-\eta\sigma\sqrt{T}\right)-\left(\frac{H}{S}\right)^{2\mu}N\left(\eta y_1-\eta\sigma\sqrt{T}\right)\right]$$

Knock-Out Rebate at Hit

If the option had been knocked-out during it's lifetime a rebate can be specified which is paid out at hit. The payout term is equivalent to a ***single barrier digital one-touch at hit***

Rebate of K units of CCY2

$$r = K\left[\left(\frac{H}{S}\right)^{2\mu+\lambda} N(\eta\zeta) - \left(\frac{H}{S}\right)^{2\mu-\lambda} N\left(\eta\zeta - 2\eta\lambda\sigma\sqrt{T}\right)\right]$$

Rebate of K units of CCY1

$$r = S\left[\left(\frac{H}{S}\right)^{2\mu+\lambda} N(\eta\zeta) - \left(\frac{H}{S}\right)^{2\mu-\lambda} N\left(\eta\zeta - 2\eta\lambda\sigma\sqrt{T}\right)\right]$$

Knock-Out Rebate at Expiry:

If the option had been knocked-out during it's lifetime a rebate can be specified which is paid out at hit. The payout term is equivalent to a ***single barrier digital one-touch at expiry***

Rebate of K units of CCY2

$$r = K\left[N\left(-\eta x_2 - \eta\sigma\sqrt{T}\right) - \left(\frac{H}{S}\right)^{2\mu-\lambda} N\left(\eta y_2 - \eta\sigma\sqrt{T}\right)\right]$$

Rebate of K units of CCY1

$$r = K\left[N\left(-\eta x_1 - \eta\sigma\sqrt{T}\right) - \left(\frac{H}{S}\right)^{2\mu-\lambda} N\left(\eta y_1 - \eta\sigma\sqrt{T}\right)\right]$$

9.4 FX Double Barrier Option

9.4.1 Description of Instrument

A double barrier option knocks in or out the underlying vanilla option if the underlying price touches an upper or lower barrier prior to expiration.

9.4.2 XML Representation

XML Schema

This product uses the fpmlFXBarrierOptionSchema.

FX Double Barrier Option Example

The following is an example of a double no touch:

```
<fxBarrierOption>
  <productType>DOUBLEBARRIER</productType>
  <buyerPartyReference href="ITE" />
  <sellerPartyReference href="BAN" />
  <expiryDateTime>
```

```
        <expiryDate>2005-03-04</expiryDate>
        <hourMinuteTime>10:00</hourMinuteTime>
    </expiryDateTime>
    <exerciseStyle>EUROPEAN</exerciseStyle>
    <fxOptionPremium>
        <payerPartyReference href="ITE" />
        <receiverPartyReference href="BAN" />
        <premiumAmount type="Money">
            <currency>USD</currency>
            <amount>192765.35</amount>
        </premiumAmount>
        <premiumSettlementDate>2005-01-07</premiumSettlementDate>
    </fxOptionPremium>
    <valueDate>2002-03-06</valueDate>
    <putCurrencyAmount>
        <currency>JPY</currency>
        <amount>2500000000</amount>
    </putCurrencyAmount>
    <callCurrencyAmount>
        <currency>USD</currency>
        <amount>23798191.34</amount>
    </callCurrencyAmount>
    <fxStrikePrice>
        <rate>105.05</rate>
    <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
    </fxStrikePrice>
    <spotRate>106</spotRate>
    <fxBarrier>
        <fxBarrierType>KNOCKOUT</fxBarrierType>
        <quotedCurrencyPair>
            <currency1>USD</currency1>
            <currency2>JPY</currency2>
            <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
        </quotedCurrencyPair>
        <triggerRate>102</triggerRate>
        <informationSource>
            <rateSource>Reuters</rateSource>
            <rateSourcePage>JPY=</rateSourcePage>
        </informationSource>
    </fxBarrier>
    <fxBarrier>
        <fxBarrierType>KNOCKOUT</fxBarrierType>
        <quotedCurrencyPair>
            <currency1>USD</currency1>
            <currency2>JPY</currency2>
            <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
        </quotedCurrencyPair>
        <triggerRate>115</triggerRate>
        <informationSource>
            <rateSource>Reuters</rateSource>
            <rateSourcePage>JPY=</rateSourcePage>
        </informationSource>
    </fxBarrier>
```

</fxBarrierOption>

9.4.3 Pricing

The pricing model uses Ikeda and Kunitomo (1992) formula. The value of a call double barrier can be expressed as follows:

$$d_1 = \frac{\ln(SU^{2n}/XL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{\ln(SU^{2n}/FL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_3 = \frac{\ln(L^{2n+2}/XSU^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad d_4 = \frac{\ln(L^{2n+2}/FSU^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$\mu_1 = \frac{2(b-\delta_2 - n(\delta_1-\delta_2))}{\sigma^2} + 1 \qquad \mu_2 = 2n\frac{(\delta_1-\delta_2)}{\sigma^2}$$

$$\mu_3 = \frac{2(b-\delta_2 + n(\delta_1-\delta_2))}{\sigma^2} + 1$$

$$F = Ue^{\delta_1 T}$$

$$c = Se^{(b-r)T}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^n}{L^n}\right)^{\mu_1}\left(\frac{L}{S}\right)^{\mu_2}(N(d_1)-N(d_2)) - \left(\frac{L^{n+1}}{U^nS}\right)^{\mu_3}(N(d_3)-N(d_4))\right\} -$$

$$Xe^{-rT}\sum_{n=-\infty}^{\infty}\left\{\begin{array}{l}\left(\frac{U^n}{L^n}\right)^{\mu_1-2}\left(\frac{L}{S}\right)^{\mu_2}(N(d_1-\sigma\sqrt{T})-N(d_2-\sigma\sqrt{T})) - \left(\frac{L^{n+1}}{U^nS}\right)^{\mu_3-2} \\ (N(d_3-\sigma\sqrt{T})-N(d_4-\sigma\sqrt{T}))\end{array}\right\}$$

The value of a double barrier put option can be expressed as follows:

$$y_1 = \frac{\ln(SU^{2n}/EL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad y_2 = \frac{\ln(SU^{2n}/XL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$y_3 = \frac{\ln(L^{2n+2}/ESU^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}} \qquad y_4 = \frac{\ln(L^{2n+2}/XSU^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$E = Le^{\delta_2 T}$$

$$p = Xe^{-rT}\sum_{n=-\infty}^{\infty}\left\{\begin{array}{l}\left(\frac{U^n}{L^n}\right)^{\mu_1-2}\left(\frac{L}{S}\right)^{\mu_2}(N(y_1-\sigma\sqrt{T})-N(y_2-\sigma\sqrt{T})) - \left(\frac{L^{n+1}}{U^nS}\right)^{\mu_3-2}(N(y_3-\sigma\sqrt{T}) - \\ N(y_4-\sigma\sqrt{T}))\end{array}\right\} -$$

$$Se^{(b-r)T}\sum_{n=-\infty}^{\infty}\left\{\left(\frac{U^n}{L^n}\right)^{\mu_1}\left(\frac{L}{S}\right)^{\mu_2}(N(y_1)-N(y_2)) - \left(\frac{L^{n+1}}{U^nS}\right)^{\mu_3}(N(y_3)-N(y_4))\right\}$$

Strike Outside the Barriers

The Ikeda and Kunitomo formula only holds when the strike price is inside the barrier range. When the strike is outside the range, we price the option in the following ways:

Double knock-Ins must be priced as the Vanilla European Option less the Double knock-out.

Notation:

In the following, C({strike},{lower barrier},{upper barrier}) refers to the existing RAZOR model for double barriers. The convention is “C” for call on the foreign currency, “P” for put on the foreign currency.

Double knock outs:

Case 1:

Where the prices are expressed in units of domestic currency per foreign currency,
e.g. 1.5554 USD per EUR,

1. $$C'(K,L,U) = C(L,L,U) + \frac{(L-K)}{(U-L)}\left(C(L,L,U) + P(U,L,U)\right)$$

where K < L. If K > U then value is zero.

2. $$P'(K,L,U) = P(U,L,U) + \frac{(K-U)}{(U-L)}\left(C(L,L,U) + P(U,L,U)\right)$$

where K > U. If K < L then value is zero.

Case 2:

Where the prices are expressed in units of foreign currency per domestic currency,
e.g. 106.92 JPY per USD,

C’(K,L,U) = C(U,L,U) + (K-U)/(U-L) * (C(U,L,U) + P(L,L,U)),

Where K > U. If K < L then value is zero.

4. P’(K,L,U) = P(L,L,U) + (L - K)/(U-L)*(C(U,L,U) + P(L,L,U)),

Where K<L. If K> H then value is zero.

9.5 FX Digital Option

9.5.1 Description of Instrument

Digital options (also known as “binary options”) are options that either pay out a fixed amount if the option expires in the money, or nothing. If the currency used to value the digital is the same as the payoff currency at expiry, the binary option will have either an exposure based on the payoff amount, or zero. If the payoff currency is different to the valuation currency, the digital

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option will have an exposure that changes with the exchange rate between the payoff currency and the valuation currency.

9.5.2 XML Representation

XML Schema

fpmlFXDigitalOption Schema			
Name: Type	**Occurs**	**Size**	**Description**
productType string	1..1	50	Indicates the type of product
buyerPartyReference fpmlPartyReference	1..1		
sellerPartyReference fpmlPartyReference	1..1		
expiryDateTimeExpiryDate date	1..1		
expiryDateTimeExpiryTime time	1..1		
expiryDateTimeCutName string	0..1		
fxOptionPremium fpmlFXOptionPremium	0..n		Premium amount or premium instalment amount for an option
valueDate date	1..1		The date on which both currencies traded will settle
quotedCurrencyPairCurrency1 string	1..1		The first currency specified when a pair of currencies is to be evaluated
quotedCurrencyPairCurrency2 string	1..1		The second currency specified when a pair of currencies is to be evaluated
quotedCurrencyPairQuoteBasis string	1..1		The method by which the exchange rate is quoted
spotRate decimal	0..1		An optional element used for FX forwards and certain types of FX OTC options. For deals consummated in the FX Forwards Market, this represents the current market rate for a particular currency pair. For barrier and digital/binary options, it can be useful to include the spot rate at the time the option was executed to make it easier to know whether the option needs to move "up" or "down" to be triggered.
fxEuropeanTrigger fpmlFXEuropeanTrigger	0..n		A European trigger occurs if the trigger criteria are met, but these are valid (and an observation is made) only at the maturity of the option

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fpmlFXDigitalOption Schema			
Name: Type	**Occurs**	**Size**	**Description**
fxAmericanTrigger fpmlFXAmericanTrigger	0..n		An American trigger occurs if the trigger criteria are met at any time from the initiation to the maturity of the option
triggerPayout fpmlFXOptionPayout	1..1		The amount of currency which becomes payable if and when a trigger event occurs

FX European Binary Option Example

```
<fxDigitalOption>
  <productType>Euro Binary</productType>
  <buyerPartyReference href="ITE" />
  <sellerPartyReference href="BAN" />
  <expiryDateTime>
     <expiryDate>2004-01-24</expiryDate>
     <expiryTime>1200</expiryTime>
     <businessCenter>AUSY</businessCenter>
     <cutName>Sydney</cutName>
  </expiryDateTime>
  <fxOptionPremium>
     <payerPartyReference href="ITE" />
     <receiverPartyReference href="BAN" />
     <premiumAmount type="Money">
        <currency>USD</currency>
        <amount>369000</amount>
     </premiumAmount>
     <premiumSettlementDate>2001-09-14</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2004-01-26</valueDate>
  <quotedCurrencyPair>
     <currency1>AUD</currency1>
     <currency2>USD</currency2>
     <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
  </quotedCurrencyPair>
  <spotRate>0.6500</spotRate>
  <fxEuropeanTrigger>
     <triggerCondition>Above</triggerCondition>
     <quotedCurrencyPair>
        <currency1>AUD</currency1>
        <currency2>USD</currency2>
        <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
     </quotedCurrencyPair>
     <triggerRate>0.6000</triggerRate>
     <informationSource>
        <rateSource>REUTERS</rateSource>
        <rateSourcePage>AUD=</rateSourcePage>
     </informationSource>
     </fxEuropeanTrigger>
  <triggerPayout>
     <currency>AUD</currency>
```

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```
    <amount>10000</amount>
    <payoutStyle>IMMEDIATE</payoutStyle>
  </triggerPayout>
</fxDigitalOption>
```

9.5.3 Pricing

Let

S = the exchange rate

r = the annualised domestic interest rate, continuously compounded

r_f = the annualised foreign interest rate, continuously compounded

σ = the exchange rate volatility

X = the strike rate

T_x = the time from today to expiry, annualised

T_d = the time from spot to delivery, annualised

$F_0 = S_0 e^{(r-r_f)T_d}$

N is the cumulative normal distribution function.

$$d_1 = \frac{\ln\left(\frac{F_0}{X}\right) + \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}} \qquad d_2 = \frac{\ln\left(\frac{F_0}{X}\right) - \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

The European digital asset price is given by:

$$d_{asset} = e^{-rT_d} F_0 N(d_1)$$

The European digital cash price is given by:

$$d_{cash} = e^{-rT_d} X N(d_2)$$

9.6 FX Digital Barrier Option

9.6.1 Description of Instrument

Digital barrier options are options containing a barrier that, when breached, either causes an underlying cash payment to occur, or knocks-out an underlying cash payment.

9.6.2 XML Representation

This product uses the FPML FXDigitalOption product schema in FPML version 3.

FX One Touch Digital Option Example

```
<fxDigitalOption>
  <productType>One Touch</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
```

```xml
      <expiryDate>2004-01-22</expiryDate>
      <expiryTime>1600</expiryTime>
      <cutName>AUSY</cutName>
    </expiryDateTime>
    <valueDate>2004-01-24</valueDate>
    <quotedCurrencyPair>
      <currency1>AUD</currency1>
      <currency2>USD</currency2>
      <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
    </quotedCurrencyPair>
    <spotRate>0.5500</spotRate>
    <fxAmericanTrigger>
      <touchCondition>Touch</touchCondition>
      <quotedCurrencyPair>
        <currency1>AUD</currency1>
        <currency2>USD</currency2>
        <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
      </quotedCurrencyPair>
      <triggerRate>0.7500</triggerRate>
      <informationSource>
        <rateSource>REUTERS</rateSource>
        <rateSourcePage>AUD=</rateSourcePage>
      </informationSource>
      <observationStartDate>2001-09-14</observationStartDate>
      <observationEndDate>2004-01-22</observationEndDate>
    </fxAmericanTrigger>
    <triggerPayout>
      <currency>AUD</currency>
      <amount>100000000</amount>
      <payoutStyle>DEFERRED</payoutStyle>
    </triggerPayout>
  </fxDigitalOption>
```

FX No Touch Digital Option Example

```xml
<fxDigitalOption>
  <productType>No Touch</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
    <expiryDate>2004-01-22</expiryDate>
    <expiryTime>1600</expiryTime>
    <cutName>AUSY</cutName>
  </expiryDateTime>
  <valueDate>2004-01-24</valueDate>
  <quotedCurrencyPair>
    <currency1>AUD</currency1>
    <currency2>USD</currency2>
    <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
  </quotedCurrencyPair>
  <spotRate>0.5500</spotRate>
  <fxAmericanTrigger>
    <touchCondition>Notouch</touchCondition>
    <quotedCurrencyPair>
```

```
        <currency1>AUD</currency1>
        <currency2>USD</currency2>
        <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
      </quotedCurrencyPair>
      <triggerRate>0.6000</triggerRate>
      <informationSource>
        <rateSource>REUTERS</rateSource>
        <rateSourcePage>AUD=</rateSourcePage>
      </informationSource>
      <observationStartDate>2001-09-14</observationStartDate>
      <observationEndDate>2004-01-22</observationEndDate>
    </fxAmericanTrigger>
    <triggerPayout>
      <currency>AUD</currency>
      <amount>100000000</amount>
      <payoutStyle>DEFERRED</payoutStyle>
    </triggerPayout>
  </fxDigitalOption>
```


9.6.3 Pricing

Riener and Rubinstein present a set of formulas that can be used to price the different types of digital barrier options.

$$x_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} \qquad x_2 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(H^2/SX)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} \qquad y_2 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T}$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad \mu = \frac{b-\sigma^2/2}{\sigma^2} \qquad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$$

$$A_1 = Se^{(b-r)T} N(\phi x_1)$$

$$B_1 = Ke^{-rT} N(\phi x_1 - \phi\sigma\sqrt{T})$$

$$A_2 = Se^{(b-r)T} N(\phi x_2)$$

$$B_2 = Ke^{-rT} N(\phi x_2 - \phi\sigma\sqrt{T})$$

$$A_3 = Se^{(b-r)T} (H/S)^{2(\mu+1)} N(\eta y_1)$$

$$B_3 = Ke^{-rT} (H/S)^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T})$$

$$A_4 = Se^{(b-r)T} (H/S)^{2(\mu+1)} N(\eta y_2)$$

$$B_4 = K_e^{-rT} (H/S)^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$A_5 = K((H/S)^{\mu+\lambda} N(\eta z) + (H/S)^{\mu-\lambda} N(\eta z - 2\eta\lambda\sigma\sqrt{T})$$

By using the formulas A_1 to A_5 and B_1 to B_4 in various combinations we can value the different types of binary barrier options.

#	Condition	Value	η	ϕ	#	Condition	Value	η	ϕ
1	$S > H$	A_5	1		18	$S < H, X > H$	$B_1 - B_2 + B_4$	−1	−1
2	$S < H$	A_5	−1			$X < H$	B_1		−1
3	$S > H, K = H$	A_5	1		19	$S > H, X > H$	$A_2 - A_3 + A_4$	1	−1
4	$S < H, K = H$	A_5	−1			$X < H$	A_1		−1
5	$S > H$	$B_2 + B_4$	1	−1	20	$S < H, X > H$	$A_1 - A_2 + A_3$	−1	−1
6	$S < H$	$B_2 + B_4$	−1	1		$X < H$	A_3		−1
7	$S > H$	$A_2 + A_4$	1	−1	21	$S > H, X > H$	$B_1 - B_3$	1	1
8	$S < H$	$A_2 + A_4$	1	−1		$X < H$	$B_2 - B_4$	1	1
9	$S > H$	$B_2 - B_4$	1	1	22	$S < H, X > H$	0		
10	$S < H$	$B_2 - B_4$	−1	−1		$X < H$	$B_1 - B_2 + B_3 - B_4$	−1	1
11	$S > H$	$A_2 - A_4$	1	1	23	$S > H, X > H$	$A_2 - A_4$	1	1
12	$S < H$	$A_2 - A_4$	−1	−1		$X < H$	$A_2 - A_4$	−1	−1
13	$S > H, X > H$	B_3	1		24	$S < H, X > H$	0		
	$X < H$	$B_1 - B_2 + B_4$	1	1		$X < H$	$A_1 - A_2 + A_3 - A_4$	−1	1
14	$S < H, X > H$	B_1		1	25	$S > H, X > H$	$B_1 - B_2 + B_3 - B_4$	1	−1
	$X < H$	$B_2 - B_3 + B_4$	−1	1		$X < H$	0		
15	$S > H, X > H$	A_3	1		26	$S < H, X > H$	$B_2 - B_4$	−1	−1
	$X < H$	$A_1 - A_2 + A_4$	1	1		$X < H$	$B_1 - B_3$	−1	−1
16	$S < H, X > H$	B_1		1	27	$S > H, X > H$	$A_1 - A_2 + A_3 - A_4$	1	−1
	$X < H$	$B_2 - B_3 + B_4$	−1	1		$X < H$	0		
17	$S > H, X > H$	$B_2 - B_3 + B_4$	1	−1	28	$S < H, X > H$	$A_2 - A_4$	−1	−1
	$X < H$	B_1		−1		$X < H$	$A_1 - A_3$	−1	−1

9.7 FX Double Digital Option

9.7.1 Description of Instrument

These derivative products have a double barrier, and a digital payoff is created if either barrier is breached, or if neither barrier is breached. The payoff may be either upon the breach of the barrier - “at hit” - or at expiry.

Comment [M.L.1]: Currently Not Supported

9.7.2 XML Representation

XML Schema

fpmlFXDigitalOption schema			
Name: Type	**Occurs**	**Size**	**Description**
productType string	1..1	50	Indicates the type of product
buyerPartyReference fpmlPartyReference	1..1		
sellerPartyReference fpmlPartyReference	1..1		

fpmlFXDigitalOption schema			
Name: Type	**Occurs**	**Size**	**Description**
expiryDateTimeExpiryDate date	1..1		
expiryDateTimeExpiryTime time	1..1		
expiryDateTimeCutName string	0..1		
fxOptionPremium fpmlFXOptionPremium	0..n		Premium amount or premium instalment amount for an option
valueDate date	1..1		The date on which both currencies traded will settle
quotedCurrencyPairCurrency1 string	1..1		The first currency specified when a pair of currencies is to be evaluated
quotedCurrencyPairCurrency2 string	1..1		The second currency specified when a pair of currencies is to be evaluated
quotedCurrencyPairQuoteBasis string	1..1		The method by which the exchange rate is quoted
spotRate decimal	0..1		An optional element used for FX forwards and certain types of FX OTC options. For deals consummated in the FX Forwards Market, this represents the current market rate for a particular currency pair. For barrier and digital/binary options, it can be useful to include the spot rate at the time the option was executed to make it easier to know whether the option needs to move ”up” or ”down” to be triggered.
fxEuropeanTrigger fpmlFXEuropeanTrigger	0..n		A European trigger occurs if the trigger criteria are met, but these are valid (and an observation is made) only at the maturity of the option
fxAmericanTrigger fpmlFXAmericanTrigger	0..n		An American trigger occurs if the trigger criteria are met at any time from the initiation to the maturity of the option
triggerPayout fpmlFXOptionPayout	1..1		The amount of currency which becomes payable if and when a trigger event occurs

FX Double One Touch Digital Option Example

```
<fxDigitalOption>
  <productType>Double one touch</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
```

```
    <expiryDate>2004-11-26</expiryDate>
    <hourMinuteTime>1400</hourMinuteTime>
    <businessCenter>AUSY</businessCenter>
    <cutName>SYDNEY</cutName>
  </expiryDateTime>
  <fxOptionPremium>
    <payerPartyReference href="ITE"/>
    <receiverPartyReference href="BAN"/>
    <premiumAmount type="Money">
      <currency>AUD</currency>
      <amount>78000</amount>
    </premiumAmount>
    <premiumSettlementDate>2001-11-14</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2004-11-28</valueDate>
  <quotedCurrencyPair>
    <currency1>AUD</currency1>
    <currency2>USD</currency2>
    <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
  </quotedCurrencyPair>
  <spotRate>0.5580</spotRate>
  <fxAmericanTrigger>
    <touchCondition>Touch</touchCondition>
    <quotedCurrencyPair>
      <currency1>AUD</currency1>
      <currency2>USD</currency2>
      <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
    </quotedCurrencyPair>
    <triggerRate>0.5800</triggerRate>
    <informationSource>
      <rateSource>REUTERS</rateSource>
      <rateSourcePage>AUD=</rateSourcePage>
    </informationSource>
    <observationStartDate>2001-11-12</observationStartDate>
    <observationEndDate>2004-11-26</observationEndDate>
  </fxAmericanTrigger>
  <fxAmericanTrigger>
    <touchCondition>Touch</touchCondition>
    <quotedCurrencyPair>
      <currency1>AUD</currency1>
      <currency2>USD</currency2>
      <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
    </quotedCurrencyPair>
    <triggerRate>0.5400</triggerRate>
    <informationSource>
      <rateSource>REUTERS</rateSource>
      <rateSourcePage>AUD=</rateSourcePage>
    </informationSource>
    <observationStartDate>2001-11-12</observationStartDate>
    <observationEndDate>2004-11-26</observationEndDate>
  </fxAmericanTrigger>
  <triggerPayout>
    <currency>AUD</currency>
    <amount>2000000</amount>
```

```
      <payoutStyle>IMMEDIATE</payoutStyle>
    </triggerPayout>
  </fxDigitalOption>
```


FX Double No Touch Digital Option Example


```
<fxDigitalOption>
  <productType>Double one touch</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
    <expiryDate>2004-11-26</expiryDate>
    <hourMinuteTime>1400</hourMinuteTime>
    <businessCenter>AUSY</businessCenter>
    <cutName>SYDNEY</cutName>
  </expiryDateTime>
  <fxOptionPremium>
    <payerPartyReference href="ITE"/>
    <receiverPartyReference href="BAN"/>
    <premiumAmount type="Money">
      <currency>AUD</currency>
      <amount>78000</amount>
    </premiumAmount>
    <premiumSettlementDate>2001-11-14</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2004-11-28</valueDate>
  <quotedCurrencyPair>
    <currency1>AUD</currency1>
    <currency2>USD</currency2>
    <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
  </quotedCurrencyPair>
  <spotRate>0.5580</spotRate>
  <fxAmericanTrigger>
    <touchCondition>Notouch</touchCondition>
    <quotedCurrencyPair>
      <currency1>AUD</currency1>
      <currency2>USD</currency2>
      <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
    </quotedCurrencyPair>
    <triggerRate>0.5800</triggerRate>
    <informationSource>
      <rateSource>REUTERS</rateSource>
      <rateSourcePage>AUD=</rateSourcePage>
    </informationSource>
    <observationStartDate>2001-11-12</observationStartDate>
    <observationEndDate>2004-11-26</observationEndDate>
  </fxAmericanTrigger>
  <fxAmericanTrigger>
    <touchCondition>Notouch</touchCondition>
    <quotedCurrencyPair>
      <currency1>AUD</currency1>
      <currency2>USD</currency2>
      <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
    </quotedCurrencyPair>
```

```
            <triggerRate>0.5400</triggerRate>
            <informationSource>
                <rateSource>REUTERS</rateSource>
                <rateSourcePage>AUD=</rateSourcePage>
            </informationSource>
            <observationStartDate>2001-11-12</observationStartDate>
            <observationEndDate>2004-11-26</observationEndDate>
        </fxAmericanTrigger>
        <triggerPayout>
            <currency>AUD</currency>
            <amount>2000000</amount>
            <payoutStyle>IMMEDIATE</payoutStyle>
        </triggerPayout>
    </fxDigitalOption>
```


9.7.3 Pricing

Razor uses the Hui (1996) closed form approximation with 5 terms.

knock-out:

$$L = \ln\left(\frac{H_2}{H_1}\right) \quad \alpha = -\frac{1}{2}\left(\frac{2b}{\sigma^2}-1\right) \quad \beta = -\frac{1}{4}\left(\frac{2b}{\sigma^2}-1\right)^2 - 2\frac{r}{\sigma^2}$$

$$c_{knock-out} = \sum_{i=1}^{\infty} \frac{2\pi i K}{L^2}\left[\frac{\left(\frac{S}{H_1}\right)^{\alpha} - (-1)^i \left(\frac{S}{H_2}\right)^{\alpha}}{\alpha^2 + \left(\frac{i\pi}{L}\right)^2}\right] \sin\left(\frac{i\pi}{L}\ln\left(\frac{S}{H_1}\right)\right) e^{-\frac{1}{2}\left[\left(\frac{i\pi}{L}\right)^2 - \beta\right]\sigma^2 T}$$

knock-in:

$$C_{knock-in} = Ke^{rT} - C_{knock-out}$$

American binary knock-out option

In this option, if S ever reaches one barrier, $H2$, then the option is worthless; thus on the line $H2$ the option value is zero. If S ever reaches another barrier, $H1$, the payment is$R$ at the time of touching the payment barrier.

$$c_{knock-out} = R\left(\frac{S}{H_1}\right)^{\alpha}\left\{\sum_{i=1}^{\infty}\frac{2}{i\pi}\left[\frac{\beta - \left(\frac{i\pi}{L}\right)^2 e^{-\frac{1}{2}\left[\left(\frac{i\pi}{L}\right)^2 - \beta\right]\sigma^2 T}}{\left(\frac{i\pi}{L}\right)^2 - \beta}\right]\sin\left(\frac{i\pi}{L}\ln\left(\frac{S}{H_1}\right)\right) + \left(1 - \frac{\ln\left(\frac{S}{H_1}\right)}{L}\right)\right\}$$

9.8 FX Average Rate Options

9.8.1 Description of Instrument

An average rate option is a derivative security that gives the buyer a payoff at the maturity of the option based on the difference between the average of exchange rates sampled at points along the life of the option, and a strike rate.

An average strike option is similar to an average rate option, but the payoff is the difference between an average of exchange rates sampled at points along the life of the option, and the exchange rate upon maturity of the option.

9.8.2 XML Representation

XML Schema

fpmlFXAverageRateOption Schema			
Name: Type	Occurs	Size	Description
productType string	1..1	50	Indicates the type of product
buyerPartyReference fpmlPartyReference	1..1		
sellerPartyReference fpmlPartyReference	1..1		
expiryDateTimeExpiryDate date	1..1		
expiryDateTimeExpiryTime time	1..1		
expiryDateTimeCutName string	0..1		
exerciseStyle string	1..1		The manner in which the option can be exercised
fxOptionPremium fpmlFXOptionPremium	0..n		Premium amount or premium instalment amount for an option
valueDate date	1..1		The date on which both currencies traded will settle
putCurrencyAmountCurrency string	1..1		The currency in which an amount is denominated
putCurrencyAmountAmount decimal	1..1		The monetary quantity in currency units
callCurrencyAmountCurrency string	1..1		The currency in which an amount is denominated
callCurrencyAmountAmount decimal	1..1		The monetary quantity in currency units
fxStrikePriceRate	1..1		

fpmlFXAverageRateOption Schema			
Name: Type	**Occurs**	**Size**	**Description**
decimal			
fxStrikePriceStrikeQuoteBasis string	1..1		The method by which the strike rate is quoted
spotRate decimal	0..1		An optional element used for FX forwards and certain types of FX OTC options. For deals consummated in the FX Forwards Market, this represents the current market rate for a particular currency pair. For barrier and digital/binary options, it can be useful to include the spot rate at the time the option was executed to make it easier to know whether the option needs to move "up" or "down" to be triggered.
payoutCurrency string	1..1		The ISO code of the currency in which a payout (if any) is to be made when a trigger is hit on a digital or barrier option
averageRateQuoteBasis string	1..1		The method by which the average rate that is being observed is quoted
roundingPrecision int	0..1		Specifies the rounding precision in terms of a number of decimal places. Note how a percentage rate rounding of 5 decimal places is expressed as a rounding precision of 7 in the FpML document since the percentage is expressed as a decimal, e.g. 9.876543.
payoutFormula string	0..1		The description of the mathematical computation for how the payout is computed
primaryRateSource fpmlInformationSource	1..1		The primary source for where the rate observation will occur. Will typically be either a page or a reference bank published rate.
secondaryRateSource fpmlInformationSource	0..1		An alternative, or secondary, source for where the rate observation will occur. Will typically be either a page or a

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fpmlFXAverageRateOption Schema			
Name: Type	**Occurs**	**Size**	**Description**
			reference bank published rate.
fixingTime fpmlBusinessCenterTime	1..1		The time at which the spot currency exchange rate will be observed. It is specified as a time in a specific business centre, e.g. 11:00am London time.
averageRateObservationSchedule fpmlFXAverageRateObservationSchedule	1..1		Parametric schedule of rate observations
averageRateObservationDate fpmlFXAverageRateObservationDate	0..n		One of more specific rate observation dates
observedRates fpmlObservedRates	0..n		Describes prior rate observations within average rate options. Periodically, an average rate option agreement will be struck whereby some rates have already been observed in the past but will become part of computation of the average rate of the option. This structure provides for these previously observed rates to be included in the description of the trade.

FX Average Rate Option Example

```
<fxAverageRateOption>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
    <expiryDate>2003-11-30</expiryDate>
    <hourMinuteTime>12:30</hourMinuteTime>
    <businessCenter>AUSY</businessCenter>
  </expiryDateTime>
  <exerciseStyle>EUROPEAN</exerciseStyle>
  <fxOptionPremium>
    <payerPartyReference href="ITE"/>
    <receiverPartyReference href="BAN"/>
    <premiumAmount type="Money">
      <currency>USD</currency>
      <amount>1750</amount>
    </premiumAmount>
    <premiumSettlementDate>2003-08-18</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2003-12-04</valueDate>
  <putCurrencyAmount>
    <currency>AUD</currency>
    <amount>10000000</amount>
```

</putCurrencyAmount>
<callCurrencyAmount>
<currency>**USD**</currency>
<amount>**5855000**</amount>
</callCurrencyAmount>
<fxStrikePrice>
<rate>**0.5855**</rate>
<strikeQuoteBasis>**CALLCURRENCYPERPUTCURRENCY**</strikeQuoteBasis>
</fxStrikePrice>
<payoutCurrency>**USD**</payoutCurrency>
<averageRateQuoteBasis>**CALLCURRENCYPERPUTCURRENCY**</averageRateQuoteBasis>
<primaryRateSource>
<rateSource>**AUUS**</rateSource>
</primaryRateSource>
<fixingTime>
<hourMinuteTime>**18:00**</hourMinuteTime>
<businessCenter>**AUSY**</businessCenter>
</fixingTime>
<averageRateObservationDate>
<observationDate>**2003-11-01**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-02**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-05**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-06**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-07**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-08**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-09**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-13**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>
</averageRateObservationDate>
<averageRateObservationDate>
<observationDate>**2003-11-14**</observationDate>
<averageRateWeightingFactor>**1**</averageRateWeightingFactor>

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```
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-15</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-16</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-19</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-20</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-21</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-23</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-26</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-27</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-28</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-29</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
    <averageRateObservationDate>
      <observationDate>2003-11-30</observationDate>
      <averageRateWeightingFactor>1</averageRateWeightingFactor>
    </averageRateObservationDate>
  </fxAverageRateOption>
```


9.8.3 Pricing

RAZOR uses the Turnbull and Wakeman approximation to price arithmetic average rate options.

Note that the formula doesn’t work when the cost of carry is zero.

$$\sigma_A = \sqrt{\frac{\ln(M_2)}{T} - 2b_A}$$

$$b_A = \frac{\ln(M_1)}{T}$$

$$d_1 = \frac{\ln(S/X) + (b_A + \sigma_A^2/2)T_2}{\sigma_A\sqrt{T_2}}$$

$$d_2 = d_1 - \sigma_A\sqrt{T_2}$$

$$c \approx Se^{(b_A - r)T_2}N(d_1) - Xe^{-rT_2}N(d_2)$$

$$p \approx Xe^{-rT_2}N(d_2) - Se^{(b_A - r)T_2}N(d_1)$$

The exact first and second moments of the arithmetic average are:

$$M_1 = \frac{e^{bT} - e^{b\tau}}{b(T-\tau)}$$

$$M_2 = \frac{2e^{(2b+\sigma^2)T}}{(b+\sigma^2)(2b+\sigma^2)(T-\tau)^2} + \frac{2e^{(2b+\sigma^2)\tau}}{b(T-\tau)^2}\left(\frac{1}{2b+\sigma^2} - \frac{e^{b(T-\tau)}}{b+\sigma^2}\right)$$

9.9 FX Fade-In Option

9.9.1 Contract Definition

A fade-in option is a type of barrier option. There is a preset barrier value for the underlying asset at a predetermined future date. There are four types of barriers:

Up-and-in
If the underlying asset value is above the barrier value on the predetermined future date, then the option turns to the plain-vanilla option. Otherwise, the option expires without value at maturity.
Up-and-Out
If the underlying asset value is above the barrier value on the predetermined future date, then the option expires immediately without any value. Otherwise, the option pays off like a plain-vanilla option at maturity.
Down-and-in
If the underlying asset value is below the barrier value on the predetermined future date, then the option turns to the plain-vanilla option. Otherwise, the option expires without value at maturity.
Down-and-out
If the underlying asset value is below the barrier value on the predetermined future date, then the option expires immediately without

any value. Otherwise, the option pays off like a plain-vanilla option at maturity.

9.9.2 Model Specification

Define

t = the valuation date.
T_1 = barrier observation date.
T_2 = option maturity date.
T_3 = option settlement date.
K = strike price.
h = barrier value.
$S(t)$ = the underlying asset value at time t.
$r(t,T)$ = the domestic risk-free rate applied from time t to T.
$q(t,T)$ = the foreign risk-free rate applied from time t to T.
$\sigma(t,T)$ = the volatility of the underlying asset applied from time t to T.

$\alpha(s,t,DC)$ = length of time from s to t in years and DC is the day count basis., e.g. $\alpha\left(s,t,\frac{actual}{365}\right)=\frac{t-s}{365}$.

$\Phi(x_1,x_2,\Sigma)$ = cumulative value of a standard 2-D normal distribution evaluated at x_1 for first variable, x_2 for the second variable and Σ is the covariance matrix.

Note that $r(t,T)$, $q(t,T)$ and $\sigma(t,T)$ can be obtained from the term structure curve.

Option Payoff

We define:
$\delta=\pm 1$ for up and down options respectively.
$\eta=\pm 1$ for in and out options respectively.

We also define the following function:

$f(S(T_1);\delta,\eta,h)=\eta$ if $\delta\cdot S(T_1)\geq\delta\cdot h$
$f(S(T_1);\delta,\eta,h)=1-\eta$ otherwise.

The payoff at maturity of the fade-in option is:
$V_T = f(S(T_1);\delta,\eta,h)\cdot[\beta\cdot\max(S(T_2)-K,0)]$

where
$\beta = \pm 1$ for call and put respectively.

9.9.3 Pricing Formulas

The price of a fade-in option at the valuation date t is:

$$V_t = df(t, T_3) \times \left\{ \left(\frac{1+\delta}{2} - \delta\eta \right) \cdot \beta \cdot [F_2 \Phi(\alpha, \gamma, \Sigma_1) - K\Phi(\alpha', \gamma', \Sigma_1)] + \right.$$

$$\left. \left(\frac{1-\delta}{2} + \delta\eta \right) \cdot \beta \cdot [F_2 \Phi(-\alpha, \gamma, \Sigma_2) - K\Phi(-\alpha', \gamma', \Sigma_2)] \right\}$$

where

$$\alpha = \frac{\ln h - m_1}{\sqrt{v_1}} - \sqrt{v_1}$$

$$\alpha' = \frac{\ln h - m_1}{\sqrt{v_1}}$$

$$\gamma = \beta \cdot \frac{\ln\left(\frac{F_2}{K}\right) + \frac{v_1 + v_2}{2}}{\sqrt{v_1 + v_2}}$$

$$\gamma' = \beta \cdot \frac{\ln\left(\frac{F_2}{K}\right) - \frac{v_1 + v_2}{2}}{\sqrt{v_1 + v_2}}$$

$$\rho = -\beta \cdot \sqrt{\frac{v_1}{v_1 + v_2}}$$

$$v_1 = \sigma^2(t, T_1)T_1$$

$$m_1 = \ln F_1 - \frac{1}{2} v_1$$

$$v_2 = \sigma^2(t, T_2)T_2 - v_1$$

$$F_1 = S(t)e^{(r(t,T_1) - q(t,T_1))\alpha(t,T_1,DC)}$$

$$F_2 = S(t)e^{(r(t,T_2) - q(t,T_2))\alpha(t,T_2,DC)}$$

Σ_1
=[1 ρ ; ρ 1] which is a 2 $\times$ 2 matrix for the standard 2-D normal distribution
Σ_2
=[1 $-\rho$; $-\rho$ 1] which is a 2 $\times$ 2 matrix for the standard 2-D normal distribution.

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Chapter 10

Base Metals

10.1 Metal Forwards

10.1.1 Description of Instrument

Spot and Forward Metal transactions involve the exchange of metal for cash in a particular currency. Because the underlying exchange is very similar to an FX spot or forward transaction, with one of the currencies being the metal, this is how the product is modelled and the exposure calculated in RAZOR.

10.1.2 XML Representation

fpmlFXLeg Schema			
Name: Type	**Occurs**	**Size**	**Description**
productType String	1..1	50	Indicates the type of product
exchangedCurrency1 fpmlCurrencyFlow	1..1		This is the first of the two currency flows that define a single leg of a standard foreign ex- change transaction
exchangedCurrency2 fpmlCurrencyFlow	1..1		This is the second of the two currency flows that define a single leg of a standard foreign exchange transaction
valueDate Date	1..1		The date on which both currencies traded will settle
exchangeRate fpmlFXRate	1..1		The rate of exchange between the two currencies
nonDeliverableForward fpmlFXCashSettlement	0..1		Used to describe a particular type of FX forward transaction that is settled in a single currency

Metal Forward Example

```
<fxleg>
  <exchangedCurrency1>
     <payerPartyReference href = "ITE"/>
     <receiverPartyReference href = "BAN"/>
     <paymentAmount>
        <currency>AUD</currency>
        <amount>10000000</amount>
     </paymentAmount>
  </exchangedCurrency1>
  <exchangedCurrency2>
     <payerPartyReference href = "BAN"/>
     <receiverPartyReference href = "ITE"/>
     <paymentAmount>
        <currency>XAU</currency>
```

```
            <amount>1852</amount>
        </paymentAmount>
    </exchangedCurrency2>
    <valueDate>2003-12-21</valueDate>
    <exchangeRate>
        <quotedCurrencyPair>
            <currency1>AUD</currency1>
            <currency2>XAU</currency2>
            <quoteBasis>CURRENCY1PERCURRENCY2</quoteBasis>
        </quotedCurrencyPair>
        <rate>539.9568</rate>
    </exchangeRate>
</fxleg>
```

10.1.3 Pricing

Let

C_{ccy} = the cash flow denominated in currency $_{ccy}$

C_{metal} = the amount of the metal $_{metal}$

df_{ccy} = the ccy discount factor from spot to settlement

df_{metal} = the $metal$ discount factor from spot to settlement

S_{ccy}^{val} = the $ccy \to val$ exchange rate

S_{metal}^{val} = the $metal \to val$ exchange rate

Then

$$V = C_{metal} df_{metal} S_{metal}^{val} + C_{ccy} df_{ccy} S_{ccy}^{val}$$

10.2 Metal Vanilla Options

10.2.1 Description of Instrument

Metal vanilla options give the purchaser the right to buy (vanilla calls) or sell (vanilla put) the metal at the given strike price. Again we can treat these products in a similar manner to FX vanilla options.

10.2.2 XML Representation

XML Schema

We are using the FPML fpmlFXSimpleOption schema to represent this product.

Metal European Option Example

```
<fxSimpleOption>
    <productType>Metal Call Option</productType>
    <buyerPartyReference href="ITE"/>
    <sellerPartyReference href="BAN"/>
    <expiryDateTime>
        <expiryDate>2004-05-31</expiryDate>
```

```xml
        <expiryTime>1200</expiryTime>
    </expiryDateTime>
    <exerciseStyle>European</exerciseStyle>
    <fxOptionPremium>
        <payerPartyReference href="ITE"/>
        <receiverPartyReference href="BAN"/>
        <premiumAmount>
            <currency>AUD</currency>
            <amount>50000</amount>
        </premiumAmount>
        <premiumSettlementDate>2003-06-02</premiumSettlementDate>
    </fxOptionPremium>
    <valueDate>2004-06-02</valueDate>
    <putCurrencyAmount>
        <currency>AUD</currency>
        <amount>1000000</amount>
    </putCurrencyAmount>
    <callCurrencyAmount>
        <currency>XAU</currency>
        <amount>1852</amount>
    </callCurrencyAmount>
    <fxStrikePrice>
        <rate>539.9568</rate>
        <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
    </fxStrikePrice>
</fxSimpleOption>
```


10.2.3 Pricing

Let

S = the current spot price of the metal

r = the continuously compounded interest rate

r_f = the continuously compounded yield on the metal

σ = the price volatility

X = the strike price

T_x = the time from today to expiry, annualised

T_d = the time from spot to delivery, annualised

$$F_0 = S_0 e^{(r-r_f)T_d}$$

$$d_1 = \frac{\ln\left(\frac{F_0}{X}\right) + \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

$$d_2 = \frac{\ln\left(\frac{F_0}{X}\right) - \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

The European call price is given by

$$c = e^{-rT_d}(F_0 N(d_1) - XN(d_2))$$

The European put price is given by

$$p = e^{-rT_d}(XN(-d_2) - F_0 N(-d_1))$$

10.3 Metal Single Barrier Options

10.3.1 Description of Instrument

Barrier options on metal products are very similar to barrier options on FX products.

10.3.2 XML Representation

XML Schema

RAZOR uses the FPML FXBarrierOption schema to represent single barrier options on metal products.

Metal Single Barrier Option Example

```
<fxBarrierOption>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
     <expiryDate>2002-02-06</expiryDate>
     <hourMinuteTime>10:00</hourMinuteTime>
     <businessCenter>AUSY</businessCenter>
  </expiryDateTime>
  <exerciseStyle>EUROPEAN</exerciseStyle>
  <fxOptionPremium>
     <payerPartyReference href="ITE"/>
     <receiverPartyReference href="BAN"/>
     <premiumAmount type="Money">
        <currency>AUD</currency>
        <amount>50000</amount>
     </premiumAmount>
     <premiumSettlementDate>2001-11-06</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2002-02-08</valueDate>
  <putCurrencyAmount>
     <currency>AUD</currency>
     <amount>1000000</amount>
  </putCurrencyAmount>
  <callCurrencyAmount>
     <currency>XAU</currency>
     <amount>1852</amount>
  </callCurrencyAmount>
  <fxStrikePrice>
     <rate>539.9568</rate>
     <strikeQuoteBasis>CALLCURRENCYPERPUTCURRENCY</strikeQuoteBasis>
  </fxStrikePrice>
```

```xml
    <spotRate>533.6190</spotRate>
    <fxBarrier>
        <fxBarrierType>KNOCKIN</fxBarrierType>
        <quotedCurrencyPair>
            <currency1>AUD</currency1>
            <currency2>XAU</currency2>
            <quoteBasis>CURRENCY1PERCURRENCY2</quoteBasis>
        </quotedCurrencyPair>
        <triggerRate>542.20</triggerRate>
        <informationSource>
            <rateSource>Reuters</rateSource>
            <rateSourcePage>XAU=</rateSourcePage>
        </informationSource>
    </fxBarrier>
</fxBarrierOption>
```


10.3.3 Pricing

Let

$$x_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} \qquad x_2 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(H^2/SX)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} \qquad y_2 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T}$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad \mu = \frac{b-\sigma^2/2}{\sigma^2} \qquad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$$

$$A = \phi S e^{(b-r)T} N(\phi x_1) - \phi X e^{-rT} N(\phi x_1 - \phi\sigma\sqrt{T})$$

$$B = \phi S e^{(b-r)T} N(\phi x_2) - \phi X e^{-rT} N(\phi x_2 - \phi\sigma\sqrt{T})$$

$$C = \phi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_1) - \phi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T})$$

$$D = \phi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_2) - \phi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$E = K e^{-rT} (N(\eta x_2 - \eta\sigma\sqrt{T}) - (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}))$$

$$F = K((\frac{H}{S})^{\mu+\lambda} N(\eta\zeta) + (\frac{H}{S})^{\mu-\lambda} N(\eta\zeta - 2\eta\lambda\sigma\sqrt{T}))$$

$$c_{di}(X > H) = C + E, \eta = 1, \phi = 1$$

$$c_{di}(X < H) = A - B + D + E, \eta = 1, \phi = 1$$

$$c_{ui}(X > H) = A + E, \eta = -1, \phi = 1$$

$$c_{ui}(X < H) = B - C + D + E, \eta = -1, \phi = 1$$

$$p_{di}(X > H) = B - C + D + E, \eta = 1, \phi = -1$$

$$p_{di}(X < H) = A + E, \eta = 1, \phi = -1$$

$$p_{ui}(X > H) = A - B + D + E, \eta = -1, \phi = -1$$

$$p_{ui}(X < H) = C + E, \eta = -1, \phi = -1$$

$$c_{do}(X > H) = A - C + F, \eta = 1, \phi = 1$$

$$c_{do}(X < H) = B - D + F, \eta = 1, \phi = 1$$

$$c_{uo}(X > H) = F, \eta = -1, \phi = 1$$

$$c_{uo}(X < H) = A - B + C - D + F, \eta = -1, \phi = 1$$

$$p_{do}(X > H) = A - B + C - D + F, \eta = 1, \phi = -1$$

$$p_{do}(X < H) = F, \eta = 1, \phi = -1$$

$$p_{uo}(X > H) = B - D + F, \eta = -1, \phi = -1$$

$$p_{uo}(X < H) = A - C + F, \eta = -1, \phi = -1$$

10.4 Metal Double Barrier Option

10.4.1 Description of Instrument

Double barrier metal options are similar to FX double barrier options, where the barrier levels are upper and lower barriers on the metal price.

10.4.2 XML Representation

XML Schema

This product uses the fpmlFXBarrierOptionSchema.

Metal Double Barrier Option Example


```xml
<fxBarrierOption>
  <productType>DOUBLEBARRIER</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
    <expiryDate>2002-03-04</expiryDate>
    <hourMinuteTime>10:00</hourMinuteTime>
    <businessCenter>AUSY</businessCenter>
  </expiryDateTime>
  <exerciseStyle>EUROPEAN</exerciseStyle>
  <fxOptionPremium>
    <payerPartyReference href="ITE"/>
    <receiverPartyReference href="BAN"/>
    <premiumAmount type="Money">
      <currency>AUD</currency>
      <amount>50000</amount>
    </premiumAmount>
    <premiumSettlementDate>2002-01-07</premiumSettlementDate>
  </fxOptionPremium>
```

```
    <valueDate>2002-03-06</valueDate>
    <putCurrencyAmount>
      <currency>AUD</currency>
      <amount>1000000</amount>
    </putCurrencyAmount>
    <callCurrencyAmount>
      <currency>XAU</currency>
      <amount>1852</amount>
    </callCurrencyAmount>
    <fxStrikePrice>
      <rate>539.9568</rate>
      <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
    </fxStrikePrice>
    <spotRate>538.9789</spotRate>
    <fxBarrier>
      <fxBarrierType>KNOCKOUT</fxBarrierType>
      <quotedCurrencyPair>
        <currency1>AUD</currency1>
        <currency2>XAU</currency2>
        <quoteBasis>CURRENCY1PERCURRENCY2</quoteBasis>
      </quotedCurrencyPair>
      <triggerRate>541.354</triggerRate>
      <informationSource>
        <rateSource>Reuters</rateSource>
        <rateSourcePage>XAU=</rateSourcePage>
      </informationSource>
    </fxBarrier>
    <fxBarrier>
      <fxBarrierType>KNOCKOUT</fxBarrierType>
      <quotedCurrencyPair>
        <currency1>AUD</currency1>
        <currency2>XAU</currency2>
        <quoteBasis>CURRENCY1PERCURRENCY2</quoteBasis>
      </quotedCurrencyPair>
      <triggerRate>534.55</triggerRate>
      <informationSource>
        <rateSource>Reuters</rateSource>
        <rateSourcePage>XAU=</rateSourcePage>
      </informationSource>
    </fxBarrier>
  </fxBarrierOption>
```


10.4.3 Pricing

The value of a call double barrier can be expressed as follows:

$$d_1 = \frac{\ln(SU^{2n} / XL^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(SU^{2n} / FL^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_3 = \frac{\ln(L^{2n+2} / XSU^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_4 = \frac{\ln(L^{2n+2} / FSU^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$\mu_1 = \frac{2(b - \delta_2 - n(\delta_1 - \delta_2))}{\sigma^2} + 1$$

$$\mu_2 = 2n\frac{(\delta_1 - \delta_2)}{\sigma^2}$$

$$\mu_3 = \frac{2(b - \delta_2 + n(\delta_1 - \delta_2))}{\sigma^2} + 1$$

$$F = Ue^{\delta_1 T}$$

$$c = Se^{(b-r)T}\sum_{n=-\infty}^{\infty}\left\{(\frac{U^n}{L^n})^{\mu_1}(\frac{L}{S})^{\mu_2}(N(d_1) - N(d_2)) - (\frac{L^{n+1}}{U^n S})^{\mu_3}(N(d_3) - N(d_4))\right\} -$$

$$Xe^{-rT}\sum_{n=-\infty}^{\infty}\left\{\begin{matrix}(\frac{U^n}{L^n})^{\mu_1 - 2}(\frac{L}{S})^{\mu_2}(N(d_1 - \sigma\sqrt{T}) - N(d_2 - \sigma\sqrt{T})) - (\frac{L^{n+1}}{U^n S})^{\mu_3 - 2} \\ (N(d_3 - \sigma\sqrt{T}) - N(d_4 - \sigma\sqrt{T}))\end{matrix}\right\}$$

The value of a double barrier put option can be expressed as follows:

$$y_1 = \frac{\ln(SU^{2n} / EL^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$y_2 = \frac{\ln(SU^{2n} / XL^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$y_3 = \frac{\ln(L^{2n+2} / ESU^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$y_4 = \frac{\ln(L^{2n+2} / XSU^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$E = Le^{\delta_2 T}$$

$$p = Xe^{-rT}\sum_{n=-\infty}^{\infty}\left\{\begin{matrix}(\frac{U^n}{L^n})^{\mu_1 - 2}(\frac{L}{S})^{\mu_2}(N(y_1 - \sigma\sqrt{T}) - N(y_2 - \sigma\sqrt{T})) - (\frac{L^{n+1}}{U^n S})^{\mu_3 - 2}(N(y_3 - \sigma\sqrt{T}) - \\ N(y_4 - \sigma\sqrt{T}))\end{matrix}\right\} -$$

$$Se^{(b-r)T}\sum_{n=-\infty}^{\infty}\left\{(\frac{U^n}{L^n})^{\mu_1}(\frac{L}{S})^{\mu_2}(N(y_1) - N(y_2)) - (\frac{L^{n+1}}{U^n S})^{\mu_3}(N(y_3) - N(y_4))\right\}$$

Chapter 11

Commodities and Energy

11.1 Commodity Forwards

11.1.1 Description of Instrument

Commodity forwards are forward agreements to buy or sell a commodity at a specific price. These types of agreements are modelled in RAZOR in a similar manner to FX forwards.

Commodity products such as oil have standard units of measurement. This is encapsulated in the asset ID.

The settlement price can be a singe-date price or an average over a pre-specified period. If the contract is to be settled against an average price, the Accumulated Average needs to be an input from the Source System.

11.1.2 XML Representation

Commodity Forward Example

```xml
<fxleg>
  <exchangedCurrency1>
    <payerPartyReference href = "ITE"/>
    <receiverPartyReference href = "BAN"/>
    <paymentAmount>
      <currency>AUD</currency>
      <amount>10000000</amount>
    </paymentAmount>
  </exchangedCurrency1>
  <exchangedCurrency2>
    <payerPartyReference href = "BAN"/>
    <receiverPartyReference href = "ITE"/>
    <paymentAmount>
      <currency>OILBRENT</currency>
      <amount>50000</amount>
    </paymentAmount>
  </exchangedCurrency2>
  <valueDate>2003-12-21</valueDate>
  <exchangeRate>
    <quotedCurrencyPair>
      <currency1>OILBRENT</currency1>
      <currency2>AUD</currency2>
      <quoteBasis>CURRENCY1PERCURRENCY2</quoteBasis>
    </quotedCurrencyPair>
    <rate>20</rate>
  </exchangeRate>
</fxleg>
```

11.1.3 Pricing

The value of a Commodity Forward is calculated as follows:

$V = D_T N(F - X)$

where:

V - commodity payer swap price

F – the projected forward commodity value derived from the underlying commodity index forward curve.

X – commodity forward contract price.

D_T - discount factor to the payment date of contract

N - contract notional.

11.2 Electricity Futures

11.2.1 Introduction

In Razor, it provides two methods to determine the price of electricity futures. One method is to obtain the futures price from the forward price curve while the other method is to use the market quoted price directly.

11.2.2 Definition

$f_t(T)$	=	futures price at time t that matures at time T.
K	=	contract price.
$df(t_1, t_2)$	=	discount factor for the period (t_1, t_2).

11.2.3 Valuation

The value of the electricity futures v_t at time t is computed as

$$v_t = df(t,T) \times (f_t(T) - K).$$

11.3 Commodity Vanilla Options

11.3.1 Description of Instrument

Vanilla commodity option gives its holder the right to buy (Call) or sell (Put) a commodity at a predetermined price (strike) at a given date (maturity).

Commodity vanilla options are treated in a similar manner to metal vanilla options.

11.3.2 XML Representation

XML Schema

Commodity vanilla options use the FX vanilla specification of the FpML version 3 standard.

Commodity European Option Example


```
<fxSimpleOption>
  <productType>Call Commodity Option</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
     <expiryDate>2003-06-04</expiryDate>
     <hourMinuteTime>1400</hourMinuteTime>
     <businessCenter>AUSY</businessCenter>
     <cutName>Sydney</cutName>
  </expiryDateTime>
  <exerciseStyle>European</exerciseStyle>
  <fxOptionPremium>
     <payerPartyReference href="ITE"/>
     <receiverPartyReference href="BAN"/>
     <premiumAmount type="Money">
        <currency>AUD</currency>
        <amount>1000</amount>
     </premiumAmount>
     <premiumSettlementDate>2002-12-06</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2003-06-06</valueDate>
  <putCurrencyAmount>
     <currency>AUD</currency>
     <amount>1000000</amount>
  </putCurrencyAmount>
  <callCurrencyAmount>
     <currency>OILBRENT</currency>
     <amount>50000</amount>
  </callCurrencyAmount>
  <fxStrikePrice>
     <rate>20.00</rate>
     <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
  </fxStrikePrice>
</fxSimpleOption>
```


11.3.3 Pricing

Vanilla Put and Call commodity options are priced using the standard Black-Scholes formula

Let

S = the current spot price of the commodity

r = the continuously compounded interest rate

r_f = the continuously compounded yield on the commodity

σ = the price volatility

X = the strike price

T_x = the time from today to expiry, annualised

T_d = the time from spot to delivery, annualised

$F_0 = S_0 e^{(r-r_f)T_d}$, where $(r - r_f)$ is the cost of carry

if the future price F_0 of the commodity is provided, then the cost of carry is zero.

$$d_1 = \frac{\ln\left(\frac{F_0}{X}\right) + \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

$$d_2 = \frac{\ln\left(\frac{F_0}{X}\right) - \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

The European call price is given by

$$c = e^{-rT_d}(F_0 N(d_1) - XN(d_2))$$

The European put price is given by

$$p = e^{-rT_d}(XN(-d_2) - F_0 N(-d_1))$$

Please refer to section 17.1.4 - generalised Black-Scholes option pricing formula for further detail and the Greeks.

11.4 Commodity Single Barrier Options

11.4.1 Description of Instrument

Barrier options on commodities and energy are treated in a similar way to barrier options on FX products.

11.4.2 XML Representation

XML Schema

RAZOR uses the FPML FXBarrierOption schema to represent single barrier options on commodities.

Commodity Single Barrier Option Example

```
<fxBarrierOption>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
    <expiryDate>2002-02-06</expiryDate>
    <hourMinuteTime>10:00</hourMinuteTime>
  </expiryDateTime>
  <exerciseStyle>European</exerciseStyle>
  <fxOptionPremium>
    <payerPartyReference href="ITE"/>
    <receiverPartyReference href="BAN"/>
    <premiumAmount type="Money">
      <currency>AUD</currency>
      <amount>10000</amount>
    </premiumAmount>
    <premiumSettlementDate>2001-11-06</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2002-02-08</valueDate>
  <putCurrencyAmount>
    <currency>AUD</currency>
    <amount>1000000</amount>
  </putCurrencyAmount>
  <callCurrencyAmount>
    <currency>OILBRENT</currency>
    <amount>31736</amount>
  </callCurrencyAmount>
  <fxStrikePrice>
    <rate>31.51</rate>
    <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
  </fxStrikePrice>
  <spotRate>30</spotRate>
  <fxBarrier>
    <fxBarrierType>KNOCKIN</fxBarrierType>
    <quotedCurrencyPair>
      <currency1>OILBRENT</currency1>
      <currency2>AUD</currency2>
      <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
    </quotedCurrencyPair>
    <triggerRate>34</triggerRate>
  </fxBarrier>
</fxBarrierOption>
```

11.4.3 Pricing

Using the Reiner and Rubinstein Single Barrier Model

Let

$$x_1 = \frac{\ln(S/X)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} \qquad x_2 = \frac{\ln(S/H)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(H^2/SX)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T} \qquad y_2 = \frac{\ln(H/S)}{\sigma\sqrt{T}} + (\mu+1)\sigma\sqrt{T}$$

$$z = \frac{\ln(H/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad \mu = \frac{b-\sigma^2/2}{\sigma^2} \qquad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$$

$$A = \phi S e^{(b-r)T} N(\phi x_1) - \phi X e^{-rT} N(\phi x_1 - \phi\sigma\sqrt{T})$$

$$B = \phi S e^{(b-r)T} N(\phi x_2) - \phi X e^{-rT} N(\phi x_2 - \phi\sigma\sqrt{T})$$

$$C = \phi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_1) - \phi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T})$$

$$D = \phi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_2) - \phi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$E = K e^{-rT} (N(\eta x_2 - \eta\sigma\sqrt{T}) - (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T}))$$

$$F = K((\frac{H}{S})^{\mu+\lambda} N(\eta\zeta) + (\frac{H}{S})^{\mu-\lambda} N(\eta\zeta - 2\eta\lambda\sigma\sqrt{T}))$$

$$c_{di}(X > H) = C + E, \eta = 1, \phi = 1$$

$$c_{di}(X < H) = A - B + D + E, \eta = 1, \phi = 1$$

$$c_{ui}(X > H) = A + E, \eta = -1, \phi = 1$$

$$c_{ui}(X < H) = B - C + D + E, \eta = -1, \phi = 1$$

$$p_{di}(X > H) = B - C + D + E, \eta = 1, \phi = -1$$

$$p_{di}(X < H) = A + E, \eta = 1, \phi = -1$$

$$p_{ui}(X > H) = A - B + D + E, \eta = -1, \phi = -1$$

$$p_{ui}(X < H) = C + E, \eta = -1, \phi = -1$$

$$c_{do}(X > H) = A - C + F, \eta = 1, \phi = 1$$

$$c_{do}(X < H) = B - D + F, \eta = 1, \phi = 1$$

$$c_{uo}(X > H) = F, \eta = -1, \phi = 1$$

$$c_{uo}(X < H) = A - B + C - D + F, \eta = -1, \phi = 1$$

$$p_{do}(X > H) = A - B + C - D + F, \eta = 1, \phi = -1$$

$$p_{do}(X < H) = F, \eta = 1, \phi = -1$$

$$p_{uo}(X > H) = B - D + F, \eta = -1, \phi = -1$$

$$p_{uo}(X < H) = A - C + F, \eta = -1, \phi = -1$$

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11.5 Commodity Double Barrier Option

11.5.1 Description of Instrument

Double barrier options on commodities and energy are treated in a similar way to double barrier options on FX products.

11.5.2 XML Representation

XML Schema

This product uses the fpmlFXBarrierOptionSchema.

Commodity Double Barrier Option Example

```
<fxBarrierOption>
  <productType>DOUBLEBARRIER</productType>
  <buyerPartyReference href="ITE"/>
  <sellerPartyReference href="BAN"/>
  <expiryDateTime>
     <expiryDate>2002-03-04</expiryDate>
     <hourMinuteTime>10:00</hourMinuteTime>
  </expiryDateTime>
  <exerciseStyle>EUROPEAN</exerciseStyle>
  <fxOptionPremium>
     <payerPartyReference href="ITE"/>
     <receiverPartyReference href="BAN"/>
     <premiumAmount type = "Money">
        <currency>AUD</currency>
        <amount>10000</amount>
     </premiumAmount>
     <premiumSettlementDate>2002-01-07</premiumSettlementDate>
  </fxOptionPremium>
  <valueDate>2002-03-06</valueDate>
  <putCurrencyAmount>
     <currency>AUD</currency>
     <amount>1000000</amount>
  </putCurrencyAmount>
  <callCurrencyAmount>
     <currency>OILBRENT</currency>
     <amount>31736</amount>
  </callCurrencyAmount>
  <fxStrikePrice>
     <rate>31.51</rate>
     <strikeQuoteBasis>PUTCURRENCYPERCALLCURRENCY</strikeQuoteBasis>
  </fxStrikePrice>
  <spotRate>30</spotRate>
  <fxBarrier>
     <fxBarrierType>KNOCKOUT</fxBarrierType>
     <quotedCurrencyPair>
        <currency1>OILBRENT</currency1>
        <currency2>AUD</currency2>
        <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
```

```xml
            </quotedCurrencyPair>
            <triggerRate>34</triggerRate>
        </fxBarrier>
        <fxBarrier>
            <fxBarrierType>KNOCKOUT</fxBarrierType>
            <quotedCurrencyPair>
                <currency1>OILBRENT</currency1>
                <currency2>AUD</currency2>
                <quoteBasis>CURRENCY2PERCURRENCY1</quoteBasis>
            </quotedCurrencyPair>
            <triggerRate>27</triggerRate>
        </fxBarrier>
    </fxBarrierOption>
```


11.5.3 Pricing

The value of a call double barrier can be expressed as follows:

$$d_1 = \frac{\ln(SU^{2n}/XL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(SU^{2n}/FL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_3 = \frac{\ln(L^{2n+2}/XSU^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_4 = \frac{\ln(L^{2n+2}/FSU^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$\mu_1 = \frac{2(b-\delta_2 - n(\delta_1-\delta_2))}{\sigma^2} + 1$$

$$\mu_2 = 2n\frac{(\delta_1-\delta_2)}{\sigma^2}$$

$$\mu_3 = \frac{2(b-\delta_2 + n(\delta_1-\delta_2))}{\sigma^2} + 1$$

$$F = Ue^{\delta_1 T}$$

$$c = Se^{(b-r)T}\sum_{n=-\infty}^{\infty}\left\{(\frac{U^n}{L^n})^{\mu_1}(\frac{L}{S})^{\mu_2}(N(d_1)-N(d_2)) - (\frac{L^{n+1}}{U^nS})^{\mu_3}(N(d_3)-N(d_4))\right\} -$$

$$Xe^{-rT}\sum_{n=-\infty}^{\infty}\left\{\begin{matrix}(\frac{U^n}{L^n})^{\mu_1-2}(\frac{L}{S})^{\mu_2}(N(d_1-\sigma\sqrt{T})-N(d_2-\sigma\sqrt{T})) - (\frac{L^{n+1}}{U^nS})^{\mu_3-2} \\ (N(d_3-\sigma\sqrt{T})-N(d_4-\sigma\sqrt{T}))\end{matrix}\right\}$$

The value of a double barrier put option can be expressed as follows:

$$y_1 = \frac{\ln(SU^{2n}/EL^{2n}) + (b+\sigma^2/2)T}{\sigma\sqrt{T}}$$

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$$y_2 = \frac{\ln(SU^{2n} / XL^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$y_3 = \frac{\ln(L^{2n+2} / ESU^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$y_4 = \frac{\ln(L^{2n+2} / XSU^{2n}) + (b + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$E = Le^{\delta_2 T}$$

$$p = Xe^{-rT} \sum_{n=-\infty}^{\infty} \left\{ \begin{array}{l} (\frac{U^n}{L^n})^{\mu_1 - 2} (\frac{L}{S})^{\mu_2} (N(y_1 - \sigma\sqrt{T}) - N(y_2 - \sigma\sqrt{T})) - (\frac{L^{n+1}}{U^n S})^{\mu_3 - 2} (N(y_3 - \sigma\sqrt{T}) - \\ N(y_4 - \sigma\sqrt{T})) \end{array} \right\} -$$

$$Se^{(b-r)T} \sum_{n=-\infty}^{\infty} \left\{ (\frac{U^n}{L^n})^{\mu_1} (\frac{L}{S})^{\mu_2} (N(y_1) - N(y_2)) - (\frac{L^{n+1}}{U^n S})^{\mu_3} (N(y_3) - N(y_4)) \right\}$$

11.6 Commodity Average Rate Swap

11.6.1 Description of Instrument

Commodity Average Swap is represented as the stream of Commodity Average Forward transactions on the same commodity with successive delivery and maturity days that can have different notional amounts on the floating side of the swap.

11.6.2 XML Representation


```
<commoditySwap>
      <productType>Commodity</productType>
     <commodityForward>
      <commodityPhysicalSettlement>
        <firmness>firm</firmness>
      </commodityPhysicalSettlement>
      <buyerParty href='EUROPE' />
      <sellerParty href='DKW' />
      <commodityBuyerPrice>
        <formulaPrice>
          <fixedPrice>
            <precision>0</precision>
            <currency>USD</currency>
            <amount></amount>
            <perUom></perUom>
          </fixedPrice>
          <indexPrice>
            <margin>1.00</margin>
```

```
      <index>WTI</index>
      <indexPct>1.0</indexPct>
      <indexTenor>
        <period>W</period>
        <periodMultiplier>1</periodMultiplier>
      </indexTenor>
      <fixingDateOffset>
        <period>M</period>
        <periodMultiplier>-2</periodMultiplier>
  <businessDayConvention>FOLLOWING</businessDayConvention>
        <dayType>Business</dayType>
      </fixingDateOffset>
    </indexPrice>
    <averagingTerms>
      <averagingMethod>Unweighted</averagingMethod>
      <averagingStartDate opt='y'>
        <anchor>FixingDate</anchor>
        <adjustment>optional not used</adjustment>
        <offset>
          <periodMultiplier>-2</periodMultiplier>
          <period>D</period>
  <businessDayConvention>FOLLOWING</businessDayConvention>
          <dayType>Business</dayType>
        </offset>
      </averagingStartDate>
      <averagingEndDate opt='y'>
        <anchor>FixingDate</anchor>
        <adjustment>optional not used</adjustment>
        <offset>
          <periodMultiplier>2</periodMultiplier>
          <period>D</period>
<businessDayConvention>FOLLOWING</businessDayConvention>
          <dayType>Business</dayType>
        </offset>
      </averagingEndDate>
    </averagingTerms>
  </formulaPrice>
</commodityBuyerPrice>
<commodityUnits>
  <commodity>WTI</commodity>
  <totalVolume>60000</totalVolume>
  <totalUom>BBL</totalUom>
  <volumeAmount>0</volumeAmount>
  <volumeUom>X</volumeUom>
  <volumePerFreq>TERM</volumePerFreq>
</commodityUnits>
<commodityDeliveryPeriod>
  <startDate>2006-07-01</startDate>
  <endDate>2006-08-01</endDate>
</commodityDeliveryPeriod>
```

```
</commodityForward>
<commodityForward>
  <commodityPhysicalSettlement>
    <firmness>firm</firmness>
  </commodityPhysicalSettlement>
  <buyerParty href='DKW' />
  <sellerParty href='EUROPE' />
  <commodityBuyerPrice>
    <formulaPrice>
      <fixedPrice>
        <precision>0</precision>
        <currency>USD</currency>
        <amount></amount>
        <perUom></perUom>
      </fixedPrice>
      <indexPrice>
        <margin>1.00</margin>
        <index>WTI</index>
        <indexPct>1.0</indexPct>
        <indexTenor>
          <period>M</period>
          <periodMultiplier>1</periodMultiplier>
        </indexTenor>
        <fixingDateOffset>
          <period>M</period>
          <periodMultiplier>-2</periodMultiplier>
     <businessDayConvention>FOLLOWING</businessDayConvention>
          <dayType>Business</dayType>
        </fixingDateOffset>
      </indexPrice>
      <averagingTerms>
        <averagingMethod>Unweighted</averagingMethod>
        <averagingStartDate opt='y'>
          <anchor>FixingDate</anchor>
          <adjustment>optional not used</adjustment>
          <offset>
            <periodMultiplier>-2</periodMultiplier>
            <period>D</period>
     <businessDayConvention>FOLLOWING</businessDayConvention>
            <dayType>Business</dayType>
          </offset>
        </averagingStartDate>
        <averagingEndDate opt='y'>
          <anchor>FixingDate</anchor>
          <adjustment>optional not used</adjustment>
          <offset>
            <periodMultiplier>2</periodMultiplier>
            <period>D</period>
     <businessDayConvention>FOLLOWING</businessDayConvention>
            <dayType>Business</dayType>
```

```
          </offset>
        </averagingEndDate>
      </averagingTerms>
    </formulaPrice>
  </commodityBuyerPrice>
  <commodityUnits>
    <commodity>WTI</commodity>
    <totalVolume>60000</totalVolume>
    <totalUom>BBL</totalUom>
    <volumeAmount>0</volumeAmount>
    <volumeUom>X</volumeUom>
    <volumePerFreq>TERM</volumePerFreq>
  </commodityUnits>
  <commodityDeliveryPeriod>
    <startDate>2006-07-01</startDate>
    <endDate>2006-08-01</endDate>
  </commodityDeliveryPeriod>
</commodityForward>
</commoditySwap>
```


11.6.3 Pricing

The value of commodity swap contract can be calculated as the sum of rolling commodity forward contracts values:

$$V_{swap} = \sum_{i=1}^{N} V_i = \sum_{i=1}^{M} (F_i - X_i) D_{T_i} N_i \ ,$$

where:

V_{swap} - commodity payer swap price

M - number of legs in commodity swap

V_i - i'th leg commodity forward contract value

F_i – the projected forward commodity value derived from the underlying commodity index forward curve.

X_i – i'th leg commodity forward contract price. This can be the same for all N legs of commodity swap.

D_{T_i} - discount factor to the payment date of i'th contract

N_i - i'th contract notional.

Average price commodity swaps also referred to as Asian type contracts are settled against the average of prices for an underlying commodity over a period of time.

The value of the average rate commodity swap contract is represented by:

$$V_{swap} = \sum_{i=1}^{N} V_i = \sum_{i=1}^{M} (A_i - X_i) D_{T_i} N_i$$

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with A_i defined as the floating arithmetic average rate accumulated through the i'th leg averaging period.
Depending if the considered averaging period is pending or current the average commodity rate is either derived from the underlying forward curve adjusted by simulated or observed values.
For pending averaging period starting after the valuation date the average rate is determined from the interpolated values derived from the forward curve

$$A_i = \frac{1}{K_i} \sum_{j=1}^{K_i} F_{t_i}$$

K_i - the number of required observations in i'th leg averaging period,

F_{t_i} - interpolated from forward curve projected observation at time t_i within i'th averaging period.
For current averaging period started before the valuation date the last formula should be adjusted:

$$A_i = \frac{mA + \sum_{j=K_i-m+1}^{K_i} F_{t_i}}{m + K_i}$$

where A is current accumulated average and m is the number of past observations.
The XML presentation of commodity swaps and forwards allows definition of the averaging period schedule of each leg.
Average price options, also referred to as Asian options or average rate options, are settled against the average of prices for an underlying commodity over a period of time.
Average price options are financially settled upon expiration and cannot be exercised into the underlying futures contract.

11.7 Commodity Average Rate Options

11.7.1 Description of Instrument

Average price options, also referred to as Asian options or average rate options, are settled against the average of prices for an underlying commodity over a period of time. These instruments have become popular in the over-the-counter markets during the past decade as a way of dampening market volatility.

Average price options are financially settled upon expiration and cannot be exercised into the underlying futures contract.

11.7.2 XML Presentation

```xml
<commodityOption>
    <commodityUnderlying>
      <commodityPhysicalSettlement>
        <firmness>firm</firmness>
      </commodityPhysicalSettlement>
      <buyerParty href='NTHAMERICA' />
      <sellerParty href='BARCAP' />
      <commodityBuyerPrice>
        <formulaPrice>
          <indexPrice>
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              <period>M</period>
              <businessDayConvention>Business</businessDayConvention>
            </fixingDateOffset>
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          <fixedPrice>
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            <currency>USD</currency>
            <amount>0</amount>
            <perUom>BBL</perUom>
          </fixedPrice>
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            <averagingEndDate>
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          </averagingTerms>
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      </commodityBuyerPrice>
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        <volumeUom>X</volumeUom>
        <volumePerFreq>TERM</volumePerFreq>
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      <commodityDeliveryPeriod>
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        <endDate>2006-07-11</endDate>
      </commodityDeliveryPeriod>
    </commodityUnderlying>
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    <sellerParty href='BARCAP' />
    <payoutFormula>AverageRate</payoutFormula>
    <optionType>CALL</optionType>
    <strike>
      <strikePrice>70</strikePrice>
      <buyer>NTHAMERICA</buyer>
      <seller>BARCAP</seller>
    </strike>
  </commodityOption>
```


11.7.3 Pricing

The Levy Approximation formula is used for pricing arithmetic average-rate commodity option.

$C_{asian} \approx S_A N(d_1) - Xe^{-rT_2} N(d_2)$

where

C_{asian} - average-rate commodity call option

X - is the commodity option strike price,

S_A - arithmetic average of the known commodity price fixing and

$S_A = \frac{S}{T * b}\left(e^{(b-r)T_2} - e^{-rT_2}\right)$,

S – representing commodity spot price, r - risk free interest rate, b - commodity cost of carry rate, T_2-remaining time to option maturity from valuation date and T - original time to option maturity.

Other components of Levy approximation formula calculated according to:

$d_1 = \frac{1}{\sqrt{V}}\left[\frac{\ln(D)}{2} - \ln(X^*)\right]$ and $d_2 = d_1 - \sqrt{V}$

The last expressions use following notations:

$D = \frac{M}{T^2}$, $V = \ln(D) - 2\left[rT_2 + \ln(S_A)\right]$ and $X^* = X - \frac{T - T_2}{T} S_A$.

$$M = \frac{2S^2}{b+\sigma^2}\left[\frac{e^{(2b+\sigma^2)T_2}-1}{2b+\sigma^2}-\frac{e^{bT_2}-1}{b}\right].$$

Chapter 12
Interest Rate Derivatives

12.1 Forward Rate Agreements

12.1.1 Description of Instrument

A FRA is the right to buy or sell a short-term money market instrument at some date in the future. It is essentially a contract that locks in a forward interest rate for a counterparty.

12.1.2 XML Representation

FRAs are represented in FPML version 3.

fpmlFRA Schema			
Name: Type	**Occurs**	**Size**	**Description**
adjustedEffectiveDate fpmlAdjustedDate	1..1		The start date of the FRA
adjustedTerminationDate fpmlAdjustedDate	0..1		The end date of the FRA
paymentDate fpmlUnadjustedDate	1..1		The date the payment occurs on the FRA
fixingDateOffset fpmlRelativeDateOffset	1..1		The date offset that the floating rate is fixed
dayCountFraction string	0..1	10	The day count fraction used to calculate the payment amount
calculationPeriodNumberOfDays int	0..1		The number of days in the calculation period
notional fpmlAmount	1..1		The notional face value of the FRA
fixedRate double	1..1		The fixed rate of the FRA
floatingRateIndex string	0..1	50	The floating rate index to use as a reference rate
indexTenor fpmlTenor	1..1		The tenor of the FRA
fraDiscounting boolean	1..1		Whether discounting is applied to the FRA

12.1.3 Pricing

There are essentially two formulas for pricing FRAs depending on the market in which the FRA is traded — the BBA formula which is typically used for FRAs traded outside of Australia, and the AFMA formula.

The AFMA formula is as follows:

Let

F_{fra} = The notional face value of the FRA

r_{fra} = The agreed FRA rate

r_m = The market benchmark rate, the forward rate derived from the index curve

r_{zero} = the zero coupon rate from pricing date to settlement

r_{mat} = the zero coupon rate from pricing date to maturity

d_{fra} = number of days from settlement to maturity

d_{fwd} = number of days from valuation to settlement

D = number of days in the year

V_{fra} = FRA exposure

Then

$$V_{fra} = \frac{\left(\frac{F_{fra} r_{fra} d_{fra} / D}{1 + r_{fra} d_{fra} / D}\right) - \left(\frac{F_{fra} r_m d_{fra} / D}{1 + r_m d_{fra} / D}\right)}{1 + r_{zero} d_{fwd} / D}$$

The BBA formula for pricing FRAs is:

$$V_{fra} = \frac{(F_{fra} r_{fra} d_{fra} / D) - (F_{fra} r_m d_{fra} / D)}{1 + r_{mat} d_{fra} / D}$$

12.2 Interest Rate and Cross Currency Swaps

12.2.1 Description of Instrument

A swap is an agreement whereby two counterparties exchange periodic interest payments. In the case of a cross currency swap, these period payments are denominated in different currencies.

12.2.2 XML Representation

Interest Rate Swaps are represented in FPML version 3.

fpmlSwap Schema			
Name: Type			Description

fpmlSwap Schema			
Name: Type			Description
swapStream fpmlInterestRateStream			The fpmlInterestRateStream structure that defines the interest rate flows for the swap


```
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 <swapStream>
   <payerPartyReference href="ITE" />
   <receiverPartyReference href="BAN" />
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     <effectiveDate id="">i
       <unadjustedDate>2003-05-29</unadjustedDate>
     </effectiveDate>
     <terminationDate id="">
       <unadjustedDate>2004-05-31</unadjustedDate>
     </terminationDate>
     <calculationPeriodDatesAdjustments>
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     </calculationPeriodDatesAdjustments>
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       <rollConvention>IMM</rollConvention>
     </calculationPeriodFrequency>
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      </principalExchanges>
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        </terminationDate>
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        </calculationPeriodDatesAdjustments>
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        </calculationPeriodFrequency>
      </calculationPeriodDates>
      <paymentDates>
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          <periodMultiplier>6</periodMultiplier>
          <period>M</period>
        </paymentFrequency>
        <payRelativeTo id="">CalculationPeriodEndDate</payRelativeTo>
        <paymentDatesAdjustments>
```


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```xml
        <businessDayConvention>MODFOLLOWING</businessDayConvention>
      </paymentDatesAdjustments>
    </paymentDates>
    <resetDates>
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          <negativeInterestRateTreatment></negativeInterestRateTreatment>
        </floatingRateCalculation>
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      <finalExchange>false</finalExchange>
      <intermediateExchange>false</intermediateExchange>
    </principalExchanges>
  </swapStream>
</swap>
```

12.2.3 Pricing

Pricing interest rate swaps is a matter of present-valuing both legs of the swap. The formula presented below gives an example of pricing a vanilla fixed-floating swap. The pricing model is very general however and will price floating-floating and cross currency swaps.

Let

F_{float} = the face value of the floating leg of the swap

F_{fixed} = the face value of the fixed leg of the swap

r_{fixed} = the coupon rate of the swap

$r_{j-1\to j}$ = the implied forward rate of the swap

p_{float} = the floating payment periods per year

p_{fixed} = the fixed payment periods per year

V_{swap} = the exposure of the swap

df_j = the discount factor from time $0 \to j$

Then

$$V_{swap} = \sum_{i=1}^{n} F_{fixed} \frac{r_{fixed}}{p_{fixed}} df_i + \sum_{j=1}^{n} F_{float} \frac{r_{j-1\to j}}{p_{float}} df_j$$

12.2.4 Other Variations of Swaps

Averaging Swaps

These contracts are similar to normal swaps except that the cashflows of the floating leg are based on the average of some index rates.

Compounding Swaps

These contracts are swaps that the cashflows of the floating leg are based on some index rates compounded over the accruing period.

Overnight Index Swap (OIS)

In a OIS transaction, the floating rate used to determine the floating leg cashflows is based on an overnight rate that is reset daily. This typically is the interbank overnight or call interest rate.

Pricing

The pricing of these variations of swaps are the same as the conventional interest rate swaps, i.e. the present value of the difference between the floating leg cashflow value and fixed leg cashflow value. The required unknown floating rates used to determine the floating leg cashflows can always be obtained from the forward rate curve as in the conventional swap case.

12.3 Interest Rate Futures

12.3.1 Description of Instrument

There are three types of interest rate futures supported in Razor, namely, 30 day interbank cash rate futures, 90 day bank bill futures and 3 and 10 year bond futures.

1. 30 Day Interbank Cash Rate Futures

   These contracts are based on the interbank overnight cash rate published by the Reserve Bank of Australia and allow users to hedge against fluctuations in the overnight cash rate and better manage their daily cash exposures. These contracts allow the buyer to fix the cash rate in a specified future date for 30 days.

2. 90 Day Bank Bill Futures

   These contracts are the major short term interest rate derivative products.These contracts allow the holder to enter into a 90 day bank bill at a predetermined price in a specified future date.

3. 3 and 10 Year Bond Futures

   These contracts are the major medium and long term interest rate derivative products in Australia.These contracts allow the buyer to enter into a 3 or 10 year bond at a predetermined price in a specified future date.

Note Razor supports futures contracts in all major economies and different maturity dates for bill futures and bond futures are also supported.

12.3.2 XML Representation

Bill Futures


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<deal>
    <trade>
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        <tradeDate>2007-09-12</tradeDate>
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        <counterparty>83976</counterparty>
        <internalUnit>GTYA</internalUnit>
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        <status>OPEN</status>
        <location>Bank1</location>
        <domicileCountry>154</domicileCountry>
      </tradeHeader>
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        <extension>
          <value>ValueByMTMYieldCurve</value>
          <name>FuturesValueStyle</name>
        </extension>
```

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      &lt;name&gt;DiscountFuturesValueToValueDate&lt;/name&gt;
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  &lt;/extensions&gt;
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    &lt;irFuture&gt;
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      &lt;/settlementDate&gt;
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            &lt;/terminationDate&gt;
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      </irFuture>
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Bond Futures


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      <dealer>84751</dealer>
      <counterparty>83976</counterparty>
      <internalUnit>CFXA</internalUnit>
      <buySell>BUY</buySell>
      <status>OPEN</status>
      <location>Bank1</location>
      <domicileCountry>154</domicileCountry>
    </tradeHeader>
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        <value>ValueByMTMYieldCurve</value>
        <name>FuturesValueStyle</name>
      </extension>
      <extension>
        <value>true</value>
        <name>DiscountFuturesValueToValueDate</name>
      </extension>
    </extensions>
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      <irFuture>
        <productType>BondFuture</productType>
        <instrumentId>SFYTB</instrumentId>
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        </settlementDate>
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  <bondStream>
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```
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Cash Rate Futures


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       <exchangeCode>SFE</exchangeCode>
       <numberContracts>400</numberContracts>
```


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<tradedPrice>93.67</tradedPrice>
<priceQuote>FuturesPrice</priceQuote>
<fixedTickValue>24.66</fixedTickValue>
<fixedTickUnit>1</fixedTickUnit>
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  <bondStream>
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    <receiverPartyReference href="GTY" />
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      <effectiveDate>
        <unadjustedDate>2007-10-01</unadjustedDate>
        <dateAdjustments>
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        </dateAdjustments>
      </effectiveDate>
      <terminationDate>
        <unadjustedDate>2007-10-31</unadjustedDate>
        <dateAdjustments>
          <businessDayConvention>NONE</businessDayConvention>
        </dateAdjustments>
      </terminationDate>
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      </paymentFrequency>
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```

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                &lt;fixedRateSchedule /&gt;
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            &lt;/calculationPeriodAmount&gt;
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              &lt;initialExchange&gt;false&lt;/initialExchange&gt;
              &lt;finalExchange&gt;true&lt;/finalExchange&gt;
            &lt;/principalExchanges&gt;
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          &lt;position&gt;1&lt;/position&gt;
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  &lt;/dealHeader&gt;
&lt;/deal&gt;
```

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12.3.3 Pricing

We define some notations before moving to pricing of each type of interest rate futures:

F = face value.

f_0 = futures price at contract initiation.

f_t = futures price at valuation date.

TV = tick value.

TS = tick size.

P = price of underlying asset, e.g. bank bill if it is a bank bill future.

N = number of futures contracts.

Note:

1. Futures prices are quoted as $1 - yield\%$.
2. $\frac{TV}{TS}$ represents the value of 0.01% of premium.

Pricing of Interest Rate Futures

1. 30 Day Interbank Cash Rate Futures

   Depend on the user's requirement, Razor can return four different types of values in regard to the values of the cash rate futures.

   (a) The Trade Price Amount

   This is equal to $N \times F \times \left(1 + \frac{100 - f_0}{100} \times \frac{30}{365}\right)$.

   (b) The Futures Price Amount

   This is equal to $N \times F \times \left(1 + \frac{100 - f_t}{100} \times \frac{30}{365}\right)$

   (c) The Net Settlement Amount

   This is equal to $N \times F \times (f_t - f_0) \times \left(\frac{TV}{TS}\right)$.

   (d) The present values of (a), (b) and (c) are also supported.

   It should be noted that for 30 day interbank cash rate futures, because the contracts always have a face value with fixed value $F = 3,000,000$ at contract initiation rather than expiry with a fixed term to maturity of 30 days, the 0.01% premium is always $\frac{TV}{TS} = 3000000 \times 0.0001 \times \frac{30}{365} = 24.66$.

2. 90 Day Bank Bill Futures

The price of a bank bill is:

$$P = \frac{F \times 365}{365 + \left(\frac{yield\% \times DaystoMaturity}{100} \right)}.$$

For a 90 day bank bill future, the price is simply:

$$P = \frac{F \times 365}{365 + \left(\frac{(100 - f) \times 90}{100} \right)}.$$

Again, the four types of values Razor can return for the bond futures are:

(a) The Trade Price Amount

This is equal to $\dfrac{N \times F \times 365}{365 + \left(\frac{(100 - f_0) \times 90}{100} \right)}$.

(b) The Futures Price Amount

This is equal to $\dfrac{N \times F \times 365}{365 + \left(\frac{(100 - f_t) \times 90}{100} \right)}$.

(c) The Net Settlement Amount

This is equal to

$$N \times F \times 365 \times \left(\frac{1}{365 + \left(\frac{(100 - f_t) \times 90}{100} \right)} - \frac{1}{365 + \left(\frac{(100 - f_0) \times 90}{100} \right)} \right).$$

(d) The present values of (a), (b) and (c) are also supported.

3. 3 and 10 Year Bond Futures

The price of bond is:

$$P = N \times F \times \left[\frac{c\left(1 - v_f^n\right)}{i_f} + v_f^n \right],$$

where

$$i_f = \frac{100 - f}{100} \times \frac{1}{2}.$$

$$v_f = \left(1 + i_f\right)^{-1}.$$

$$n = \begin{cases} 6 & \text{if 3 year bond} \\ 20 & \text{if 10 year bond} \end{cases}.$$

$$c = \frac{CouponRate\%}{100} \times \frac{1}{2}.$$

The values can be returned by Razor are:

(a) The Trade Price Amount

This is equal to $N \times F \times \left[\frac{c\left(1 - v_{f_0}^n\right)}{i_{f_0}} + v_{f_0}^n \right]$.

(b) The Futures Price Amount

This is equal to $N \times F \times \left[\frac{c\left(1 - v_{f_t}^n\right)}{i_{f_t}} + v_{f_t}^n \right]$.

(c) The Net Settlement Amount

The net settlement amount of a bond future is:

$$N \times F \times \left[\left(\frac{c\left(1 - v_{f_t}^n\right)}{i_{f_t}} + v_{f_t}^n \right) - \left(\frac{c\left(1 - v_{f_0}^n\right)}{i_{f_0}} + v_{f_0}^n \right) \right].$$

(d) The present values of (a), (b) and (c) are also supported.

Note that we have semi-annual compounding in the examples above. Razor is able to set frequencies in other conventional intervals, e.g. quarterly and yearly.

12.4 Bond Options, Options on Bond Futures, and Options on Money Market or Bill Futures

12.4.1 Description of Instrument

Options on interest rate products are the right given to the holder of the options to enter into the interest rate products in the future at a predetermined rate or price.

12.4.2 XML Representation

Option on Bond Future: - note for Option on Bill Future - just replace bond section with example bond section from bill future.


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<deal>
    <trade>
      <tradeHeader>
        <tradeId>OPT:1289</tradeId>
        <tradeDate>2007-09-07</tradeDate>
        <tradeType>SFYTO</tradeType>
```

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      <counterparty>78907</counterparty>
      <internalUnit>TDXA</internalUnit>
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      <status>OPEN</status>
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      <domicileCountry>154</domicileCountry>
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    <product>
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        <optionType>CALL</optionType>
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          <strikePrice>93</strikePrice>
          <priceQuote>FuturesPrice</priceQuote>
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          <seller>TDX</seller>
        </strike>
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          </paymentDate>
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  </calculation>
</calculationPeriodAmount>
```

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          <position>1</position>
          <paymentAmount>
            <paymentType>SETTLEMENT</paymentType>
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            </paymentAmount>
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          </paymentAmount>
          <exInterestDays>0</exInterestDays>
        </bond>
        <optionWriterReference href="0" />
        <optionHolderReference href="78907" />
      </bondOption>
    </product>
  </trade>
  <dealHeader>
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    <dealType>SFYTO</dealType>
    <dealDate>2007-09-07</dealDate>
    <status>OPEN</status>
  </dealHeader>
</deal>
```


12.4.3 European Exercise Pricing

European bond options, options on bond futures, and options on money market or bill futures are all priced using the Black-76 model.

This is achieved by converting strike quotes to a consistent dollar price quote, and converting yield volatilities if provided to price volatilities. These conversions are outlined below the model details.

The Black-76 Model

Let

$F =$ the forward market price

$\sigma =$ the price volatility

$X =$ the strike price

$T_x =$ the time from today to expiry, annualised

$T_S =$ the time from today to settlement, annualised

$r =$ the continuous yield to settlement

$N =$ the cumulative normal distribution function.

$$d_1 = \frac{\ln(\frac{F}{X}) + \frac{\sigma^2 T_x}{2}}{\sigma\sqrt{T_x}}$$

$$d_2 = d_1 - \sigma\sqrt{T_x}$$

The European call price is given by

$$c = e^{-rT_S}\left[FN(d_1) - XN(d_2)\right]$$

The European put price is given by

$$p = e^{-rT_S}\left[XN(-d_2) - FN(-d_1)\right]$$

Please refer to section 17.1.4 - generalised Black-Scholes option pricing formula for further detail and the Greeks.

Strike Quote Conversions

Futures Price = 100 - Yield

Dollar Price = ForwardPrice(Yield, T), where T is forward valuation date for the underlying

Yield to Price Volatility Conversion

$\sigma_P = \sigma_Y yD$ where

$\sigma_P =$ price volatility

$\sigma_Y =$ yield volatility

$y =$ forward yield (continuous)

$D =$ MacCaulay Duration

12.4.4 American Exercise Pricing

American bond options, options on bond futures, and options on money market or bill futures are all priced using the Binomial CRR model. For details, please refer to Binomial Model on page 264. Pricing uses the same model as an equity option, with coupons instead of discrete dividends. Due to the recursive binomial calls, performance with many coupons can be an issue. This can be avoided by using no more steps than coupons. Volatility is assumed to be yield vol, and is converted into price vol by multiplying by MacCaulay duration and YTM.

12.5 Caps and Floors

12.5.1 Description of Instrument

Caps and floors are akin to a swap with embedded options on the floating rate payments, which in effect "cap" the floating rate exposure, or limit the downside liability.

12.5.2 XML Representation

fpmlCapFloor Schema			
Name: Type			Description
capFloorStream fpmlInterestRateStream			

fpmlInterestRateStream Schema			
Name: Type	Occurs	Size	Description
payerPartyReference fpmlPartyReference	1..1		The identifier of the paying party
receiverPartyReference fpmlPartyReference	1..1		The identifier of the receiving party
calculationPeriodDates fpmlCalculationPeriodDates	1..1		Indicates the schedule that the floating rate calculations occurs
paymentDates fpmlPaymentDates	1..1		Indicates the schedule that the date payments occur
resetDates fpmlResetDates	1..1		This structure indicates when floating rate resets occur
calculationPeriodAmount fpmlCalculationPeriodAmount	1..1		This structure indicates how the amounts to be paid is determined
stubCalculationPeriodAmount fpmlStubCalculationPeriodAmount	0..1		This structure indicates how the stub period amounts to be paid are determined

fpmlInterestRateStream Schema			
Name: Type	Occurs	Size	Description
principalExchanges fpmlPrincipalExchanges	0..1		Determines if and when the exchange
cashflows fpmlCashflows	0..1		This type gives us the fixed cash flows represented by the product

12.5.3 Pricing

Each cap or floor is considered to be made up of “caplets” or “floorlets”, that limit the liability or exposure for each floating rate payment. The caplets or floorlets are valued separately as options and then summed to produce the overall value for the cap or floor.

We can use the Black model to allow us to price caps by inputting rf as the current forward price, and rx as the strike price. The output of the Black model is in terms of a yield percentage per anum. To convert this into a dollar amount, we need to multiply the yield by the interest sensitivity of the option. This interest sensitivity will be determined by the principal amount of the option and the term of the underlying interest period.

Let

$S =$ the forward market rate

$X =$ the cap/floor rate

$\sigma =$ the yield volatility

F = the implied forward rate at each caplet maturity as the underlying asset.

τ = number of days in the forward rate period.

$basis$ = number of days per year used in the market.

$T =$ the time until expiry, annualised

$$d_1 = \frac{\ln(\frac{S}{X}) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\frac{S}{X}) - \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

$$V_{floorlet} = \frac{Notional \times \frac{\tau}{basis}}{1 + F\frac{\tau}{basis}} e^{-rT}\left(XN(-d_2) - SN(-d_1)\right)$$

$$V_{floor} = \sum_{i=1}^{n} V_{floorlet}$$

$$V_{caplet} = \frac{Notional \times \frac{\tau}{basis}}{1 + F\frac{\tau}{basis}} e^{-rT}\left(SN(d_1) - XN(d_2)\right)$$

$$V_{cap} = \sum_{i=1}^{n} V_{caplet}$$

12.6 Collars

12.6.1 Description of Instrument

A collar is like an agregation of a cap and floor, in that the interest rate exposure is limited to a certain range, with both a cap and a floor.

12.6.2 XML Representation

The collar product makes use of the FPML cap and floor product specifications.

12.6.3 Pricing

Let

$S =$ the underlying market par rate

$X_{cap} =$ the cap rate

$X_{floor} =$ the floor rate

$\sigma =$ the yield volatility

$T =$ the time until expiry, annualised

$$d_1 = \frac{\ln(\frac{S}{X}) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(\frac{S}{X}) - \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

The collar premium is given by:

$$V_{collar} = \sum_{i=1}^{n} \frac{Notional \times \frac{\tau_i}{basis}}{1 + F_i \frac{\tau_i}{basis}} e^{-rT_i} \left(S_i N(d_1) - X_{cap,i} N(d_2) \right)$$

$$- \sum_{i=1}^{n} \frac{Notional \times \frac{\tau_i}{basis}}{1 + F_i \frac{\tau_i}{basis}} e^{-rT_i} \left(X_{floor,i} N(-d_2) - S_i N(-d_1) \right).$$

Note that a collar can be structured as buying a cap and selling a floor simultaneously with the same underlying details and expiry.

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12.7 Swaption

12.7.1 Description of Instrument

A swaption is an agreement between two counterparties to enter into a currency or interest rate swap at an agreed fixed rate at a date in the future.

12.7.2 XML Representation

fpmlSwaption Schema			
Name: Type	Occurs	Size	Description
premium fpmlPremium	1..1		The premium for the swaption
europeanExercise fpmlEuropeanExercise	1..1		The exercisable period
calculationAgentPartyReference fpmlRef	1..1		
cashSettlement fpmlCashSettlement	0..1		
swaptionStraddle boolean	0..1		
swap ftmlSwap	1..1		The underlying swap

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  <europeanExercise>
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      <unadjustedDate>2001-05-31</unadjustedDate>
    </commencementDate>
    <expirationDate id="">
      <unadjustedDate>2003-05-29</unadjustedDate>
    </expirationDate>
  </europeanExercise>
  <swap>
    <swapStream>
      <payerPartyReference href="ITE" />
      <receiverPartyReference href="BAN" />
      <calculationPeriodDates id="CalcPeriodDates0">
        <effectiveDate id="">
          <unadjustedDate>2003-05-29</unadjustedDate>
        </effectiveDate>
        <terminationDate id="">
          <unadjustedDate>2004-05-31</unadjustedDate>
        </terminationDate>
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          <period>M</period>
```

```xml
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         <paymentFrequency>
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            <period>M</period>
         </paymentFrequency>
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            <businessDayConvention>MODFOLLOWING</businessDayConvention>
         </paymentDatesAdjustments>
      </paymentDates>
      <resetDates>
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            <period>D</period>
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         </fixingDates>
         <resetFrequency>
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            <period>M</period>
         </resetFrequency>
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      </resetDates>
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            </notionalSchedule>
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            </fixedRateSchedule>
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         </calculation>
      </calculationPeriodAmount>
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         <finalExchange>false</finalExchange>
         <intermediateExchange>false</intermediateExchange>
      </principalExchanges>
   </swapStream>
   <swapStream>
      <payerPartyReference href="BAN" />
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```

```xml
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    </effectiveDate>
    <terminationDate id="">
        <unadjustedDate>2004-05-31</unadjustedDate>
    </terminationDate>
    <calculationPeriodDatesAdjustments>
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    </calculationPeriodDatesAdjustments>
    <calculationPeriodFrequency>
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    </calculationPeriodFrequency>
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    </paymentFrequency>
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      <intermediateExchange>false</intermediateExchange>
  </principalExchanges>
 </swapStream>
</swap>
</swaption>
```


12.7.3 Pricing

European swaptions are priced using the Black-76 model. The Black-76 value is multiplied by a factor adjusting for the tenor of the swaptions as shown by Smith (1991). American and Bermudan swaptions can be priced either using the Black model or using a numerical approximation (such as HGM/J). Most numerical approximations for Bermudan swaptions will have a large detrimental impact on the simulation process, so directing these trades to the PELookup Server is recommended unless a fast analytical solution is used.

The Black-76/Smith Model

Let

t_1 = Tenor of swap in years.

F = Forward rate of underlying swap.

X = Strike rate of swaption.

r = Risk-free interest rate.

T = Time to expiration in years.

σ = Volatility of the forward-starting swap rate.

m = Compoundings per year in swap rate.

Price of payer swap is:

$$c = \left[\frac{1 - \frac{1}{\left(1+\frac{F}{m}\right)^{t_1 \times m}}}{F} \right] e^{-rT} \left[FN(d_1) - XN(d_2) \right]$$

Price of receiver swap is:

$$p = \left[\frac{1 - \frac{1}{\left(1+\frac{F}{m}\right)^{t_1 \times m}}}{F} \right] e^{-rT} \left[XN(-d_2) - FN(-d1) \right],$$

where

$$d_1 = \frac{\ln\left(\frac{F}{X}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T} .$$

12.7.4 Calculating Greeks

The Greeks are calculated numerically using central finite difference approximations for both the first and second derivatives. Central differences are used since the order of accuracy $O(h^2)$ (for the first order) is greater than single (forward or backward) differences which is $O(h)$

The first and second derivatives in relation to the central finite differences are:

$$f^{'}(x) = \frac{\partial}{\partial x} = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h}$$

$$f^{''}(x) = \frac{\partial^2}{\partial x^2} = \lim_{h \to 0} \frac{f(x+h) + f(x-h) - 2f(x)}{h^2}$$

where

$$\left. \frac{\partial}{\partial x} \approx \frac{f(x+h) - f(x-h)}{2h} \right\} h \in \mathbb{R}, h = (0, \varepsilon)$$

The limit can be approximated by selecting a sufficiently small non-zero value for h. In the case of delta,gamma,vega and rho, h is set to 1 basis point or 0.0001. Theta h is chosen as 1 day in units of years or $1/365 \approx 0.00274$. For theta we use only the backward finite difference defined as:

$$f^{'}(x) = \frac{\partial}{\partial x} = \lim_{h \to 0} \frac{f(x) - f(x-h)}{h}$$

Delta

Delta is defined as the difference in value of the swaption by perturbing the forward swap rate F by $\pm 1bp$ where all other parameters remain constant.

Let
f = a function defined as the value of the payer or receiver swaption
F = Forward rate of underlying swap
$1bp$ = 1 basis point or 0.0001

$$\Delta = \frac{\partial V}{\partial F} \approx \frac{f(F+1bp) - f(F-1bp)}{2bp}$$

Gamma

Gamma is defined as the difference in delta by perturbing the forward swap rate F by $\pm 1bp$ where all other parameters remain constant. In Razor we approximate gamma by using the second order finite central difference.

Let
f = a function defined as the price of the payer or receiver swaption
F = Forward rate of underlying swap
$1bp$ = 1 basis point or 0.0001

$$\Gamma = \frac{\partial \Delta}{\partial F} = \frac{\partial^2 V}{\partial F^2} \approx \frac{f(F+1bp) + f(F-1bp) - 2f(F)}{1bp^2}$$

Vega

Vega is defined as the difference in value of the swaption by perturbing the forward swap volatility σ by $\pm 1\%$ where all other parameters remain constant.

Let
f = a function defined as the price of the payer or receiver swaption
σ = Volatility of the forward-starting swap rate.
1% = 1 percent as a decimal or 0.01

$$\Lambda = \frac{\partial V}{\partial \sigma} \approx \frac{f(\sigma+1\%) - f(\sigma-1\%)}{2\%}$$

Rho

Rho is defined as the change in value of the swaption by perturbing the risk free rate r by $\pm 1bp$ where all other parameters remain constant.

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In Razor, pricing formulas accept the risk free rate in terms of a continuously compounded discount factor.

Let
f = a function defined as the price of the payer or receiver swaption
df = a continuously compounded discount factor of r in terms of T
dfs = discount factor shift amount as $dfs = e^{-0.0001 T}$
$1bp$ = 1 basis point or 0.0001

Here we use a modified version of the central derivative. Since the continuously compounded discount factor is logarithmic we substitute in the following relations:

$$\ln a + \ln b = \ln(ab)$$
$$\ln a - \ln b = \ln(a/b)$$
$$\rho = \frac{\partial V}{\partial r} \approx \frac{f(df \times dfs) - f(df / dfs)}{2bp}$$

Theta

Theta is defined as the difference in value by shifting the value date forward by one day where all other parameters remain constant. Since time moves in a forward direction, for theta we use the backward finite difference which is defined as:

$$\left.\frac{\partial}{\partial x} \approx \frac{f(x) - f(x-h)}{h}\right\} h > 0$$

In Razor, pricing formulas accept the risk free rate in terms of a continuously compounded discount factor. Therefore, an adjustment to the discount factor is made to account for the decay of 1 day.

Let
f = a function defined as the price of the payer or receiver swaption
T = Time to expiration in years.
$T-1$ = Time to expiration minus 1 day in years.
df = a continuously compounded discount factor of r in terms of T
$df_{T_{-1}}$ = a continuously compounded discount factor of r in terms of T_{-1}
$1dy$ = 1 day in years or 1/365

Where

$$df_{T_{-1}} = df^{\frac{T-1}{T}}$$

$$\Theta = -\frac{\partial V}{\partial T} \approx \frac{f(T_{-1}, df_{T_{-1}}) - f(T)}{1dy}$$

12.8 European Swaption

12.8.1 Contract Definition

The owner of a payer (receiver, respectively) swaption has the right to enter at option maturity time the underlying forward payer (receiver, respectively) swap settled in arrears. A forward payer swap pays fixed rate and receives floating rate while a forward receiver swap pays float rate and receives fixed rate.

Define

β = 1 for payer swaption; -1 for receiver swaption.

N = notional amount.

$t =$ valuation date.

T = maturity date of the option.

T_0 = the underlying swap start date, $T_0 \geq t$.

T_n = the underlying swap end date.

T_i^{float} = $i = 1,\ldots,n^{float}$, the floating side payment dates of the underlying swap.

T_i^{fix} = $i = 1,\ldots,n^{fix}$, the fixed side payment dates of the underlying swap.

T_i = $i = 0,\ldots,n^{float} - 1$, the forward period start dates of the swap's reference index.

κ = the strike rate.

δ_i^{float} = the day count fraction between T_{i-1}^{float} and T_i^{float}.

δ_i^{fix} = the day count fraction between T_{i-1}^{fix} and T_i^{fix}.

$B(t,T)$

=the price of zero-coupon bond at time t paying $1 at time T.

δ_{tenor} = the tenor of the reference index of the underlying swap.

$L(t, T_i, T_i + \delta_{tenor})$

=the simple-compounded forward interest rate prevailing at time t, starting at time T_i and ending at time $T_i + \delta_{tenor}$.

$$= \frac{1}{\delta_{tenor}} \left(\frac{df\,(t, T_i)}{df\,(t, T_i + \delta_{tenor})} - 1 \right) + r_{spread}$$

r_{spread} = the spread added to the reference index of the underlying swap.

$df\,(t, t_j)$

= the discount factor from time t to time t_j.

Swaption Payoff

The swaption payoff at maturity time T is:

$$N \cdot \beta \left[\sum_{i=1}^{n^{float}} B\left(T, T_i^{float}\right) \cdot \delta_i^{float} \cdot L\left(T; T_{i-1}, T_{i-1} + \delta_{tenor}\right) - \sum_{i=1}^{n^{fix}} B\left(T, T_i^{fix}\right) \cdot \delta_i^{fix} \kappa \right].$$

12.8.2 Pricing Formulas

Using Black's model, the price of payer swaption at the valuation date is:

$$PS_t^{black} = N \cdot G(t) \left\{ \kappa(t,T) N(d_1) - \kappa N(d_2) \right\}.$$

The price of receiver swaption at the valuation date is:

$$RS_t^{black} = N \cdot G(t) \left\{ \kappa N(-d_2) - \kappa(t,T) N(-d_1) \right\}.$$

$$d_1 = \frac{\ln\left(\frac{\kappa(t,T)}{\kappa}\right) + \sigma^2_{T-t,T_n-T_0} \cdot (T-t)}{\sigma_{T-t,T_n-T_0} \cdot \sqrt{T-t}}.$$

$$d_2 = d_1 - \sigma_{T-t,T_n-T_0} \sqrt{T-t}.$$

σ_{T-t,T_n-T_0} is the implied volatility with option term $T-t$ and swap term $T_n - T_0$.

$$G(t) = \sum_{i=1}^{n^{fix}} \delta_i^{fix} df\left(t, T_i^{fix}\right).$$

$\kappa(t,T)$ is the forward swap rate and is equal to:

$$\kappa(t,T) = \frac{\sum_{i=1}^{n^{float}} L\left(t, T_{i-1}, T_{i-1} + \tau_{tenor}\right) df\left(t, T_i^{float}\right)}{G(t)}.$$

12.9 Bermudan Swaption

12.9.1 Contract Definition

For the time $T_k < T_h < T_n$, a Bermudan swaption is contract that gives the holder the right to enter at any time T_l $(T_k \le T_l \le T_n)$ into an interest-rate swap with first reset in T_l, last reset in T_n and fixed rate K.

Define

$\{l_1, l_2, \ldots, l_{n-1}, l_n\}$ = the vector of time nodes at the swap reset dates.

$\{c_1, c_2 \ldots, c_{n-1}, c_n\}$ = the vector of time nodes at the exercise dates.

t_k = the actual time at time node k.

$B_{i,j}(q)$ = zero-coupon bond price at node (i, j) valued with maturity at time node q.

$r_{i,j}$ = interest rate at node (i, j) implied by the interest rate tree.
$IRS_{i,j}$ = the interest rate swap value at node (i, j).
$CC_{i,j}$ = the rolling back value at node (i, j).
f = the fraction of a year for the swap payment, e.g. if the swap is paying semi- annually then $f = \frac{1}{2}$.

Note that the actual time of time node l_1 (t_{l_1}) is T_l and the actual time of time node l_n (t_{l_n}) is T_n since T_l and T_n are the first swap reset time and last swap reset time respectively by definition. Also the actual time of time node c_1 (t_{c_1}) is T_l since the swap payments start at the first exercise date.

12.9.2 Pricing

Bermudan swaption has an early exercise feature. Unlike American-style options, Bermudan swaption only allows early exercises at some discrete point time, i.e. on the swap reset dates. Representing Bermudan swaption price in closed-form solutions is very difficult. However, it can be priced numerically using the interest rate tree approach through backward induction.

Step 1:
Construct an interest-rate tree starting today to the last swap reset date with interest rate nodes covering all interim swap reset dates.

Step 2:
Introduce a vector of bond prices $B_{l_n,j}(l_n) = 1$ for all the nodes at time node l_n, i.e. at time T_n.

Step 3:
Calculate the vector of bond prices $B_{l_{n-1},j}(l_n)$ for all the nodes at time node l_{n-1} using backward induction:

$$B_{l_{n-1},j}(l_n) = e^{-r_{l_{n-1},j}\left(t_{l_n} - t_{l_{n-1}}\right)}\left(p_u B_{l_n,j+1}(l_n) + p_m B_{l_n,j}(l_n) + p_d B_{l_n,j-1}(l_n)\right)$$

Introduce a new vector of bond prices $B_{l_{n-1},j}(l_{n-1}) = 1$ for all the nodes at time node l_{n-1}.

Step 4:
Calculate both bond vector prices $B_{l_{n-2},j}(l_n)$ and $B_{l_{n-2},j}(l_{n-1})$ from the bond vector prices $B_{l_{n-1},j}(l_n)$ and $B_{l_{n-1},j}(l_{n-1})$ respectively using the backward induction similar to the above step.

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Introduce a new vector of bond prices $B_{l_{n-2},j}(l_{n-2})=1$ for all the nodes at time node l_{n-2}.

Step 5:
Such backward induction and new vector bond prices introduction will continue until the first swap reset date at time node l_1, i.e. at time T_l.

Step 6:
During the rolling back process, if the nodes hit the last exercise date at time node c_n, we calculate the interest rate swap (IRS) value:

$$IRS_{c_n,j} = 1 - B_{c_n,j}(l_n) - \sum_{k=c_n+1}^{l_n} fKB_{c_n,j}(k).$$

Note that $B_{i,j}(q)$ can be obtained from the rolling back bond vector prices.
We also define:
$CC_{c_n,j} = IRS_{c_n,j}$.

Step 7:
We calculate $CC_{c_n-1,j}$ for all nodes at time node $c_n - 1$ using backward induction:

$$CC_{c_n-1,j} = e^{-r_{c_n-1,j}\left(t_{c_n}-t_{c_n-1}\right)}\left(p_u CC_{c_n,j+1} + p_m CC_{c_n,j} + p_d CC_{c_n,j-1}\right).$$

Step 8:

Such rolling back process of CC continues until it reaches node $(0,0)$. However, during any of the time nodes that match the exercise dates, we need to calculate the $IRS_{i,j}$ values:

$$IRS_{i,j} = 1 - B_{i,j}(l_n) - \sum_{k=i+1}^{l_n} fKB_{i,j}(k).$$

Note that $B_{i,j}(q)$ can be obtained from the rolling back bond vector prices.

We then compare $CC_{i,j}$ and $IRS_{i,j}$. If $CC_{i,j} \geq IRS_{i,j}$, we keep $CC_{i,j}$ as its original value. If $CC_{i,j} < IRS_{i,j}$ then we set $CC_{i,j}$ to $IRS_{i,j}$ and all the rolling back process of CC will use the new $CC_{i,j}$ value instead of the old one.

Step 9:
The rolling back $CC_{0,0}$ is the Bermudan swaption price.

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Pricing can also be done using an older method called Geske Approximation (please refer to section 12.19). This model will be used if pricing parameter or trade extension “ModelGeskeBermudan” is set with value 1.

12.10 Constant Maturity Swap

12.10.1 Description of Instrument

A Constant Maturity Swap, also known as a CMS, is a swap that allows the purchaser to fix the duration of received flows on a swap. The floating leg of an interest rate swap typically resets against a published index. The floating leg of a constant maturity swap fixes against a point on the swap curve on a periodic basis.

Constant Maturity Swaps can be either single currency or cross currency swaps. The prime factor therefore for a constant maturity swap is the shape of the forward implied yield curves. A single currency constant maturity swap versus LIBOR is similar to a series of differential interest rate fix (or “DIRF”) in the same way that an interest rate swap is similar to a series of forward rate agreements.

12.10.2 XML Representation

The CMS uses the fpmlInterestRateStream with the additional of a few elements. A *constantMaturity* element was added as part of the *floatingRateCalculation*. This consisted of the *term* of the swap/treasury rate and its *frequency*.

12.10.3 Pricing

Brotherton-Ratcliffe and Iben show that the convexity adjustment that must be made to the forward rate (f) is:

$$-0.5 f^2 \sigma^2 T \left[B''(f) / B'(f) \right]$$

Where

f = forward CMS/CMT rate

σ = volatility

T = time to maturity of the forward

$B(f)$ = the price at time t of a security that provides coupons equal to the
forward CMS/CMT rate over the life of the bond as a function of yield (Y)
.

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$B^{'}(f)$ = first derivative of the bond price (B) with respect to yield (Y).

$B^{''}(f)$ = second derivative of the bond price (B) with respect to yield (Y).

To cater for the convexity effect on the forward rate, the adjusted forward rate becomes:

$$f - 0.5 f^2 \sigma^2 T \left[B^{''}(f) / B^{'}(f) \right].$$

This means that to obtain the expected forward rate, the convexity adjustment should be added to the forward swap rate (for CMS) or bond yield (for CMT).

Example:

f = 5% pa
Y = 5% pa
σ = 10% pa

Assume that the instrument provides a payoff in 3 years time linked to the 3 year rate.

The value of the instrument is given by:
$B(Y) = f/(1+Y) + f/(1+Y)^2 + (1+f)/(1+Y)$

The first and second derivatives are given by:
$B^{'}(f) = -f/(1+Y)^2 - 2f/(1+Y)^3 - 3(1+f)/(1+f)^4$ = -2.7232
$B^{''}(f) = 2f/(1+Y)^3 + 6f/(1+Y)^4 + 12*(1+f)/(1+Y)^5$ = 10.2056

The convexity adjustment is given by:
$(-0.5)(0.05)^2(0.10)^2 3(11.8469/-2.7232) = 0.0001631$ or 1.631 bps.

To adjust the forward rate with the convexity effect, the adjusted forward rate is 5.01631% pa instead of 5.00% pa.

The volatility to use for the convexity adjustment for a specific forward forecast swap rate will be dependent on the swap rate term, and for which forward start date. Finding a swaption volatility for expiry matching the forward start date and underlying maturity matching the swap rate term would be the ideal.

A practical compounding solution in determining $B^{'}(f)$ and $B^{''}(f)$ is shown below:

Consider one cashflow where:

m	=	compounding frequency per annum
n	=	number of years
x	=	rate per annum
c	=	cashflow amount (coupon payment equivalent)

Then:

$$B\ (x) = \frac{c}{\left(1+\frac{x}{m}\right)^{mn}}$$

The first derivative:

$$B^{`}(x) = \frac{-c.n}{\left(1+\frac{x}{m}\right)^{mn+1}}$$

And the second derivative:

$$B^{``}(x) = \frac{c.n.(m.n+1)}{m.\left(1+\frac{x}{m}\right)^{mn+2}}$$

Hence the convexity adjustment can be found by the summation of these cashflows along with the principal.

12.11 Power Reverse Dual Currency Swap

A power-reverse dual-currency swap, or PRDC, pays FX-linked coupons in exchange for floating-rate payments.

- Leg 1: Stream of coupons linked with FX options.
- Leg 2: Fixed or Floating rates.

12.11.1 Pricing FX-Linked Coupon Leg

Let us define the given inputs:

- A tenor structure $0 < T_1 < \ldots < T_n$.
- A tenor period $\tau_i = T_i - T_{i-1}$.

- S_t the spot FX rate at time t.
- r_d domestic risk-free rate at time t.
- r_f foreign risk-free rate at time t.
- σ volatility of foreign exchange rate.
- w_1 the value generated by some deterministic function, e.g. deterministic foreign coupon rate.
- w_2 the value generated by some deterministic function, e.g. deterministic domestic coupon rate.
- b_l the floor rate.
- b_u the cap rate.
- t the valuation date.

An FX-linked, or PRDC, coupon for the period $[T_{i-1}, T_i]$, $i = 1,..,n$, pays the amount:

$$(T_i - T_{i-1}) C_i \left(S_{T_i}\right)$$

at time T_i, where:

$$C_i \left(S_{T_i}\right) = \min\left(\max\left(w_1 S_{T_i} - w_2, b_l\right), b_u\right).$$

In the classical structure, $b_l = 0$ and $b_u = \infty$ (payoff floored at 0 and no cap) the payoff is simply:

$$C_i \left(S_{T_i}\right) = h \cdot \max\left(S_{T_i} - K, 0\right)$$

where the strike price is defined as $K = \frac{w_2}{w_1}$ and the payment rate is $h = w_1$.

If we assume a deterministic risk-free rate[1], w_1 and w_2 are values generated by some deterministic functions then a single cash flow of the option leg paid for tenor time interval (T_{i-1}, T_i) paid on time T_i evaluated at time t, call it CF_{t_i}, is:

$$E^Q\left[\frac{B(t)}{B(T_{i-1})} h \max\left(S_{T_i} - K, 0\right) | F_t\right] = h\left(S_t e^{-r_f(T_i - t)} N(d_1) - K e^{-r_d(T_i - t)} N(d_2)\right),$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r_d - r_f + \frac{\sigma^2}{2}\right)(T_i - t)}{\sigma\sqrt{T_i - t}},$$

$$d_2 = d_1 - \sigma\sqrt{T_i - t}.$$

Note that all r_d, r_f and σ can be obtained from the term structure curve.

The value of the option leg, VL_j^P, can be priced as:

$$VL_j^p = \sum_{i>j}\left(T_i^f - T_{i-1}^f\right) CF_{t_i}.$$

Important Assumptions

- Risk-free rates are deterministic.
- w_1 and w_2 are values generated by some deterministic functions.

If any of the above “deterministic” assumptions does not hold, then this pricing method does not work.

12.11.2 Pricing Floating Leg

The floating leg is:

[1] It is not really realistic because the floating interest rate in the floating leg is stochastic.

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$$VL_J^R = E^Q\left[\sum_{i>j}\frac{B(t)}{B(T_i)}N\left(U\left(T_{i-1},T_i\right)+s_i\right)\left(T_{i-1},T_i\right)\middle|F_t\right]$$
$$= N\sum_{i>j}B\left(t,T_i\right)\left(T_i-T_{i-1}\right)\left(E^{Q^{T_i}}\left[U\left(T_{i-1},T_i\right)\middle|F_t\right]+s_i\right)$$

Note that $E^{Q^{T_i}}\left[\left(U\left(T_{i-1},T_i\right)\right)|F_t\right]$ can be obtained from section 17.1 depending on what forward floating rate it is.

12.12 Cancellable Yield Curve Basket Swap with Floor

A yield curve basket swap with floor is a swap contract type with the following payoff:

- Leg 1: Rate paid based on the basket of constant maturity treasury (CMT) rate.
- Leg 2: Fix / Floating rate.

12.12.1 Pricing Option Leg

The option leg consists of a basket of rates.

Define:

$U^k\left(t,T_i\right)$ = the k^{th} CMS forward rate in the basket.
K = the floor level.
$\mu^k\left(t,T\right)$ = drift function of the k^{th} CMS rate at time t.
$\sigma^k\left(t,T\right)$ = volatility function of the k^{th} CMS rate at time t.
F_{T_j} = the floor price at date T_j evaluated at valuation date t.
f_{T_i} = the floorlet price at tenor date T_i evaluated at valuation date t.
n = number of tenor dates.
w^k = weighting for the k^{th} CMS in the basket.
t = valuation date.
ρ_{ij} = correlation between rate i and j.
$\mu^b\left(t,T\right)$ = basket drift function at time t.
$\sigma^b\left(t,T\right)$ = basket volatility function of basket at time t.
$U^b\left(t,T_i\right)$ = the basket CMF forward fate.
$W_t^{Q^{T_i},k}$ = Wiener process $W_t^{Q_i^T}$ for the k^{th} asset.
$W_t^{Q^{T_i},b}$ = Wiener process $W_t^{Q_i^T}$ for the basket.

We need to find the distribution of $U^b(t,T_i)$ to find the basket floor value.

We know that each individual floating rate under the forward martingale measure is distributed as:

$$\frac{dU^k(t,T_i)}{U^k(t,T_i)} = \mu^k(t,T_i)\,dt + \sigma^k(t,T_i)\,dW^{Q^{T_i},k}.$$

Thus the basket floating rate under Q^{T_i} is distributed as:

$$\begin{aligned}
&\frac{dU^b(t,T_i)}{U^b(t,T_i)} \\
&= \frac{\sum_k w^k dU^k(t,T_i)}{\sum_k w^k U^k(t,T_i)} \\
&= \frac{\sum_k w^k U^k(t,T_i)\mu^k(t,T_i)}{\sum_k w^k U^k(t,T_i)}dt + \frac{\sum_k w^k U^k(t,T_i)\sigma^k(t,T_i)\,dW_t^{Q^{T_i},k}}{\sum_k w^k U^k(t,T_i)} \\
&= \frac{\sum_k w^k U^k(t,T_i)\mu^k(t,T_i)}{\sum_k w^k U^k(t,T_i)}dt + \sum_k \lambda^k(t,T_i)\sigma^k(t,T_i)\,dW_t^{Q^{T_i},k}\ .
\end{aligned}$$

It is clearly not lognormally distributed. However, to retain the lognormal property for the basket rates, we need to freeze the drift and volatility.

$$\begin{aligned}
&\frac{dU^b(t,T_i)}{U^b(t,T_i)} \\
&\approx \mu^b(t,T_i)\,dt + \sum_k \lambda^k(0,T_i)\sigma^k(t,T_i)\,dW_t^{Q^{T_i},k} \\
&= \mu^{b^*}(t,T_i)\,dt + \sqrt{\sum_k\sum_l \lambda^k(0,T_i)\lambda^l(0,T_i)\rho_{ij}\sigma^k(t,T_i)\sigma^l(t,T_i)}\,dW_t^{Q^{T_i},b} \\
&= \mu^{b^*}(t,T_i)\,dt + \sigma^b(t,T_i)\,dW_t^{Q^{T_i},b}.
\end{aligned}$$

It is assumed that $\sigma^b\left(t,T^f\right)$ is provided by the user.

Note that $\mu^{b^*}(t,T_i)$ represents the basket drift under measure Q^{T_i}.

$\mu^{b^*}(t,T_i)$ is approximated by:

$$\begin{aligned}
&E^{Q^{T_i}}\left[U^b(T_{i-1},T_i)\,|\,F_t\right] \\
&= E^{Q^{T_i}}\left[\sum_k w^k U^k(T_{i-1},T_i)\,|\,F_t\right]
\end{aligned}$$

$$=\sum_k w^k E^{Q^{T_i}}\left[U^k\left(T_{i-1},T_i\right)|F_t\right].$$

Note that the computation of $E^{Q^{T_i}}\left[U^k\left(T_{i-1},T_i\right)|F_t\right]$ is given in section 17.1. For example, if the rate is LIBOR rate, then $E^{Q^{T_i}}\left[U^k\left(T_{i-1},T_i\right)|F_t\right]=U^k\left(t,T_i\right)$. If the rate is CMS rate, then we need convexity adjustment.

The basket rate can now be treated as a single rate.
An interest rate floor is a series of floorlets:

$$F_{T_j}=\sum_{i>j}^{n} f_{T_i}.$$

The value of a floorlet is:

$$f_{T_i}=\left(T_i-T_{i-1}\right)B\left(t,T_i\right)E^{Q^{T_i}}\left[\max\left(K-U^b\left(T_{i-1},T_i\right)\right)|F_t\right].$$

Note that $U^b\left(t,T_i\right)$ is lognormally distributed and we know the drift and volatility of $U^b\left(t,T_i\right)$ from above. Thus:

$$E^{Q^{T_i}}\left[\max\left(K-U^b\left(T_{i-1},T_i\right)\right)|F_t\right]$$

$$=KN\left(-d_2\right)-E^{Q^{T_i}}\left[U^b\left(T_{i-1},T_i\right)|F_t\right]N\left(-d_1\right),$$

where:

$$E^{Q^{T_i}}\left[U^b\left(T_{i-1},T_i\right)|F_t\right]=U^b\left(t,T_i\right)e^{\mu^{b^*}(t,T_i)(T_i-t)},$$

$$U^b\left(t,T_i\right)=\sum_{k=1}^{n} w^k U^k\left(t,T_i\right),$$

$$\mu^{b^*}\left(t,T_i\right)=\sum_{k=1}^{n} w^k E^{Q^{T_i}}\left[U^k\left(T_{i-1},T_i\right)|F_t\right].$$

Note that the computation of $E^{Q^{T_i}}\left[U^k\left(T_{i-1},T_i\right)|F_t\right]$ is given in section 17.1. For example, if the rate is LIBOR rate, then $E^{Q^{T_i}}\left[U^k\left(T_{i-1},T_i\right)|F_t\right]=U^k\left(t,T_i\right)$. If the rate is CMS rate, then we need convexity adjustment.

$$d_1=\frac{\ln\left(\frac{E^{Q^{T_i}}\left[U^b\left(T_i,T_{i-1}\right)|F_t\right]}{K}\right)+\frac{\sigma^b\left(t,T_i\right)^2\left(T_i-t\right)}{2}}{\sigma^b\left(t,T_i\right)\sqrt{T_i-t}},$$

$$d_2=\frac{\ln\left(\frac{E^{Q^{T_i}}\left[U^b\left(T_{i-1},T_i\right)|F_t\right]}{K}\right)-\frac{\sigma^b\left(t,T_i\right)^2\left(T_i-t\right)}{2}}{\sigma^b\left(t,T_i\right)\sqrt{T_i-t}}.$$

12.12.2 Pricing Floating Fix Leg

Pricing floating and fix leg can be done using the usual forward martingale methods.

The floating leg value is:

$$VL_j^P = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(E^{Q^{T_i}}\left[\left(U\left(T_{i-1},T_i\right)\right) | F_t\right] + s_i\right).$$

Note that $E^{Q^{T_i}}\left[\left(U\left(T_{i-1},T_i\right)\right) | F_t\right]$ can be obtained from section 17.1 depending on what forward floating rate it is.

The fix leg value is:

$$VL_j^p = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(fixed_{T_i} + s_i\right).$$

12.13 Cancellable Swap with Equity Trigger

Cancellable swap with equity trigger is an interest rate swap contract where the underlying swap can be cancelled by either the booking-party or counter-party or automatically terminated earlier than maturity depending on the series of specified equity trigger levels. The underlying interest rate swap can be receiving floating interest rate payments on one side and paying fixed interest payments on another side, or receiving fixed interest payments and paying floating interest payments. Either party has the right to cancel the swap:

- At the strike price on the exercise date.
- Automatically terminated according to the equity termination rate and termination date.

The underlying swap is a standard swap (as the equity trigger is only considered for early termination in simulation, and not for MTM). The two legs are:

- Leg 1: It is the fixed leg and is based on fixed rate
- Leg 2: It is the floating rate and is based on LIBOR with a spread.

12.13.1 Pricing Fixed Leg

The fixed leg is a stream of fixed rate plus a spread. The value of the fixed leg is:

$$VL_j^p = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(fixed_{T_i} + s_i\right).$$

12.13.2 Pricing Floating Leg

The floating leg is:

$$VL_j^P = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(E^{Q^{T_i}}\left[\left(U\left(T_{i-1},T_i\right)\right) | F_t\right] + s_i\right).$$

Note that $E^{Q^{T_i}}\left[\left(U\left(T_{i-1},T_i\right)\right) | F_t\right]$ can be obtained from section 17.1 depending on what forward floating rate it is.

12.13.3 Cancellable Swap

It is the same as early exericisable swaps and can be structured as a Bermudan swaption and the non-cancellable swap.

12.13.4 Cancellable Swap with Equity Trigger

The early termination for the cancellable swap is subject to an equity trigger. The evaluation of this callable swap is as for standard callable swap so without taking into account the equity trigger for the MtM. This equity trigger only has an impact in the simulation: if the path simulated implies a termination then the MtM becomes 0 (like for the PRDC with early terminations).

12.14 Cancellable Reverse Interest Rate Swap

A swap contract type where the rate paid on one side increases as market floating rates decline and the rate on another side is based on fixed rate.

The two legs are:

- Leg 1:Reverse floating leg.
- Leg 2: Fixed or floating rates.

12.14.1 Pricing Reverse Floating Side

The rate paid on the reverse floating side is set by the fixed rate minus the multiple of floating reference rate for each payment period.

We write:

f_{T_i}	=	fixed rate at time T_i.
w_{T_i}	=	weight at time T_i.

Hence for each call date T_j:

$$
\begin{aligned}
&VL_j^P \\
&= E^Q\left[\sum_{i>j}\frac{B(t)}{B(T_i)}N\left(f_{T_i}-\left(w_{T_i}\cdot U(T_{i-1},T_i)+s_{T_i}\right)\right)(T_i-T_{i-1})\mid F_t\right] \\
&= N\sum_{i>j}B(t,T_i)(T_i-T_{i-1})\left(f_{T_i}-w_{T_i}\cdot E^{Q^{T_i}}\left[U(T_{i-1},T_i)\mid F_t\right]-s_{T_i}\right).
\end{aligned}
$$

Note that $E^{Q^{T_i}}\left[\left(U(T_{i-1},T_i)\right)\mid F_t\right]$ can be obtained from section 17.1 depending on what forward floating rate it is.

12.14.2 Reverse Floating Side with Floor

The reverse floating side can also have a floor. If the floor rate is K, the value of this single cash flow at time t_i, call it CF_{t_i} is:

$$
\begin{aligned}
CF_{t_i} &= B(t,T_i)E^{Q^{T_i}}\left[\max\left\{K-f_{T_i}+w_{T_i}\cdot\left(U(T_{i-1},T_i)-s_{T_i}\right),0\right\}\mid F_t\right] \\
&= w_{T_i}E^{Q^{T_i}}\left[\max\left\{U(T_{i-1},T_i)-\left(s_{T_i}-\frac{K}{w_{T_i}}+\frac{f_{T_i}}{w_{T_i}}\right),0\right\}\mid F_t\right].
\end{aligned}
$$

The above can be evaluated using Black's call option pricing formula with $E^{Q^{T_i}}\left[U(T_{i-1},T_i)\mid F_t\right]$ to replace the current stock price and $s_{T_i}-\frac{K}{w_{T_i}}+\frac{f_{T_i}}{w_{T_i}}$ to replace the strike price. Note that $E^{Q^{T_i}}\left[U(T_{i-1},T_i)\mid F_t\right]$ can be evaluated as in section 17.1 depending on what type of rate it is.

Note that we did not include notional amount to keep the length of the formula shorter. It can be incorporated by multiplying the notional amount to the formula as a scalar constant.
The value for floating leg at callable time t_j evaluated at time t is:

$$VL_j^P=\sum_{i>j}CF_{t_i}.$$

12.14.3 Pricing Fixed-Floating Leg

The floating leg value is:

$$VL_j^P=N\cdot\sum_{i>j}B(t,T_i)(T_i-T_{i-1})\left(E^{Q^{T_i}}\left[\left(U(T_{i-1},T_i)\right)\mid F_t\right]+s_i\right).$$

Note that $E^{Q^{T_i}}\left[\left(U(T_{i-1},T_i)\right)\mid F_t\right]$ can be obtained from section 17.1 depending on what forward floating rate it is.

The fix leg value is:

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$$VL_j^p = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(f_{T_i} + s_i\right).$$

12.15 Interest Rate Cliquet

An interest rate cliquet consists of a series of forward starting options where the strike price for the next exercise date is set equal to a positive constant times the asset price as of the previous exercise date.

12.15.1 Payoff of Interest Rate Cliquet

The payoff of interest rate cliquet for the tenor period (T_{i-1}, T_i) paid on payment date T_i is:

$$(T_i - T_{i-1})\left(\max\left(U(T_{i-1}, T_i) - U(T_{i-2}, T_{i-1}), 0\right)\frac{B(t)}{B(T_i)}\right).$$

The above is only the payoff for a single period. The whole interest rate cliquet payoff is the sum of each individual payoff (T_{i-1}, T_i) for $i = 1, 2, ..., n$, assuming there are n tenor dates in total.

12.15.2 Pricing

We first focus on the pricing of the claim for the tenor period (T_{i-1}, T_i). The price of the claim, f_{T_i} evaluated on the valuation date t is:

$$\begin{aligned} f_{T_i} &= E^{Q_0}\left[(T_i - T_{i-1})\left(\max\left(U(T_{i-1}, T_i) - U(T_{i-2}, T_{i-1}), 0\right)\frac{B(T_i)}{B(t)}\right) \mid F_t\right] \\ &= (T_i - T_{i-1}) B(t, T_i) E^{Q^{T_i}}\left[\left(\max\left(U(T_{i-1}, T_i) - U(T_{i-2}, T_{i-1}), 0\right)\right) \mid F_t\right] \end{aligned}$$

For the first payment, $U(T_0, T_1)$ is observable. Thus it is a standard cap with strike rate $U(T_0, T_1)$. For later payments, by assuming $U(T_{i-1}, T_i)$ and $U(T_{i-2}, T_{i-1})$ are bivariate-lognormally distributed,

$$\begin{aligned} &E^{Q^{T_i}}\left[\left(\max\left(U(T_{i-1}, T_i) - U(T_{i-2}, T_{i-1}), 0\right)\right) \mid F_t\right] \\ &= B(t, T_i)\left(E^{Q^{T_i}}\left[U(T_{i-1}, T_i) \mid F_t\right] N(d_1) - E^{Q^{T_i}}\left[U(T_{i-2}, T_{i-1}) \mid F_t\right] N(d_2)\right), \end{aligned}$$

where

$E^{Q^{T_i}}\left[U(T_{i-1}, T_i) \mid F_t\right]$ can be obtained from section 17.1,

$E^{Q_0^{T_i^p}}\left[U(T_{i-2}, T_{i-1}) \mid F_t\right]$ can be obtained from section 17.1 (we can do this because we assume the correlation of rates between different tenors is 0),

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$$d_1 = \frac{\ln\left(\frac{E^{Q^{T_i}}\left[U\left(T_{i-1},T_i\right)|F_t\right]}{E^{Q^{T_i}}\left[U\left(T_{i-2},T_{i-1}\right)|F_t\right]}\right)}{\sqrt{\sigma_i^2\cdot\left(T_i-t\right)-\sigma_{i-1}^2\cdot\left(T_{i-1}-t\right)}}+\frac{1}{2}\sqrt{\sigma_i^2\cdot\left(T_i-t\right)-\sigma_{i-1}^2\cdot\left(T_{i-1}-t\right)},$$

$$d_2 = d_1-\sqrt{\sigma_i^2\left(T_i-t\right)-\sigma_{i-1}^2\left(T_{i-1}-t\right)},$$

σ_i and σ_{i-1} can be calibrated from market data and the values of σ_i and σ_{i-1} provided by the user.

The price of the cliquet at the valuation date t is:

$$C_t = \sum_i f_{T_i}.$$

12.16 Range Accrual Swap

For range accrual swap, one leg is the range accrual note and the other leg is either a floating rate leg or fixed rate leg. Range accrual notes are similar to fixed rate bonds except that the coupons are only paid when some reference rate falls within a particular range.

The legs for range accrual swap are:

- Leg 1: Range accrual note.
- Leg 2: Fixed or floating rate.

12.16.1 Pricing Range Accrual Note Leg

Range accrual notes can be treated as structured products that the payoffs of range accrual notes can be replicated by a series of digital floorlets.

Define:

a_j = the accrual factor for the period t_{j-1} and t_j.

$B_{\max}$ = the upper bound of reference rate range for coupon payment.

$B_{\min}$ = the lower bound of reference rate range for coupon payment.

$D(t,T,K)$ = the price of digital floorlet on the reference rate at time t maturing at time T with strike rate K.

t_j = time for the j^{th} coupon.

t = the valuation date.

Step 1
We first consider the coupon contribution to the j^{th} coupon by a single day say at time t_i where $t < t_i < t_j$.

The coupon contribution for the j^{th} coupon by time t_i is:

$$a_j \frac{1}{365} \begin{cases} U(t_{i-1}, t_i) & if \;\; B_{\min} \leq U(t_{i-1}, t_i) \leq B_{\max} \\ 0 & otherwise \end{cases}.$$

The above simply means that for the j^{th} coupon, an amount of $a_j \frac{U(t_{i-1}, t_i)}{365}$ is contributed by time t_i if the reference at time t_{i-1} falls within the target bound. Note that the accrual factor a_j is there to keep the generosity because we don't always start the range accrual note on the coupon payment day.

Step 2:
The payoff of a digital floorlet $D(t, t_i, K)$ is:

$$\begin{cases} U(t_{i-1}, t_i) & if \;\; U(t_{i-1}, t_i) \leq K \\ 0 & otherwise \end{cases}.$$

By longing $D(t, t_i, B_{\max})$ and shorting $D(t, t_i, B_{\min})$ with notional amount $a_j \frac{1}{365}$, this structured payoff at time t_i is:

$$a_j \frac{1}{365} \begin{cases} U(t_{i-1}, t_i) & if \;\; B_{\min} \leq U(t_{i-1}, t_i) \leq B_{\max} \\ 0 & otherwise \end{cases}.$$

This completely replicates the range accrual notes except that the timing of this payoff for range accrual note is at time t_j while the structured payoff is at time t_i. Time correction is required for an exact replication. We now look at the present values of these two payoffs:

We assume the discount factor is non stochastic, the expected present value for the range accrual note is:

$$a_j \frac{1}{365} E^Q \left[\frac{B(t)}{B(t_j)} U(t_{i-1}, t_i) 1_{\{B_{\min} \leq U(t_{i-1}, t_i) \leq B_{\max}\}} \mid F_t \right]$$

$$= B(t, t_j) a_j \frac{1}{365} E^{Q^{t_i}} \left[U(t_{i-1}, t_i) 1_{\{B_{\min} \leq U(t_{i-1}, t_i) \leq B_{\max}\}} \mid F_t \right].$$

The expected present value for the structured digital floorlets is:

$$a_j \frac{1}{365} E^Q \left[\frac{B(t)}{B(t_i)} U(t_{i-1}, t_i) 1_{\{B_{\min} \le U(t_{i-1}, t_i) \le B_{\max}\}} \mid F_t \right]$$

$$= B(t, t_j) a_j \frac{1}{365} E^{Q^{t_i}} \left[U(t_{i-1}, t_i) 1_{\{B_{\min} \le U(t_{i-1}, t_i) \le B_{\max}\}} \mid F_t \right].$$

Thus by holding a notional of $\frac{B(t,t_j)}{B(t,t_i)} a_j \frac{1}{365}$ for the digital floorlets, we can fix the time mismatch problem and completely replicate the range accrual note claim.

Step 3
We need to know the price of digital floorlet. The payoff of digital floorlet is:

$$\begin{cases} U(t_{i-1}, t_i) & if \ U(t_{i-1}, t_i) \le K \\ 0 & otherwise \end{cases}.$$

It is known that $U(t_{i-1}, t_i)$ is lognormally distributed under Q^{t_i}, the price of the floorlet can be evaluated using the normal asset-or-nothing digital floorlet, i.e.:

$D(t, t_i, K) = B(t, t_i) E^{Q^{t_i}} \left[U(t_{i-1}, t_i) \mid F_t \right] N(-d)$ with

$E^{Q^{t_i}} \left[U(t_{i-1}, t_i) \mid F_t \right]$ can be computed as in section 17.1,

$$d = \frac{\ln\left(\frac{E^{Q^{t_i}} \left[U(t_{i-1}, t_i) \mid F_t \right]}{K} \right) + \frac{\sigma_e^2 \cdot (t - t_i)}{2}}{\sigma_e \sqrt{t_i - t}}.$$

Note that σ_e here is the volatility of the LIBOR rate and can be obtained from the term structure curve.

Step 4
The above two steps demonstrated the replication of range accrual note for a single day contribution by time t_i using floorlets maturing at time t_i. To replicate the whole range accrual, we simply use a series of long and short floorlets maturing for each of the time $t \le t_i \le t_j$, where $j = 1, 2, 3, \ldots, n$ with n denoting the number of coupon payments.

Step 5
We need to evaluate the digital floorlets. If the reference rate is LIBOR rate, we can construct a measure such that the forward LIBOR rate is a geometric Brownian distributed martingale. If the reference rate is not LIBOR rate then one way we can do is to make an assumption that the reference rate is geometric Brownian distributed. In either way, the digital floorlets can be evaluated using Black's formula.

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Note that if the coupon payment is based on fixed rate rather than floating rate, then we have cash-or-nothing digital floorlet formula instead of asset-or-nothing digital floorlet formula.

12.16.2 Pricing Floating-Fix Leg

Pricing floating and fix leg can be done using the usual forward martingale methods.

The floating leg value is:

$$VL_j^p = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(E^{Q^{T_i}}\left[\left(U(T_{i-1},T_i)\right) | F_t\right] + s_i\right).$$

The fix leg value is:

$$VL_j^p = N \cdot \sum_{i>j} B(t,T_i)(T_i - T_{i-1})\left(f_{T_i} + s_i\right).$$

The forward LIBOR rates $U(t,T_i)$ can be obtained from the LIBOR term structure curve.

12.17 Constant Maturity Cap and Floor

Constant maturity cap and floor are like normal cap and floor except for that the underlying rate becomes constant maturity swap rae. The pricing can be done using Black-Scholes' formula.

12.17.1 Pricing

The price of constant maturity cap is:

$$B(t,T_i)\left(E_{P^{Q^{T_i}}}\left[U(T_{i-1},T_i) | F_t\right] N(d_1) - KN(d_2)\right).$$

The price of constant maturity floor is:

$$B(t,T_i)\left(KN(-d_2) - E_{P^{Q^{T_i}}}\left[U(T_{i-1},T_i) | F_t\right] N(-d_1)\right).$$

Note that $E_{P^{Q^{T_i}}}\left[U(T_{i-1},T_i) | F_t\right]$ can be calculated as in section 17.1.

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12.18 Early Exercisable Interest Rate Derivatives

Some of the interest rate derivatives from section 12.12 to 12.17 (except for section 12.16 on interest rate cliquet) can have early exercise features. This section discusses on how to cater for early exercise features for those contracts.

Step 1
For any interest rate swap product, there is a payer leg and a receiver leg. The payer leg generates a series of cash flows and the receiver leg generates a different set of cash flows. We treat each leg separately and the value of the swap is simply the difference between the values of the two legs.

Step 2
We need to determine the value of each leg. To calculate the value of the payer leg, we find a market proxy such that the proxy generates similar cash flows as the payer leg. The value of the payer leg is approximated by the price of the proxy. The same procedure is used to calculate the value of the receiver leg. The value of the interest rate swap is difference between the values of the two legs.

Step 3
The cash flows of longing the interest rate swap with early exercise features can be structured as longing a Bermudan swaption on the interest rate swap (with pay and receive legs reversed) with the corresponding early exercise dates and longing the interest rate swap without the early exercise features. The value on the valuation date for the interest rate swap with early exercise features is the sum of the value of the Bermudan swaption and the value of the interest rate swap without early exercise features on the valuation date. The value of Bermudan swaption can be approximated using Geske's approximation and the value of interest rate swap without early exercise features can be calculated as described in Step 2.

Note that if the other party also has the right to early exercise, then this early exercise right can be structured as shorting the Bermudan swaption.

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If both parties have callable rights on an underlying swap, then there will be both a long and a short Bermudan swaption with explicit exercise dates specific to each party.

12.19 Bermudan Swaption – Geske's Approximation

In this section, we provide a closed form solution for valuing Bermudan swaptions. A Bermudan swaption is an option to exercise into an interest rate swap at a finite number of callable dates $T_1^c, T_2^c, \ldots, T_n^c$. In this document all Bermudan Swaptions are priced with Geske's approximation[2].

Without loss of generality, we assume the valuation date $t=0$. Let us denote VL_j^P and VL_j^R the value of the time T_j^c **residual legs** paid (resp. received) by the option holder **being evaluated on the valuation date**. The European option maturing at time T_j^c is given by the following formula:

$$V_j = VL_j^R \Phi\left(d_{1,j}\right) - VL_j^P \Phi\left(d_{2,j}\right),$$

and the formula for computing a general Bermudan swaption:

$$V = V_{j\max} + \left(1 - \Phi\left(d_{2,j\max}\right)\right) \cdot \frac{\sum_{j \neq j\max} V_j \Phi\left(d_{2,j}\right)}{\sum_{j \neq j\max} \Phi\left(d_{2,j}\right)},$$

where

$$d_{1,j} = \frac{\ln\left(\frac{VL_j^R}{VL_j^P}\right) + \frac{1}{2}\sigma_j^2 T_j^c}{\sigma_j \sqrt{T_j^c}},$$

$$d_{2,j} = d_{1,j} - \sigma_j \sqrt{T_j^c},$$

[2] Callable feature in Triple-A program, Christophe Michel, Oct. 23, 2007

and $\Phi(\cdot)$ is the cumulative density function of the normal centred distribution.

$j\max$ is the index for which $V_{j\max}$ is the maximum (is the maximum European price). The volatility σ, used to compute the probabilities to be ITM and OTM in $d_{1,j}$ and $d_{2,j}$, is assumed ex ante (given as a term structure).

12.20 Numerical Interest Rate Models

Very often when valuing financial instruments, we assume the underlying interest rate to be deterministic. It is a reasonable assumption if the major risk components inherited in the instruments are not the interest rate risk. By assuming deterministic interest rates, we actually assume the price for the interest rate risk is zero. However, deterministic interest rate is clearly not valid when valuing interest rate products as it does not make any sense to price a product with a zero price for the inherited risk. It is then natural to assume the interest rate to follow some stochastic processes and it leaves with the problem of choosing a suitable model to describe the interest rate process. There are many literatures on modelling interest rate. Some focused on modelling the short rate (e.g. Hull-White Model), some focused on modelling the forward rate (e.g. HJM approach) and also some attempted to model in the context of observable market rates (e.g. BGM Model). In this document, we will concentrate on the method of modelling short rate using Hull-White one factor model. We will first state some definitions.

Define:

r_t = the short rate at time t.

$B(t,T)$ = the zero-coupon bond price at time t that matures at time T.

B_t = the savings account at time t.

W_t = the Weiner Process at time t.

We should note that by definition, the following equality should hold:

$$B_t = e^{\int_{u=0}^{t} r_j du}$$

$$B(t,T) = B_t E_{P^*}\left(B_T^{-1} \mid F_t\right)$$

where

P^* is a probability measure such that the discounted bond price is a martingale under this measure (called the spot-martingale measure) and F_t is the filtration to time t.

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12.20.1 Hull-White One Factor Interest Rate Tree Model (Equal Time Step)

Hull and White (1990) proposed a model that used the following stochastic differential equation to model the short rate process:

$$dr_t = (\theta(t) - ar_t)dt + \sigma dW_t$$

where

$\theta(t)$	=	time-dependent deterministic function.
a	=	constant mean-reversion rate of the short rate.
σ	=	constant volatility of the short rate.

The Hull-White model has a mean reversion feature and the speed of mean reversion of the short rate is determined by the parameter a. The time-dependent deterministic function $\theta(t)$ is a free function allowing us to fit our interest rate model to the initial term-structure curve (i.e. it is an arbitrage-free model).

Using the Hull-White one factor model, closed-form solutions can be derived for some relatively more simple interest rate products, e.g. zero-coupon bonds and bond options. However, for more exotic and complicated products (e.g Bermudan swaptions), it is often difficult to obtain closed-form solutions. Hull and White (1994) proposed a numerical model for the short rate by discretising the analytical Hull-White one factor model using interest rate tree.

Hull-White Interest Rate Tree
Hull and White (1994) proposed a trinomial interest tree that is essentially a discrete-time, discrete-state version of the Hull-White model. The interest rate can move in three different ways in the next time period. The interest rate model is very useful when valuing complicated interest rate products especially those with early exercise features.

Steps to construct the trinomial tree
Step1:
Recall

$$dr_t = (\theta(t) - ar_t)dt + \sigma dW_t.$$

We define another process

$$dr_t^* = -ar_t^* dt + \sigma dW_t.$$

We need to construct the tree for r_t^* first.
We discretise the interest rate movement and time period so that we have Δr_t^* and Δt instead of dr_t^* and dt.

Step2:

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We set $\Delta r_t^* = \sigma\sqrt{3\Delta t}$. This choice of Δr_t^* is good for error minimisation. We define node (i, j) as the node where $t = i\Delta t$ and $r_t^* = j\Delta r_t^*$ (i implies the time point and j implies the interest rate state). We can then construct a tree for the Δr_t^* process. The following table is a demonstration:

					j =
			$3\Delta r_t^*$	$3\Delta r_t^*$	3
		$2\Delta r_t^*$	$2\Delta r_t^*$	$2\Delta r_t^*$	2
	Δr_t^*	Δr_t^*	Δr_t^*	Δr_t^*	1
0	0	0	0	0	0
	$-\Delta r_t^*$	$-\Delta r_t^*$	$-\Delta r_t^*$	$-\Delta r_t^*$	-1
		$-2\Delta r_t^*$	$-2\Delta r_t^*$	$-2\Delta r_t^*$	-2
			$-3\Delta r_t^*$	$-3\Delta r_t^*$	-3
i = 0	1	2	3	4	

Note that the above demonstration is capped and floored at $j = \pm 3$. It is a good practice that we impose cap and floor for the interest rate tree to emphasize its mean reversion feature. Hull and White showed that probabilities are always positive if we set the maximum j value (denoted $j_{\max}$) equal to the smallest integer greater than $0.184/\Delta t$ and the minimum j value (denoted $j_{\min}$) equal to $-j\max$.

Step 3:
Define p_u, p_m and p_d be the probabilities that the interest rates move up one state, remain stable at the current state and drop down one state respectively. Note that if we are at the upper bound rate, i.e. at $j = j_{\max}$, then p_u, p_m and p_d become the probabilities that the interest rates remain stable at the current state, drop down one state and drop down two states respectively. Similar adjustments but in different directions can be followed when we are at the lower bound, i.e. at $j = j_{\min}$.

Hull and White proposed the following probabilities:

At normal state:

$$p_u = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 - aj\Delta t}{2} \qquad p_m = \frac{2}{3} - a^2 j^2 \Delta t^2 \qquad p_d = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 + aj\Delta t}{2}$$

At upper bound state:

$$p_u = \frac{7}{6} + \frac{a^2 j^2 \Delta t^2 - 3aj\Delta t}{2}$$

$$p_m = -\frac{1}{3} - a^2 j^2 \Delta t^2 + 2aj\Delta t$$

$$p_d = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 - aj\Delta t}{2}$$

At lower bound state:

$$p_u = \frac{1}{6} + \frac{a^2 j^2 \Delta t^2 + aj\Delta t}{2}$$

$$p_m = -\frac{1}{3} - a^2 j^2 \Delta t^2 - 2aj\Delta t$$

$$p_d = \frac{7}{6} + \frac{a^2 j^2 \Delta t^2 + 3aj\Delta t}{2}.$$

Step 4:
We now need to choose the free deterministic function $\theta(t)$ in the Hull-White Model to make sure that our interest rate model is consistent with the initial term structure curve (i.e. to ensure the model is arbitrage-free).

Define:
$\alpha_t = r_t - r_t^*$.

It can be shown that
$d\alpha_t = [\theta(t) - a\alpha(t)]dt$ and then

$$\alpha_t = F(0,t) + \frac{\sigma^2}{2a^2}\left(1 - e^{-at}\right)^2$$

where
$F(0,t)$ = the forward rate implied at time t from time 0.

We need to calculate α_t in the discrete state and time setting. Once we can calculate α_t, we can then construct the arbitrage-free interest rate tree for r_t from r_t^*. Note that $r_t = r_t^* + \alpha_t$.

Step 5:
Hull and White proposed the following formulas to calculate α_t. In the discrete time setting, α_t is a step function.

Define n_m to be the number of nodes on each side of the central node at time $m\Delta t$ for m to be integers ≥ 0.

$$\alpha_m = \frac{\ln \sum_{j=-n_m}^{n_m} Q_{m,j} e^{-j\Delta r_t^* \Delta t} - \ln B(0, m+1)}{\Delta t}$$

$$Q_{m+1,j} = \sum_k Q_{m,k}\, p(k,j) \exp[-(\alpha_m + k\Delta R)\Delta t]$$

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where
$p(k,j)$ is the probability of moving from node (m,k) to node $(m+1,j)$ and the summation is taken over all values of k for which this is nonzero.

Note that both α_0 and $B(0,m+1)$ are immediately available from the initial term structure. α_0 is simply the continuous rate of the zero-coupon bond $B(0,\Delta t)$.

12.20.2 Hull-White One Factor Interest Rate Tree Model and Calibration Method (Unequal Time Step)

Summary
This document specifies the procedures on generalising the equal time step Hull-White tree to the unequal time step cases. This generalised version of Hull-White tree is important for pricing financial products with early exercise features because exercise dates do not usually fall on equal time step nodes. The tree construction method is based on the book by Brigo and Mercurio. Calibration of model is also mentioned in this section.

Definitions and notations
We need to redefine the notations slightly for unequal time step cases:

$x_{i,j}$ = the interest rate at node (i,j).

Δt_i = $t_{i+1}-t_i$.

Δx_i = change in interest rate in vertical direction at time t_i.

p_u, p_m and p_d are probabilities of moving up, middle and down as usual.

$E\{X(t_{i+1}) \mid X(t_i)=x_{i,j}\}=M_{i,j}$.

$Var\{X(t_{i+1}) \mid X(t_i)=x_{i,j}\}=V_{i,j}^2$.

Assumption
Δx_i is assumed to be constant across time t_i.

With the above assumption, we can write $x_{i,j} = j\Delta x_i$.

Methodology
The Hull-White one factor model is by definition:

$dr=\left(\theta(t)-ar\right)dt+\sigma dW_t^*$.

In the equal time step case, we start by constructing the tree for the process $dx=-axdt+\sigma dW_t^*$ and then determine $\theta(t)$ by fitting the initial term structure curve. For unequal time step case, we use the same approach. This document only discusses on how to construct

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$dx = -axdt + \sigma dW_t^*$ since the fitting of initial term structure is the same as equal time step case.

Note that $x_{i+1,j+1} = x_{i+1,j} + \Delta x_{i+1}$ and $x_{i+1,j-1} = x_{i+1,j} - \Delta x_{i+1}$ as in the equal time step case. However, in the equal time step case, the following also hold:

$x_{i+1,j+1} = x_{i,j} + \Delta x_{i+1}$.

$x_{i+1,j} = x_{i,j}$.

$x_{i+1,j-1} = x_{i,j} - \Delta x_{i+1}$.

The three equations above are no longer true for the unequal time step case, thus the formula for p_u, p_m and p_d are also different. By setting $\Delta x_{i+1} = V_i\sqrt{3}$ (this minimises the variability of the tree, please refer to Hull and White 1994), we can find the following formula for unequal time step cases:

$$p_u = \frac{1}{6} + \frac{\eta_{j,k}^2}{6V_i^2} + \frac{\eta_{j,k}}{2\sqrt{3}V_i}.$$

$$p_m = \frac{2}{3} - \frac{\eta_{j,k}^2}{3V_i^2}.$$

$$p_d = \frac{1}{6} + \frac{\eta_{j,k}^2}{6V_i^2} - \frac{\eta_{j,k}}{2\sqrt{3}V_i}.$$

$\eta_{j,k} = M_{i,j} - x_{i+1,k}$ by definition.

It can be shown for the Hull-White model, the following are true:

$M_{i,j} = x_{i,j} - ax_{i,j}\Delta t_i$.

$V_{i,j} = \sigma\sqrt{\Delta t_i}$.

Note that in the above formulas, we did not specify what is k. We now explain the meaning of k and how to determine the value of k.

As we have mentioned before, for equal time step case, $x_{i+1,j} = x_{i,j}$, i.e., if we branch to the next time period through the middle path, the vertical node in the next time period we have branched to is still j and the rate stays the same as the previous time period. However, it is not true for the unequal time step case. Although we branch to the next node at time t_{i+1} from time t_i through the middle path, the vertical node at time t_{i+1} does not have to be j but can be some other number k. k is chosen in a way such that the central node we branch to at time t_{i+1}, i.e., $x_{i+1,k}$, is closest to $M_{i,j}$. It is now obvious that

$$k = round\left(\frac{M_{i,j}}{\Delta x_{i+1}}\right).$$

We then have the full specification of the model.

We can now construct the whole tree for $x_{i,j}$ of the process $dx = -axdt + \sigma dW_t^*$. As we mentioned before, the free parameter $\theta(t)$ can be fitted using initial term structure curve and the method is similar to the equal time step case using the following formulas recursively:

$$Q_{i+1,j} = \sum_h Q_{i,h} p_{h,j} \exp\left(-\left(\alpha_i + h\Delta x_i\right)\Delta t_i\right)$$ and

$$\alpha_i = \frac{1}{\Delta t_i} \ln \frac{\sum_j Q_{i,j} \exp\left(-j\Delta x_i \Delta t_i\right)}{P\left(0, t_{i+1}\right)}.$$

where

$P\left(0, t_{i+1}\right)$

=the initial bond price at time 0 maturing at time t_{i+1} and can be observed from the initial term structure curve.

$p_{h,j}$ = probability moving from state h to j.

The final fitted interest rate tree can be calculated as:

$r_{i,j} = x_{i,j} + \alpha_i$.

Thus we can construct the whole tree for $r_{i,j}$ of the process

$dr = \left(\theta(t) - ar\right)dt + \sigma dW_t^*$.

Calibration

We have assumed the parameters a and σ are known in the interest tree model. However, in practice, we do not know these two values. We should find a way to determine these parameters or in another words, to calibrate the model. The usual way to calibrate the model is to use the market instruments to find the market implied values for the parameters.

Step One:

Find a set of 'calibrating instruments', i.e. the instruments that are used to estimate the parameter values. These calibrating instruments must be instruments that the prices can be calculated from the calibrated model, i.e. the Hull-White unequal time step tree in this case. For example, we can choose European swaptions as our calibrating instruments. The number of calibrating instruments must be no less than the number of parameters to be estimated. The calibrating instruments chosen should be as similar as possible to the instrument being valued. For example, if we want to calculate the value for a Bermudan swaption, we then should use say European swaptions as the calibrating instruments.

Step Two:

We calculate the calibrating instruments values using the calibrated model and we denote V_i to be the model value of the i^{th} calibrating instrument. Note that V_i is a function of a and σ.

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Step Three:
We can observe the actual market values for the calibrating instruments and we denote U_i to be the observed market value of the i^{th} calibrating instrument. Assuming there are n calibrating instruments, we calculate the sum of least squares between the observed market values and model values of the calibrating instruments using the formula:

$$SSE = \sum_{i=1}^{n} \left(U_i - V_i\right)^2 .$$

Step Four:
We choose a and σ such that the SSE is minimised. It can be done using constrained non-linear optimisation algorithm (Box Constrained Levenberg-Marquardt algorithm).

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Chapter 13

Commercial Lending

13.1 Collateral

Collateral refers to the practice of providing assets to secure an obligation. Collateral can take many forms: property, inventory, equipment, receivables, oil reserves, etc.

Collateralisation agreements are often used to secure repo, securities lending and derivatives transactions. Under this agreement, a party who owes an obligation to another party posts collateral, usually cash or securities, to secure the obligation. In the event that the party defaults on the obligation, the secured party may seize the collateral.

The arrangement can be unilateral where only one party is obliged to post collateral, or bilateral where both parties may be obliged to post collateral. Alternatively, the net obligation may be collateralised, in which case the party who is the net obligator posts collateral for the value of the net obligation.

Periodically, the secured obligation is revalued and the collateral is adjusted to reflect changes in value. The securing party adjusts the collateral holdings depending on the current revaluation of the security.

13.2 Cash Advance Facility

A cash advance facility is where the bank gives a company a facility to draw money from – essentially a loan with an agreed set of terms and conditions. Cash Advance Facilities may be *committed* or *uncommitted*. Committed facilities give the company guaranteed access to the full amount of the facility. Cash Advance Facilities may be *revolving* or *non-revolving*. Revolving facilities are ones where the borrow can repay some or all of the principal and then draw back down on it again.

13.2.1 Cash Advance Facility XML Representation

Cash Advance Facilities are represented as a “CreditLine” product, which uses the following XML Schema.

XML Schema

CreditLine Schema			
Name: Type	**Occurs**	**Size**	**Description**
currency string	1..1	3	
drawnAmount decimal	1..1		
startDate date	1..1		

CreditLine Schema			
Name: Type	**Occurs**	**Size**	**Description**
periods CreditLinePeriods	1..1		
floatingRateDefinition fpmlFloatingRateDefinition	0..1		
calculationPeriodFrequency fpmlCalculationPeriodFrequency	0..1		
fixedRate double	0..1		

CreditLinePeriod Schema			
Name: Type			**Description**
lineAmount decimal			
maturityDate date			
committed boolean			
capitaliseInterest boolean			

Credit Line Example

```
<creditLine>
  <currency>USD</currency>
  <drawnAmount>100000000.000000</drawnAmount>
  <startDate>2003-05-29</startDate>
  <periods>
     <period>
        <lineAmount>200000000.000000</lineAmount>
        <maturityDate>2009-05-29</maturityDate>
        <committed>true</committed>
        <capitaliseInterest>false</capitaliseInterest>
```

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```
        </period>
    </periods>
    <calculationPeriodFrequency>
        <periodMultiplier>3</periodMultiplier>
        <period>M</period>
        <rollConvention>MF</rollConvention>
    </calculationPeriodFrequency>
    <fixedRate>0.050000</fixedRate>
</creditLine>
```

The CreditLine contains a list of CreditLinePeriod structures that allows the user to define amortization periods or complex structured loans.

13.2.2 Cash Advance Facility Credit Issues

Committed facilities give the borrower a credit exposure equal to the maximum amount of the facility. *Uncommitted* facilities give the borrower a credit exposure equal to the amount they are currently drawn.

13.2.3 Cash Advance Facility Pricing

The exposure on an *uncommitted* cash advance facility is considered to be simply the amount drawn.

The exposure on a *committed* cash advance facility is considered to be the full limit of the facility.

13.3 Generic Interest Rate

13.3.1 Description of Instrument

A number of financial instruments can be represented using the GenericInterestRate product.

13.3.2 XML Representation

GenericInterestRate Schema			
Name: Type			Description
referenceParty fpmlPartyReference			
interestRateStream fpmlInterestRateStream			

The Generic Interest Rate product encapsulates the FPML InterestRateStream representation.

Chapter 14
Securities

14.1 Promissory Note

14.1.1 Description of Instrument

A Promissory Note or PN is a document issued by a borrower promising to repay a loan under agreed-upon terms.

14.1.2 XML Representation

This product is represented using the GenericInterestRate schema.

Pricing

Let

$F_{pn} =$ the face value of the promissory note

$r_{pn} =$ the annualised yield to maturity of the promissory note

$d_{pn} =$ the days to maturity of the promissory note

$D =$ the number of days in the year

$V_{pn} =$ the exposure of the promissory note

Then

$$V_{pn} = \frac{F_{pn}}{1 + r_{pn} \frac{d_{pn}}{D}}$$

14.2 Commercial Bill

14.2.1 Description of Instrument

Commercial Bills are a short term security issued by a company. The holder of the instrument at maturity receives the face value of the bill from the acceptor who in turn seeks repayment from the drawer.

14.2.2 XML Representation

This product is represented using the GenericInterestRate schema.

Credit Implications

Should the acceptor default, the holder may seek recourse from previous endorsers.

14.2.3 Pricing

Let

F_{pn} = the face value of the bill

r_{pn} = the annualised yield to maturity of the bill

d_{pn} = the days to maturity of the bill

D = the number of days in the year

V_{pn} = the exposure of the bill

Then

$$V_{pn} = \frac{F_{pn}}{1 + r_{pn} \frac{d_{pn}}{D}}$$

14.3 Certificate of Deposit

14.3.1 Description of Instrument

Certificates of Deposit (or *CD*) are short term securities issued at a discount to face value by a bank. The holder of the security receives from the bank the face value of the CD at maturity.

14.3.2 XML Representation

This product is represented using the GenericInterestRate schema.

14.3.3 Pricing

Let

F_{pn} = the face value of the bill

r_{pn} = the annualised yield to maturity of the bill

d_{pn} = the days to maturity of the bill

D = the number of days in the year

V_{pn} = the exposure of the bill

Then

$$V_{pn} = \frac{F_{pn}}{1 + r_{pn} \frac{d_{pn}}{D}}$$

14.4 Commercial Paper

14.4.1 Description of Instrument

Commercial Paper is a discount instrument issued by a company to raise funds. It is unsecured, and thus generally trades at a higher required rate of return.

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The company has the obligation to repay the face value of the paper to the bearer at maturity.

14.4.2 XML Representation

This product is represented using the GenericInterestRate schema.

14.4.3 Credit Implications

As commercial paper is an unsecured investment instrument, the amount at risk is the discounted face value of the paper held.

14.4.4 Pricing

The value of the Commercial Paper is the present value of the face value of the paper held.

Let

P_{cp} = the face value of the commercial paper

df_n = the discount factor at time n

V_{cp} = the value of the commercial paper

Then

$$V_{cp} = P_{cp} df_n$$

14.5 Bank Accepted Bill

14.5.1 Description of Instrument

A bank accepted bill is a discount instrument redeemable for the face value of the bill upon maturity. A bank accepted bill provides security as the bank has undertaken to pay the bearer of the bill the face value at maturity.

14.5.2 XML Representation

This product is represented using the GenericInterestRate schema.

14.5.3 Pricing

The value of the bank accepted bill is the present value of the face value of the bank bill.

Let

P_{bab} = the face value of the bank accepted bill

df_n = the discount factor at time n

V_{bab} = the value of the bank accepted bill

Then

$$V_{cp} = P_{bab} df_n$$

14.6 Bonds

14.6.1 Description of Instrument

Bonds are secured loans that investors make to corporations and governments. Corporations and governments issue bonds when they want to raise capital. Bonds typically pay out a stream of cash-flows to the bearer (the coupon payments), including repayment of the face value at maturity.

A bond can be described by the following attributes:

The *issue date* is the day on which the life of a bond starts. The *term to maturity* defines the period of time, or the life of the bond. The bond's *maturity date* is the date on which the last payment is due.

The *face value* (also called par value or principal sum) of a bond represents the amount that will be repaid to the bondholder at maturity.

The *coupon* is the nominal annual rate of interest that is paid to the bondholder on a regular basis. It is usually expressed as a percentage of the face value (*coupon rate*). The coupon rate is either fixed or variable. The coupon rate is given as an annualised percentage of the face value. For example, a 7% semi-annual bond pays 3.5% of the face value twice a year.

The *purchase price* is the price the investor pays to buy the bond, i.e., to receive this series of cash flows (coupon and face value).

The *coupon period* is not necessarily the same for all bonds. The coupon payments are made semi-annually, which is common in the USA, or annually, which is more common in Europe. The coupon payment dates are fixed.

There are many different types of bonds being traded in the market. Bonds can pay interest that can be fixed (Coupon Bond), floating (Floating Rate Bond) or payable at maturity (Zero Coupon Bond). Other types of bonds can be converted into stock at maturity rather than paid interest (Convertible Bond). Still other bonds can be called back by the issuing company before maturity (Callable Bond).

14.6.2 XML Representation

As there is no bond schema yet defined in FpML, RAZOR's bond schema has been developed by IT&e, but using base components defined by FPML version 3.

The RAZOR bond schema supports the parametric representation of a bond. The parametric information is designed to capture all the economic information required to calculate the exposure of the bond the dates, amounts and rates that imbue the bond with value.

A bond is considered as being a product that contains FPML's Interest Rate Stream structure. This provides the parametric representation of the bond.

fpmlBond Schema			
Name: Type	**Occurs**	**Size**	**Description**
fpmlPartyReference	1..1		
calculationPeriodDates fpmlCalculationPeriodDates	1..1		

fpmlBond Schema			
Name: Type	Occurs	Size	Description
paymentDates fpmlPaymentDates	1..1		
calculationPeriodAmount fpmlCalculationPeriodAmount	1..1		
paymentAmount fpmlAmount	0..1		

14.6.3 Credit Implications

There are different contexts against which the credit risk of a bond can be measured. *Issuer risk* is the risk the institution has to the issuer of the bond. This is the risk that the issuer will default on the obligations imbued in the bond. *Counterparty risk* is the risk that the counterparty that the bond has been bought from defaults on the exchange, so that the purchaser doesn't end up owning the bond.

14.6.4 Pricing

The value of a bond is equal to the present value of the expected cash flows. The interest rate or discount rate used to compute the present value depends on the yield offered on comparable securities in the market. The term structure used to present value the bond's cash flows is assumed to be bootstrapped from comparable securities.

The first step in determining the value of a bond is to work out what it's cash flows are. The cash flows of a non-callable fixed rate coupon bond consist of periodic coupon payments to the maturity date and the payment of the face value at maturity. Determining the number and timing of cash flows that a bond has is based on the roll convention and date basis convention of the bond.

We can now calculate the bond price when we have the cash amounts to be paid and the dates when they are to be paid. Each cashflow is valued to today using our bootstrapped yield curve (the yield curves in RAZOR store spot discount factors to enable faster cash flow discounting).

In RAZOR we take the pessimistic view with callable bonds, assuming that they will not be called.

Let

$C_i =$ the cashflow at time i

$P =$ the bond's principal

$r =$ the coupon rate of the bond

d_i = the number of days in the coupon period i

$D =$ the number of days in the year

$n =$ the number of coupon payments

$V_{bond} =$ the value of the bond

$$C_i = \Pr \frac{d_i}{D}$$

Then

$$V_{bond} = Pdf_n + \sum_{i=0}^{n} C_i df_i$$

14.6.5 Spread Calculation

If a bond is priced against a yield curve, a spread is needed so that the bond can be properly priced to the market. This is due to the yield curve being typically composed of interest rate instruments that have a different credit profile from the bond in question for example: LIBOR versus the US treasury rate. The spread can be recovered from the bond's market price and the yield curve using an iterative solver method.

Define:

P^c	=	clean price of a bond given explicitly from the market
$P(y)$	=	market bond price based on yield curve
ai	=	accrued interest per $1 notional or face value
s	=	spread
N	=	accrued interest per $1 notional or face value

Method:

From the yield curve, the market value of the bond price is: $NP(y)$. The value of the given bond with accrued interest is: $N(P^c + ai)$. To calculate the spread, we solve for s iteratively using a bisection method such that: $NP(y+s) = NP(P^c + ai)$.

Unless it is provided the initial guess for spread is fixed at 1%.

14.7 Bonds - Capital Indexed

14.7.1 Description of Instrument

A capital indexed bond is a bond where the PVd cashflows of the bond is indexed in line with an increase in the Consumer Price Index from the date of issue of the security to the settlement date. I.e, the yield is expressed in real terms. Thus, an indexation factor is required.

14.7.2 XML Representation

Using the bond schema for the Vanilla bond, the floating rate multiplier schema is used to represent the historical inflation rates.

An inflation Adjustment indicator is set to determine which type of inflation adjusted bond it is.

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14.7.3 Credit Implications

The credit implications are the same as a normal bond.

14.7.4 Pricing

The indexation factor is determined by the percentage change of the CPI from the date of issue of the security to the settlement date.

$K_t = K_{t-1}(1+P)$ K is the indexation factor at next interest payment date.

P is the average percentage change in the Consumer Price Index over the two quarters ending in the quarter which is two quarters prior to that in which the next interest payment falls (for example, if the next interest payment is in November, *p* is based on the average movement in the Consumer Price Index over the two quarters ending the preceding June quarter)

$\dfrac{\left(\dfrac{CPI_t}{CPI_{t-2}} - 1\right)}{2}$ where CPI_t is the Consumer Price Index for the second quarter of the relevant two quarter period, and CPI_{t-2} is the Consumer Price Index two quarters previously.

If the settlement date occurs between two payment dates then the indexation factor is discounted back from the next payment date.

i.e. $= K_t(1+P)^{-\frac{f}{d}}$ where f is number of days from next payment date and d is the number of days between next payment date and previous payment date.

Pricing Formula:

The Market Value of a CIB is calculated according to the formula:

$$MV = \left(\frac{v}{1+\frac{p}{100}}\right)^{f/d} * \left[g(x+a_n)+100v^n\right] * \frac{K_t}{100} * \frac{FV}{100}$$

where

v	=	$1/(1+i)$
i	=	Market Yield / (Frequency*100)
f	=	Number of days from settlement to next coupon date
d	=	Number of days between previous and next coupon dates
p	=	Average percentage change in CPI over the two quarters ending in the quarter two quarters before that in which the next interest payment occurs = $[CPI_t / CPI_{t-2} - 1] * 100/2$. For example: if next coupon payment is in August 2004, CPI_t will be the 2004 March quarter CPI and CPI_{t-2} will be the 2003 September quarter CPI.

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g	=	Coupon rate / Frequency
x	=	1 if there is an interest payment at the next coupon date; 0 otherwise
n	=	Number of coupon periods from next coupon date to maturity
a_n	=	$(1-v^n)/i$
K_t	=	Nominal value of principal at next coupon date = $K_{t-1} * [1+p/100]$.
K_{t-1}	=	Nominal value of principal at previous coupon date. If there was no previous coupon date, $K_{t-1} = \$100$. Rounded to 2 decimal places.
FV	=	Face Value

14.8 Bonds - Indexed Annuity

14.8.1 Description of Instrument

An Indexed Annuity bond is one where the future annuities are multiplied by an indexation factor, and present valued.

14.8.2 XML Representation

Using the bond schema for the Vanilla bond, the floating rate multiplier schema is used to represent the historical inflation rates.

An inflation Adjustment indicator is set to determine which type of inflation adjusted bond it is.

14.8.3 Credit Implications

The credit implications are the same as a normal bond.

14.8.4 Pricing

The indexation factor is $IF = \dfrac{CPI_t}{CPI_{t-1}}$. If the settlement date occurs between two payment dates then the indexation factor is discounted back from the next payment date.

i.e. $= \left(\dfrac{CPI_t}{CPI_{t-1}}\right)^{-\frac{f}{d}}$ where f is number of days from next payment date and d is the number of days between next payment date and previous payment date.

Pricing Formula:

The Market Value of an IAB is calculated according to the formulae:

$$MV = \left(\frac{v}{q}\right)^{f/d} * \left(Z + a_n\right) * B_{t-1} * q * \frac{FV}{100}$$ if the CPI value for the next coupon payment is not yet known.

$$MV = \left(\frac{v}{q}\right)^{f/d} * \left(Z + a_n\right) * B_t * \frac{FV}{100}$$ if the CPI value for the next coupon payment is known

where

v	=	$1/(1+i)$
i	=	Market Yield / (Frequency*100)
q	=	Quarterly inflation factor = CPI_t / CPI_{t-1} (with a minimum value of 1)
CPI_0	=	Base CPI
f	=	Number of days from settlement to next coupon date
d	=	Number of days between previous and next coupon dates
Z	=	1 if the price includes the next payment; 0 otherwise
n	=	Number of coupon periods from next coupon date to maturity
a_n	=	$(1 - v^n)/i$
B_t	=	Annuity Payment at time $t = B_0 * CPI_t / CPI_0$
B_0	=	Base Annuity Payment
FV	=	Face Value

14.9 Inflation Linked Bond with Indexation Lag

14.9.1 UK ILG Eight-Month Indexation Lag

Valuation of UK ILG Eight-month Indexation Lag

These are bonds whose coupon and principal payments are linked to the UK RPI index, assuming an eight-month lag.

Valuation Method

This method relates to UK ILGs issued prior to 1 January 2005.

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a) The MTM calculation is similar to a regular coupon paying bond with the difference being that the coupon and principal repayments are scaled by an “Indexation Coefficient” based on the UK Retail Price Index (RPI).

b) In order to calculate the Indexation Coefficient, the following are required:

   i. The RPI applicable when the bond was originally issued, the “Base RPI”. This will be sourced from bond static data.

   ii. The RPI applicable for the coupon payment concerned. Note the 8-month lag means that the RPI quote is taken for the month 8-months prior to the coupon payment. RPI quotes will be sourced from market data.

c) For the current coupon period the RPI is known. For future coupons and the principal repayment, the RPI is forecast using an assumed annual rate of inflation. This is the parameter value to be modelled, for example, the forecast inflation rate is assumed to be 3% pa.

d) The NPV for an ILG with Eight-Month Indexation Lag is calculated as follows:

$$Trade\ NPV_{ILG8} = \sum_{i=1}^{n} I_i.C_i.F_i.DF_i + P.F_n.DF_n$$

where,

F_i	=	Indexation Coefficient for ith coupon (rounded to 5 decimal places)
	=	$\dfrac{RI_i^{m_i-8}}{RI_{Base}}$
$RI_i^{m_i-8}$	=	Reference Index Value for ith coupon, being 8 months prior to m_i (if quote is not available, then value is approximated as described below)
m_i	=	month/year in which the ith coupon payment falls
RI_{Base}	=	Reference Index Value applicable at bond issue date
n	=	number of future coupon payments
C_i	=	ith coupon payment based on appropriate accrual basis
I_i	=	ex-interest indicator for ith coupon payment; I_i=0 for ex-interest, I_i=1 for cum-interest.
P	=	bond par amount
DF_i	=	bond curve discount factor from valuation date to ith coupon date

e) For clarification, the Reference Index Value relating to a coupon payment in month m_i, is the index value for month m_i *-8*. For example, the Reference Index Value applicable to a coupon payment date on any day in Sep-09 is the index value for Jan-09.

f) For future coupon payments where the relevant Reference Index Value has not yet been published, $RI_i^{m_i-8}$ is approximated using a growth rate assumption applied to the most recently published index value, as follows:

$$RI_i^{m_i-8} = RI^M (1+z)^{\frac{m_i-8-M}{12}}$$

where,

RI^M	=	most recently published RPI quote
m_i	=	month/year in which the *i*th coupon payment falls
M	=	month/year of the most recently published RPI
z	=	assumed index annualized growth rate (3%)

g) Accrued Interest for an ILG with Eight-Month Indexation Lag is calculated as follows:

$AccInt_{ILB8}$ = Real Accrued Interest x Index Coeff for next coupon

$$= \left[C_1 . \frac{\text{No. of actual accrued days up to settlement date}}{\text{No. of actual days in coupon period}} \right] * F_1$$

where,

C_1	=	next coupon payment (ie. coupon rate / coupon frequency x notional)
$F1$	=	Indexation Coefficient for the next coupon

h) UK ILGs issued prior to 1 July 2005 (ie. 8 month index lag) trade on an inflation-adjusted basis. Accordingly, the price of such gilts reflects inflation since it was originally issued, eg. most of these ILGs trade at a price well in excess of £100 per £100 nominal.

i) The following examples describe the application of the rules for determining the appropriate Indexation Coefficient:

i. Given a Valuation Date of 10 April 2009, a Base RPI of 102, the most recent RPI published for index month Feb-09 of 125, and a next coupon payment date of 1 September 2009 (or any other day in Sep-09):

m_i	=	Sep-09
m_i-8	=	Jan-09
$RI_i^{m_i-8}$	=	RPI for index month Jan-09 (published during Feb-09), eg. 120
M	=	Feb-09
F_i	=	120 / 102 = 1.176471

ii. Given a Valuation Date of 10 April 2009, a Base RPI of 102, the most recent RPI published for index month Feb-09 of 125, and an *i*th coupon payment date of 1 March 2012 (or any other day in Mar-12):

m_i	=	Mar-12
m_i -8	=	Jul-11
M	=	Feb-09
RI^M	=	125
z	=	0.03
$RI_i^{m_i-8}$	=	125 * (1 + 0.03) ^ (29 / 12)
	=	134.255877
F_i	=	134.255877 / 102 = 1.316234

14.9.2 Inflation-Linked Bonds with Three-Month Indexation Lag (“Canadian Model”)

Valuation of ILBs with Three-Month Indexation Lag

These are inflation-linked bonds (ILBs) whose coupon and principal payments follow the standard “Canadian Model” which assumes a three-month indexation lag.

For example, the following method applies to the following securities:

Country	Reference Index	Issuing Authority
UK (issued after 2005)	Aggregate RPI	ONS
France	CPI ex-tobacco	INSEE
Germany	HICP ex-tobacco	EUROSTAT
Italy	HICP ex-tobacco	EUROSTAT
Greece	HICP ex-tobacco	EUROSTAT

RPI refers to the Retail Price Index in the UK.
CPI refers to the Consumer Price Index in France.
HICP refers to the Harmonized Index of Consumer Prices in the Euro-Zone.

Valuation Method

a) The valuation method for ILBs with three-month indexation follows the de facto industry standard and is commonly referred to as the “Canadian Model”.

b) The Indexation Coefficient under the Canadian Model:

   (i) is based on index quotes 3-months and 2-months prior to the coupon payment,

   (ii) is calculated using linear interpolation.

c) The NPV for an ILB with Three-Month Indexation Lag is calculated as follows:

$$Trade\ NPV_{ILB3} = \sum_{i=1}^{n} I_i.C_i.F_i.DF_i + P.F_n.DF_n$$

where,

F_i = Indexation Coefficient for *i*th coupon (rounded to 5 decimal places)

$$= \frac{RI_i^{\text{interp}}}{RI_{Base}}$$

RI_i^{interp}	=	Reference Index Value for *i*th coupon based on linear interpolation as described below
RI_{Base}	=	Reference Index Value applicable at bond issue date
n	=	number of future coupon payments
C_i	=	*i*th coupon payment based on appropriate accrual basis
I_i	=	ex-interest indicator for *i*th coupon payment; I=0 for ex-interest, I=1 for cum-interest.
P	=	bond par amount
DFi	=	bond curve discount factor from valuation date to ith coupon date

d) The interpolated Reference Index Value for the *i*th coupon, RI_i^{interp}, is determined using the following rules:

1. The Reference Index Value applicable to the first day of the month in which the coupon falls, m_i, is the index value for month m_i -3. For example, the Reference Index Value applicable to June 1 is the index value for March.
2. The Reference Index Value for any other day in month m_i is calculated by linear interpolation between the index values for month m_i -3 and month m_i -2, according to the following formula:

$$RI_i^{\text{interp}} = \left(\frac{d_i - 1}{ND_i^{m_i}}\right)\left(RI_i^{m_i-2} - RI_i^{m_i-3}\right) + RI_i^{m_i-3}$$

where,

m_i	=	month/year in which the *i*th coupon payment falls
d_i	=	day of month in which the *i*th coupon payment falls
$ND_i^{m_i}$	=	number of days in month, m_i, in which the *i*th coupon payment falls
$RI_i^{m_i-3}$	=	Reference Index Value for month m_i -3
$RI_i^{m_i-2}$	=	Reference Index Value for month m_i -2

3. When calculating the interpolated Reference Index Value, if any $RI_i^{m_i-3}$ or $RI_i^{m_i-2}$ have not yet been published, these values are approximated using a growth rate assumption applied to the most recently published index value, as follows:

$$RI_i^{m_i-Lag} = RI^M (1+z)^{\frac{m_i-Lag-M}{12}}$$

where,

RI^M	=	Most recently published RPI quote
m_i	=	month/year in which the *i*th coupon payment falls

Lag	=	number of months prior to m_i for which the index value is required, ie. either 2 or 3
M	=	Month/Year of the most recently published RPI
z	=	assumed index annualized growth rate (3%)

e) Accrued Interest for an ILB with Three-Month Indexation Lag (Canadian Model) is calculated as follows:

$$AccInt_{ILB3} = \text{Real Accrued Interest x Current Index Coefficient}$$

$$AccInt_{ILB3} = \left[C_1 .* \frac{\text{No. of actual accrued days up to settlement date}}{\text{No. of actual days in coupon period}}\right] * F_{RS}$$

where,

C_1	=	next coupon payment (ie. coupon rate / coupon frequency x notional)
F_{RS}	=	Indexation Coefficient as at Regular Settlement date
	=	$\dfrac{RI_{RS}^{\text{interp}}}{RI_{Base}}$
RI_{RS}^{interp}	=	Interpolated Reference Index Value as at Regular Settlement date based on linear interpolation
	=	$\left(\dfrac{d_{RS}-1}{ND_{RS}^{m_{RS}}}\right)\left(RI_{RS}^{m_{RS}-2} - RI_{RS}^{m_{RS}-3}\right) + RI_{RS}^{m_{RS}-3}$

f) ILBs based on the Canadian Model trade on a real price basis. That is, the impact of inflation since the bond was first issued is effectively stripped out from the price for trading purposes, and hence will typically have a quoted price around par. However, settlement prices will be on the basis of the inflation-adjusted price. The relationship between traded real price and settlement inflation-adjusted prices is as follows:

Inflation-Adjusted Dirty Price = (Real Clean Price + Real Acc Int) x F_{RS}

where,

F_{RS}	=	Indexation Coefficient as at Regular Settlement date, calculated as per (e) above

g) The following examples describe the application of the rules for determining the appropriate Indexation Coefficient:

i. Given a Valuation Date of 25 March 2009, a Base RPI of 102, the most recent RPI published for index month Feb-09 of 125, and a next coupon payment date of 1 April 2009:

m_i	=	Apr-09
m_i -3	=	Jan-09
$RI_i^{m_i-3}$	=	RPI for index month Jan-09 (published during Feb-09), eg. 120
RI_i^{interp}	=	$RI_i^{m_i-3}$ = 120
F_i	=	120 / 102 = 1.176471

ii. Given a Valuation Date of 25 March 2009, a Base RPI of 102, the most recent RPI published for index month Feb-09 of 125, and a next coupon payment date of 20 April 2009:

m_i = Apr-09
m_i -3 = Jan-09
m_i -2 = Feb-09
$RI_i^{m_i-3}$ = RPI for index month Jan-09, eg. 120
$RI_i^{m_i-2}$ = RPI for index month Feb-09, eg. 125
d_i = 20
$ND_i^{m_i}$ = 30
RI_i^{interp} = (20 -1) / 30 * (125 - 120) + 120
= 123.166667
F_i = 123.166667 / 102 = 1.207516

iii. Given a Valuation Date of 25 March 2009, a Base RPI of 102, the most recent RPI published for index month Feb-09 of 125, and a next coupon payment date of 21 July 2009:

m_i = Jul-09
m_i -3 = Apr-09
m_i -2 = May-09
M = Feb-09
RPI^M = 125
z = 0.03
$RI_i^{m_i-3}$ = 125 * (1 + 0.03) ^ (2 / 12) = 125.617328
$RI_i^{m_i-2}$ = 125 * (1 + 0.03) ^ (3 / 12) = 125.927134
d_i = 21
$ND_i^{m_i}$ = 31
RI_i^{interp} = (21 -1) / 31 * (125.927134 - 125.617328) + 125.617328
= 125.817203
F_i = 125.817203 / 102 = 1.233502

14.10 Perpetuity Bond

Valuation of Perpetuity

Razor system shall value a bond that pays coupons in perpetuity using the Net Present Value method (NPV).

Valuation Method

a) Future cashflows up to the last discount curve point will be discounted directly using curve DFs.

b) Coupons beyond the last curve point will be valued using a yield-to-perpetuity calculation.

c) The NPV for a perpetuity is calculated as follows:

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$$Trade\ NPV_{Perp} = \sum_{i=1}^{n} C_i DF_i + \frac{C_n}{r}.DF_n$$

where,

C_i = ith next coupon payment based on appropriate accrual basis.
n = number of coupon payments up to last bond discount curve point.
DF_i = bond curve discount factor from valuation date to ith coupon date.
r = $DF_{Last}^{-1/YF_{Last} \cdot f} - 1$ (using DF) *or*
r = $(1+z_{Last})^{1/f} - 1$ (using annualized zero rate).
YF_{Last} = year fraction between valuation date and last date on bond discount curve.
DF_{Last} = bond curve discount factor from valuation date to last date on bond discount curve.
z_{Last} = annual compounding zero rate at last date on bond zero curve.
f = coupon frequency, ie. Number of coupons per year.

14.11 Cash Bond

We first define some notation.

Terminology - Dates

$TD = Trade\ \ Date$

$V = Valuation\ \ Date$

$TS = Trade\ \ Settlement\ \ Date$

$RS = Regular\ Settlement\ Date$

$CD_j = Exdividend\ \ or\ \ coupon\ \ date\ \ if\ \ exdivided\ \ date\ \ equals\ \ coupon\ \ date$

Terminology - Accruals

$$\alpha_{V,RS} = Appropriate\ \ Accrual\ \ from\ V\ to\ \ RS = \frac{(RS - V)}{\lambda}$$

$$\alpha_{V,TS} = Appropriate\ \ Accrual\ \ from\ V\ to\ TS = \frac{(TS - V)}{\lambda}$$

$$\alpha_{CD_j TS} = Appropriate\ \ Accrual\ \ from\ CD_j\ \ to\ \ TS = \frac{(TS - \ \ CD_j)}{\lambda}$$

$$\alpha_{RS,TS} = Appropriate\ \ Accrual\ \ from\ \ RS\ \ to\ \ TS = \frac{(TS - \ \ RS)}{\lambda}$$

$\lambda = Appropriate\ Currency\ \ Accrual\ \ Basis$ (this should be driven by the bond Trade XML)

The difference between the days is driven by the daycount convention.
Terminology - Prices

P_V = Bond Clean Price(s) at the Valuation Date

$AI_{RS} = Accrued\ \ Interest\ \ to\ \ RS$

Terminology - Rates

$LDF_t = Libor\ Discount\ Function\ at\ t$

$RIF_t = Repo\ Interest\ Function\ at\ t$

$$Libor_{V,TS} = \text{Libor Curve Derived Rate from V to TS} = \left[\frac{LDF_V}{LDF_{TS}} - 1\right]\left[\frac{1}{\alpha_{V,TS}}\right]$$

$$Libor_{V,RS} = Libor\ Curve\ Derived\ Rate\ from\ V\ to\ RS = \left[\frac{LDF_V}{LDF_{RS}} - 1\right]\left[\frac{1}{\alpha_{V,RS}}\right]$$

$$R_{RS,TS} = \text{Repo Curve Derived ForwardRate from RS to TS} = \left[\frac{RIF_{RS}}{RIF_{TS}} - 1\right]\left[\frac{1}{\alpha_{RS,TS}}\right]$$

$$R_{CD,TS} = \text{Repo Curve Derived Forward Rate from CD to TS} = \left[\frac{RIF_{CD}}{RIF_{TS}} - 1\right]\left[\frac{1}{\alpha_{CD,TS}}\right]$$

Terminology - Cashflows

$C_j = Cashflow\ \ j$

$n = Number\ of\ Cashflows\ \ from\ V\ to\ TS$

$N = Bond\ Nominal\ Amount$

$D_{RS} = \text{Bond Dirty Value at RS} = \left[N\left(P_V + AI_{RS}\right)\right]$

$B = \text{Forward Value of Bond Dirty Value, at TS} = D_{RS}\left[1 + R_{RS,TS}\alpha_{RS,TS}\right]$

$FC = \text{Sum of Forward Values of Interim Coupons, at TS} = \sum_{j}^{n} C_j\left[1 + R_{CD_j,TS}\alpha_{CD_j,TS}\right]$

For clarity purposes: if RS < exDivDate < TS then FC <>0 otherwise FC=0

$FCV =\ \ Future\ Cash\ Value(s)$ - this is equivalent to the bond cash settlement/payment amount.

Cash Bond

This is a separate valuation methodology for forward settling bonds. Cash bonds settled on or after the ex-dividend, or coupon date if no ex-dividend date exists, will be deemed to be ex-coupon. Therefore, if the ex-divided/coupon date occurs between *RS* and TS, then FC will have some value otherwise $FC = 0$. If both the ex-dividend date and the coupon date occur between RS and TS then the coupon must obviously only accrue interest from the coupon date to TS rather than from the ex-dividend date. However if the ex-divided date falls before TS while the coupon date falls after TS then the coupon will need to be discounted back to TS.

NPV for the buyer of the cash bond trade is:

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$$Trade\ NPV = \left[\left(\frac{1}{1+Libor_{V,TS}\alpha_{V,TS}}B\right)-\left(\frac{1}{1+Libor_{V,TS}\alpha_{V,TS}}FC\right)\right]-\left(\frac{1}{1+Libor_{V,TS}\alpha_{V,TS}}FCV\right)$$

where:

$P_V\ in\ D_{RS} = P_V^{Bid} = \text{Bond Clean Bid Price at the Valuation Date}$

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NPV for the seller of the cash bond trade is:

$$Trade\ NPV = \left(\frac{1}{1+Libor_{V,TS}\alpha_{V,TS}}FCV\right)-\left[\left(\frac{1}{1+Libor_{V,TS}\alpha_{V,TS}}B\right)-\left(\frac{1}{1+Libor_{V,TS}\alpha_{V,TS}}FC\right)\right]$$

where:

$P_V\ in\ D_{RS} = P_V^{Ask} = \text{Bond Clean Ask Price at the Valuation Date}$

14.12 Floating Rate Notes

14.12.1 Description of Instrument

A floating rate note (FRN) is a debt instrument similar to a bond in that it makes regular interest payments. Unlike a bond, however, the FRN pays an amount based on a floating benchmark rate and a fixed margin.

14.12.2 XML Representation

The FRN is represented using FPML's InterestRateStream structure.

14.12.3 Pricing

In the case of a bond, valuation consists of present valuing a series of known cash flows. With an FRN, the only cash flows we know are the coupon interest payment at the end of the current interest payment period and the repayment of the principal at maturity — all of the intermediate coupon payments are unknown.

There are two methods to evaluate the price of floating rate notes. The first method is to use the discounting of cash flows similar to the floating side of an interest rate swap and another method is to use a closed form pricing formula. Note that MBS has the same payment structure as FRN except that MBS has a chance of early repayment. Due to the similarity in the payment structure of these two types of contracts, the pricing methodologies are similar. We shall discuss the pricing of both FRN and MBS in the following.

1. Discounting Cash Flows

For FRN, the valuation is the same as floating leg of an interest rate swap. The intermediate coupon payments can be calculated from the forward curve. For MBS, it is the same as FRN except that we do not know the length of the contract. We can simply use the weighted average life (WAL) as a proxy for the length of MBS. The maturity date used for the MBS is the WAL if it falls on a coupon date, otherwise it will be set to the coupon date following the WAL. The valuation is then carried out the same way as the FRN. The only difference is that we multiply the final discounted value of the MBS by the bond factor (BF) to obtain the final price.

2. Closed Form Pricng Formula

We first define some notations:

FV = Face value.

C = Next coupon amount per \$100 FV , $\left(BBSW + \frac{IM}{100}\right)\times\frac{d}{365}$.

IM = Issue margin.

TM = Trading margin.

f = Number of days from settlement to next coupon date.

d = Number of days between previous and next coupon dates.

a = Annuity factor $\frac{1-(1+i)^{-n}}{i}$.

i = $\frac{s+\frac{TM}{100}}{freq\times 100}$.

s = Swap rate.

r = Discount rate.

n =

Number of coupon periods between the next coupon date and the WAL date with the appropriate n rounding convention $=\frac{WAL-NCD}{365.25}\times Frequency$.

The value of an MBS is:

$$\frac{100+C+\left(\frac{IM-TM}{100\times freq}\right)\times a}{1+\left(r+\frac{TM}{100}\right)\times\left(\frac{f}{36500}\right)}\times\frac{FV}{100}\times BF .$$

The value of a FRN is by substituting time to maturity for WAL and 1 for BF .

14.13 Repurchase Agreements

14.13.1 Description of Instrument

Repurchase Agreements involve the selling of securities to a counterparty and simultaneously agreeing to buy the securities back on a predetermined date at a predetermined price based on the Repo rate. Effectively this results in borrowing cash for the period and providing the bond as collateral.

A reverse Repo is the transaction of opposite side.

If the repo includes an underlying security, it is priced separately first, then included in the pricing of the repo. At the moment, we support repo with an underlying irBond. Users can specify a curve (called the bond curve) to use for pricing the underlying bond instead of the reval curve that is being used currently. One additive and one multiplicative factor can be specified to be applied to the bond curve before pricing takes place.

Razor values the following Eligible Securities for specific or general collateral:

a) Bonds
    i. Fixed rate.
    ii. Zero Coupon.
    iii. Perpetual.
b) Strips.
c) Bills
    i. Discount yield-quoted.
    ii. Money market yield-quoted.
d) Inflation -Linked Bonds
    i. UK ILG Eight-Month Indexation Lag.
    ii. UK ILG Three-Month Indexation Lag (Canadian Model).
    iii. EU ILB Three-Month Indexation Lag (Canadian Model).

14.13.2 XML Representation

Repurchase agreements are modelled using two trades The purchase/sale of a bond on the near date and the sale/purchase of the bond on the far date.

Repo is implemented as a structural deal combining the short position in underlying asset and long cash position reflecting the borrowed amount at Repo start date and a forward transaction of underlying asset repurchase at repo termination date. Coupon payments during the life of the Repo Transaction are optional

14.13.3 Credit Implications

Repurchase agreements have similar credit implications to the underlying security. In the case where the underlying security is a debt security (bond),

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there is counterparty risk with seller of Repo and an underlying security issuer risk. Under a classical Repo agreement the borrower of the cash (lender of the underlying asset) may still receive any cash flows generated by the asset prior to termination of the deal and that passed on by Repo seller.

14.13.4 Pricing

Define

S1 = Cash payment for the spot leg at t1.
S2 = Cash payment for the forward leg at t1.
B1 = Bond price for the spot leg per 100 at t0.
B2 = Bond price for the forward leg per 100 at t0.
R_R = Repo rate from repo curve.
R_L = Discount rate from RF curve.

The repo valuation formula is

$$MTM = MTM_{Leg1} + MTM_{Leg2}$$

$$MTM_{Leg1} = \left[S1 - N\frac{B1}{100}\left(1 + R_R\frac{t_1 - t_0}{360}\right)\right] \times \left[\frac{1}{1 + R_L\frac{t_1 - t_0}{360}}\right]$$

$$MTM_{Leg2} = \left[-S2\left(1 + R\frac{t_2 - t_1}{360}\right) + N\frac{B2}{100}\left(1 + R_R\frac{t_2 - t_0}{360}\right)\right] \times \left[\frac{1}{1 + R_L\frac{t_2 - t_0}{360}}\right]$$

Points to note:

Although the above uses ACT/360 the day count convention should follow trade and market configuration as usual.

The bond price B is obtained by pricing the bond (discounted cashflows) using the bond curve and value date t0. It is the dirty price at t0.

Given the selling and purchase prices of the bond are not the same due to bid-ask spread, the above pricing formula has made spread adjustment to cater for this mismatch.

14.14 General Collateralised Repo

There are two parties to a repo trade: A (the seller) and B (the buyer). On the trade date, the two parties enter into an agreement whereby on a set date, the settlement date, A will sell to bank B a nominal amount of securities in exchange for cash. The agreement also demands that on the termination date B will sell identical stock back to A at the previously agreed price, and consequently Bank B will have its cash returned with interest at the agreed repo rate. If a coupon is paid it will be handed over to the seller on the coupon payment date.

14.14.1 Sterling GC Trades

Valuation of Sterling GC trades

a) Razor system shall value Sterling GC trades using the NPV method.

b) For the long collateral holder of a Sterling GC trade, the NPV is calculated follows:

$$Trade\ NPV_{GC} = S_1\left[R\frac{TS_2 - TS_1}{Y} - r_{\max(VD,TS_1),TS_2}\frac{TS_2 - \max(VD,TS_1)}{Y}\right].DF_2$$

where,

VD	=	valuation date
TS_1	=	trade opening leg date
TS_2	=	trade maturity leg date
S_1	=	GC trade notional, ie. Initial cash payment value
R	=	trade repo rate (retrieved from the xml trade)
$r_{i,j}$	=	market forward repo rate between time i and j
Y	=	appropriate year accrual basis
DF_2	=	libor discount factor from VD to TS_2

c) For the short collateral holder of a Sterling GC trade, the NPV is calculated follows:

$$Trade\ NPV_{GC} = -S_1\left[R\frac{TS_2 - TS_1}{Y} - r_{\max(VD,TS_1),TS_2}\frac{TS_2 - \max(VD,TS_1)}{Y}\right].DF_2$$

14.14.2 Euro GC Trades

Valuation of Euro GC trades

a) Razor system shall value Euro GC trades using the NPV method.

b) For the long collateral holder of a Euro GC trade, the NPV is calculated exactly as per a Sterling GC trade, given as follows:

$$Trade\ NPV_{GC} = S_1\left[R\frac{TS_2 - TS_1}{Y} - r_{\max(VD,TS_1),TS_2}\frac{TS_2 - \max(VD,TS_1)}{Y}\right].DF_2$$

where,

VD	=	valuation date
TS_1	=	trade opening leg date
TS_2	=	trade maturity leg date
S_1	=	GC trade notional, ie. initial cash payment value
R	=	trade repo rate (retrieved from the xml trade)
$r_{i,j}$	=	market forward repo rate between time i and j
Y	=	appropriate year accrual basis
DF_2	=	euribor discount factor from VD to TS_2

c) For the short collateral holder of a Euro GC trade, the NPV is calculated exactly as per a Sterling GC trade, given as follows:

$$Trade\ NPV_{GC} = -S_1\left[R\frac{TS_2 - TS_1}{Y} - r_{\max(VD,TS_1),TS_2}\frac{TS_2 - \max(VD,TS_1)}{Y}\right].DF_2$$

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14.15 Bill Index Deposits

Product Decscription

14.15.1 Guaranteed Bill Index Deposits

The Guaranteed Bill Index Deposit (GBID) is a short-term deposit product offering guaranteed returns every 90 days determined by reference to an index. If the index performance is positive for the period of the deposit, the return will be positive. However, if the index performance is negative for the period of the deposit, the return will be negative and an amount equivalent to the negative return will become payable to the issuer. The GBID provides a money market return without the need to actively manage the cash and bill market investment alternatives.

14.15.2 Protected Bill Index Deposits

The Protected Bill Index Deposit (GBID) is similar to the Guaranteed Bill Index Deposit (GBID) but which also protects the investors capital, by flooring any return to zero and so protecting the investors from paying for any negative return on the index .

GBID maturity dates will be the quarter end dates of March, June, September and December. Interest is calculated by reference to the daily rate of return of the UBS Australian Bank Bill Index. Interest is paid quarterly in arrears on the maturity date.

Finmark Schema and Example

This finmark schema is supported by the bond schema.

Example


```
- <deal>
- <trade>
- <tradeHeader>
  <tradeDate>2007-09-13</tradeDate>
  <tradeType>CDBD</tradeType>
  <counterparty>56789</counterparty>
  <internalUnit>1234</internalUnit>
  <buySell>SELL</buySell>
  <status>OPEN</status>
  </tradeHeader>
- <extensions>
- <extension>
```

```xml
<value>HISTORICAL</value>
<name>HistoricalRateSet</name>
</extension>
- <extension>
<value>UBSBBI</value>
<name>AUD BBI 1D</name>
</extension>
</extensions>
- <product>
- <bond>
- <bondStream>
<payerPartyReference href="56789" />
<receiverPartyReference href="1234" />
- <calculationPeriodDates id="CalcPeriodDates0">
- <effectiveDate>
<unadjustedDate>2007-08-13</unadjustedDate>
- <dateAdjustments>
<businessDayConvention>MODFOLLOWING</businessDayConvention>
</dateAdjustments>
</effectiveDate>
</calculationPeriodDates>
- <calculationPeriodAmount>
- <calculation>
- <notionalSchedule>
- <notionalStepSchedule>
<currency>AUD</currency>
<initialValue>100000</initialValue>
</notionalStepSchedule>
</notionalSchedule>
- <floatingRateCalculation>
<floatingRateIndex>BillIndex</floatingRateIndex>
</floatingRateCalculation>
<dayCountFraction>ACT/360</dayCountFraction>
</calculation>
</calculationPeriodAmount>
- <principalExchanges>
<intermediateExchange>false</intermediateExchange>
<initialExchange>false</initialExchange>
<finalExchange>false</finalExchange>
</principalExchanges>
</bondStream>
<productType>GBID</productType>
<securityId>GBID</securityId>
</bond>
</product>
</trade>
- <dealHeader>
<dealType>CDBD</dealType>
<dealDate>2009-02-19</dealDate>
<status>OPEN</status>
</dealHeader>
</deal>
```

Calculation of UBS Warburg Bank Bill Index

The UBS Warburg Bank Bill Index is based on a series of 13 91-day bank bills that mature at 7 day intervals (every Tuesday). Upon maturity, the face value of the maturing bill is reinvested in a new 91-day bill at its discounted price.

The Index is calculated daily by valuing these 13 bank bills. The yield for each bill is determined by interpolation between the 10:15 am cash rate, 1-month BBSW and 3-month BBSW rates, as shown in the table below. The Index is calculated by summing the discounted values of the 13 bills and dividing by 10000.

Days to Maturity	Interpolated Rate
0-7	R1
8-14	2/3**R1** + 1/3**R2**
15-21	1/3**R1** + 2/3**R2**
22-28	R2
29-35	8/9**R2** + 1/9**R3**
36-42	7/9**R2** + 2/9**R3**
43-49	6/9**R2** + 3/9**R3**
50-56	5/9**R2** + 4/9**R3**
57-63	4/9**R2** + 5/9**R3**
64-70	3/9**R2** + 6/9**R3**
71-77	2/9**R2** + 7/9**R3**
78-84	1/9**R2** + 8/9**R3**
85-91	R3

R1 = cash rate.
R2 = 1 month BBSW rate.
R3 = 3 month BBSW rate.

Example
Cash Rate = 6.5%.
1 Month BBSW Rate = 7.07%.
3 Month BBSW Rate = 7.0983%.
Current Date = 2007-9-13.
Yield Rate = Simple Interest.
Day Convention = ACT/365.

Maturity Date	No. of Days	Yield Rate	Face Value	DF	PV
2007-11-20	68	0.0708886667	4599097	0.9869654866	4539150.01
2007-11-27	75	0.0709201111	4590987	0.9856366856	4525045.21
2007-12-04	82	0.0709515556	4595705	0.9843102894	4523599.72
2007-12-11	89	0.0709830000	4782951	0.9829862928	4701575.27
2007-9-18	5	0.0650000000	4678551	0.9991103812	4674388.87
2007-9-25	12	0.0669000000	4646588	0.9978053749	4636390.48
2007-10-2	19	0.0688000000	4599276	0.9964314106	4582863.07

2007-10-9	26	0.0707000000	4632012	0.9949890715	4608801.32
2007-10-16	33	0.0707314444	4590717	0.9936457371	4561546.38
2007-10-23	40	0.0707628889	4587825	0.9923048376	4552520.94
2007-10-30	47	0.0707943333	4598925	0.9909663678	4557380.00
2007-11-6	54	0.0708257778	4620371	0.9896303227	4572459.24
2007-11-13	61	0.0708572222	4593062	0.9882966973	4539308.01
Sum					59563722.01

Valuation

There are 4 methods (parameter "MTMMethod") for modeling GBID/PBID products.

For the get MTM function - the value returned for MTMMETHOD of …

MTM – cashflow based, notional and interest – overnight or at maturity - pv'd to value date of flows

FACE – cashflow based, notional and interest – overnight or at maturity - sum of face value of flows

STEP – step schedule – returns current balance (notional outstanding)

BBI – method calculates return on BBI Index

Interest Calculation

The GBID/PBID net interest calculation is the sum of the historic accrued interest and forecast interest calculation.

The balance at time t will be calculated from the original deposit and any notional adjustments which have occurred before time t

The daily interest is calculated as follows

$$Interest_T = Balance_T \times \left(\frac{Daily\ return}{100} \right)$$

$$Daily\ GBID\ return = \left(\frac{BBI_T}{BBI_{T-1}} - 1 \right) \times 100, \quad Daily\ PBID\ return = \max(Daily\ GBID\ return, 0)$$

Where

$$BBI_T = UBS\ Australian\ Bank\ Index\ for\ Day\ T$$

$$BBI_{T-1} = UBS\ Australian\ Bank\ Index\ for\ Day\ T-1$$

The BBI Index is only calculated and provided by TCV for business days. Interest is calculated daily for each day of the month including non-business days. For non business days the interest is calculated from the last known daily return.

Today's interest amount will be calculated from the BBI observation as set yesterday (normally 11am).

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All previous daily interest amounts are calculated from the historical observations for the BBI. If the BBI Index History is not provided then accrued interest will be set to zero.

For historic interest to be calculated for time t – the rate of return currently is BBI(t)/BBI(t-1) BUT should be BBI(t-1)/BBI(t-2) – so interest is P(t-1)*(BBI(t-1)/BBI(t-2) – 1)
No interest is calculated for start date – ie on day of initial deposit the user does not earn interest.
On start date the first days interest can only be forecast.
On first days interest day – the interest is calculated from BBIs observed at COB on start date ie t-1 and previous day ie t-2
Forecast accrued interest is implemented in the setting up of the transition cache – so there is no impact from forecast accrued interest for VaR or stress/sensitivity – ie its not dependent on the current scenario and only the base scenario – needs to be fixed – but lower priority
Forecast and Historic daily interest mostly caters for changing notional correctly – except on the days of notional change – interest should be calculated on previous notional
Weekend handling for historic interest calculation – if observation for t-1 is not known due to being a weekend the previous return is applied for interest at time t

The interest accrued today is calculated from yesterdays BBI return applied to yesterdays outstanding principal (not today's).

Calculation of Interest

t - daily interest calculation for day at time t
P_t - outstanding balance at time t
s - deposit start date
v - value date
m - deposit maturity date
$r_{s,t}$ - annualised rate of return for period from s to t
df_t - discount factor from time t from the BBI curve

$df^f_{s,t}$ - forward discount factor from time t to time s from the BBI curve

NOTE: $df_t = df^f_{0,t}$

NOTE: for PBID the daily return is floored at 1 - ie calculated daily interest is floored at zero

Historical Accrued: for time $i = s + 1$ to v

$$I_i^h = P_{i-1} * \left(\frac{BBI_{i-1}}{BBI_{i-2}} - 1\right)$$

Forecast: for time $j = v + 1$ to m

$$I^{f}_{j-1,j} = P_{j-1} * \left(\frac{df_{j-1}}{df_j} - 1\right)$$

Net Interest: is the sum of daily interest already accrued plus sum of the forecast daily interest

$$AI = \sum_{i=s+1}^{v} I^{h}_{i} + \sum_{j=v+1}^{m} I^{f}_{j-1,j}$$

Annex:
In case of interest to anyone, justification for this calculation for forward forecast daily interest is …

For $df^{f}_{s,t}$ …

$$df^{f}_{s,t} = \frac{df_t}{df_s}$$

$$df^{f}_{s,t} = 1 \Big/ \left(1 + r_{s,t} * \frac{t-s}{365}\right)$$

Hence …

$$\frac{df_t}{df_s} = 1 \Big/ \left(1 + r_{s,t} * \frac{t-s}{365}\right)$$

And …

$$r_{s,t} = \frac{365}{t-s} * \left(\frac{df_s}{df_t} - 1\right)$$

The calculated interest for a forward period from time s to time t is

$$I^{f}_{s,t} = P_t * r_{s,t} * \frac{t-s}{365}$$

By substituting for $r(s,t)$ above, we get

$$I^{f}_{s,t} = P_t * \left(\frac{df_s}{df_t} - 1\right)$$

The calculated interest for a forward daily period from time $t-1$ to time t is then found by setting $s = t-1$

$$I^{f}_{t-1,t} = P_t * \left(\frac{df_{t-1}}{df_t} - 1\right)$$

Example of Interest Calculation

BBI on 24/01/2001 = 5000.

Opening Balance = $10,000,000.

Transaction Details:

Date	Transaction	Amount	Balance
25/01/2001	Deposit	10,000,000	$10,000,000
28/01/2001	Deposit	5,000,000	$5,000,000
31/01/2001	Interest	1,869.81	$1,869.81

Principal = $15,000,000.
Interest = $1,869.81.
Closing Balance = $15,001,869.81.

Calculation of Interest Payable

Date	BBI	Daily Return	Balance	Interest
25/01/2001	5000.10	0.002%	$10,000,000	$200.00
26/01/2001	5000.15	0.001%	$10,000,000	$100.00
27/01/2001	5000.35	0.004%	$10,000,000	$399.99
28/01/2001	5000.40	0.001%	$15,000,000	$149.99
29/01/2001	5000.80	0.008%	$15,000,000	$119.99
30/01/2001	5000.90	0.002%	$15,000,000	$299.95
31/01/2001	5001.10	0.004%	$15,000,000	$599.89
Total Interest				$1869.81

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Chapter 15
Equities

15.1 Ordinary Shares

15.1.1 Description of Instrument

Ordinary shares give the holder limited liability ownership of the company.

15.1.2 XML Representation

The asset representation of shares allows us to specify either discrete dividends, or a dividend yield.

Equity Example

```xml
<equity>
  <instrumentId>ITE</instrumentId>
  <description>ITE Ordinary</description>
  <effectiveDate>
     <unadjustedDate>2003-12-12</unadjustedDate>
  </effectiveDate>
  <numberShares>100000</numberShares>
  <exchangeCode>ASX</exchangeCode>
  <currency>AUD</currency>
</equity>
```

15.1.3 Credit Implications

RAZOR will simulate the price of the stock. Fixed dividend payments may also be assigned to the stock, or an annualised dividend yield. Because the stock price is quoted in terms of the currency where the exchange is based that the stock is traded on, the holding could also incur exchange rate risk.

15.1.4 Pricing

Let

P_{ccy} = price per share in currency *ccy*

N = number of shares held

V_{ccy} = Exposure in currency ccy

Then

$$V_{ccy} = NP_{ccy}$$

15.2 Equity Forwards

15.2.1 Basket Equity Forward

A Basket Equity Forward is an agreement to purchase a specified basket of equity shares for at an agreed future time for an agreed value.

To support this product, Razor requires updates to the following components:
Pricing Adapter
XML Schema
Deal Entry Screen

XML Representation


```xml
<equity>
  <instrumentId>Basket</instrumentId>
  <forwardType>Vanilla</forwardType>
<basket>
<basketAsset>
  <id>ANZ</id>
  <weightingFactor>0.3</weightingFactor>
  </basketAsset>
<basketAsset>
  <id>TEL</id>
  <weightingFactor>0.2</weightingFactor>
  </basketAsset>
<basketAsset>
  <id>BHP</id>
  <weightingFactor>0.5</weightingFactor>
  </basketAsset>
  </basket>
  <numberShares>100000</numberShares>
  <exchangeCode>ASX</exchangeCode>
  <currency />
<paymentAmount>
  <paymentType>SETTLEMENT</paymentType>
<paymentAmount>
  <currency>AUD</currency>
  <amount>1666000</amount>
  </paymentAmount>
  <adjustedPaymentDate>2005-12-09</adjustedPaymentDate>
  </paymentAmount>
</equity>
```


Pricing

The Razor pricing adapter for Equity Forwards currently supports the following methods of pricing for a single share:

Future Equity Price Calculation
Future Equity Price Calculation - Forward Equity Price on Trade
PV(Future Equity Price Calculation - Forward Equity Price on Trade)

Please Note that the Forward Equity Price on the Trade is expected to be specified as an average equity price across the basket, which is multiplied by the number of shares to get the total basket value.

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Where Future Equity Price Calculation ($F(t_j)$) at the Forward Settlement Date t_j is defined as:

$$F(t_j) = S(t)e^{b(t_j)(t_j - t)}$$

Where: $S(t)$ = Current Market Price of Equity

$b(t_j) = r_{t_j} - q$

And: r_{t_j} = zero rate at t_j

q = annualized dividend yield

In order to cater for Basket Equity Forwards, the Future Equity Price is now a Future Equity Basket Price and is required to take all equity shares that make up the basket into account, hence the Future Equity Basket Price Calculation becomes:

$$F_b(t_j) = \sum_{i}^{N} w_i F_i(t_j)$$

Where: N = Number of Equities that make up the basket

w_i = weight of Equity i (S_i) in the basket

$F_i(t_j) = S_i(t)e^{b_i(t_j)(t_j - t)}$

And: $S_i(t)$ = Current Market Price of Equity (i)

$b_i(t_j) = r_{t_j} - q_i$

q_i = annualized dividend yield for Equity (i)

XML Schema

The Current Equity Schema (used for Equity Forwards) does not support basket definitions. An additional tag is required in the Equity schema to enable the basket definition as shown below, where subsequent underlying equities and their respective weights toward the basket can be defined.

Please Note: The weights should be normalized and the number of shares specified on the trade is required to be equal to the total number of shares that make up the basket across all underlying equities. Note this is the same implementation as for Equity Basket Asian Options.

fpmlEquity Schema			
Name: Type	Occurs	Size	Description

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fpmlEquity Schema			
Name: Type	Occurs	Size	Description
instrumentId string	1..1		Equity Id or "Basket"
description string	0..1		The long name of the security
basket rzmlBasket	0..1		Basket, grouping of assets and weightings
forwardType string	0..1		The type of the equity forward
settlementDays positiveInteger	0..1		The number of settlement days for this equity
numberShares double	1..1		Number of shares
exchangeCode ExchangeCode	1..1		Stock Exchange Code eg:ASX
currency fpmlCurrency	1..1		Currency
paymentAmount fpmlPayment	0..1		Payment amount

15.2.2 Foreign Equity Forward

A foreign equity forward contract is an agreement to purchase a certain number of shares of an individual equity at an agreed future time, at an agreed price, or probably with agreed cash dividends, which are in a different currency than the equity.

We will consider three types of equity forwards, namely: The quanto equity forwards, the domestic-valued foreign equity forwards (DVFEF) and the basket version of foreign equity forwards.

Quanto
In quanto equity forward, the exchange rate is fixed.

XML Representation

```
<equity>
  <instrumentId>Basket</instrumentId>
  <forwardType>Quanto</forwardType>
<basket>
<basketAsset>
  <id>ANZ</id>
  <weightingFactor>0.3</weightingFactor>
  <guaranteedFXRate>1.2</guaranteedFXRate>
  </basketAsset>
```

```
<basketAsset>
  <id>TEL</id>
  <weightingFactor>0.2</weightingFactor>
  <guaranteedFXRate>1.4</guaranteedFXRate>
  </basketAsset>
<basketAsset>
  <id>BHP</id>
  <weightingFactor>0.5</weightingFactor>
  <guaranteedFXRate>1</guaranteedFXRate>
  </basketAsset>
  </basket>
  <numberShares>100000</numberShares>
  <exchangeCode>ASX</exchangeCode>
  <currency/>
<paymentAmount>
  <paymentType>SETTLEMENT</paymentType>
<paymentAmount>
  <currency>USD</currency>
  <amount>1666000</amount>
  </paymentAmount>
  <adjustedPaymentDate>2005-12-09</adjustedPaymentDate>
  </paymentAmount>
<settlementDays>4</settlementDays>
</equity>
```


Continuous Dividend Yield

Define

t = maturity date of the contract.

ρ = the correlation between the underlying stock and FX rate.

σ_S = the equity price volatility coming from the volatility matrix and is a deal specified quantity.

σ_X = the volatility of FX rate.

X_{GER} = the guaranteed exchange rate.

$Q(t)$ = the continuous dividend yield of foreign equity.

$zcdf_f(0,t)$ = discount factor on time period $(0,t)$ from the foreign zero curve.

$zcdf_d(0,t)$ = discount factor on time period $(0,t)$ from the domestic zero curve (USD).

MSD = the market settlement date.

$zcdf_f(t,t+MSD)$

=forward discount factor on time period $(t, t+MSD)$ from the foreign zero curve which is defined as:

$$zcdf_f(t, t+MSD) = \frac{zcdf_f(0, t+MSD)}{zcdf_f(0, t)}.$$

S = spot price denominated in specified foreign currency.

The forward price denominated in USD is:

$$F_{GER}(t) = X_{GER} S e^{-(Q(t)+\rho\sigma_S\sigma_X)\cdot t}\left(\frac{zcdf_f(t, t+MSD)}{zcdf_f(0, t+MSD)}\right).$$

Discrete Dividends

Define
d_i = represent decalred and projected discrete dividends going ex-dividend
on or before t.
$B(t)$ = the basis function to time t.
ρ = the correlation between the foreign equity and the exchange rate
(domestic/foreign).
σ_X = the volatility of the exchange rate (domestic/foreign).
$FX_{S/D}$ = the spot FX rate used to convert discounted dividends from their
currency to that of the underlier.
$q(t_N, t)$
= forward dividend yield in time period (t_N, t) which is defined as:

$$q(t_{i-1}, t_i) = \frac{(Q(t_i)\cdot t_i) - (Q(t_{i-1})\cdot t_{i-1})}{t_i - t_{i-1}}.$$

$zcdf_D(0, D_i)$
= the discount factors on time period $(0, T_i)$ from the zero curve of the
currency of each discrete cash dividend.
N = index reference for the last discrete dividend which will go ex-dividend
on or before time t.

The forward price denominated in USD is:

$$F_{GER(t)}$$
$$= X_{GER}\cdot$$
$$\left\{\left[S\cdot zcdf_f(t, t+MSD) - \left(FX_{S/D}\sum_{i=1}^{N} zcdf_D(0, PD_i)\cdot d_i\right)\right]\cdot\frac{e^{B(t)\cdot t - \rho\sigma_S\sigma_X\cdot t - q(t_N, t)\cdot(t-t_N)}}{zcdf_f(0, t+MSD)}\right\}.$$

Domestic – Valued Foreign Equity Forward (DVFEF)
In DVFEF, the exchange rate is not fixed.

XML Representation


```
<equity>
  <instrumentId>Basket</instrumentId>
  <forwardType>DVFEF</forwardType>
<basket>
<basketAsset>
  <id>ANZ</id>
  <weightingFactor>0.3</weightingFactor>
  </basketAsset>
<basketAsset>
  <id>TEL</id>
  <weightingFactor>0.2</weightingFactor>
  </basketAsset>
<basketAsset>
  <id>BHP</id>
  <weightingFactor>0.5</weightingFactor>
  </basketAsset>
  </basket>
  <numberShares>100000</numberShares>
  <exchangeCode>ASX</exchangeCode>
  <currency/>
<paymentAmount>
  <paymentType>SETTLEMENT</paymentType>
<paymentAmount>
  <currency>USD</currency>
  <amount>1666000</amount>
  </paymentAmount>
  <adjustedPaymentDate>2005-12-09</adjustedPaymentDate>
  </paymentAmount>
<settlementDays>4</settlementDays>
</equity>
```


Define

X	=	the spot exchange rate (domestic/foreign).
f	=	foreign currency.
d	=	domestic currency.
D	=	dividends.

The DVFEF price is:

$$F_{DV}(t) = XSe^{-Q(t)\cdot t}\left(\frac{zcdf_d(0,t+MSD)}{zcdf_d(0,t)}\right) / zcdf_d(0,t+MSD) = XSe^{-Q(t)\cdot t}\left(\frac{1}{zcdf_d(0,t)}\right)$$

.

If there is a basis function, then it should be applied as in the quanto case. Also, if there are discrete dividend payments, then the DVFEF price is:

$$F_{DV}(t) = X \cdot \left(S \cdot \left(\frac{zcdf_d(0, t+MSD)}{zcdf_d(0,t)} \right) - \left[FX_{S/D} \cdot \sum_{i=1}^{N} zcdf_D(0, PD_i) \cdot d_i \right] \right) \cdot \frac{e^{B(t)\cdot t - q(t_N, t)\cdot(t-t_N)}}{zcdf_d(0, t+MSD)} .$$

Basket Equity Forward

Define

n = number of forward contracts in the basket.

The basket price is simply a linear combination of the individual forward contract in the basket:

$$F_B = \sum_{i=1}^{n} w_i F_i .$$

15.3 Equity Options

15.3.1 Description of Instrument

Razor supports European and American call options. American call options on company stock give the holder of the option the right but not the obligation to purchase the stock at the strike price at any time up until the expiry date of the option.

European call options on company stock give the holder of the option the right but not the obligation to purchase (Call) the stock or to sell the stock (Put) at the predetermined strike price at the date of option maturity.

American call options on company stock give the holder of the option the right but not the obligation to purchase or sell the stock at the strike price at any time up until the expiry date of the option.

15.3.2 XML Representation

fpmlEquityOption Schema			
Name: Type	**Occurs**	**Size**	**Description**
productType fpmlProductType	0..1		Indicates the type of product
optionType fpmlOptionType	1..1		Indicates the type of option
underlying fpmlInstrumentRef	1..1		The underlying equity
Strike	1..1		The option strike price

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fpmlEquityOption Schema			
Name: Type	Occurs	Size	Description
fpmlStrike			
numberOfOptions Int	1..1		The number of options traded
optionEntitlement Double	1..1		The entitlement per option
equityExercise fpmlEquityExercise	1..1		The option details
equityOptionFeatures fpmlEquityOptionFeatures	0..1		The index option features
equityPremium fpmlPremium	1..1		The option premium

15.3.3 Pricing

European Style

Vanilla Call and Put options are priced using the standard Black-Scholes formula

Vanilla Call price is:

$$C = Se^{-qT} N(d_1) - Xe^{-rT} N(d_2),$$

Vanilla Put price is:

$$P = Xe^{-rT} N(-d_2) - Se^{-qT} N(-d_1)$$

with d_1 and d_2 defined as:

$$d_1 = \frac{\ln(\frac{S}{X}) + (r - q + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = \frac{\ln(\frac{S}{X}) + (r - q - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}.$$

and $N(\ )$ - cumulative normal distribution function.

Other variables in formulas represent:

S - stock spot price,

X - option strike price,

r - continuously compounded risk free rate until option maturity

σ - volatility of the relative price change of the underlying stock price,

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T – time to expiration in years.

q - a known dividend yield that underlying stock pays during option life.

Please refer to section 17.1.4 - generalised Black-Scholes option pricing formula for further detail and the Greeks.

If a European option is written on stock that is due to pay dividends during option life that are presented in discrete form (t_i, D_i) then the stock price S in the above formulas should be adjusted:

$$S = S - \sum_i D_i e^{-rt_i}$$

where

$t_i < T$ - is time in years to i'th dividend payout

D_i - the known value of i'th dividend.

American Style

American options on stock can be priced in various ways, using either closed-form approximations or numerical solutions – binomial tree model or finite differences method. All of which are implemented in Razor.

Bjerksund and Stensland

The Bjerksund and Stensland approximation is an efficient model for pricing American stock options in which case an annualised dividend yield is applied as input to the pricing model.

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$$\alpha = (I - X)I^{-\beta}$$

$$\beta = \left(\frac{1}{2} - \frac{b}{\sigma^2}\right) + \sqrt{\left(\frac{b}{\sigma^2} - \frac{1}{2}\right)^2 + 2\frac{r}{\sigma^2}}$$

$$\phi(S,T,\lambda,\gamma,H,I) = e^{\lambda} S^{\gamma}(N(d) - (\frac{I}{S})^{k} N(d - \frac{2\ln(I/S)}{\sigma\sqrt{T}}))$$

$$\lambda = (-r + \gamma b + \frac{1}{2}\gamma(\gamma-1)\sigma^2)T$$

$$d = -\frac{\ln(S/H) + (b + (\gamma - \frac{1}{2})\sigma^2)T}{\sigma\sqrt{T}}$$

$$\kappa = \frac{2b}{\sigma^2} + (2\gamma - 1)$$

$$I = B_0 + (B_\infty - B_0)(1 - e^{h(T)})$$

$$h(T) = -(bT + 2\sigma\sqrt{T})\left(\frac{B_0}{B_\infty - B_0}\right)$$

$$B_\infty = \frac{\beta}{\beta - 1}X$$

$$B_0 = \max(X, (\frac{r}{r-b})X)$$

$$C = \alpha S^{\beta} - \alpha\phi(S,T,\beta,I,I) + \phi(S,T,1,I,I) - \phi(S,T,1,X,I) - X\phi(S,T,0,I,I) + X\phi(S,T,0,X,I)$$

$$P(S,X,T,r,b,\sigma) = C(X,S,T,r-b,-b,\sigma)$$

Black Scholes

The generic Black-Scholes model can be extended to cover options on a cash SPI. The major difference is that we must now incorporate an asset income in the form of dividends. We will assume the underlying index is broad-based and thus the payment of dividends is approximately continuous.

Binomial (Cox-Ross-Rubinstein Binomial Tree)

Razor's Binomial model is based on the Cox-Ross-Rubinstein Binomial Tree. This numerical model is most useful for American options where no closed form solution is possible. The impact of this computational intensive model is a detrimental effect on performance.

For a background to the binomial model see Options, Futures, and Other Derivatives (fifth International edition) by John C. Hull pages 392 to 403, Chapter 12 Paul Wilmott on Quantitative Finance, or alternatively pages 123-140 of Modelling Derivatives in C++ by Justin London.

Non dividend paying stock:

Let

$r =$ the continuously compounded risk free rate

$X =$ the strike of the option

T = time to maturity

$\delta t =$ time interval length

$n =$ number of tree steps

$R =$ the tree's growth factor

$u =$ the tree's up step size

$d =$ the tree's down step size

$pUp =$ probability of an up movement

$i =$the i'th time step

$j =$ the j'th node

$S_{ij} =$ the price at the i'th step and the j'th node

$V_{ij} =$ the price of the option at the i'th step and the j'th node

Then the parameters to construct the tree are:

$$\delta t = \frac{T}{n}$$

$$u = e^{\sigma\sqrt{\delta t}}$$

$$d = 1/u$$

$$R = e^{r\delta t}$$

$$pUp = \frac{R-d}{u-d}$$

$$S_{ij} = S_{00}u^{j}d^{i-j}$$

For a call at the terminal nodes $i = n$:

$$V_{nj} = \max(0, S_{00}u^{j}d^{i-j} - X)$$

For a call at each node where $0 \le i \le (n-1) and\, 0 \le j \le i$:

$$V_{ij} = \max(S_{00}u^{j}d^{i-j} - X, (pUp * V_{i+1\,j+1} + (1-pUp) * V_{i+1\,j})/R)$$

For a put at the terminal nodes $i = n$:

$$V_{nj} = \max(0, X - S_{00}u^{j}d^{i-j})$$

For a put at each node where $0 \le i \le (n-1) and 0 \le j \le i$:

$$V_{ij} = \max(X - S_{00}u^{j}d^{i-j}, (pUp * V_{i+1\,j+1} + (1 - pUp) * V_{i+1\,j}) / R)$$

Dividend yield paying stock:

Let

$r =$ the continuously compounded risk free rate

$X =$ the strike of the option

T = time to maturity

$\delta t =$ time interval length

$n =$ number of tree steps

$R =$ the tree's growth factor

$u =$ the tree's up step size

$d =$ the tree's down step size

$pUp =$ probability of an up movement

$i =$the i'th time step

$j =$ the j'th node

$S_{ij} =$ the price at the i'th step and the j'th node

$V_{ij} =$ the price of the option at the i'th step and the j'th node

$q =$ the dividend yield

Then the parameters to construct the tree are:

$$\delta t = \frac{T}{n}$$

$$u = e^{\sigma \sqrt{\delta t}}$$

$$d = 1/u$$

$$R = e^{r\delta t}$$

$$pUp = \frac{\dfrac{R}{e^{q\sqrt{\delta t}}} - d}{u - d}$$

$$S_{ij} = S_{00}u^{j}d^{i-j}$$

At the terminal nodes $i = n$:

For a call: $V_{nj} = \max(0, S_{00}u^j d^{i-j} - X)$

For a put: $V_{nj} = \max(0, X - S_{00}u^j d^{i-j})$

At intermitted nodes $0 \le i \le (n-1) and\, 0 \le j \le i$:

For a call: $V_{ij} = \max(S_{00}u^j d^{i-j} - X, (pUp * V_{i+1\,j+1} + (1 - pUp) * V_{i+1\,j}) / R)$

For a put:

$V_{ij} = \max(X - S_{00}u^j d^{i-j}, (pUp * V_{i+1\,j+1} + (1 - pUp) * V_{i+1\,j}) / R)$

Discrete Dividend paying stock:

Let

$r =$ the continuously compounded risk free free rate

$X =$ the strike of the option

T = time to maturity

$\delta t =$ time interval length

$n =$ number of tree steps

$R =$ the tree's growth factor

$u =$ the tree's up step size

$d =$ the tree's down step size

$pUp =$ probability of an up movement

$i =$the i'th time step

$j =$ the j'th node

$S_{ij} =$ the price at the i'th step and the j'th node

$V_{ij} =$ the price of the option at the i'th step and the j'th node

$k =$ the k'th dividend

$D_k =$ the dividend amount at time k

$t_k =$ the time to the k'th ex-dividend date

Then the parameters to construct the tree are:

$$\delta t = \frac{T}{n}$$

$$u = e^{\sigma \sqrt{\delta t}}$$

$$d = 1/u$$

$$R = e^{r\delta t}$$

$$pUp = \frac{R-d}{u-d}$$

At the terminal nodes $i = n$:

$$S_{nj} = S_{00}u^j d^{n-j}$$

For a call: $V_{nj} = \max(0, S_{00}u^j d^{n-j} - X)$

For a put: $V_{nj} = \max(0, X - S_{00}u^j d^{n-j})$

At other intermitted nodes $0 \le i \le (n-1) and 0 \le j \le i$ but not where $\delta t * i \le t_k \le \delta t * (i+1) and 0 \le j \le i$:

$$S_{ij} = S_{00}u^j d^{i-j}$$

For a call: $V_{ij} = \max(S_{00}u^j d^{i-j} - X, (pUp * V_{i+1 j+1} + (1-pUp) * V_{i+1 j}) / R)$

For a put:

$$V_{ij} = \max(X - S_{00}u^j d^{i-j}, (pUp * V_{i+1 j+1} + (1-pUp) * V_{i+1 j}) / R)$$

With the discrete dividend model, the binomial tree does not linkup and jumps are required from the last node prior to an ex-dividend date to the ex-dividend date. From the ex-dividend date a new tree is created with the following parameters as before with the exception that now:

$$\delta t = \frac{T - t_k}{n - i}$$

The new tree then continues as per normal until the next ex-dividend date where a new jump and tree needs to be created.

At a jump point however, where $\delta t * i \le t_k \le \delta t * (i+1) and 0 \le j \le i$:

$$S_{t_k j} = S_{ij} - D_k$$

Imposing the no-arbitrage constraint that the option value is either the option value with the dividend, or the payoff:

For a call:

$$V_{t_k j} = \max(S_{00}u^j d^{i-j} - X, (pUp * V_{i+1 j+1} + (1-pUp) * V_{i+1 j}) / R)$$

For a put:

$$V_{t_k j} = \max(X - S_{00}u^j d^{i-j}, (pUp * V_{i+1 j+1} + (1-pUp) * V_{i+1 j}) / R)$$

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And just prior to the jump:

For a call:

$$V_{ij} = \max(S_{00}u^{j}d^{i-j} - X, V_{t_k j} - D_k)$$

For a put:

$$V_{ij} = \max(X - S_{00}u^{j}d^{i-j}, V_{t_k j} - D_k)$$

Since the binomial tree does not recombine, and a new tree has to be started for each ex-dividend date, recursion is used to simplify the implementation. Considering the vast number of extra nodes that this non recombining model generates a significant impact on performance will occur, particularly for frequent dividends on a long dated expiry option.

For further reading on this discrete dividend payment model see pages 129-131 and 146 to 147 of Paul Wilmott on Quantitative Finance. For an implementation visit Financial Principle's Discrete Dividend model by Bernt Arne Odegaard.

15.3.4 Equity Option Greeks

For American equity options, greek sensitivities are calculated numerically using a finite difference scheme. European option sensitivities are provided as analytical solutions to the Black-Scholes model (refer to section 17.2.4 for details).

Delta

Delta is defined as the difference in value of the option by perturbing the underlying equity price S by $\pm 1bp$ where all other parameters remain constant.

Let

f	=	a function defined as the price of the option
S	=	Underlying equity spot price
$1bp$	=	1 basis point or 0.0001

$$\Delta = \frac{\partial P}{\partial S} \approx \frac{f(S+1bp) - f(S-1bp)}{2bp}$$

Gamma

Gamma is defined as the difference in delta by perturbing the underlying equity spot price S by $\pm 1bp$ where all other parameters remain constant. In Razor we approximate gamma by using the second order finite central difference.

Let

f	=	a function defined as the price of the option
S	=	Underlying equity spot price
$1bp$	=	1 basis point or 0.0001

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 P}{\partial S^2} \approx \frac{f(S+1bp)+f(S-1bp)-2f(S)}{1bp^2}$$

Vega

Vega is defined as the difference in value of the option by perturbing the equity volatility σ by $\pm 1\%$ where all other parameters remain constant.

Let

f	=	a function defined as the price of the option
σ	=	Implied volatility of the underlying equity
1%	=	1 percent as a decimal or 0.01

$$\Lambda = \frac{\partial P}{\partial \sigma} \approx \frac{f(\sigma+1\%)-f(\sigma-1\%)}{2\%}$$

Theta

Theta is defined as the difference in value by shifting the value date forward by one day where all other parameters remain constant. Since time moves in a forward direction, for theta we use the backward finite difference which is defined as:

$$\left.\frac{\partial}{\partial x} \approx \frac{f(x)-f(x-h)}{h}\right\} h > 0$$

In Razor, pricing formulas accept the risk free rate in terms of a continuously compounded discount factor. Therefore, an adjustment to the discount factor is made to account for the decay of 1 day.

Let

f	=	a function defined as the price of the option
T	=	Time to expiration in years.
T_{-1}	=	Time to expiration minus 1 day in years.
df	=	a continuously compounded discount factor of r in terms of T
$df_{T_{-1}}$	=	a continuously compounded discount factor of r in terms of T_{-1}
$1dy$	=	1 day in years or 1/365

Where

$$df_{T_{-1}} = df^{\frac{T_{-1}}{T}}$$

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$$\Theta = -\frac{\partial P}{\partial T} \approx \frac{f(T_{-1}, df_{T_{-1}}) - f(T)}{1dy}$$

15.4 Equity Index Options

15.4.1 Description of Instrument

The Equity Index Option is a variant of the Equity Option. The differences between equity and index options occur primarily in the underlying instrument and the method of settlement. Equity Index options are predominately traded via exchanges hence the other common term used are equity ETO's, Exchange Traded Options.

15.4.2 XML Representation

The index option is represented similarly to the equity option. It uses the fpmlEquityOption schema with the optional equityOptionFeatures element made mandatory as below:

```
<equityOptionsFeatures>
  <etoFeatures>
        <indexOptionTickUnit>0.5</indexOptionTickUnit>
  </etoFeatures>
</equityOptionsFeatures>
```

The other slight differences occur in the numberOfOptions and the optionEntitlement elements. In an index option sense the numberOfOptions implies the number of contracts, while the optionEntitlement element implies the Tick Value.

15.4.3 Pricing

The index Option is priced exactly the same as the equity option above. The only factor that needs to considered is in the amount or position of the trade where the Tick Value needs to be factored in.

$$EquityIndexOptionPosition = EquityOptionPrice * numberOfOptions * optionEntitlement * TickUnit$$

15.5 Equity Basket Option

15.5.1 Contract Definition

An equity basket option is an option contract where the underlying asset is a basket of individual equities. There are two valuation methods for equity basket options.

15.5.2 Valuation 1

The basket option price in Razor is approximated using single-equity Black-Scholes formula with adjustments to the input variables. The spot rate, cost of carry and volatility in the single-equity Black-Scholes formula are replaced by basket spot rate, basket cost of carry and basket volatility.

Define:

t = the valuation date.
T = the option expiration date.
S_t^i = the price of asset i in the basket at time t.
$b_{t,T}^i$ = the cost of carry of asset i in the basket for time T implied at time t.
$\sigma_{t,T}^i$ = volatility of asset i in the basket for time T implied at time t.
K = the strike price.
S_t^b = the basket price at time t.
$b_{t,T}^b$ = the basket cost of carry for time T implied at time t.
$\sigma_{t,T}^b$ = the basket volatility for time T implied at time t.
$r_{t,T}$ = the risk-free rate for time T implied at time t.
w_i = weighting of asset i in the basket.
ρ_{ij} = correlation coefficient between asset i and asset j.
n = number of assets in the basket.

The basket variables are approximated as:

$$S_t^b = \sum_{i=1}^{n} w_i S_t^i .$$

$$b_{t,T}^b = \sum_{i=1}^{n} w_i b_{t,T}^i .$$

$$\sigma_{t,T}^b = \sqrt{\sum_{i=1}^{n} w_i^2 \left(\sigma_{t,T}^i\right)^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{i-1} w_i w_j \rho_{ij} \sigma_{t,T}^i \sigma_{t,T}^j} .$$

Note that $b_{t,T}^i$, $r_{t,T}$ and $\sigma_{t,T}^i$ can be obtained from the term-structure curve for each individual asset i.

Pricing Formulas:

Call option price is given as:

$$c = S_t^b e^{\left(b_{t,T}^b - r_{t,T}\right)T} N\left(d_1\right) - K e^{-r_{t,T}T} N\left(d_2\right).$$

Put option price is given as:

$$p = K e^{-r_{t,T}T} N\left(-d_2\right) - S_t^b e^{\left(b_{t,T}^b - r_{t,T}\right)T} N\left(-d_1\right).$$

$$d_1 = \frac{\ln\left(\frac{S_t^b}{K}\right) + \left(b_{t,T}^b + \frac{\left(\sigma_{t,T}^b\right)^2}{2}\right)T}{\sigma\sqrt{T}}.$$

$$d_2 = d_1 - \sigma_{t,T}^b\sqrt{T}.$$

This valuation method only supports single currency. All assets must be denominated in the same currency as the strike price. It also supports continuous dividend yield but not discrete dividend payout.

The Case of Single Asset
If we only have one asset in the basket, Razor calculates the price using the single asset option pricing formula instead of the basket formula.

15.5.3 Valuation 2

Razor also supports another method of calculating equity basket option. We can use the Asian quanto basket formula to calculate the equity basket option price. This valuation method has the advantage that it supports cross currencies for the asset basket. We first recall the formula for Asian quanto basket.

Define:

t = the valuation date.
T = the option expiration date.
n_a = number of underlying assets.
n_f = number of fixings.
m = number of observed fixings.
C = payoff currency.
$S_i(t)$ = i by 1 array of asset prices i at time t.
C_i = currency i for asset i.
$\sigma_{S_i}(t_j)$ = i by j matrix of volatilities of asset i at time t_j.
X_i
= i by 1 array of exchange rate quoted as payoff currency per currency i ,i.e. C/C_i.
$\sigma_{X_i}(t_j)$ = i by j matrix of volatilities X_i at time t_j.

ρ_{ij} = i by i matrix of instantaneous correlations of $\ln S_i$ and $\ln S_j$.
ρ_i = i by 1 array of instantaneous correlations of S_i and X_i.
$r_{C_i}(t_j)$ =
i by j matrix of instantaneous risk-free interest rates in C_i market.
$b_i(t_j)$ = i by j matrix of cost of carries for asset i at time t_j.

K	=	strike price is payoff currency.
W	=	Wiener process.

We further define:

$$\bar{S}(t_n) = \sum_{i=1}^{n_a} \sum_{j=1}^{n_f} w_{ij} S_i(t_j).$$

Option Payoff:

Call: $\max(\bar{S}(t_{n_f}) - K, 0)$

Put: $\max(K - \bar{S}(t_{n_f}), 0)$.

Pricing Formulas:

We have assets denominated in different currencies. We need to find a measure say C, such that all assets under this measure are risk-neutral. It can be shown (Datey, Gauthier and Simonato) that in such risk-neutral measure, the asset prices are distributed as:

$$dS_i(t) = (b_i(t) - \rho_i \cdot \sigma_{S_i}(t) \cdot \sigma_{X_i}(t)) \cdot S_i(t) \cdot dt + \sigma_{S_i}(t) \cdot S_i(t) \cdot dW_i^C(t), \quad \forall i = 1,2,\dots,n_a$$

.

Regardless all the technical terms, the important thing is that the underlying asset price still has the form of geometric Brownian motion with different drift and diffusion term. It means that we can use similar approaches in Asian non-quanto options for Asian quanto options.

$$\bar{S}(t_n) = \sum_{i=1}^{n_a} \sum_{j=1}^{n} w_{ij} S_i(t_j) = \bar{S}(t_m) + \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} w_{ij} S_i(t_j) = \bar{S}(t_m) + M_t$$

where

$$M_t = \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} w_{ij} S_i(t_j).$$

We denote $b_i^*(t_j) = b_i(t_j) - \rho_i \cdot \sigma_{S_i}(t_j) \cdot \sigma_{X_i}(t_j)$

and define:

$$F_i(t_j) = S_i(t) e^{b_i^*(t_j)(t_j - t)}$$

$$E^*(M_t) = \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} w_{ij} F_i(t_j).$$

$$E^*(M_t^2) = \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} \sum_{k=1}^{n_a} \sum_{l=m+1}^{n_f} w_{ij} w_{kl} F_i(t_j) F_k(t_l) \times$$

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$\exp\left(\rho_{ik}\sigma_{S_i}\left(\min\left(t_j,t_l\right)\right)\sigma_{S_k}\left(\min\left(t_j,t_l\right)\right)\min\left(t_j-t,t_l-t\right)\right)$.

We can then use the fixed strike formulas with all $K-\sum_{i=1}^{m}\alpha_i S_{t_i}$ replaced by $K-\bar{S}(t_m)$. Exercise for certain and out-of-money for certain cases are the same by replacing $K-\sum_{i=1}^{m}\alpha_i S_{t_i}$ with $K-\bar{S}(t_m)$.

To find the price for equity basket option, we simply assume there is only one fixing date, i.e. $n_f=1$, on the option expiration date and the correlation between the underlying asset and its denominated currency to be zero, i.e. $\rho_i=0$.

15.5.4 Equity Basket Option with Quanto Feature

If one or more assets in the basket are denominated in any currency different from the paying currency, then we cannot use the method proposed in Valuation 1. We need to use method proposed in Valuation 2 by taking the correlation between the underlying asset and its denominated currency into account. It can be achieved using the above Asian quanto basket formula by assuming there is only one fixing date, i.e. $n_f=1$, on the option expiration date and input the required correlation coefficients.

This valuation method supports continuous dividend yield but not discrete dividend payout.

15.5.5 Summary

In summary, we have two different valuation methods to calculate the equity basket price for non-quanto case and one method (the Asian quanto basket option formula) for the quanto case. Currently, both valuation methods support for continuous dividend yield but not discrete dividend payout.

15.6 Equity Futures

15.6.1 Description of Instrument

A Futures are legally binding contracts to buy or sell a particular asset (or cash equivalent) on a specified future date. Equity futures are futures traded in the equity market on organized exchanges.

Equity Futures can consist of contracts of an individual equity, a basket of equities or an equity index.

15.6.2 XML Representation

An example of the XML product section of an equity index future is as follows:

```xml
<equityFuture>
  <instrumentId>XJO7U</instrumentId>
  <currency>AUD</currency>
  <numberShares>1</numberShares>
  <exchangeCode>ASX</exchangeCode>
  <numberContracts>2</numberContracts>
  <futurePrice>58050.00</futurePrice>
  <indexTickUnit>10</indexTickUnit>
  <settlementDate id="">
       <unadjustedDate>2007-09-20</unadjustedDate>
</settlementDate>
</equityFuture>
```

15.6.3 Pricing

The individual equity, basket, or equity index are all similarly priced. It's in the input values in the 3 above fields, numberShares, numberContracts and indexTickUnit where their values vary from the default of 1.0.

Equity Future:

Let

V_{ccy} = value of equity future in currency *ccy*

F_{ccy} = the projected forward price derived from the underlying stock/index forward curve in currency ccy.

X = equity future contract price.

df = discount factor to the payment date of contract

N = number of shares

Then:

$$V_{ccy} = df * N * (F_{ccy} - X)$$

Equity Index Future:

Let

V_{ccy} = value of equity future in currency *ccy*

F_{ccy} = the projected forward price derived from the underlying stock/index forward curve in currency ccy.

X = equity future contract price.

df = discount factor to the payment date of contract

C = number of contracts

k = index tick unit

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Then:

$$V_{ccy} = df * C * (F_{ccy} - X) / k$$

15.7 Asian Options: Introduction

Asian Options encompass various types of average options including Average Rate (Fixed Strike) and Average Strike (Floating Strike) Options. Asian options are options where the payoff depends on the average price of the underlying asset during at least some part of the life of the option.

This document specifies the various pricing models used for valuation of a subset of Asian options. The first section will focus on Asian options with geometric average features. The second section will concentrate on Asian options with continuous arithmetic average features. The third section will also be on Asian options with discrete arithmetic average features but with equal fixing length and weights and the final section will be on Asian options with discrete arithmetic average features with unequal fixing length, weights and time-dependent underlying parameters.

Define:

$S_t =$ the current underlying asset price at time t.

$\bar{S} =$ the average price of the underlying asset during some part of the life of the
option.

$K =$ the strike price of the option.

$T =$ the original maturity time of the option.

$\tau =$ $T - t$ is the remaining time to maturity.

$r =$ the risk-free rate.

$\sigma =$ the volatility of the underlying asset.

$b =$ the cost of carry

$N(\cdot) =$ the cumulative normal distribution function.

Unless otherwise stated, the above definitions will be assumed for all the formulae.

Example: The four fundamental types of payoff for Asian Options:

Type	Payoff	Name
(1)	$\max(\bar{S} - K, 0)$	Fixed Strike Call
(2)	$\max(K - \bar{S}, 0)$	Fixed Strike Put
(3)	$\max(S_T - \bar{S}, 0)$	Floating Strike Call
(4)	$\max(\bar{S} - S_T, 0)$	Floating Strike Put

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15.7.1 Types of Averaging and Fixing

We can see from the payoff table that the payoff of Asian options depends on $\bar{S}$, the average price of the underlying asset S . Two approaches to taking the average of the asset prices are the geometric average and the arithmetic average.

Geometric Average

1. Assume we have n asset prices $S_1, S_2, ..., S_n$.
The discrete geometric average of these n asset prices is defined as:

$$\bar{S} = (S_1 S_2 ... S_n)^{\frac{1}{n}}.$$

2. The continuous geometric average over the time period, say T , is defined as:

$$\bar{S} = \exp\left(\frac{1}{T}\int_0^T \log S_u du\right).$$

Arithmetic Average

1. Again, assume we have n asset prices $S_1, S_2, ..., S_n$.
The discrete arithmetic average of these n asset prices is defined as:

$$\bar{S} = \frac{S_1 + S_2 + ... + S_n}{n}.$$

2. The continuous arithmetic average over the time period, say T , is defined as:

$$\bar{S} = \frac{1}{T}\int_0^T S_u du.$$

Types of Fixing

Fixings are stock prices used in averaging. In discrete arithmetic average, fixings can be assumed to have equal weights, i.e. $\bar{S} = \frac{S_1 + S_2 + ... + S_n}{n}$ or unequal weight, i.e. $\bar{S} = \sum_{i=1}^{n} w_i S_i$ where w_i is the normalised weight for the i^{th} fixing. Fixings can also be equally spaced (e.g monthly) or they can be unequally spaced in an average period.

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15.7.2 Geometric Average Asian Option Pricing Models

Fixed Strike Continuous Geometric Asian Option

Define:

$S_t =$ the current underlying asset price at time t.

$\bar{S} =$ the continuous geometric average price of the underlying asset during some part of the life of the option.

$K =$ the strike price of the option.

$T =$ the original maturity time of the option.

$\tau =$ $T-t$ is the remaining time to maturity.

$r =$ the risk-free rate.

$\sigma =$ the volatility of the underlying asset.

$b =$ the cost of carry.

$N(\cdot) =$ the cumulative normal distribution function.

Option Payoff:

Call: $\max(\bar{S}-K,0)$

Put: $\max\left(K-\bar{S},0\right)$

Pricing Model:

Case1: Valuation date on or before average start date

Kemma Vorst (1990) derived the following formulae for pricing fixed strike Asian options:

$$c_{fix}(t)=S_0 e^{(b_A-r)\tau}N(d_1)-Ke^{-r\tau}N(d_2)$$

$$p_{fix}(t)=Ke^{-r\tau}N(-d_2)-S_0 e^{(b_A-r)\tau}N(-d_1)$$

where

$$d_1=\frac{\ln\left(\frac{S_0}{K}\right)+\left(b_A+\frac{1}{2}\sigma_A^2\right)\tau}{\sigma_A\sqrt{\tau}}$$

$$d_2=d_1-\sigma_A\sqrt{\tau}$$

$$\sigma_A=\frac{\sigma}{\sqrt{3}}$$

$$b_A=\frac{1}{2}\left(b-\frac{\sigma^2}{6}\right).$$

Case 2: Valuation date in the averaging

We further define:

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$G_t =$ the time t observed geometric mean of the underlying asset.

Pricing Model:

$$c_{fix}(t) = S_q N(d_1) - Ke^{-r\tau} N(d_2)$$

$$p_{fix}(t) = Ke^{-r\tau} N(-d_2) - S_q N(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S_q}{K}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)\tau}{\sigma_A\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma_A\sqrt{\tau}$$

$$S_q = S_t\left(\frac{G_t}{S_t}\right)^{\frac{t}{T}} e^{-\bar{q}(t,T)\tau}$$

$$\sigma_A = \frac{\sigma}{\sqrt{3}} \cdot \frac{\tau}{T}$$

$$\bar{q}(t,T) = r - \frac{1}{2}\left(b - \frac{1}{2}\sigma^2\right) \cdot \frac{\tau}{T} - \frac{1}{6}\sigma^2\frac{\tau^2}{T^2}.$$

Reference: Generalisation of Peter Buchen's method.

Fixed Strike Discrete Geometric Asian Option

Define:

$S_t =$ the current underlying asset price at time t.

$\bar{S} =$ the discrete geometric average price of the underlying asset during some part of the life of the option.

$K =$ the strike price of the option.

$T =$ the original maturity time of the option.

$\tau =$ $T - t$ is the remaining time to maturity.

$r =$ the risk-free rate.

$\sigma =$ the volatility of the underlying asset.

$b =$ the cost of carry.

$t_i =$ time to the i^{th} fixing.

$\sigma_i =$ the implied global volatility for an option expiring at time t_i.

$v_i =$ the local volatility between each time fixing.

$n =$ total number of fixings.

$N(\cdot) =$ the cumulative normal distribution function.

Option Payoff:

Call: $\max(\bar{S} - K, 0)$

Put: $\max(K-\bar{S},0)$

Pricing Model:

$$c_{fix}(t)=S_0e^{(b_G-r)\tau}N(d_1)-Ke^{-r\tau}N(d_2)$$
$$p_{fix}(0)=Ke^{-r\tau}N(-d_2)-S_0e^{(b_G-r)\tau}N(-d_1)$$

where

$$d_1=\frac{\ln\left(\frac{S_0}{K}\right)+\left(b_G+\frac{\sigma_G^2}{2}\right)T}{\sigma_G\sqrt{T}}$$
$$d_2=d_1-\sigma_G\sqrt{T}$$
$$b_G=\frac{\sigma_G^2}{2}+\frac{1}{nT}\sum_{i=1}^{n}\left(b-\frac{\sigma_i^2}{2}\right)t_i$$

From Levy (1997), the σ_G is calculated using the Levy Approach which uses global volatility as an input.

$$\sigma_G^2=\frac{1}{n^2T}\left[\sum_{i=1}^{n}\sigma_i^2t_i+2\sum_{i=1}^{n-1}(n-i)\sigma_i^2t_i\right]$$

15.7.3 Arithmetic Average Asian Option Pricing Models (Continuous Averaging)

Fixed Strike Continuous Arithmetic Asian Option

Define:

$S_t=$ the current underlying asset price at time t.

$\bar{S}=$ the continuous arithmetic average price of the underlying asset during some part of the life of the option.

$K=$ the strike price of the option.

$T=$ the original maturity time of the option.

$\tau=$ $T-t$ is the remaining time to maturity.

$r=$ the risk-free rate.

$\sigma=$ the volatility of the underlying asset.

$b=$ the cost of carry.

$N(\cdot)=$ the cumulative normal distribution function.

Option Payoff:

Call: $\max(\bar{S}-K,0)$

Put: $\max(K-\bar{S},0)$

Pricing Models:

Turnbull and Wakeman Approximation

Define:

$T_A =$ the length of the total average period.

$t_1 =$ time to the beginning of the average period from current time t.

$\tau_A =$ $T_A - \tau$.

Pricing Formulas:

Case1 : Valuation date on or before average start date

$$c_{fix}(t) \approx S_t e^{(b_A - r)\tau} N(d_1) - K e^{-r\tau} N(d_2)$$
$$p_{fix}(t) \approx K e^{-r\tau} N(d_2) - S_t e^{(b_A - r)\tau} N(d_1)$$

$$d_1 = \frac{\ln(S_t / K) + (b_A + \frac{\sigma_A^2}{2})\tau}{\sigma_A \sqrt{\tau}}$$
$$d_2 = d_1 - \sigma_A \sqrt{\tau}$$

$$\sigma_A = \sqrt{\frac{\ln(M_2)}{\tau} - 2b_A}$$
$$b_A = \frac{\ln(M_1)}{\tau}$$

The exact first and second moments of the arithmetic average are:

$$M_1 = \frac{e^{b\tau} - e^{bt_1}}{b(\tau - t_1)}$$
$$M_2 = \frac{2e^{(2b+\sigma^2)\tau}}{(b+\sigma^2)(2b+\sigma^2)(\tau - t_1)^2} + \frac{2e^{(2b+\sigma^2)t_1}}{b(\tau - t_1)^2}\left(\frac{1}{2b+\sigma^2} - \frac{e^{b(\tau - t_1)}}{b+\sigma^2}\right)$$

When we have zero cost of carry (i.e. $b = 0$):

$$M_1 = 1$$
$$M_2 = \frac{2e^{\sigma^2\tau} - 2e^{\sigma^2 t_1}\left[1 + \sigma^2(\tau - t_1)\right]}{\sigma^4(\tau - t_1)^2}.$$

Case 2: Valuation date in averaging

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It is clear from the above formula that it does not handle the situation when we need to calculate option price at times when the average period already began. However, this problem can be solved by a slight modification of the above formula.

We denote S_A to be the average asset price realised or observed. To adjust the formula for the in progress case (i.e. $\tau_A > 0$), we simply replace the strike price K by $\hat{K}$ and the option value must be multiplied by $\frac{T_A}{\tau}$, where

$$\hat{K} = \frac{T_A}{\tau} K - \frac{\tau_A}{\tau} S_A.$$

Also we set t_1 to be zero for all cases when $\tau_A > 0$.

Case 3: Exercise for certain

If we are inside the average period (i.e. $\tau_A > 0$), and $\frac{T_A}{\tau} K - \frac{\tau_A}{\tau} S_A < 0$, then a call option is for certain to be in-the-money and the put option is for certain to be out-of-the-money at the maturity. This leads to the following results:

$$c_{fix}(t) = e^{-r\tau}\left(E\left[S_A^*\right] - K\right)$$

$$p_{fix}(t) = 0$$

where

$$E\left[S_A^*\right] = S_A \frac{T_A - \tau}{T_A} + SM_1 \frac{\tau}{T_A}$$

t_1 is zero for this case because $\tau_A > 0$.

Levy Approximation

Define:

$S_A =$ the arithmetic average of the known asset price fixings.

Pricing Formulas:

$$c_{fix}(t) \approx S_E N(d_1) - K^* e^{-r\tau} N(d_2)$$

$$p_{fix}(t) \approx c_{fix} - S_E + K^* e^{-r\tau}$$

$$d_1 = \frac{1}{\sqrt{V}}\left[\frac{\ln(D)}{2} - \ln(K^*)\right]$$

$$d_2 = d_1 - \sqrt{V}$$

$$S_E = \frac{S_t}{T*b}\left(e^{(b-r)\tau} - e^{-r\tau}\right)$$

$$K^* = K - \frac{T-\tau}{T} S_A$$

$$V = \ln(D) - 2\left[r\tau + \ln(S_E)\right]$$

$$D = \frac{M}{T^2}$$

$$M = \frac{2S_t^2}{b+\sigma^2}\left[\frac{e^{(2b+\sigma^2)\tau} - 1}{2b+\sigma^2} - \frac{e^{b\tau} - 1}{b}\right]$$

Floating Strike Continuous Arithmetic Asian Option

Define:

$S_t =$ the current underlying asset price at time t.

$\overline{S} =$ the continuous arithmetic average price of the underlying asset during some part of the life of the option.

$K =$ the strike price of the option.

$T =$ the original maturity time of the option.

$\tau =$ $T-t$ is the remaining time to maturity.

$r =$ the risk-free rate.

$\sigma =$ the volatility of the underlying asset.

$b =$ the cost of carry.

$N(\cdot) =$ the cumulative normal distribution function.

Option Payoff:

Call: $\max(S - \overline{S}, 0)$

Put: $\max(\overline{S} - S, 0)$

Pricing Models:

Henderson-Wojakowski Identity

Define:

$\lambda =$ some constants $\in \Re$.

Different Asian option prices can be computed as:

$$c_{float}\left(S_t, \lambda, r, b, \sigma, t, T\right) = c_{float} = e^{-r(T-t)} E^*\left(\lambda S_T - \overline{S}\right)^+$$

$$p_{float}\left(S_t, \lambda, r, b, \sigma, t, T\right) = p_{float} = e^{-r(T-t)} E^*\left(\overline{S} - \lambda S_T\right)^+$$

$$c_{fix}(S_t, K, r, b, \sigma, t, T) = c_{fix} = e^{-r(T-t)}E^*\left(\overline{S} - K\right)^+$$
$$p_{fix}(K, S_t, r, b, \sigma, t, T) = p_{fix} = e^{-r(T-t)}E^*\left(K - \overline{S}\right)^+$$

Note that E^* is the expectation taken under risk-neutral measure. c_{fix} and p_{fix} are the prices of fixed strike Asian call and put options respectively. When $\lambda = 1$, c_{float} and p_{float} are the prices of floating strike Asian call and put options respectively.

Pricing Formula:

Henderson and Wojakowski proved the following identity:

$$c_{float}(S_t, \lambda, r, b, \sigma, t, T) = p_{fix}(\lambda S_t, S_t, r-b, -b, \sigma, t, T).$$

$$p_{float}\left(S_t, \frac{K}{S_t}, r, b, \sigma, t, T\right) = c_{fix}(K, S_t, r+b, -b, \sigma, t, T).$$

We assume $\lambda = 1$, the above identities can be summarised in words as:
The first identity simply says that the price of a floating strike call option with current stock price S_t at time t, risk-free rate r, cost of carry b, volatility σ that matures at time T is equal to the price of an at-the-money fixed strike put option with the risk free rate replaced by $r-b$, cost of carry replaced by $-b$ and rest of variables holding constant.
The second identity says that an at-the-money floating strike put option with current stock price S_t at time t, risk-free rate r, cost of carry b, volatility σ that matures at time T is equal to a fixed strike call option with the risk-free rate replaced by $r+b$, cost of carry replaced by $-b$ and rest of the variables holding constant.

With the above two identities, we can compute the floating strike options once we know the fixed strike options.

The above identities can be extended to forward-starting options at some time $t > 0$ for the option prices at the times before the averaging begins. However, it does not work for in-progress case. It also only works for continuous arithmetic averages.

Bouaziz, Briys and Crouhy Model

Define:

$T_A =$ the total average time period.

$\tau_A = T_A - \tau$.

Pricing Formulas:

Case1: Valuation date on or before average start date

$$c_{float}(t) \approx S_t \exp(-rT_A)\left[\hat{r}\cdot\frac{T_A}{2}\cdot N\left(\frac{\sqrt{3}\hat{r}\sqrt{T_A}}{2\sigma}\right)+\sqrt{\frac{\sigma^2 T_A}{6\pi}}\cdot e^{\frac{-3\hat{r}^2 T_A}{8\sigma^2}}\right].$$

$$p_{float} \approx \left[\frac{\left(1-e^{-rT_A}\right)}{rT_A}-1\right]S_t + c_{float}$$

where

$$\hat{r} = r - \frac{1}{2}\sigma^2 .$$

Set $T_A = T$, then we obtain the case for valuation date on the beginning of average period.

Case 2: Valuation date in averaging

Define:

$$M_t = \frac{1}{T_A}\int_{T-T_A}^{t} S_u du$$ and note that M_t is known at time t.

$$c_{float}(t)$$

$$\approx S_t \exp(-r\tau)\left[mN\left(\frac{m}{\sqrt{v}}\right)+\sqrt{\frac{v}{2\pi}}\exp\left(\frac{-m^2}{2v}\right)\right]$$

where

$$m = 1 - \frac{\tau}{T_A} + \hat{r}\tau - \frac{\hat{r}\tau^2}{2T_A} - M_t .$$

$$v = \left[\tau + \frac{\tau^3}{3T_A^2} - \frac{\tau^2}{T_A}\right]\sigma^2 .$$

Set $T = T_A$ leads to the case when averaging was taken from the initiation of the option contract.

By put-call parity:

$$p_{float}(t) \approx M_t e^{-r\tau} + S_t\left[\frac{1-e^{-r\tau}}{rT_A}-1\right]+c_{float}$$

Continuous Reciprocal Asian Option

Define:

$S_t =$ the current underlying asset price at time t.

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$\bar{S} =$ the continuous arithmetic average price of the underlying asset during some part of the life of the option.
$K =$ the strike price of the option.
$T =$ the original maturity time of the option.
$\tau =$ $T-t$ is the remaining time to maturity.
$r =$ the risk-free rate.
$\sigma =$ the volatility of the underlying asset.
$b =$ the cost of carry.
$N(\cdot) =$ the cumulative normal distribution function.

Option Payoff:

Call: $$\max\left[\frac{1}{\frac{1}{T}\int_0^T S_u du} - \frac{1}{S_T}, 0\right]$$

Put: $$\max\left[\frac{1}{S_T} - \frac{1}{\frac{1}{T}\int_0^T S_u du}, 0\right]$$

Pricing Model:

Dufresne (2000) proposed a model to evaluate reciprocal Asian options with continuous arithmetic average using the Laguerre series. Dufresne model works for the case when average period and the option maturity coincides exactly, i.e. the payoff for the call and put options are respectively:
and
.

The generalised Laguerre polynomials are defined as:

$$L_n^a(x) = \frac{x^{-a}}{n!} e^x \frac{d}{dx^n}\left(x^{n+a} e^{-x}\right) = \sum_{k=0}^{n} \frac{\Gamma(n+a+1)}{\Gamma(k+a+1)} \frac{(-x)^k}{k!(n-k)!}.$$

We further define:
$\Gamma(\cdot)$ to be the gamma function.

$$v = b - \frac{1}{2}\sigma^2.$$

$$A_t^{(\mu)} = \int_0^t e^{2\mu s + 2W_s} ds, \quad t \geq 0, \quad \mu \in \Re.$$

Dufresne proved the following expectation:

$$E\left(2A_t^{(\mu)}\right)^{-k} = \int_0^\infty \phi_\mu(k,t,y)\psi_\mu(-1,y)\,dy \quad , \quad k = 1,2,\ldots$$

$$\phi_\mu(1,t,y) = \frac{ye^{-\frac{\mu^2 t}{2}-\frac{y^2}{2t}}}{\sqrt{2\pi t^3}} .$$

$$\phi_\mu(k,t,y) = \frac{1}{2(1-k)}\frac{\partial}{\partial t}\phi_\mu(k-1,t,y) + (k-\mu-1)\phi(k-1,t,y) \quad , \quad k = 2,3,\ldots$$

$$\psi_\mu(-1,y) = \frac{\cosh[(\mu-1)y]}{\sinh(y)} .$$

The no-arbitrage price for the fixed strike reciprocal Asian call option is equal to:

$$e^{-rT}\frac{\sigma^2 T}{2S_0}C^R(\mu,T,x) .$$

where

$$C^R(\mu,t,x) = c^{a+1}x^b e^{-cx}\sum_{n=0}^{\infty} a_n(t)L_n^a(cx) \quad , \quad 0 < x < \infty ,$$

$$a_n(t) = \sum_{k=0}^{n}\frac{n!(-c)^k}{\Gamma(k+a+1)k!(n-k)!}\frac{E\left(2A_t^{(\mu)}\right)^{-(a-b+k+2)}}{(a-b+k+1)(a-b+k+2)} \quad , \quad n = 0,1,\ldots$$

$$t = \frac{\sigma^2 T}{4} .$$

$$\mu = \frac{2v}{\sigma^2} .$$

$$x = \frac{2S_0}{\sigma^2 KT} .$$

a , b and c are constants and we can set to zero for computational convenience.

$$E\left(2A_t^{(\mu)}\right)^{-k} = \int_0^\infty \phi_\mu(k,t,y)\psi_\mu(-1,y)\,dy .$$

can be evaluated using numerical integration, for example, Gauss-Legendre Quadrature.

$\phi_\mu(k,t,y) = \frac{1}{2(1-k)}\frac{\partial}{\partial t}\phi_\mu(k-1,t,y) + (k-\mu-1)\phi(k-1,t,y)$ can be calculated using symbolic packages such as Mathematica or it can be done manually. Usually we require only 10 terms by truncating at the point $k = 10$. Thus we only need $\phi_\mu(1,t,y), \phi_\mu(2,t,y) \ldots, \phi_\mu(10,t,y)$.

$\Gamma(z) = z!$ if z is an integer ≥ 0.

Generally, if z is a complex number with positive real part, then

$\Gamma(z)=\int_0^\infty t^{z-1}e^{-z}dt$. This covers the non-integer cases of z and this integral can be shown to converge definitely. However, we should choose a and b in a way such that non-integer value of z is not present for easier computation.

The no-arbitrage price for the fixed strike reciprocal Asian put option is equal to:

$$e^{-rT}\left(\frac{1}{K}-E\left[\frac{1}{T}\int_0^T S_0 e^{vt+\sigma W_t}dt\right]^{-1}\right)$$

where

$$E\left[\frac{1}{T}\int_0^T S_0 e^{vt+\sigma W_t}dt\right]^{-1}$$

$$=\frac{\sigma^2 T}{2S_0}E\left(\frac{1}{2A_t^{(\mu)}}\right)$$

$$=\frac{\sigma^2 T}{2S_0}\frac{e^{\frac{-\mu^2 t}{2}}}{\sqrt{2\pi t^3}}\int_0^\infty y e^{\frac{-y^2}{2t}}\frac{\cosh[(\mu-1)y]}{\sinh(y)}dy.$$

and again

$$v=b-\frac{1}{2}\sigma^2.$$

$$t=\frac{\sigma^2 T}{4}.$$

$$\mu=\frac{2v}{\sigma^2}.$$

$$x=\frac{2S_0}{\sigma^2 KT}.$$

a, b and c are constants and we can set to zero for computational convenience.

15.7.4 Arithmetic Average Asian Option Pricing Models (Discrete Averaging, Equal Fixing Weighting and Constant Underlying Variables)

Fixed Strike Discrete Arithmetic Asian Option

Define:

$S_t =$ the current underlying asset price at time t.

$\bar{S} =$ the discrete arithmetic average price of the underlying asset during some part of the life of the option.

$K =$ the strike price of the option.

$T =$ the original maturity time of the option.

$\tau =$ $T - t$ is the remaining time to maturity.

$r =$ the risk-free rate.

$\sigma =$ the volatility of the underlying asset.

$b =$ the cost of carry.

$N(\cdot) =$ the cumulative normal distribution function.

Option Payoff:

Call: $\max(\bar{S} - K, 0)$

Put: $\max\left(K - \bar{S}, 0\right)$

Pricing Models:

Turnbull-Wakeman Model

Define:

$t_1 =$ time to the beginning of average period from current time t.
$n =$ number of average points.

Pricing Formulas:

Case 1: Valuation date on or before average start date

The valuation formula (Levy 1997 and Haug, Haug and Margrabe 2003) is:

$$c_{fix}(t) \approx e^{-r\tau}\left[F_A N(d_1) - KN(d_2)\right]$$
$$p_{fix}(t) \approx e^{-r\tau}\left[KN(-d_2) - F_A N(-d_1)\right]$$

where

$$d_1 = \frac{\ln\left(\frac{F_A}{K}\right) + \frac{\sigma_A^2}{2}\tau}{\sigma_A\sqrt{\tau}}$$
$$d_2 = d_1 - \sigma_A\sqrt{\tau}$$

$$h = \frac{\tau - t_1}{n-1}$$
$$F_A = E[A_\tau] = \frac{S_t}{n} e^{bt_1} \frac{1-e^{bhn}}{1-e^{bh}}$$
$$\sigma_A = \sqrt{\frac{\ln\left(E\left[A_\tau^2\right]\right) - 2\ln\left(E[A_\tau]\right)}{\tau}}$$
$$E\left[A_\tau^2\right] = \frac{S_t^2 e^{(2b+\sigma^2)t_1}}{n^2}\left[\frac{1-e^{(2b+\sigma^2)hn}}{1-e^{(2b+\sigma^2)h}} + \frac{2}{1-e^{(b+\sigma^2)h}}\left(\frac{1-e^{bhn}}{1-e^{bh}} - \frac{1-e^{(2b+\sigma^2)hn}}{1-e^{(2b+\sigma^2)h}}\right)\right].$$

In the case when cost of carry $b = 0$,

$$E[A_\tau] = S_t$$
$$E\left[A_\tau^2\right] = \frac{S_t^2 e^{\sigma^2 t_1}}{n^2}\left[\frac{1-e^{\sigma^2 hn}}{1-e^{\sigma^2 h}} + \frac{2}{1-e^{\sigma^2 h}}\left(n - \frac{1-e^{\sigma^2 hn}}{1-e^{\sigma^2 h}}\right)\right]$$

Case 2: Valuation date in averaging
We now look at the case when we want to find a price at a time inside the average period.

Define:

$m =$ number of observed average points.
$S_A =$ the arithmetic average of the known asset price fixings.

If we are inside the average period, i.e. $m > 0$, then the strike price is replaced by:

$$\hat{K} = \frac{nK - mS_A}{n-m} n - \frac{m}{n-m}$$

Also we set $t_1 = 0$ for all cases $m > 0$.

Case 3: Exercise for certain

If $S_A > \frac{n}{m} K$, then the exercise is certain for a call, and put is certain to be out-of-the-money. This leads to following results:

$$c_{fix}(t) = e^{-rT}\left(\hat{S}_A - K\right)$$

$$p_{fix}(t) = 0$$

where

$$\hat{S}_A = S_A \frac{m}{n} + E[A]\frac{n-m}{n}.$$

Note $E[A]$ can be calculated as $E[A_\tau]$ with $t_1 = 0$ because $m > 0$ in this case.

Case 4: One fixing left:

If there is only one more fixing before maturity, the following formula is used:

$$c_{fix}(t) = \frac{1}{n}\left[Se^{(b-r)\tau}N(d_1) - K^* e^{-r\tau}N(d_2)\right]$$

$$p_{fix}(t) = \frac{1}{n}\left[K^* e^{-r\tau}N(-d_2) - Se^{(b-r)\tau}N(-d_1)\right]$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K^*}\right) + \left(b + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$K^* = nK - (n-1)S_A.$$

Curran's Approximation

Define:

$t_1 =$ time to the beginning of average period from current time t.

$n =$ number of averaging points.

Pricing Formulas:

Case1: Valuatin date on or before average start date

$$c_{fix}(t) \approx e^{-r\tau}\left[\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i + \frac{\sigma_i^2}{2}} N\left(\frac{\mu - \ln(\hat{K})}{\sigma_x} + \frac{\sigma_{xi}}{\sigma_x}\right) - KN\left(\frac{\mu - \ln(\hat{K})}{\sigma_x}\right)\right]$$

where

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$$h = \frac{\tau - t_1}{n-1}$$

$$\mu_i = \ln(S_t) + \left(b - \frac{\sigma^2}{2}\right) t_i$$

$$\sigma_i = \sqrt{\sigma^2 [t_1 + (i-1)h]}$$

$$\sigma_{xi} = \sigma^2 \left\{ t_1 + h\left[(i-1) - \frac{i(i-1)}{2n} \right] \right\}$$

$$\mu = \ln(S_t) + \left(b - \frac{\sigma^2}{2}\right)\left[t_1 + \frac{(n-1)h}{2}\right]$$

$$\sigma_x = \sqrt{\sigma^2 \left[t_1 + \frac{h(n-1)(2n-1)}{6n} \right]}$$

$$\hat{K} = 2K - \frac{1}{n}\sum_{i=1}^{n} \exp\left\{ \mu_i + \frac{\sigma_{xi}[\ln(K) - \mu]}{\sigma_x^2} + \frac{\sigma_i^2 - \frac{\sigma_{xi}^2}{\sigma_x^2}}{2} \right\}$$

Case 2: Valuation date in averaging

Define:
$m =$ number of observed average points.
$S_A =$ the arithmetic average of the known asset price fixings.

If the average already began at the time when we want to find the option price (i.e. $m > 0$), then the strike price is replaced by:

$$\hat{K} = \frac{nK - mS_A}{n-m} n - \frac{m}{n-m}.$$

Also we set $t_1 = 0$ for all cases $m > 0$.

Case 3: Exercise for certain

If $S_A > \frac{n}{m}K$, then call is for certain going to be in-the-money and put is for certain to be out-of-money. This leads to the following result:

$$c_{fix} = e^{-r\tau}\left(\hat{S}_A - K\right)$$

$$p_{fix} = 0$$

where

$$\hat{S}_A = S_A \frac{m}{n} + E[A]\frac{n-m}{n}$$

$$E[A_\tau] = \frac{S_t}{n} e^{bt_1} \frac{1 - e^{bhn}}{1 - e^{bh}}$$

.

Note $E[A]$ can be calculated as $E[A_\tau]$ with $t_1 = 0$ because $m > 0$ in this case.

Case 4: One fixing left:
If there is only one more fixing before maturity, the following formula is used:

$$c_{fix} = \frac{1}{n}\left[S_t e^{(b-r)\tau} N(d_1) - K^* e^{-r\tau} N(d_2)\right]$$

$$p_{fix} = \frac{1}{n}\left[K^* e^{-r\tau} N(-d_2) - S_t e^{(b-r)\tau} N(-d_1)\right]$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K^*}\right) + \left(b + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

$$K^* = nK - (n-1)S_A.$$

Levy's Approximation
We will adapt some slightly different notations and conventions for Levy's model.

Define:

$S_t =$ the current asset price at time t.

$S_{t_i} =$ the asset price of the $(i+1)^{th}$ fixing (i.e. S_{t_0} is the asset price of the first fixing).

$K =$ the strike price of option.

$r =$ the risk-free rate.

$b =$ the cost of carry.

$T =$ the original time to maturity.

$\tau =$ $T - t$, the remaining time to maturity.

$t_i =$ time to the $(i+1)^{th}$ fixing from current time t (i.e. t_0 is the time to the first fixing).

$\sigma =$ volatility of asset.

$m =$ number of observed average points after the first fixing.

$n =$ total number of fixings minus one.

$S_A =$ the arithmetic average of the known asset price fixings.

$E^* =$ the expectation under risk-neutral measure.

We assume that there are $n+1$ number of fixings in total at times $t_0, t_1, \ldots t_n$. Thus t_0 is the time to the first fixing and t_n is the time to the $(n+1)^{th}$ fixing (the last fixing) from current time t.

We further define:

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$$h = \frac{t_n - t_0}{n}.$$

$$\zeta = \frac{t - t_m}{h}.$$

$$M_t = \bar{S} - S_A \frac{m+1}{n+1} = \frac{1}{n+1}\sum_{i=0}^{n} S_{t_i} - \frac{1}{m+1}\sum_{i=0}^{m} S_{t_i} \frac{m+1}{n+1} = \frac{1}{n+1}\sum_{i=m+1}^{n} S_{t_i}.$$

If $K - \frac{S_A(m+1)}{n+1} < 0$ then call will be in-the-money for certain and put will be out-of-money for certain, thus:

$$c_{fix}(t) \approx e^{-r\tau}\left(\hat{S} - K\right)$$
$$p_{fix}(t) \approx 0$$

where

$$\hat{S} = S_A \frac{m+1}{n+1} + E^*[M_t].$$

$$E^*[M_t] = \frac{S_t}{n+1} e^{b(1-\zeta)h}\left[\frac{1-e^{b(n-m)h}}{1-e^{bh}}\right].$$

If $K - \frac{S_A(m+1)}{n+1} \geq 0$, then the discrete average version of Levy's approximation for the case where $t \geq t_0$ is:

$$c_{fix}(t) \approx e^{-r\tau}\left\{E^*[M_t]N(d_1) - \left[K - \frac{S_A(m+1)}{n+1}\right]N(d_2)\right\}$$

$$p_{fix}(t) \approx e^{-r\tau}\left\{E^*[M_t]\cdot[N(d_1)-1] - \left[K - \frac{S_A(m+1)}{n+1}\right]\cdot[N(d_2)-1]\right\}$$

where

$$d_1 = \frac{\frac{1}{2}\ln E^*\left[M_t^2\right] - \ln\left[K - \frac{S_A(m+1)}{n+1}\right]}{v_t}$$

$$d_2 = d_1 - v_t$$

$$v_t = \sqrt{\ln E^*\left[M_t^2\right] - 2\ln E^*\left[M_t\right]}.$$

$$E^*[M_t] = \frac{S_t}{n+1} e^{b(1-\zeta)h}\left[\frac{1-e^{b(n-m)h}}{1-e^{bh}}\right]$$

$$E^*\left[M_t^2\right] = \frac{S_t^2}{(n+1)^2} e^{-2\zeta\left(b+\frac{1}{2}\sigma^2\right)h}\left(A_1 - A_2 + A_3 - A_4\right)$$

where

$$A_1 = \frac{e^{\left(2b+\sigma^2\right)h} - e^{\left(2b+\sigma^2\right)(n-m+1)h}}{\left(1-e^{bh}\right)\left(1-e^{\left(2b+\sigma^2\right)h}\right)}$$

$$A_2 = \frac{e^{\left[b(n-m+2)+\sigma^2\right]h} - e^{\left(2b+\sigma^2\right)(n-m+1)h}}{\left(1-e^{bh}\right)\left(1-e^{\left(b+\sigma^2\right)h}\right)}$$

$$A_3 = \frac{e^{\left(3b+\sigma^2\right)h} - e^{\left[b(n-m+2)+\sigma^2\right]h}}{\left(1-e^{bh}\right)\left(1-e^{\left(b+\sigma^2\right)h}\right)}$$

$$A_4 = \frac{e^{2\left(2b+\sigma^2\right)h} - e^{\left(2b+\sigma^2\right)(n-m+1)h}}{\left(1-e^{\left(b+\sigma^2\right)h}\right)\left(1-e^{\left(2b+\sigma^2\right)h}\right)}$$

For the case when $t < t_0$, we need to modify $E^*[M_t]$ and $E^*\left[M_t^2\right]$.

$$E^*[M_t] = \frac{S_t}{n+1} e^{b(t_0-t)}\left[\frac{1-e^{b(n+1)h}}{1-e^{bh}}\right]$$

$$E^*\left[M_t^2\right] = \frac{S_t^2}{(n+1)^2} e^{\left(2b+\sigma^2\right)(t_0-t)}\left(B_1 - B_2 + B_3 - B_4\right)$$

$$B_1 = \frac{1-e^{\left(2b+\sigma^2\right)(n+1)h}}{\left(1-e^{bh}\right)\left(1-e^{\left(2b+\sigma^2\right)h}\right)}$$

$$B_2 = \frac{e^{b(n+1)h} - e^{\left(2b+\sigma^2\right)(n+1)h}}{\left(1-e^{bh}\right)\left(1-e^{\left(b+\sigma^2\right)h}\right)}$$

$$B_3 = \frac{e^{bh} - e^{b(n+1)h}}{\left(1-e^{bh}\right)\left(1-e^{\left(b+\sigma^2\right)h}\right)}$$

$$B_4 = \frac{e^{\left(2b+\sigma^2\right)h} - e^{\left(2b+\sigma^2\right)(n+1)h}}{\left(1-e^{\left(b+\sigma^2\right)h}\right)\left(1-e^{\left(2b+\sigma^2\right)h}\right)}.$$

Levy's model covers all plain-vanilla, forward start and in progress cases. It is also not limited by the case where the last fixing date does not coincide with the option maturity date.

15.7.5 Arithmetic Average Asian Option Pricing Models (Discrete Averaging, Variable Fixing Weighting and Time-Dependent Underlying Parameters)

Fixed Strike Asian Option

Define:

$S_t =$ the asset price at current time (or valuation date) t.

$S_{t_i} =$ the asset price of the i^{th} fixing.

$\alpha_i =$ the normalised weight of the i^{th} fixing.

$\bar{S}_{t_i} = \sum_{u=1}^{i} \alpha_u S_{t_u}$.

$n =$ total number of fixings.

$m =$ number of observed fixings.

$K =$ the strike price.

$r_{t_i} =$ the risk-free rate at time t_i.

$b_{t_i} =$ the cost of carry at time t_i.

$\sigma_{t_i} =$ the volatility of the underlying asset at time t_i.

$T =$ the original time to maturity.

$\tau =$ the remaining time to maturity.

$df_{t,T} =$ the discount factor for the period from t to T.

$N(\cdot) =$ the cumulative normal distribution function.

$E^* =$ the expectation under risk-neutral measure.

Note:

$$\sum_{i=0}^{n} \alpha_i = 1$$
.

Similarly to the equal time interval case, we define:

$$M_t = \bar{S}_{t_n} - \bar{S}_{t_m} = \sum_{i=m+1}^{n} \alpha_i S_{t_i}$$
.

Further, we define:

$$F_{t_i} = E^*\left[S_{t_i}\right] = S_t e^{b_{t_i}(t_i - t)}$$

Option Payoff:

Call: $\max(\bar{S} - K, 0)$

Put: $\max\left(K - \bar{S}, 0\right)$

Derivations:

$$\begin{aligned}
& E^*[M_t] \\
& = E^*\left[\sum_{i=m+1}^{n} \alpha_i S_{t_i}\right] \\
& = \sum_{i=m+1}^{n} \alpha_i E^*\left[S_{t_i}\right] \\
& = \sum_{i=m+1}^{n} \alpha_i F_{t_i}
\end{aligned}$$

$$\begin{aligned}
& E^*\left[M_t^2\right] \\
& = E^*\left[\sum_{i=m+1}^{n} \alpha_i S_{t_i} \sum_{j=m+1}^{n} \alpha_j S_{t_j}\right] \\
& = E^*\left[\sum_{i=m+1}^{n} \sum_{j=M=1}^{n} \alpha_i \alpha_j S_{t_i} S_{t_j}\right] \\
& = \sum_{i=m+1}^{n} \sum_{j=m+1}^{n} \alpha_i \alpha_j E^*\left[S_{t_i} S_{t_j}\right] \\
& = \sum_{i=m+1}^{n} \sum_{j=m+1}^{n} \alpha_i \alpha_j F_{t_i} F_{t_j} e^{\sigma^2_{\min(t_i, t_j)} \min\left(t_i - t, t_j - t\right)}
\end{aligned}$$

$$\begin{aligned}
& c_{fix}(t) \\
& \approx df_{t,T} E^*\left[\max\left(\bar{S}_{t_n} - K, 0\right)\right]
\end{aligned}$$

$$= df_{t,T} E^* \left[\max\left(\sum_{i=1}^{n} \alpha_i S_{t_i} - K, 0 \right) \right]$$

$$= df_{t,T} E^* \left[\max\left(\sum_{i=1}^{m} \alpha_i S_{t_i} + \sum_{i=m+1}^{n} \alpha_i S_{t_i} - K, 0 \right) \right]$$

$$= df_{t,T} E^* \left[\max\left(\sum_{i=1}^{m} \alpha_i S_{t_i} + M_t - K, 0 \right) \right]$$

$$= df_{t,T} E^* \left[\max\left(M_t - \left(K - \sum_{i=1}^{m} \alpha_i S_{t_i} \right), 0 \right) \right].$$

Pricing Formulas:

Case 1: Exercise for certain and out-of-money for certain

If $K - \sum_{i=1}^{m} \alpha_i S_{t_i} < 0$ then call will be in-the-money for certain and put will be out-of-money for certain, thus:

$$c_{fix}(t) \approx df_{t,T} \left(\hat{S} - K \right)$$
$$p_{fix}(t) \approx 0$$

where

$$\hat{S} = \sum_{i=1}^{m} \alpha_i S_{t_i} + E^*[M_t] = \bar{S}_{t_m} + E^*[M_t].$$

Case 2: Valuation date before last fixing

If $K - \sum_{i=1}^{m} \alpha_i S_{t_i} \geq 0$ then we assume M_t is lognormally distributed like the equal time interval case, the resulted formulas for variable fixing time intervals are:

$$c_{fix}(t) \approx df_{t,T} \left\{ E^*[M_t] N(d_1) - \left[K - \sum_{i=1}^{m} \alpha_i S_{t_i} \right] N(d_2) \right\}$$

$$p_{fix}(t) \approx df_{t,T} \left\{ E^*[M_t] \cdot [N(d_1) - 1] - \left[K - \sum_{i=1}^{m} \alpha_i S_{t_i} \right] \cdot [N(d_2) - 1] \right\}$$

where

$$d_1 = \frac{\frac{1}{2} \ln E^*\left[M_t^2\right] - \ln\left[K - \sum_{i=1}^{m} \alpha_i S_{t_i} \right]}{v_t}$$

$$d_2 = d_1 - v_t$$

$$v_t = \sqrt{\ln E^*\left[M_t^2\right] - 2 \ln E^*[M_t]}.$$

$$E^*[M_t] = \sum_{i=m+1}^{n} \alpha_i F_{t_i}$$

$$E^*\left[M_t^2\right] = \sum_{i=m+1}^{n} \sum_{j=m+1}^{n} \alpha_i \alpha_j F_{t_i} F_{t_j} e^{\sigma^2_{\min(t_i,t_j)} \min(t_i - t, t_j - t)}.$$

Case 3: Valuation date on or after last fixing

$$c_{fix}(t) \approx df_{t,T}\left\{\bar{S}_{t_m} e^{r_t (T-t)} - K\right\}$$

$$p_{fix}(t) \approx df_{t,T}\left\{K - \bar{S}_{t_m} e^{r_t (T-t)}\right\}.$$

Floating Strike Asian Option

Define:

$S_t =$ the asset price at current time (or valuation date) t.

$S_{t_i} =$ the asset price of the i^{th} fixing (i.e. S_{t_0} is the asset price of the first fixing).

$\alpha_i =$ the normalised weight of the i^{th}.

$\bar{S}_{t_i} = \sum_{u=1}^{i} \alpha_u S_u$.

$n =$ total number of fixings.

$m =$ number of observed fixings.

$K =$ the strike price.

$r_{t_i} =$ the risk-free rate at time t_i.

$b_{t_i} =$ the cost of carry at time t_i.

$\sigma_{t_i} =$ the volatility of the underlying asset at time t_i.

$T =$ the original time to maturity.

$\tau =$ the remaining time to maturity.

$df_{t,T} =$ the discount factor for the period from t to T.

$N(\cdot) =$ the cumulative normal distribution function.

$E^* =$ the expectation under risk-neutral measure.

$$\sum_{i=1}^{n} \alpha_i = 1.$$

$$M_t = \bar{S}_{t_n} - \bar{S}_{t_m} = \sum_{i=m+1}^{n} \alpha_i S_{t_i}.$$

$$F_{t_i} = E^*\left[S_{t_i}\right] = S_t e^{b_{t_i}(t_i - t)}.$$

Assumptions:

1. $t_0 = 0$.
2. $t_n = T$.
3. $\left[\ln\left(\bar{S}_{t_n}\right), \ln(S_T)\right]$ follows a bivariate normal distribution.

Option Payoff:

Call: $\max(\bar{S} - K, 0)$

Put: $\max\left(K - \bar{S}, 0\right)$

Pricing Formulas:

Case 1: Valuation date before last fixing and last fixing matches the option expiry date

The pricing formulas are:

$$c_{float}(t) \approx df_{t,T}\left\{F_T N(d_1) - E^*\left[\bar{S}_{t_n}\right] N(d_2)\right\}$$
$$p_{float}(t) \approx df_{t,T}\left\{E^*\left[\bar{S}_{t_n}\right] N(-d_2) - F_T N(-d_1)\right\}$$

where

$$d_1 = \frac{\ln\left(\dfrac{F_T}{E^*\left[\bar{S}_{t_n}\right]}\right)}{\sigma_S} + \frac{1}{2}\sigma_S$$
$$d_2 = d_1 - \sigma_S$$
$$E^*\left[\bar{S}_{t_n}\right] = E^*\left[\bar{S}_{t_m} + M_t\right] = \bar{S}_{t_m} + E^*[M_t]$$
$$E^*[M_t] = \sum_{i=m+1}^{n} \alpha_i F_{t_i}$$
$$\sigma_S^2 = \sigma^2(T) + \sigma_x^2\left(\frac{E^*[M_t]}{E^*\left[\bar{S}_{t_n}\right]}\right)^2 - 2\ln\left(\frac{\sum_{i=m+1}^{n} w_i F_{t_i}\exp\left(\sigma_{t_i}^2 \cdot (t_i - t_0)\right)}{E^*[M_t]}\right) \times \left(\frac{E^*[M_t]}{E^*\left[\bar{S}_{t_n}\right]}\right)$$

$$\sigma^2(\tau) = \sigma_T^2(T - t_0)$$

$\sigma_x^2 = v^2$ in the fixed strike case.

Case 2: Valuation date after the last fixing

The payoffs for call and put are respectively:

$\max(S_T - \bar{S}_{t_n})$ and $\max(\bar{S}_{t_n} - S_T)$.

The option prices can be evaluated using standard Black-Scholes model since $\bar{S}_{t_n}$ is known and can be treated as a constant strike.

Case 3: Maturity Mistmatch
The Levy model assumes the last fixing time is on the maturity day. In reality, the last fixing can happen before the maturity day. In such a case, we can use the double average rate option (DARO) model to approximate the option price as floating strike Asian option is just a special case of DARO.

To find $c^*_{float}(t)$, we set $\beta = 1$ $w_1 = 1$, $w_2 = 1$, and $K = 0$ in the DARO model. We also assume there is only one asset fixing for $S^{(1)}$ and the fixing point is at option maturity.

To find $p^*_{float}(t)$, we set $\beta = 1$ $w_1 = 1$, $w_2 = 1$, and $K = 0$ in the DARO model. We also assume there is only one asset fixing for $S^{(2)}$ and the fixing point is at option maturity.

For details on DARO model, please refer to the section on DARO.

Asian Basket Options 1

Define:

$w_i^s =$	weight of asset i in the basket
$S_i(t) =$	price of asset i at time t.
$b_i(t) =$	cost of carry of asset i at time t.
$\sigma_i(t) =$	volatility of asset i at time t.
$\rho_{ij} =$	correlation between asset i and j
$S_b(t) =$	price of the basket at time t.
$\sigma_b(t) =$	volatility of the basket at time t.
$n_a =$	number of assets in the basket.
$n_f =$	number of fixings.

Option Payoff:

Call: $$\max\left(\sum_{i=1}^{n_a}\sum_{j=1}^{n_f} S_i(t_j) - K, 0\right)$$

Put: $$\max\left(K - \sum_{i=1}^{n_a}\sum_{j=1}^{n_f} S_i(t_j), 0\right)$$

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Pricing Formulas:

$$S_b(t) = \sum_{i=1}^{n_a} w_i^s S_i(t).$$

$$b_b(t) = \sum_{i=1}^{n_a} w_i^s b_i(t).$$

$$\sigma_b(t) = \sqrt{\sum_{i=1}^{n_a} \left(w_i^s\right)^2 \sigma_i^2(t) + 2\sum_{i=1}^{n_a}\sum_{j=1}^{i-1} w_i^s w_j^s \rho_{ij}\sigma_i(t)\sigma_j(t)}$$

Then we can apply the normal fixed strike or floating strike Asian option formulas to calculate the Asian basket option price.

Asian Quantos Basket Options

Quantos (also called fixed exchange rate foreign equity options) are option contracts that are denominated in another currency than that of the underlying equity exposure. Asian Quanto incorporates both the average rate and cross-currency natures.

Define:

$n_a =$ number of underlying assets.

$n_f =$ number of fixings.

$m =$ number of observed fixings.

$C =$ payoff currency.

$S_i(t) =$ i by 1 array of asset prices i at time t.

$C_i =$ currency i for asset i.

$\sigma_{S_i}(t_j) =$ i by j matrix of volatilities of asset i at time t_j.

$X_i =$ i by 1 array of exchange rate quoted as payoff currency per currency i,i.e. C/C_i.

$\sigma_{X_i}(t_j) =$ i by j matrix of volatilities X_i at time t_j.

$\rho_{ij} =$ i by i matrix of instantaneous correlations of $\ln S_i$ and $\ln S_j$.

$\rho_i =$ i by 1 array of instantaneous correlations of S_i and X_i.

$r_{C_i}(t_j) =$ i by j matrix of instantaneous risk-free interest rates in C_i market.

$b_i(t_j) =$ i by j matrix of cost of carries for asset i at time t_j.

$K =$ strike price is payoff currency.
$W =$ Wiener process.

We further define:

$$\bar{S}(t_n) = \sum_{i=1}^{n_a} \sum_{j=1}^{n_f} w_{ij} S_i(t_j).$$

Option Payoff:

Call: $\max\left(\bar{S}\left(t_{n_f}\right) - K, 0\right)$

Put: $\max\left(K - \bar{S}\left(t_{n_f}\right), 0\right)$.

Pricing Formulas:

We have assets denominated in different currencies. We need to find a measure say C, such that all assets under this measure are risk-neutral. It can be shown (Datey, Gauthier and Simonato) that in such risk-neutral measure, the asset prices are distributed as:

$$dS_i(t) = \left(b_i(t) - \rho_i \cdot \sigma_{S_i}(t) \cdot \sigma_{X_i}(t)\right) \cdot S_i(t) \cdot dt + \sigma_{S_i}(t) \cdot S_i(t) \cdot dW_i^C(t), \quad \forall i = 1,2,\ldots,n_a$$

.

Regardless all the technical terms, the important thing is that the underlying asset price still has the form of geometric Brownian motion with different drift and diffusion term. It means that we can use similar approaches in Asian non-quanto options for Asian quanto options.

$$\bar{S}(t_n) = \sum_{i=1}^{n_a} \sum_{j=1}^{n} w_{ij} S_i(t_j) = \bar{S}(t_m) + \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} w_{ij} S_i(t_j) = \bar{S}(t_m) + M_t$$

where

$$M_t = \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} w_{ij} S_i(t_j).$$

We denote $b_i^*(t_j) = b_i(t_j) - \rho_i \cdot \sigma_{S_i}(t_j) \cdot \sigma_{X_i}(t_j)$
and define:

$$F_i(t_j) = S_i(t) e^{b_i^*(t_j)(t_j - t)}$$

$$E^*(M_t) = \sum_{i=1}^{n_a} \sum_{j=m+1}^{n_f} w_{ij} F_i(t_j).$$

$$E^*\left(M_t^2\right)=\sum_{i=1}^{n_a}\sum_{j=m+1}^{n_f}\sum_{k=1}^{n_a}\sum_{l=m+1}^{n_f} w_{ij}w_{kl}F_i\left(t_j\right)F_k\left(t_l\right)\times$$
$$\exp\left(\rho_{ik}\sigma_{S_i}\left(\min\left(t_j,t_l\right)\right)\sigma_{S_k}\left(\min\left(t_j,t_l\right)\right)\min\left(t_j-t,t_l-t\right)\right).$$

We can then use the fixed strike formulas with all $K-\sum_{i=1}^{m}\alpha_i S_{t_i}$ replaced by $K-\overline{S}(t_m)$. Exercise for certain and out-of-money for certain cases are the same by replacing $K-\sum_{i=1}^{m}\alpha_i S_{t_i}$ with $K-\overline{S}(t_m)$.

Asian Basket Options 2

Another method to price the Asian basket options is similar to Asian quanto basket. We simply set the the array ρ_i to 0, the matrix $\sigma_{X_i}\left(t_j\right)$ to 0 and use $b_i\left(t_j\right)$ instead of $b_i^*\left(t_j\right)$ in the Asian quanto basket options formula.

Double Average Rate Options

Double average rate options are also called Asian spread options. The payoff is dependent on the differences of two averages of the underlying asset. The underlying asset can be a single asset or a basket of assets. For this documentation, we will assume that there is only a single underlying asset.

Define:

$S_t^{(1)} =$ the asset price at time t for the first average.

$S_t^{(2)} =$ the asset price at time t for the second average.

$S_{t_i}^{(1)} =$ the asset price of the i^{th} fixing for the first average.

$S_{t_i}^{(2)} =$ the asset price of the i^{th} fixing for the second average.

$\alpha_i^{(1)} =$ the normalised weight of the i^{th} fixing for the first average.

$\alpha_i^{(2)} =$ the normalised weight of the i^{th} fixing for the second average.

$\overline{S}_{t_i}^{(1)} = \sum_{u=1}^{i}\alpha_u^{(1)}S_{t_u}^{(1)}$.

$\overline{S}_{t_i}^{(2)} = \sum_{u=1}^{i}\alpha_u^{(2)}S_{t_u}^{(2)}$.

$n_1 =$ total number of fixings in the first average.

$n_2 =$ total number of fixings in the second average.

$m_1 =$ number of observed fixings in the first average.

$m_2 =$ number of observed fixings in the second average.
$K =$ the strike price.
$r_{t_i} =$ the risk-free rate at time t_i.
$b_{t_i} =$ the cost of carry at time t_i.
$\sigma_{t_i} =$ the volatility of the underlying asset at time t_i.
$T =$ the original time to maturity.
$\tau =$ the remaining time to maturity.
$df_{t,T} =$ the discount factor for the period from t to T.
$N(\cdot) =$ the cumulative normal distribution function.
$E^* =$ the expectation under risk-neutral measure.

Note $S_t^{(1)} = S_t^{(2)}$ for single asset case.

Further, we define:

$$F_{t_i}^{(i)} = E^*\left[S_{t_i}^{(i)}\right] = S_t^{(i)} e^{b_{t_i^{(i)}}\left(t_i^{(i)} - t\right)}$$

Note:

$$\sum_{i=1}^{n_1} \alpha_i^{(1)} = 1.$$

$$\sum_{i=1}^{n_2} \alpha_i^{(2)} = 1.$$

We further define:
$\mathrm{T}_1 = \left\{t_1^{(1)} < t_2^{(2)} < \ldots < t_{n_1}^{(1)}\right\}$ and $\mathrm{T}_2 = \left\{t_1^{(2)} < t_2^{(2)} < \ldots < t_{n_2}^{(2)}\right\}$, $\mathrm{T}_1 \cap \mathrm{T}_2$ is a null set, where
$t_i^{(1)}$'s , $t_i^{(2)}$'s are the time to the average points (i.e. fixing points) in the first and second average respectively.

Also:
$t_{n_i}^{(1)} \le T$ and $t_{n_2}^{(2)} \le T$ so that the last fixings of the two averages can be either before or on the option maturity date.

Option Payoff:

Call: $\max\left[w_1 \overline{S}_{n_1}^{(1)} - w_2 \overline{S}_{n_2}^{(2)} - K, 0\right]$

Put: $\max\left[K + w_2 \overline{S}_{n_2}^{(2)} - w_1 \overline{S}_{n_1}^{(1)}, 0\right]$

where $w_1, w_2 > 0$.

Derivations:

To price the DARO, we assume $X = w_1\overline{S}_{n_1}^{(1)} - w_2\overline{S}_{n_2}^{(2)}$ is normally distributed with mean μ and variance v. The probability density function of X becomes $f_X(x) = \frac{1}{\sqrt{2\pi v}}\exp\left(-\frac{(x-\mu)^2}{2v}\right)$.

We need to find the parameters μ and v.

The following equalities can be shown (the derivations are in appendix):

$$E^*\left(S_{t_i}^{(1)}\right) = F_{t_i}^{(1)}$$

$$E^*\left(S_{t_i}^{(2)}\right) = F_{t_i}^{(2)}$$

$$E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right) = \overline{S}_{t_{m_1}}^{(1)} + \sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right)$$

$$E^*\left(\overline{S}_{t_{m_2}}^{(2)}\right) = \overline{S}_{t_{m_2}}^{(2)} + \sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}E^*\left(S_{t_i}^{(2)}\right)$$

$$E^*\left(S_{t_i}^{(1)}S_{t_j}^{(1)}\right) = F_{t_i}^{(1)}F_{t_j}^{(1)}e^{\sigma^2_{\min\left(t_i^{(1)},t_j^{(1)}\right)}\min\left(t_i^{(1)}-t,t_j^{(1)}-t\right)}$$

$$E^*\left(S_{t_i}^{(2)}S_{t_j}^{(2)}\right) = F_{t_i}^{(2)}F_{t_j}^{(2)}e^{\sigma^2_{\min\left(t_i^{(2)},t_j^{(2)}\right)}\min\left(t_i^{(2)}-t,t_j^{(2)}-t\right)}$$

$$E^*\left(S_{t_i}^{(1)}S_{t_j}^{(2)}\right) = F_{t_i}^{(1)}F_{t_j}^{(2)}e^{\sigma^2_{\min\left(t_i^{(1)},t_j^{(2)}\right)}\min\left(t_i^{(1)}-t,t_j^{(2)}-t\right)}$$

Note $S_t^{(1)} = S_t^{(2)}$ for single asset case.

$$E^*\left(\overline{S}_{t_{n_1}}^{(1)^2}\right) = \overline{S}_{t_{m_1}}^{(1)^2} + 2\overline{S}_{t_{m_1}}^{(1)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right) + \sum_{i=m_1+1}^{n_1}\sum_{j=m_1+1}^{n_1}\alpha_i^{(1)}\alpha_j^{(1)}E^*\left(S_{t_i}^{(1)}S_{t_j}^{(1)}\right)$$

$$E^*\left(\overline{S}_{t_{n_2}}^{(2)^2}\right) = \overline{S}_{t_{m_2}}^{(2)^2} + 2\overline{S}_{t_{m_2}}^{(2)}\sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}E^*\left(S_{t_i}^{(2)}\right) + \sum_{i=m_2+1}^{n_2}\sum_{j=m_2+1}^{n_2}\alpha_i^{(2)}\alpha_j^{(2)}E^*\left(S_{t_i}^{(2)}S_{t_j}^{(2)}\right)$$

$$E^*\left(\overline{S}_{t_{n_1}}^{(1)}\overline{S}_{t_{n_2}}^{(2)}\right)$$
$$= \overline{S}_{t_{m_1}}^{(1)}\overline{S}_{t_{m_2}}^{(2)} + \overline{S}_{t_{m_1}}^{(1)}\sum_{j=m_2+1}^{n_2}\alpha_j^{(2)}E^*\left(S_{t_j}^{(2)}\right) + \overline{S}_{t_{m_2}}^{(2)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right) + \sum_{i=m_1+1}^{n_1}\sum_{j=m_2+1}^{n_2}\alpha_i^{(1)}\alpha_j^{(2)}E^*\left(S_{t_i}^{(1)}S_{t_j}^{(2)}\right)$$

We can obtain parameters μ and v:

$$\mu = w_1E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right) - w_2E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)$$

$$v$$
$$= w_1^2\left(E^*\left(\overline{S}_{t_{m}}^{(1)^2}\right) - E^*\left(\overline{S}_{t_{m}}^{(1)}\right)^2\right) + w_2^2\left(E^*\left(\overline{S}_{t_{n_2}}^{(2)^2}\right) - E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)^2\right)$$
$$- 2w_1w_2\left(E^*\left(\overline{S}_{t_{n_1}}^{(1)}\overline{S}_{t_{n_2}}^{(2)}\right) - E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right)E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)\right)$$

The price of DARO is:

$$c_{DARO}(t)$$
$$= e^{-r\tau}E^*\left(\max[\beta(X-K,0)]\right)$$

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Pricing Formulas:

We assumed $X \sim N(\mu, v)$, it follows that:

$$c_{DARO}(t) \approx df_{t,T}\left\{\sqrt{\frac{v}{2\pi}}\exp\left(-\frac{1}{2}\frac{(K-\mu)^2}{v}\right)+(\mu-K)\left[\frac{1+\beta}{2}-N\left(\frac{K-\mu}{\sqrt{v}}\right)\right]\right\}.$$

β is the call-put index (1 for a call, -1 for a put).

Reciprocal Asian Options

Asian options are options where the payoff depends on the average price of the underlying asset during at least some part of the life of the option. Reciprocal Asian options are a particular class of Asian options and the payoff depends on the reciprocal of the average price of the underlying asset. In this document, we will demonstrate the methods to value fixed strike European reciprocal Asian call and put options for both discrete and continuous arithmetic averages.

Define:

$S_t =$ the asset price at current time t.

$S_{t_i} =$ the asset price of the i^{th} fixing.

$\alpha_i =$ the normalised weight of the i^{th} fixing.

$\bar{S}_{t_i} = \sum_{u=1}^{i}\alpha_u S_{t_u}$.

$n =$ total number of fixings minus one.

$m =$ number of observed fixings minus one.

$K =$ the strike price.

$r_{t_i} =$ the risk-free rate at time t_i.

$b_{t_i} =$ the cost of carry at time t_i.

$\sigma_{t_i} =$ the volatility of the underlying asset at time t_i.

$T =$ the original time to maturity.

$\tau =$ the remaining time to maturity.

$df_{t,T} =$ the discount factor for the period from t to T.

$N(\cdot) =$ the cumulative normal distribution function.

$E^* =$ the expectation under risk-neutral measure.

Note:

$$\sum_{i=1}^{n}\alpha_i = 1.$$

Further, we define:

$$M_t = \bar{S}_{t_n} - \bar{S}_{t_m} = \sum_{i=m+1}^{n} \alpha_i S_{t_i}$$

$$F_{t_i} = E^*\left[S_{t_i}\right] = S_t e^{b_{t_i}(t_i - t)}.$$

Option Payoff:

Call: $N \max\left[\frac{1}{\bar{S}_{t_n}} - \frac{1}{S_T}, 0\right]$.

Put: $N \max\left[\frac{1}{S_T} - \frac{1}{\bar{S}_{t_n}}, 0\right]$.

N is the notional amount.

$$\bar{S}_{t_n} = \sum_{i=1}^{n} \alpha_i S_{t_i}.$$

Derivations:

Assume $t_m \le \tau \le t_{m+1}$, we know all the fixings up to time t_m.

$$\bar{S}_{t_n} = \sum_{i=1}^{m} \alpha_i S_{t_i} + \sum_{i=m+1}^{n} \alpha_i S_{t_i} = \bar{S}_{t_m} + \sum_{i=m+1}^{n} \alpha_i S_{t_i} = \bar{S}_{t_m} + M_t.$$

$$M_t = \bar{S}_{t_n} - \bar{S}_{t_m} = \bar{S}_{t_n} - c.$$

We define $c = \bar{S}_{t_m}$ and it is a known constant.

$$c_{fix}(t)$$

$$= Ndf_{t,T} E^*\left[\max\left(\frac{1}{\bar{S}_{t_n}} - \frac{1}{K}, 0\right)\right]$$

$$= Ndf_{t,T} E^*\left[\max\left(\frac{1}{M_t + c} - \frac{1}{K}, 0\right)\right]$$

For the call option to have a non-zero payoff, $\frac{1}{M_t + c} > \frac{1}{K} \Rightarrow M_t < K - c$. We also know that the average of the underlying asset price cannot be negative. It suggests that the call option has non-zero values when $0 < M_t < K - c$.

$$c_{fix}(t)$$

$$\approx Ndf_{t,T} E^*\left[\max\left(\frac{1}{M_t + c} - \frac{1}{K}, 0\right)\right]$$

$$= Ndf_{t,T}\int_{x=0}^{K-c}\left(\frac{1}{x+c}-\frac{1}{K}\right)f_{M_t}(x)dx$$
.

$f_{M_t}(x)$ is the probability density function of M_t.

Similarly, for put option to have a non-zero payoff,

$\frac{1}{K} > \frac{1}{M_t + c} \Rightarrow M_t > K - c$ and we the average of the underlying asset price cannot be negative.

$$p_{fix}(t)$$

$$\approx Ndf_{t,T}E^*\left[\max\left(\frac{1}{K}-\frac{1}{M_t+c},0\right)\right]$$

$$= Ndf_{t,T}\int_{x=\max(0,K-c)}^{\infty}\left(\frac{1}{K}-\frac{1}{x+c}\right)f_{M_t}(x)dx$$

$f_{M_t}(x)$ is the probability density function of M_t. It is sufficient for the upper limit for the integral to be

$$E^*(M_t)+10\sqrt{Var(M_t)} = E^*(M_t)+10\sqrt{E^*(M_t^2)-E^*(M_t)^2}$$
.

We assume $\ln(M_t)$ is normally distributed with mean α_t and standard deviation v_t. α_t and v_t can be approximated using Levy (1992) approach.

Pricing Formulas:

$$c_{fix}(t) \approx Ndf_{t,T}\int_{x=0}^{K-c}\left(\frac{1}{x+c}-\frac{1}{K}\right)f_{M_t}(x)dx$$
.

$$p_{fix}(t) \approx Ndf_{t,T}\int_{x=\max(0,K-c)}^{E^*(M_t)+10\sqrt{E^*(M_t^2)-E^*(M_t)^2}}\left(\frac{1}{K}-\frac{1}{x+c}\right)f_{M_t}(x)dx.$$

$$f_{M_t}(x) = \frac{1}{xv_t\sqrt{2\pi}}\exp\left\{-\frac{(\ln x-\alpha_t)^2}{2v_t^2}\right\}, \quad x>0.$$

$$\alpha_t = 2\ln E^*[M_t]-\frac{1}{2}\ln E^*\left[M_t^2\right]$$
.

$$v_t = \sqrt{\ln E^*\left[M_t^2\right]-2\ln E^*[M_t]}$$
.

$$E^*[M_t] = \sum_{i=m+1}^{n}\alpha_i F_{t_i}$$

$$E^*\left[M_t^2\right] = \sum_{i=m+1}^{n}\sum_{j=m+1}^{n}\alpha_i\alpha_j F_{t_i}F_{t_j}e^{\sigma^2_{\min(t_i,t_j)}\min(t_i-t,t_j-t)}$$
.

Note the integral can be evaluated numerically using Gauss-Legendre Quadrature.

Callable Asian Options

Note that support is only required for arithmetic averaging, with fixed strike.

Let $S(t)$ be a price process of a given underlying asset, $\{t_1 < \cdots < t_n\}$ be a set of reset dates and $T \geq t_n$ be a payoff settlement date. A callable Asian option with the underlying, S, is a derivative security whose matured payoff at the settlement date is given by

$$I_N \cdot N + N \cdot \max\{0, \beta(A-K)/K\} \quad (1)$$

where

N is the notional principal, I_N (= 0 or 1) is the notional payment indicator function, β is the call-put index, K is the strike, and A is the arithmetic average of $S(t_i),\ i = 1, \cdots, n$, with equal weight.

Equation (1) indicates that the principal is protected if $I_N = 1$. A payoff type without notional is also allowed, which removes the first part of equation (1) at maturity, but the owner of the option can still receive the call amount if the option is called on the call date.

Call Feature

Let t_c be the call date. At time t_c, $t_c < t_1$, if the underlying stock price $S(t_c)$ is above a barrier, P_c, the deal will be cancelled, either automatically or optimally by the underwriter, in which case the owner of the option will receive a call premium, C (automatic cancellation) or the minimum of C and the value of (1) at time t_c (optimal cancellation); if the deal will not be cancelled, the payoff at the settlement date is (1).

Forward Start Feature

The callable Asian option can also allow a forward start feature. This option specifies that at time t_f, the strike price of the Asian option, K, will be set equal to the current spot price, $S(t_f)$. With the forward start feature the payoff at the settlement date is

$$I_N \cdot N + N \cdot \max\{0, \beta(A - S(t_f))/S(t_f)\} \quad (2)$$

if the deal will not be cancelled. Otherwise, the owner of the option will receive a call premium, C (automatic cancellation) or the minimum of C and the value of (2) at time t_c (optimal cancellation).

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Pricing for the Call Feature Case

Let t be the current valuation date and assume $t < t_c$. Also denote $V_{t_c}(S(t_c))$ to be the value of the option at time t_c.

Automatic Cancellation

$$V_{t_c}(S(t_c)) = I(S(t_c) < P_c) \times df_{t_c,T} \times E_{t_c}\left[I_N \cdot N + N \times \max\left\{0, \frac{\beta(A-K)}{K}\right\}\right]$$
$$+ I(S(t_c) \geq P_c) \times C$$

where $df_{t_c,T} \times E_{t_c}\left[I_N \cdot N + N \times \max\left\{0, \frac{\beta(A-K)}{K}\right\}\right]$ is the expression for the value of a vanilla Asian option which can be approximated with Michael Curran formula and $df_{t_c,T}$ is the discount factor for the period from time t_c to T.

We denote $MC(S(t_c)) = df_{t_c,T} \times E_{t_c}\left[I_N \cdot N + N \times \max\left\{0, \frac{\beta(A-K)}{K}\right\}\right]$ where, $MC(S(t_c))$ is the Michael Curran formula including the notional adjustments. We can then write:

$$V_{t_c}(S(t_c)) = I(S(t_c) < P_c) \times MC(S(t_c)) + I(S(t_c) \geq P_c) \times C.$$

Optimal Cancellation

With optimal cancellation, we can write:

$$V_{t_c}(S(t_c)) = I(S(t_c) < P_c) \times MC(S(t_c)) + I(S(t_c) \geq P_c) \times \min\{C, MC(S(t_c))\}.$$

For both cancellation conditions, the value of the option at time t, is:

$$V_t(S(t)) = df_{t,t_c} \times E_t\left[V_{t_c}(S(t_c))\right]$$

where

$$E_t\left[V_{t_c}(S(t_c))\right] = \int_0^{\infty} V_{t_c}(S(t_c)) f(S(t_c)) dS(t_c),$$

$f(S(t_c))$ is the density function of $S(t_c)$.

Derivation of $f(S(t_c))$:

Under risk-neutral world,

$$dS(t) = b_{t_c} S(t)dt + \sigma_{t_c} S(t)dW_t \, .$$

Taking Ito's Lemma, and integrates, we then get:

$$\begin{aligned} &\ln S(t_c) \\ &= \ln S(t) + \left(b_{t_c} - \frac{1}{2}\sigma_{t_c}^2\right)(t_c - t) + \sigma_{t_c}^2 W_{t_c - t} \\ &\sim N\left(\ln S(t) + \left(b_{t_c} - \frac{1}{2}\sigma_{t_c}^2\right)(t_c - t), \sigma_{t_c}^2 (t_c - t)\right), \end{aligned}$$

i.e. $S(t_c)$ is lognormally distributed with

$$f(S(t_c)) = \frac{1}{S(t_c)v\sqrt{2\pi}} \exp\left\{-\frac{(\ln S(t_c) - \mu)^2}{2v^2}\right\}; \quad S(t_c) > 0,$$

where

$$\mu = \ln S(t) + \left(b_{t_c} - \frac{1}{2}\sigma_{t_c}^2\right)(t_c - t)$$

$$v^2 = \sigma_{t_c}^2 (t_c - t)$$

b_{t_c} and σ_{t_c} can be obtained from the term structure for cost of carries and volatilities with maturity t_c.

The integral can be evaluated using Gauss-Legendre quadrature with the care to divide the integral at the non-differentiable points of the integrand. For automatic cancellation, the integrand is non-differentiable at P_c. For optimal cancellation, the integrand is non-differentiable at P_c and/or the point $S^*(t_c)$, such that $C = MC(S^*(t_c))$.

Forward cost-of-carries and volatilities
It is important to note that the callable Asian options are evaluated at time t_c first and then brought back to time t. Thus we need the term structure curve for cost of carries and volatilities starting from time t_c when applying the Michael Curran formula. The formulas for transforming term structure curves are:

$$b_{t_c,t_i} = \frac{b_{t_i} t_i - b_{t_c} t_c}{t_i - t_c},$$

$$\sigma_{t_c,t_i}^2 = \frac{\sigma_{t_i}^2 t_i - \sigma_{t_c}^2 t_c}{t_i - t_c}.$$

Quanto case
Callable Asian option with quanto feature is the same except the cost of carries needs to be adjusted as in the quanto Asian basket options case,

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i.e. $b_i^*(t_j) = b_i(t_j) - \rho_i \cdot \sigma_{S_i}(t_j) \cdot \sigma_{X_i}(t_j)$. Note that cost of carries should be adjusted for quanto before converting to forward rates.

Pricing for the Forward Start Case

If the option is forward starting, both the future asset price and strike price are random variables at the valuation time t. We can write the Michael Curran formula as:

$$MC(S(t_f), S(t_c)) = MC(K = S(t_f), S(t_c)).$$

The value the option, $V_{t_c}(S(t_f), S(t_c))$, at time t_c, is given by:

Automatic Cancellation

$$V_{t_c}(S(t_f), S(t_c)) = I(S(t_c) < P_c) \times MC(S(t_f), S(t_c)) + I(S(t_c) \geq P_c) \times C.$$

Optimal Cancellation

$$\begin{aligned} V_{t_c}(S(t_f), S(t_c)) = {} & I(S(t_c) < P_c) \times MC(S(t_f), S(t_c)) + \\ & I(S(t_c) \geq P_c) \times \min\{C, MC(S(t_f), S(t_c))\}. \end{aligned}$$

Therefore, given $V_{t_c}(S(t_f), S(t_c))$, the value of the option, $V_t(S(t))$, at time t is given by:

$$V_t(S(t)) = df_{t,t_c} \times E_t[V_{t_c}(S(t_f), S(t_c))]$$

where

$$\begin{aligned} & E_t[V_{t_c}(S(t_f), S(t_c))] \\ & = E_t[E_{t_f}[V_{t_c}(S(t_f), S(t_c))]] \\ & = \int_0^\infty \int_0^\infty V_{t_c}(S(t_f), S(t_c)) \cdot f_{S_{t_c}|S_{t_f}}(S(t_c)) f_{S_{t_f}}(S(t_f)) dS(t_c) dS(t_f) \end{aligned},$$

$f_{S_{t_c}|S_{t_f}}(S(t_c))$ is the conditional density function of $S(t_c)$ given $S(t_f)$, and $f_{S_{t_f}}(S(t_f))$ is the density function of $S(t_f)$.

With similar derivations, we can show:

$$f_{S_{t_c}|S_{t_f}}(S(t_c)) = \frac{1}{S(t_c) v \sqrt{2\pi}} \exp\left\{-\frac{(\ln S(t_c) - \mu)^2}{2v^2}\right\}; \quad S(t_c) > 0,$$

where

$$v^2 = \sigma^2_{t_f,t_c}\left(t_c - t_f\right)$$

$$\sigma^2_{t_f,t_c} = \frac{\sigma^2_{t_c} t_c - \sigma^2_{t_f} t_f}{t_c - t_f}$$

σ_{t_f} can be obtained from the volatility structure with maturity t_f and σ_{t_c} can be obtained from the volatility structure with maturity t_c.

$$f\left(S\left(t_f\right)\right) = \frac{1}{S\left(t_f\right)v\sqrt{2\pi}} \exp\left\{-\frac{\left(\ln S\left(t_f\right) - \mu\right)^2}{2v^2}\right\};\ S\left(t_f\right) > 0,$$

where

$$\mu = \ln S(t) + \left(b_{t_f} - \frac{1}{2}\sigma^2_{t_f}\right)\left(t_f - t\right)$$

$$v = \sigma^2_{t_f}\left(t_f - t\right)$$

σ_{t_f} can be obtained from the volatility structure with maturity t_f.

We can approximate the double integral using two-dimensional Gauss-Legendre quadrature and tensor product. The non-differentiable points for the integrand in automatic cancellation is at P_c. In optimal cancellation, the non-differentiable points for the integrand are at P_c and/or the point $\left[S^*\left(t_c\right), S^*\left(t_f\right)\right]$, such that $C = MC\left(S^*\left(t_c\right), S^*\left(t_f\right)\right)$.

Tensor Product Rule

Tensor product is a technique used in multi-dimensional numerical integration. For the following, we will focus on two-dimensional integral since it is highest dimension we need in the callable model. However, the same concept can be applied to higher dimensional problems.

Define $f\left(x_1, x_2\right)$ be a function of two variables. We want to evaluated the following two-dimensional integral:

$$I = \int_{x_1}\int_{x_2} f\left(x_1, x_2\right) dx_1 dx_2 .$$

For each x_1, the inner integral can be approximated as:

$$I_2 = \int_{x_2} f\left(x_1, x_2\right) dx_2 \approx \sum_j \hat{w}_j f\left(x_1, \hat{x}_{2,j}\right)$$

where $\hat{w}_j$ and $\hat{x}_{2,j}$ are Gauss-Legendre weights and points.

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The outer integral can be evaluated similarly as:
$\sum_i \hat{w}_i \sum_j \hat{w}_j f\left(\hat{x}_{1,i}, \hat{x}_{2,j}\right)$
where again
$\hat{w}_i$ and $\hat{x}_{1,j}$ are Gauss-Legendre weights and points.

The two-dimensional integral is:
$$I = \int_{x_1}\int_{x_2} f(x_1, x_2) dx_1 dx_2 \approx \sum_i \sum_j \hat{w}_i \hat{w}_j f\left(\hat{x}_{1,i}, \hat{x}_{2,j}\right) = \sum_l \hat{w}_l f(\hat{x}_l)$$
.

It is obvious that $\hat{w}_l$ contain all the combinations of products of $\hat{w}_i$ and $\hat{w}_j$ and $\hat{x}_l$ contain all the combinations of the pair $\left(\hat{x}_i, \hat{x}_j\right)$.

Non-smooth points
It is important to note that Gauss-Legendre Quadrature can only be applied to smooth functions. It is clearly that for callable payoff, the integrands are not smooth functions for both call-feature and forward-start cases. We need to split the integrand into sections so that the integrand in each section is smooth. We can then integrate each smooth integrand using Gauss-Legendre Quadrature and then sum each section together to obtain the final integral result.

Call feature case
The integrand for call feature case is not smooth at the point when $S(t_c) = P_c$ with automatic cancellation and $MC(S(t_c)) = C$ for the optimal cancellation case. We need to find those $S^*(t_c)$ such that $S^*(t_c) = P_c$ and $MC(S^*(t_c)) = C$. We then split the integral at those $S^*(t_c)$ points so that the integrand between two $S^*(t_c)$ points is smooth.
Forward start case
Forward start case
Forward start case is more complicated because of the double integral. However, the idea is similar. We first find out the Gauss-Legendre points for both the outer and inner integrals. We then have a set of Gauss points $S(t_f)$ for the outer integral and $S(t_c)$ for the inner integral. For each of the $S(t_f)$ point, we fix it in the inner integral and treat it as a single integral. We can then obtain the non-smooth points and calculate the integral just as we did in the call feature case. We then repeat for each of the $S(t_f)$ points. We obtain the final double integral by summing each of the inner integral each with the corresponding Gauss point $S(t_f)$.

Upper bound
Theoretically, we need to integrate the integral to infinity for both call feature and forward start cases. However, it is unnecessary to integrate

over the values above say 15 standard deviations as the probabilities of reaching those values are very trivial. The following are examples of truncating the integral at proper points:

$$E^*(S) = e^{\mu+\frac{1}{2}v^2}.$$
$$Var^*(S) = e^{(2\mu+v^2)}\left(e^{v^2}-1\right).$$

We can set upper limit to be:

$$upper = E^*(S) + 15 \cdot \sqrt{Var^*(S)}.$$

We can apply this idea for both call feature and forward start case with corresponding μ and v for each integral.

Valuation date on or after the forward start date or the call date

If the valuation date is on or after the forward start date but before the call date then the strike price is known and the option can be treated as a non-forward start callable Asian option with the strike price to be set to the observed spot price on the forward start date. If the valuation date is after the call date then the callable Asian option becomes a standard Asian option with price equal to

$$
\begin{aligned}
&V(t) \\
&= df_{t,T} \times E^*\left[I_N \cdot N + \frac{N}{K} \times \max\{0, \beta(A-K)\} \mid F_t\right] \\
&= df_{t,T} \times I_N \cdot N + \frac{N}{K} \times E^*\left[\max\{0, \beta(A-K)\} \mid F_t\right] \\
&= df_{t,T} \times I_N \cdot N + \frac{N}{K} \times MichaelCurranFormula(t)
\end{aligned}
$$

Valuation date on or after the last fixing date

$$c(t) \approx df_{t,T}\left\{\bar{S}_t e^{r_t(T-t)} - K\right\}$$
$$p(t) \approx df_{t,T}\left\{K - \bar{S}_t e^{r_t(T-t)}\right\}.$$

Forward Start Asian Options

Forward start Asian option is like a standard Asian option but with the strike price being determined as a predetermined multiple of the underlying asset price at some predetermined date (the strike reset date).

Define:

$n_a =$ number of underlying assets.

$n_f =$ number of fixings.

$m =$ number of observed fixings.

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$S_i(t) =$ i by 1 array of asset prices i at time t.
$\sigma_{S_i}(t_j) =$ i by j matrix of volatilities of asset i at time t_j.
$\rho_{ij} =$ i by i matrix of instantaneous correlations of $\ln S_i$ and $\ln S_j$.
$r_{C_i}(t_j) =$ i by j matrix of instantaneous risk-free interest rates in C_i market.
$b_i(t_j) =$ i by j matrix of cost of carries for asset i at time t_j.
$t_f =$ time of the strike reset date.
$\lambda_i =$ the strike reset ratio of asset i.
$K =$ strike price is payoff currency.
$W =$ Wiener process.

Option Payoff:

$$\max\left(\beta \cdot \sum_{j=1}^{n_a} w_j^a \left(\frac{\sum_{i=1}^{n_f} w_i^f S_j(t_i)}{K_j} - 1\right), 0\right).$$

$\beta = 1$ if it is a call and $\beta = -1$ if it is a put.

Pricing Formulas:

Case 1: $t_0 < t_f$

Define:

$$W_1 = \sum_{j=1}^{n_a} w_j^a \lambda_j.$$

$$\overline{w}_j = \frac{w_j \lambda_j}{W_1}.$$

$$R_j(t_f, t_i) = \frac{S_j(t_i)}{S_j(t_f)}.$$

$$K_1 = \frac{1}{W_1}.$$

The payoff in this case can be expressed as:

$$\max\left(W_1 \cdot \left[\beta \cdot \left(\sum_{j=1}^{n_a} \sum_{i=1}^{n_f} \overline{w}_j w_i^a R_j(t_f, t_i) - K_1\right)\right], 0\right).$$

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$R_j(t_f, t_i)$ can be expressed as:

$$R_j(t_f, t_i) = e^{\int_{t_f}^{t_i}\left(b_j(s)-\frac{1}{2}\sigma_j^2(s)\right)ds+\int_{t_f}^{t_i}\sigma_j(s)dW_j(s)}.$$

The relative price $R_j(t_f, t_i)$ can be viewed as the price of an asset with same dynamics as the asset price $S_j(t)$ but with the price starts from 1 at time t_f. The above payoff is exactly the same as the Asian basket option and can be priced using the Asian basket option formula.

Case 2: $t_0 \geq t_f$

Define:

$$W_2 = \sum_{j=1}^{n_a} \frac{w_j \lambda_j}{S_j(t_f)}.$$

$$\hat{w}_j = \frac{w_j \lambda_j}{W_2 S_j(t_f)}$$

$$K_2 = \frac{1}{W_2}.$$

The payoff in this case can be expressed as:

$$\max\left(W_2 \cdot \left[\beta \cdot \left(\sum_{j=1}^{n_a}\sum_{i=1}^{n_f} \hat{w}_j w_i^a S_j(t_i) - K_2\right)\right], 0\right).$$

The payoff is the same as the Asian basket case. The price can be evaluated using the Asian basket case formula.

Note for Asian forward starting option formula can be applied to the case with quanto features. Same adjustment for the drift term as in the quanto case is used to modify the formula.

Forward cost-of-carries and volatilities

Since $R_j(t_f, t_i)$ is from time t_f, we need the forward cost of carries and volatilities. The formulas to obtain these numbers are:

$$b_{t_f, t_i} = \frac{b_{t_i} t_i - b_{t_f} t_f}{t_i - t_f}$$

$$\sigma_{t_f, t_i}^2 = \frac{\sigma_{t_i}^2 t_i - \sigma_{t_f}^2 t_f}{t_i - t_f}$$

Quanto case

Callable Asian option with quanto feature is the same except the cost of carries needs to be adjusted as in the quanto Asian basket options case, i.e. $b_i^*(t_j) = b_i(t_j) - \rho_i \cdot \sigma_{S_i}(t_j) \cdot \sigma_{X_i}(t_j)$. Note that cost of carries should be adjusted for quanto before converting to forward rates.

Foreign Exchange Fixed Strike/Floating Strike/Double Average Rate Asian Options

The pricing formula is exactly the same as equity Asian options with the cost of carry replaced by $r_d - r_f$ (if there is no dividend payout) where r_d is the domestic risk-free rate and r_f is the foreign risk-free rate.

Foreign Exchange Reciprocal Averaging Asian Options
The above adjustment for foreign exchange Asian options cannot be used for reciprocal averaging Asian options. The reason is because the numeraire used in foreign exchange reciprocal averaging Asian options is foreign currency rather than the domestic currency. However, the adjustments required for reciprocal averaging are still small.

Recall that in equity reciprocal Asian option model,
$F_{t_i} = E^*[S_{t_i}] = S_t e^{b_{t_i}(t_i - t)}$.
The first adjustment is that there is an extra term for futures price, i.e. $F_{t_i} = E^*[S_{t_i}] = S_t e^{(b_{t_i} + \sigma_{t_i}^2)(t_i 0 t)}$, where $b_{t_i} = r_{t_i}^d - r_{t_i}^f$ for the foreign exchange case if there is not dividend payout and r^d and r^f are domestic and foreign risk-free rates respectively. We then use the equity reciprocal formula to calculate the option price with the changes of the above adjustments.

The second adjustment required is the resultant price based on equity reciprocal formula needs to multiply the spot exchange rate on the valuation date and the foreign discount factor to obtain the final foreign exchange reciprocal Asian option price. The exchange rate is quoted as domestic/foreign.

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15.7.7 Appendix (Proofs of the Identities for Double Average Rate Options):

1. $$E^*\left(\bar{S}^{(1)}_{t_{n_1}}\right) = \bar{S}^{(1)}_{t_{m_1}} + \sum_{i=m_1+1}^{n_1} \alpha_i^{(1)} E^*\left(S^{(1)}_{t_i}\right)$$

$$E^*\left(\bar{S}^{(1)}_{t_{n_1}}\right)$$

$$= E^*\left(\sum_{i=1}^{n_1} \alpha_i S^{(1)}_{t_i}\right)$$

$$= E^*\left(\sum_{i=1}^{m_1} \alpha_i S^{(1)}_{t_i} + \sum_{i=m_1+1}^{n_1} \alpha_i S^{(1)}_{t_i}\right)$$

$$= S^{(1)}_{t_{m_1}} + \sum_{i=m_1+1}^{n_1} \alpha_i E^*\left(S^{(1)}_{t_i}\right)$$

2. $$E^*\left(\bar{S}_{t_{n_1}}^{(1)^2}\right)=\bar{S}_{t_{m_1}}^{(1)^2}+2\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right)+\sum_{i=m_1+1}^{n_1}\sum_{j=m_1+1}^{n_1}\alpha_i^{(1)}\alpha_j^{(1)}E^*\left(S_{t_1}^{(1)}S_{t_1}^{(1)}\right)$$

$$E^*\left(\bar{S}_{t_{n_1}}^{(1)^2}\right)$$

$$=E^*\left(\left(\sum_{i=1}^{m_1}\alpha_i^{(1)}S_{t_i}^{(1)}+\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\right)^2\right)$$

$$=E^*\left(\left(\bar{S}_{t_{m_1}}^{(1)}+\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\right)^2\right)$$

$$=E^*\left(\bar{S}_{t_{m_1}}^{(1)^2}+2\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}+\left(\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\right)^2\right)$$

$$=\bar{S}_{t_{m_1}}^{(1)^2}+2\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right)+E^*\left(\left(\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\right)^2\right)$$

$$=\bar{S}_{t_{m_1}}^{(1)^2}+2\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right)+E^*\left(\sum_{i=m_1+1}^{n_1}\sum_{j=m_1+1}^{n_1}\alpha_i^{(1)}\alpha_j^{(1)}S_{t_i}^{(1)}S_{t_j}^{(1)}\right)$$

$$=\bar{S}_{t_{m_1}}^{(1)^2}+2\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right)+\sum_{i=m_1+1}^{n_1}\sum_{j=m_1+1}^{n_2}\alpha_i^{(1)}\alpha_j^{(1)}E^*\left(S_{t_i}^{(1)}S_{t_j}^{(1)}\right)$$

3.

$$E^*\left(\bar{S}_{t_{n_1}}^{(1)}\bar{S}_{t_{n_2}}^{(2)}\right)$$

$$=\bar{S}_{t_{m_1}}^{(1)}\bar{S}_{t_{m_2}}^{(2)}+\bar{S}_{t_{m_1}}^{(1)}\sum_{j=m_2+1}^{n_2}\alpha_j^{(2)}E^*\left(S_{t_j}^{(2)}\right)+\bar{S}_{t_{m_2}}^{(2)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left(S_{t_i}^{(1)}\right)+\sum_{i=m_1+1}^{n_1}\sum_{j=m_2+1}^{n_2}\alpha_i^{(1)}\alpha_j^{(2)}E^*\left(S_{t_i}^{(1)}S_{t_j}^{(2)}\right)$$

$$E^*\left(\bar{S}_{t_{n_1}}^{(1)}\bar{S}_{t_{n_2}}^{(2)}\right)$$

$$=E^*\left(\left(\sum_{i=1}^{m_1}\alpha_i^{(1)}S_{t_i}^{(1)}+\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\right)\left(\sum_{i=1}^{m_2}\alpha_i^{(2)}S_{t_i}^{(2)}+\sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}S_{t_j}^{(2)}\right)\right)$$

$$=E^*\left(\sum_{i=1}^{m_1}\alpha_i^{(1)}S_{t_i}^{(1)}\sum_{i=1}^{m_2}\alpha_i^{(2)}S_{t_i}^{(2)}+\sum_{i=1}^{m_1}\alpha_i^{(1)}S_{t_i}^{(1)}\sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}S_{t_i}^{(2)}+\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\sum_{i=1}^{m_2}\alpha_i^{(2)}S_{t_i}^{(2)}\right.$$

$$\left.+\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}\sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}S_{t_i}^{(1)}\right)$$

$$=E^*\left(\bar{S}_{t_{m_1}}^{(1)}\bar{S}_{t_{m_2}}^{(2)}+\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}S_{t_i}^{(2)}+\bar{S}_{t_{m_2}}^{(2)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}S_{t_i}^{(1)}+\sum_{i=m_1+1}^{n_1}\sum_{i=m_2+1}^{n_2}\alpha_i^{(1)}\alpha_i^{(2)}S_{t_i}^{(1)}S_{t_i}^{(2)}\right)$$

$$=\bar{S}_{t_{m_1}}^{(1)}\bar{S}_{t_{m_2}}^{(2)}+\bar{S}_{t_{m_1}}^{(1)}\sum_{i=m_2+1}^{n_2}\alpha_i^{(2)}E^*\left(S_{t_i}^{(2)}\right)+\bar{S}_{t_{m_2}}^{(2)}\sum_{i=m_1+1}^{n_1}\alpha_i^{(1)}E^*\left[S_{t_i}^{(1)}\right]+\sum_{i=m_1+1}^{n_1}\sum_{i=m_2+1}^{n_2}\alpha_i^{(1)}\alpha_i^{(2)}E^*\left[S_{t_i}^{(1)}S_{t_j}^{(2)}\right]$$

4. $$\mu=w_1E^*\left(\bar{S}_{t_{n_1}}^{(1)}\right)-w_2E^*\left(\bar{S}_{t_{n_2}}^{(2)}\right)$$

$$\mu$$

$$=E^*(X)$$

$$= E^*\left(w_1\overline{S}_{t_{n_1}}^{(1)} - w_2\overline{S}_{t_{n_2}}^{(2)}\right)$$
$$= w_1E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right) - w_2E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)$$

5.

v

$$= w_1^2\left(E^*\left(\overline{S}_{t_{n_1}}^{(1)^2}\right) - E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right)^2\right) + w_2^2\left(E^*\left(\overline{S}_{t_{n_2}}^{(2)^2}\right) - E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)^2\right)$$
$$- 2w_1w_2\left(E^*\left(\overline{S}_{t_{n_1}}^{(1)}\overline{S}_{t_{n_2}}^{(2)}\right) - E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right)E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)\right)$$

v

$$= \operatorname{var}(X)$$
$$= \operatorname{var}\left(w_1\overline{S}_{t_{n_1}}^{(1)} - w_2\overline{S}_{t_{n_2}}^{(2)}\right)$$
$$= w_1^2 \operatorname{var}\left(\overline{S}_{t_{n_1}}^{(1)}\right) + w_2^2 \operatorname{var}\left(\overline{S}_{t_{n_2}}^{(2)}\right) - 2w_1w_2Cov\left(\overline{S}_{t_{n_1}}^{(1)}, \overline{S}_{t_{n_2}}^{(2)}\right)$$
$$= w_1^2\left(E^*\left(\overline{S}_{t_{n_1}}^{(1)^2}\right) - E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right)^2\right) + w_2^2\left(E^*\left(\overline{S}_{t_{n_2}}^{(2)^2}\right) - E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)^2\right)$$

$$- 2w_1w_2\left(E^*\left(\overline{S}_{t_{n_1}}^{(1)}\overline{S}_{t_{n_2}}^{(2)}\right) - E^*\left(\overline{S}_{t_{n_1}}^{(1)}\right)E^*\left(\overline{S}_{t_{n_2}}^{(2)}\right)\right)$$

15.8 Equity Swap

15.8.1 Contract Definition

An equity swap is an agreement between two parties where one party agrees to make payments based on the return of equities and another party agrees to make payments based on the return of a fixed/floating interest rate plus a spread (or less usually based on the return of other equities). There is no exchange of notional at start or end.

15.8.2 Definition for Notations

t = valuation date.

S_t = price of the equity at time t.

T_p^m = the payment date associated with the m^{th} reset period.

T_s^m = the reset date associated with the m^{th} reset period.

$r_{t \to m}$
=risk-free rate applied for time m based on the simulated term structure at time t.

$q_{t \to m}$
=dividend yield applied for time m based on the term structure at time t.

df_{t,t_j} = discount factor from time t to time t_j.

T_s^0 = the last reset date before time t.
S_0
= the last reset price of the equity before time t.
$f_{t,t_1 \to t_2}$ =
the forward interest rate applied for time t_2 based on the term structure at time t_1.
f_{fix} = fixed interest rate.
r^0 = the last reset interest rate before time t.
N = the notional amount.
fx_t = spot foreign exchange rate (Domestic/Foreign) at valuation date t.
n = number of cash flows.
$E^*[\ ]$ = expectation under risk-neutral measure.
$E^f[\ ]$ = expectation under forward martingale measure.
B_t = savings account at time t.

Note that $f_{t,t_1 \to t_2} = E^f\left[r_{t_1 \to t_2} \mid F_t\right]$.

15.8.3 Valuation

Equity swaps consist of two streams of cash flows: cash flows from equity (equity leg) and cash flows from interest rate (interest rate leg). The parties under the contract receive one stream of cash flows and pay the other stream of cash flows. Thus the value of the equity swaps at any valuation date can be calculated as the difference between the values of the equity leg and interest rate leg at the valuation date. We demonstrate on how to calculate the values of the two legs in the following.

Equity Leg

The return of the equity leg for the m^{th} reset period for is $\frac{S_{T_s^m}}{S_{T_s^{m-1}}} - 1$.

The value of $\frac{S_{T_s^m}}{S_{T_s^{m-1}}} - 1$ at the valuation date t is:

$$E^*\left[\frac{B_t}{B_{T_p^m}}\left(\frac{S_{T_s^m}}{S_{T_s^{m-1}}} - 1\right) \mid F_t\right]$$

$$= df_{t,T_p^m} E^f\left[\frac{S_{T_s^m}}{S_{T_s^{m-1}}} - 1 \mid F_t\right]$$

$$\approx df_{t,T_p^m}\left[\exp\left(\left(r_{t \to T_s^m} - q_{t \to T_s^m}\right)\left(T_s^m - t\right) - \left(r_{t \to T_s^{m-1}} - q_{t \to T_s^{m-1}}\right)\left(T_s^{m-1} - t\right)\right) - 1\right].$$

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Note that df_{t,T_s^m} is derived from the zero-coupon bond price.

If $m=1$ and $T_s^0 \le t < T_s^1$ then the value for the first reset period of the equity leg can be derived similarly as:

$$df_{t,T_p^1}\left[\frac{S_t \exp\left(\left(r_{t\to T_s^1} - q_{t\to T_s^1}\right)\left(T_s^1 - t\right)\right)}{S_0} - 1\right].$$

To get the value of the whole equity leg, we simply sum each equity return value for each reset period. We also need to multiply by the foreign exchange rate if cross currency is present.

Thus, to summarise, the present value of the equity leg at valuation date t taking the cross currency into consideration is:

Case 1: $T_m^0 \le t < T_m^1$

$$\begin{aligned}
& pv_{equity}(t) \\
& = \sum_{k=2}^{n} df_{t,T_p^m} \times N \times fx_t \\
& \times \left(\exp\left(\left[\left(r_{t\to T_s^m} - q_{t\to T_s^m} \right)\left(T_s^m - t\right) \right] - \left[\left(r_{t\to T_s^{m-1}} - q_{t\to T_s^{m-1}} \right)\left(T_s^{m-1} - t\right) \right] \right) - 1 \right) \\
& + df_{t,T_p^1} \times N \times \left(\frac{S_t \exp\left(\left(r_{t\to T_s^1} - q_{t\to T_s^1}\right)\left(T_s^1 - t\right)\right)}{S_0} - 1 \right) \times fx_t .
\end{aligned}$$

Case 2: $t < T_s^0$

$$\begin{aligned}
& pv_{equity}(t) \\
& = \sum_{m=1}^{n} df_{t,T_p^m} \times N \times fx_t \\
& \times \left(\exp\left(\left[\left(r_{t\to T_s^m} - q_{t\to T_s^m} \right)\left(T_s^m - t\right) \right] - \left[\left(r_{t\to T_s^{m-1}} - q_{t\to T_s^{m-1}} \right)\left(T_s^{m-1} - t\right) \right] \right) - 1 \right).
\end{aligned}$$

Interest Rate Leg

The way to calculate the value of interest rate leg is similar to equity leg, and is the same as the valuation of such a leg as part of an interest rate swap.

The return of the interest rate leg for the m^{th} reset period for is $\left(r_{T_s^{m-1}\to T_s^m} + spread\right)\left(T_m^s - T_m^{s-1}\right)$.

The value of the of $\left(r_{T_s^{m-1}\to T_s^m} + spread\right)\left(T_m^s - T_m^{s-1}\right)$ at the valuation date t is:

$$E^*\left[\frac{B_t}{B_{T_p^m}}\left(r_{T_s^{m-1}\to T_s^m}+spread\right)\left(T_m^s-T_m^{s-1}\right)|F_t\right]$$
$$=df_{t,T_p^m}E^f\left[\left(r_{T_s^{m-1}\to T_s^m}+spread\right)\left(T_s^m-T_s^{m-1}\right)|F_t\right]$$
$$=df_{t,T_p^m}\left(f_{t,T_s^{m-1}\to T_s^m}+spread\right)\left(T_s^m-T_s^{m-1}\right)$$

If $m=1$ and $T_s^0\le t<T_s^1$ then the value for the first reset period of the interest rate leg can be modified as:
$df_{t,T_{1,p}}\times N\times\left(r^0+spread\right)\times\left(T_s^1-T_s^0\right)\times fx_t$.

To get the value of the whole interest rate leg, we simply sum each interest rate return value for each reset period. We also need to multiply by the foreign exchange rate if cross currency is present.

Thus, to summarise, the present value of the interest rate leg at valuation date t taking the cross currency into consideration is:

Case1: $T_s^0\le t<T_s^1$
The present value if it is floating interest rate equals:
$$pv_{float}(t)$$
$$=\sum_{m=2}^{n}df_{t,T_p^m}\times N\times\left(\left(f_{t,T_s^{m-1}\to T_s^m}+spread\right)\times\left(T_s^m-T_s^{m-1}\right)\right)\times fx_t$$
$$+df_{t,T_{1,p}}\times N\times\left(r^0+spread\right)\times\left(T_s^1-T_s^0\right)\times fx_t\,.$$

Case 2: $t<T_s^0$
$$pv_{float}(t)$$
$$=\sum_{m=1}^{n}df_{t,T_p^m}\times N\times\left(\left(f_{t,T_s^{m-1}\to T_s^m}+spread\right)\times\left(T_s^m-T_s^{m-1}\right)\right)\times fx_t\,.$$

If the interest rate leg pays fixed interest rate, then it is much simpler.

The present value if it is fixed interest rate equals:
$$pv_{fix}(t)$$
$$=\sum_{m=1}^{n}df_{t,T_p^m}\times N\times\left(\left(f_{t,T_s^{m-1}\to T_s^m}+spread\right)\times\left(T_s^m-T_s^{m-1}\right)\right)\times fx_t\,.$$

In general, the present value of the equity swap at time t is:
$pv(t)=pv_{receivingLeg}(t)-pv_{payingLeg}(t)$.

For example, if a party enters an equity swap by paying floating interest rate and receiving equity return, the preset value of the equity swap at time t is:

$$pv(t) = pv_{equity}(t) - pv_{floating}(t).$$

Within Razor current support for equity swaps, there is consideration for discrete dividend payments associated with equity leg and similarly no support for compounding interest rate leg.

Chapter 16
Credit Derivatives

16.1 Credit Default Swaps

16.1.1 Description of Instrument

Credit Default Swaps are products in which the purchaser of protection makes periodic payments in exchange for the payment of the protection amount contingent on there being a default event. What constitutes a default event is defined within the CDS documentation.

16.1.2 XML Representation

Credit Default Swaps became a standard in FPML version 4.

The schema for fpmlCreditDefaultSwap is defined as follows

fpmlCreditDefaultSwap Schema			
Name: Type	**Occurs**	**Size**	**Description**
productType fpmlProductType	0..1		Indicates the type of product
generalTerms fpmlGeneralTerms	1..1		Defines dates and reference obligation of the CDS.
feeLeg fpmlFeeLeg	1..1		Premium payment definition.
protectionTerms fpmlProtectionTerms	1..1		This is where the credit events and obligations that are applicable to the credit default swap trade are specified.
cashSettlementTerms fpmlCashSettlementTerms	0..1		Cash settlement details.
physicalSettlementTerms fpmlPhysicalSettlementTerms	0..1		Physical settlement details.

Note the fee leg contains the stub position implicitly (as per fpml spec). There are 5 stub position types: none, ShortFirst, LongFirst, ShortFinal, LongFinal.

ShortFirst and LongFirst require a firstPaymentDate.

ShortFinal and LongFinal require lastRegularPaymentDate.

A sample representation is as follows:

```
  <creditDefaultSwap>
<generalTerms>
 <effectiveDate>
```

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```
        <unadjustedDate>2006-10-18</unadjustedDate>
    <dateAdjustments>
        <businessDayConvention>NONE</businessDayConvention>
      </dateAdjustments>
    </effectiveDate>
    <scheduledTerminationDate>
        <adjustableDate>
        <unadjustedDate>2007-01-18</unadjustedDate>
         <dateAdjustments>
         <businessDayConvention>NONE</businessDayConvention>
      </dateAdjustments>
     </adjustableDate>
    </scheduledTerminationDate>
     <sellerPartyReference href="ECHIDNA" />
     <buyerPartyReference href="PLATYPUS" />
     <dateAdjustments>
      <businessDayConvention>NONE</businessDayConvention>
    </dateAdjustments>
    <referenceInformation>
     <referenceObligation>
      <primaryObligor id="KANGAROO" />
     </referenceObligation>
    </referenceInformation>
  </generalTerms>
  <feeLeg>
    <periodicPayment>
     <paymentFrequency>
      <period>M</period>
      <periodMultiplier>3</periodMultiplier>
     </paymentFrequency>
     <rollConvention>NONE</rollConvention>
     <fixedAmount>
      <currency>EUR</currency>
      <amount>0</amount>
     </fixedAmount>
     <fixedAmountCalculation>
      <dayCountFraction>ACT/360</dayCountFraction>
      <calculationAmount>
       <currency>EUR</currency>
       <amount>0</amount>
      </calculationAmount>
      <fixedRate>0.017200</fixedRate>
     </fixedAmountCalculation>
    </periodicPayment>
  </feeLeg>
 <protectionTerms>
  <calculationAmount>
   <currency>EUR</currency>
 <amount>10000000.00</amount>
  </calculationAmount>
 </protectionTerms>
</creditDefaultSwap>
```

16.1.3 Credit Implications

Pricing Contexts

There are two or more contexts that a credit default swap has when pricing. There is an exposure to the issuer of the Credit Default Swap contract, and there are exposures to the counterparties that the swap is providing the cover against.

Timing of payment on default

Counterparties typically wait up to three months after the default event has occurred in order to give the price of the reference security time to settle at a new level.

Physical delivery

Often there will be physical delivery of the reference security upon default, and there could be liquidity risk that the writer of the credit default swap incurs upon trying to purchase the reference security in order to satisfy this physical delivery requirement.

16.1.4 CDS Pricing

A credit default swap (CDS) is a financial contract in which the buyer of credit protection pays a periodic premium in return for a promise from the credit protection seller to compensate the default losses on some agreed upon third party reference credit obligation.

CDS's are by far the most important instruments enabling credit risk transfer. Although a CDS is now viewed as a relatively simple instrument, it continues to play the essential role as the key building block in a wide range of more complex credit structures.

The cash flow profile of a single name credit defaults swap consists of two legs:

Premium Leg: the buyer of protection pays periodic payments to the protection seller until the earlier of a credit event or maturity of the CDS contract.

Protection Leg: the seller of protection pays the difference between par and the recovery value of the referenced portfolio exposure value should a credit event occur during the contract.

The value of a CDS protection seller is given as the risk-neutral expected value of the present value of premium leg less the protection leg.

At the inception of a CDS, the premium is set such that the value of the Premium Leg must equal the Protection Leg. i.e., on-market CDS must have zero net present value.

Mathematically, the value of two legs can be expressed as:

The risk-neutral expected value of present value of the Premium Leg:

$$\text{Premium\_Leg} = \sum_{i=1}^{M} \left[\{ d_{T_i} \, p(T_0, T_i) \} \tau_i S \{ N(T_{i-1}) \alpha(T_{i-1}, T_i) \} \right]$$

(1)

Where:

T_0	- is the date of CDS valuation,
$T_0 < T_1 < \ldots < T_M$	- in arrears premium payment dates,
τ_i	- i^{th} payment period as a fraction of years,
S	- per-annum spread for this CDS,
$p(T_0,T_i)$	- is the risk-neutral survival probability of the reference entity from time T_0 to T_i, by convention $p(T_0,T_i) = p(T_i)$ if T_0 is the valuation date.
$N(T_i)$	- Notional value of the CDS at time T_i,
$\alpha(T_{i-1},T_i)$	- is the accrual factor from T_{i-1} to T_i,
d_T	- risk free discount factor from time T_0 to T

The risk-neutral expected value of present value of the Protection Leg:

$$\text{Protection\_Leg} = \sum_{i=1}^{D} \left[d_{t_i} \{ (p(T_0, t_{i-1}) - p(T_0, t_i) \} Loss(VR(t_i)) \right]$$

(2)

Where:

$T_0 < t_1 < \ldots\ldots < t_D$	- credit event dates and their modeled periodicity can be different from the premium payment dates.
$Loss(VR(t_i))$	- the amount of loss occurring on reference credit value at time t_i.

The value of CDS for protection seller is then the difference between a premium (fee) leg and protection (loss) leg:

$$ValueCDS(T_0) = \text{Premium\_Leg} - \text{Protection\_Leg} \,. \qquad (3)$$

The price of a credit default swap will reflect several factors. The key inputs would include the following:

The risk-neutral probability of default of the reference asset or alternatively its survival probabilities $p(T_0,T_i)$ should be provided as an input term structure.

In practice these probabilities are derived from the CDS premium spread values $S(T_i)$ - constant or time dependent and known for existing CDS contracts.

Risk free discount rates determined from the appropriate curve.

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$Loss(VR(t_i))$ - loss of an underlying referenced asset defined as some deterministic function that is usually modeled as:

$$Loss(VR(t_i)) = (1 - R(t_i))N(t_i)$$

where $R(t_i)$ - modeled recovery rate at time t_i.

Recovery rate must be assumed or estimated separately as the (ideal) joint calibration on default rates and recovery. In Razor it is currently assumed to be constant.

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Calibration.

In the pricing of credit derivatives it is assumed that default events follow a Poisson distribution and hence the survival probability can be expressed as:

$$SP(t_0, T) = e^{-\int_{t_0}^{T} \lambda(s)\,ds}$$

Where $\lambda(t)$ is called the default intensity, or the instantaneous forward default rate, or hazard rate.

The CDS instrument prices this default risk. Hence, from the observed CDS spreads in the market we can derive the risk-neutral survival probabilities.

There are two approaches to tackling this problem, the parametric bootstrapping and piece wise constant bootstrapping. Both methods are described below. Currently Razor implements the parametric bootstrapping for CDS curve calibration.

Parametric Bootstrapping

It is assumed there exists a functional form of the survival probability curve. Inputting observed CDS market prices results in a system of linear equations. This can be solved to determine the parameters of the original functional form of the survival probability curve.

This approach has the advantage of controlling the possible resulting curve shapes combined with its smoothing features.

Razor employs the following functional form:

$$S(t) = \sum_{j=1}^{N} \frac{\alpha_j}{\sqrt{(t - t_j)^2 + \xi_j^{\,2}}} \qquad (4)$$

where

$t_1, t_2, \ldots, t_N$ - tenors of observed CDS spreads quotes,

N - number of quotes,

$\xi_1, \ldots, \varsigma_N$ - weights that are used to control a degree of confidence in the accuracy to target CDS spread observations.

The direct substitution of (4) into equations (1)-(3) leads to a system of linear algebraic equations that can be easily solved against unknown parameters $\alpha_1, \ldots, \alpha_N$ using standard methods of linear algebra.

Such a representation allows for the exact fitting of the term structure to the underlying CDS spread quotes, delivers a smooth solution even for a limited number of market observations and allows for the avoidance of kinks especially at the short and long ends of credit curve where market observations are often unavailable.

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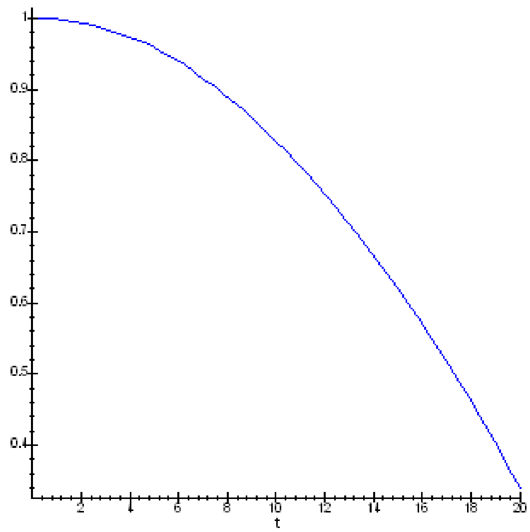
As an example, the following survival probability curve is built from three CDS spreads of 3, 5 and 7 years maturity:

$S_3 = 34.76,$

$S_5 = 57.27,$

$S_7 = 82.42$



Piece-Wise Constant Hazard Rate Bootstrapping

The hazard rate is assumed piece-wise constant and starting from the shortest maturity is 'bootstrapped' similarly to the bootstrapping procedure for the interest rate term structure.

The iterative algorithm can be described by the following procedure:

- Assume a constant hazard rates $\lambda(t)$ for all time intervals $t_i - t_{i-1}$ with $t_i, i = 1,..., M$

representing intermediate payment dates and maturity dates of all CDS quotes.

Assume that we successfully determined values of $\lambda_1,...,\lambda_k, k < M$ hazard rates.

This also guarantees the knowledge of all survival probabilities $S(t_0, t_i), i = 0,1,..., k$.

Note that:

$$S(t_0, t_{i+1}) = \frac{B_{i+1}}{A_{i+1}},$$

with

$$A_{k+1} = \bar{S}_{k+1}\tau_{k+1}d_{k+1}p(t_k) + (1-R)d_{k+1}p(t_k)\ ,$$

$$B_{k+1} = -\bar{S}_{k+1}\tau_{k+1}\sum_{i=1}^{k} d_{t_i} p(t_i) + (1-R)\left(\sum_{i=1}^{k} d_{t_i}\left(p(t_{i-1}) - p(t_i)\right) + d_{t_k} p(t_k)\right),$$

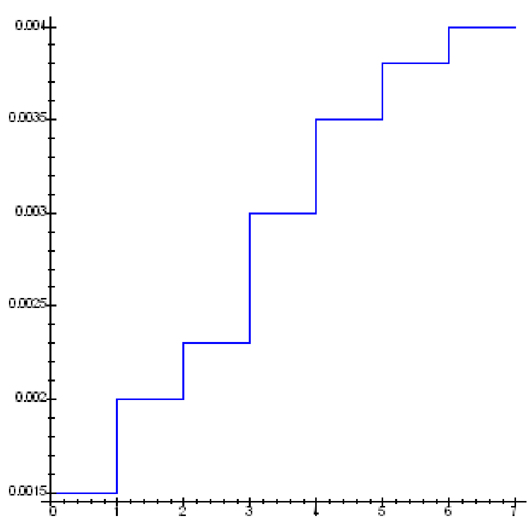
where

τ_{k+1}	- fee payment period of the k+1 CDS spread,
d_t	- risk free discount factor up to t,
p(t)	- survival probabilities up to time t,
$\bar{S}_{k+1}$	- k+1 (st) CDS spread value taken either from the available market quotes or linearly interpolated from them
R	- constant recovery rate.

Then λ_{k+1} hazard value can be found as the solution of the following equation:

$$\lambda_{k+1} = -\ln\left(\frac{S(t_0, t_{k+1})}{S(t_0, t_k)}\right) / \tau_{k+1}\ .$$

The resulting hazard curve can have the following step ladder form:



16.1.5 Configuration of CDS Spread Curve

The user has the option of specifying a CDS Spread Curve as containing CDS spreads, or to specify the probabilities directly as either survival or default rates. In the case of survival or default rates no market calibration is performed. A CDS curve must contain only one CDS Quote Type within its structure. A CDS curve has a recovery rate associated with it.

The bootstrapping method for a CDS curve is derived from the QUOTE TYPE of the “CdsSpread“ asset as follows:

SURVIVAL_RATE - actual survival rates are taken directly

DEFAULT_RATE - survival rate = 1 – default rate

CDS_SPREAD - standard market CDS spread quotes used to compute survival rates

For example:

```xml
<asset id="DefaultProbability" type="cdsSpread">
   <cdsSpread>
         <cdsQuoteType>DEFAULT_RATE</cdsQuoteType>
         <frequency>
            <months>3</months>
         </frequency>
  </cdsSpread>
</asset>
```

and

```xml
<asset id="CdsSpread" type="cdsSpread">
   <cdsSpread>
      <cdsQuoteType>CDS_SPREAD</cdsQuoteType>
      <frequency>
         <months>3</months>
      </frequency>
    </cdsSpread>
</asset>
```

16.1.6 Pricing Parameters

To price a CDS the following pricing parameters must be set.

Name: Type	Description
CreditCurveName	The family of credit curves to use. Typically it would be the industry of the name. This is optional.
CreditRating*	The rating of the name. This effective adds another specifier the credit curve name. This is optional.

* If this parameter is not present it will attempt to obtain the credit rating from the trade value:

creditDefaultSwap/generalTerms/referenceInformation/referenceObligationAt[0]/seniority

The CDS curve selected from the market can depend on the name and its rating. The curve name will be built according to the base CDS type, CreditCurveName, and CreditRating. If either parameter is omitted it does not impact the requested curve.

"CdsSpread [CreditCurveName] [CreditRating]"

Examples:

"CdsSpread FINANCE A+"
"CdsSpread MINING EUR"
"CdsSpread MANUFACTORING"
"CdsSpread"

16.2 Collateralised Debt Obligation

16.2.1 Description of Instrument

A Collateralized Debt Obligation is a securitization product backed by an underlying portfolio of credit products called the collateral consisting of instruments such as loans or CDS. The resulting cashflows from this collateral is repackaged into a number of tranches providing a variety of risk return profiles to investors.

16.2.2 XML Representation

Collateralized Debt Obligations are not covered by FpML version 4 and form a razor extension.

The schema for rzmlCollateralizedDebtObligation is defined as follows:

fpmlCollateralisedDebtObligation Schema			
Name: Type	Occurs	Size	Description
productType fpmlProductType	0..1		Indicates the type of product
tranche rzmlTranche	1..1		Tranche being priced
paymentFrequency fpmlCalculationPeriodFrequency	1..1		Frequency of premium payment
effectiveDate xsd:date	1..1		CDO start date.
maturityDate xsd:date	1..1		CDO termination date.
referencePool rzmlReferencePool	1..1		Reference pool of collateral

A sample representation is as follows:

```
<collateralizedDebtObligation>
<effectiveDate>2006-02-01</effectiveDate>
```

```
<maturityDate>2011-02-01</maturityDate>
        <tranche>
              <attachment>0.05</attachment>
              <detachment>0.1</detachment>
<spreadRate>0.005</spreadRate>
<notional>
<amount>1000000.0</amount>
              <currency>EUR</currency>
              </notional>
        </tranche>
        <paymentFrequency>
<periodMultiplier>M</periodMultiplier>
              <period>3</period>
              <rollConvention>NONE</rollConvention>
        </paymentFrequency>
        <referencePool>
              <collateral>
                <creditDefaultSwap>
                        <generalTerms>
                                <effectiveDate>
                                <unadjustedDate>2006-10-18</unadjustedDate>
                                        <dateAdjustments>
                                        <businessDayConvention>
                                                              NONE
                                        </businessDayConvention>
                                </dateAdjustments>
                        </effectiveDate>
                                <scheduledTerminationDate>
                                <adjustableDate>
                                        <unadjustedDate>2007-01-18</unadjustedDate>
                                                <dateAdjustments>
                                                <businessDayConvention>
              NONE
</businessDayConvention>
                                        </dateAdjustments>
                                </adjustableDate>
                        </scheduledTerminationDate>
                        <sellerPartyReference href="ECHIDNA" />
                        <buyerPartyReference href="PLATYPUS" />
                                <dateAdjustments>
                                <businessDayConvention>
              NONE
</businessDayConvention>
                        </dateAdjustments>
                                <referenceInformation>
                                        <referenceObligation>
                                                <primaryObligor id="KANGAROO" />
                                </referenceObligation>
                        </referenceInformation>
              </generalTerms>
                        <feeLeg>
                                <periodicPayment>
                                        <paymentFrequency>
                                        <period>M</period>
```

```xml
&lt;periodMultiplier&gt;3&lt;/periodMultiplier&gt;
&lt;/paymentFrequency&gt;
&lt;rollConvention&gt;NONE&lt;/rollConvention&gt;
&lt;fixedAmount&gt;
&lt;currency&gt;EUR&lt;/currency&gt;
&lt;amount&gt;0&lt;/amount&gt;
&lt;/fixedAmount&gt;
&lt;fixedAmountCalculation&gt;
&lt;dayCountFraction&gt;
ACT/360
&lt;/dayCountFraction&gt;
&lt;calculationAmount&gt;
&lt;currency&gt;EUR&lt;/currency&gt;
&lt;amount&gt;0&lt;/amount&gt;
&lt;/calculationAmount&gt;
&lt;fixedRate&gt;0.017200&lt;/fixedRate&gt;
&lt;/fixedAmountCalculation&gt;
&lt;/periodicPayment&gt;
&lt;/feeLeg&gt;
&lt;protectionTerms&gt;
&lt;calculationAmount&gt;
&lt;currency&gt;EUR&lt;/currency&gt;
&lt;amount&gt;10000000.00&lt;/amount&gt;
&lt;/calculationAmount&gt;
&lt;/protectionTerms&gt;
&lt;/creditDefaultSwap&gt;
&lt;/collateral&gt;
&lt;/referencePool&gt;
&lt;/collateralizedDebtObligation&gt;
```

16.2.3 Credit Implications

CDO are structured using a Special Purpose Vehicle (SPV) removing any credit exposure to the organizing entity. The CDO has a reference pool of assets with an income stream and exposure to potential losses due to credit events. The CDO defines tranches with attachment and detachment points [a, b] which reimburse credit event losses from a% to b% of the notional value of the CDO. The losses of a tranche of a lower attachment point must be fully absorbed before losses are allocated to the next tranche.

In return for this protection each tranche receives a premium on the non-defaulted notional amount it covers. This rate is higher for tranches with a lower attachment point due to the increased credit risk exposure.

The method for the allocation of premium and losses is fully defined in the waterfall documentation of the CDO.

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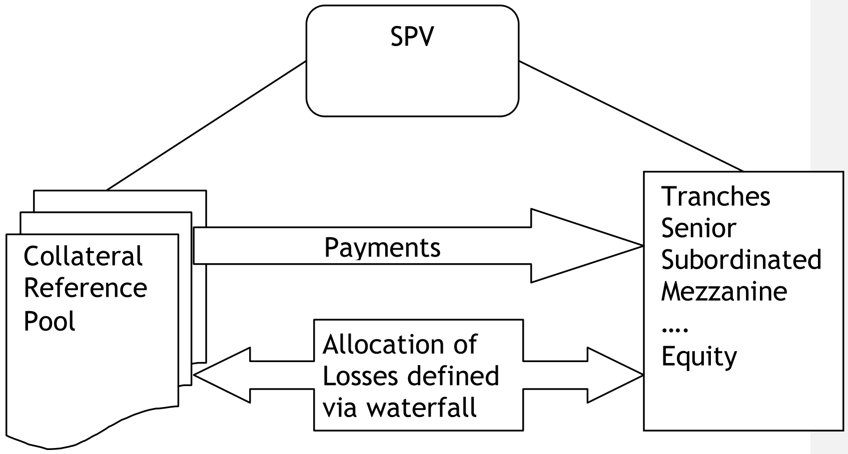

Originally CDO's were used to securitize a bank's portfolio of loans removing them from the balance sheet. All income such as interest payments are forwarded on to the holders of the tranches in the CDO. Any losses from defaults are allocated to the tranches defined by the waterfall structure.

The loan backed CDO form is now a small market which is now dominated by the Synthetic CDO which is a CDO whose collateral pool consists entirely of CDS. Razor CDO pricing assumes a Synthetic and static collateral pool, where the reference pool may not vary throughout the life of the CDO.

16.2.4 CDO Pricing

In this pricing framework, it is taken that the risk neutral probabilities can be bootstrapped from the CDS spreads (see CDS product). However in CDO pricing we consider the time to default τ and whether it is before a given time.

$P(\tau \leq t)$ - unconditional risk neutral probability the time to default is $\leq$ t.

The following definitions are required.

$t_1, \ldots, t_n = T$	- premium dates
n	- number of premium dates
t_0	- valuation date
$t_n = T$	- maturity
s_i	- effective premium rate paid over period $(t_{i-1}, t_i]$
$d_1, \ldots, d_n = T$	- discount factor corresponding to premium date
K	- number of names in collateral pool
$N^{(k)}$	- recovery adjusted notional for name k ($1 \leq k \leq K$)
J	- number of tranches
l	- attachment point of tranche being valued

u	- detachment point of tranche being valued
S = (u - l)	- size of tranche being valued (assuming a notional of 1)
L_i	- cumulative losses of the tranche being valued up to time t_i.
Λ_i	- cumulative losses of the entire reference pool up to time t_i.

We can express the losses allocated to a tranche at time t_i as:

$L_i = \min(S, \max(\Lambda_i - l, 0))$

Therefore the tranche losses is a function of the pool losses, we define this function to be $\Psi(\Lambda)$.

$\Psi(x) = \min(S, \max(x - l, 0))$

To price a CDO tranche, the risk neutral expected value of the discounted premiums, must equal to the risk neutral expected value of the discounted losses. This can be expressed as:

$$E^Q\left[\sum_{i=1}^{n} s_i(t_i - t_{i-1})(S - L_i)d_i\right] = E^Q\left[\sum_{i=1}^{n}(L_i - L_{i-1})d_i\right]$$

Hence during the life of the CDO when the premium has been set the value for a tranche buyer is:

$$V_{buy} = \sum_{i=1}^{n} s_i(t_i - t_{i-1})(S - E^Q[L_i])d_i - \sum_{i=1}^{n}(E^Q[L_i] - E^Q[L_{i-1}])d_i$$

It remains to determine the expected accumulated losses $E[L_i]$. The default process will be specified using a single factor Gaussian copula model to describe the correlation structure, with the credit events simulated independently.

Denote unconditional risk neutral probability the time to default of name k:

$$\hat{\pi}_i^{(k)} = P(\tau^{(k)} \le t_i)$$

Then conditional on the single credit driver x, the risk-neutral distribution of default time is given by:

$$\hat{\pi}_i^{(k)} = \Phi\left(\frac{\Phi^{-1}\left(\hat{\pi}_i^{(k)}\right) - \beta^{(k)}x}{\sigma^{(k)}}\right)$$

Where:

$$\int_{-\infty}^{\infty} \hat{\pi}_i^{(k)} d\Phi(x) = \hat{\pi}_i^{(k)}$$

Φ is the standard normal cumulative distribution function and Φ^{-1} is its inverse.

Define: $\overline{\pi}_i^{(k)}(x) = 1 - \hat{\pi}_i^{(k)}(x)$

Assuming the names are conditionally independent the mean tranche loss is:

$$E[L_i] = \int_{-\infty}^{\infty} E_x[L_i] d\Phi(x)$$

Where E_x is the expectation conditional on X = x.

Now dropping the i notation:

$$E[L] = E[\psi(\Lambda)]$$

$$E[L] = \sum_{l<y<u} [y - l] P(\Lambda = 1) + s\left[1 - \sum_{y<u} P(\Lambda = 1)\right]$$

Homogenous Model

Assuming a homogenous pool the number of defaults in (t_0, t_i] is binomially distributed.

Furthermore, assuming a large pool size, the binomial distribution can be approximated by a Poisson distribution.

Now defining $\hat{\lambda} = \sum_{k=1}^{K} \hat{\pi}^{(k)}$

Leads to the result:

$$E[L] = S\left(1 - e^{-\hat{\lambda}}\right) - e^{-\hat{\lambda}} \left\{ S \sum_{1<m<l/N^{(1)}} \frac{\hat{\lambda}^m}{m!} + \sum_{l/N^{(1)}<m<u/N^{(1)}} \frac{\hat{\lambda}^m}{m!} \left[u - mN^{(1)}\right] \right\}$$

Heterogenous Model

Extending this to the general case requires the following definitions:

$$N_* = \min_k N^{(k)}$$

$$N^* = \max_k N^{(k)}$$

$$f(N) = \sum_{k:N^{(k)}=N} \frac{\hat{\pi}^{(k)}}{\hat{\lambda}}$$

Where:

$$N_* \leq N \leq N^*$$

f^{*m} is the m-fold convolution of f with itself.

Then we arrive to the final result:

$$E[L] = S\left(1 - e^{-\hat{\lambda}}\right) - e^{-\hat{\lambda}} \left\{ S \sum_{1<m<l/N_*} \frac{\hat{\lambda}^m}{m!} \sum_{mN_*<N<l} f^{*m}(N) + \sum_{1<m<u/N_*} \frac{\hat{\lambda}^m}{m!} \sum_{l<N<u} [u - N] f^{*m}(N) \right\}$$

References

The pricing model was derived from the following sources:

J Hull and A White, 2003
Valuation of a CDO and an nth to default CDS without Monte Carlo simulation
Chaplin, 2005
Credit Derivatives

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I Iscoe, A Krenin and B De Prisco, 2005
Loss in translation
Risk June 2005

16.2.5 Pricing Parameters

The CDO pricing adapter uses the following pricing parameters.

Name: Type	Description
AttachmentCorrelation	Base correlation at the tranche attachment point. Default 0.
DetachmentCorrelation	Base correlation at the tranche detachment point. Default 0.
CdoModel	HomogenousPool - use the homogenous model. Otherwise (default) - use the heterogenous model.

The Homogenous model is faster but is restricted to cases where the reference pool is homogenous in nature. The heterogenous model whilst being more general is also slower to process.

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Chapter 17
Pricing

17.1 Distribution of Forward Rates

This section is a fundamental building block for pricing various interest rate derivatives in the following sections. We shall discuss on the dynamics of two types of forward rates, i.e. LIBOR rate and CMS rate, in this section and these are important for the pricing of interest rate derivative contracts in later sections.

17.1.1 Definition

$U(t,T_i)$ = any forward floating rate for the tenor period (T_{i-1},T_i) implied by the structure at time t.

$\sigma^U(t,T_i)$ = volatility function of the forward floating rate $U(t,T_i)$.

E^Q = expectation under risk-neutral measure for currency e.

$E^{Q^{T_i}}$ = expectation under forward martingale measure for currency e with respect to time T_i.

$\mathrm{F}=\{F_u\}_{0\le t\le T}$ = filtration generated by the Wiener process.

$B(t)$ = savings account at time t.

$B(t,T)$ = zero-coupon bond price at time t maturing at time T.

$\sigma(t,T)$ = zero-coupon bond volatility function.

$r(t)$ = short rate at time t.

W_t^Q = Wiener process under risk-neutral measure at time t.

$W_t^{Q^{T_i}}$ = Wiener process for currency e under forward martingale measure with respect to time T at time t.

N = notional amount.

s_i = spread of tenor period T_{i-1} to T_i.

T_i = i^{th} tenor date.

t = the valuation date.

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17.1.2 Pricing

For any interest rate derivative, we need to calculate the present value of the cashflow $U(T_{i-1},T_i)(T_i-T_{i-1})$ for the floating rate leg.

The expected present value of the cashflow is:

$$E^{Q_0}\left[\frac{B(t)}{B(T_i)}U(T_{i-1},T_i)(T_i-T_{i-1})\,|\,F_t\right]=(T_i-T_{i-1})B(t,T_i)E^{Q^{T_i}}\left[U(T_{i-1},T_i)\,|\,F_t\right].$$

Thus if we know $E^{Q^{T_i}}\left[U(T_{i-1},T_i)\,|\,F_t\right]$, we can find out the expected present value of the required cash flow for the floating rate leg of the swap. The computation of the expectation $E^{Q^{T_i}}\left[U(T_{i-1},T_i)\,|\,F_t\right]$ depends on whether the underlying forward rate is a LIBOR or CMS rate.

17.1.3 Forward LIBOR Rate

The distribution of forward LIBOR rate is:

$$dU(t,T_i)=\sigma^U(t,T_i)U(t,T_i)\,dW_t^{Q^{T_i}}.$$

Therefore:

$$E^{Q_0^{T_i}}\left[U(T_{i-1},T_i)\,|\,F_t\right]=U(t,T_i).$$

17.1.4 Forward CMS Rate

For CMS rate, we need convexity adjustment. Thus the expectation of CMS rate is:

$$E^{Q^{T_i}}\left[U(T_{i-1},T_i)\,|\,F_t\right]\approx U(t,T_i)-\frac{\sigma^U(t,T_i)^2}{2}\frac{f''(U(t,T_i))}{f'(U(t,T_i))}U^2(t,T_i)(T_i-t).$$

Note that $f(x)$ is the price-to-yield function and can be defined as:

$$f(x)=x_0\sum_{i=1}^{n}\frac{T_i-T_{i-1}}{(1+x)^{T_i-T_0}}+\frac{1}{(1+x)^{T_n-T_0}}.$$

The above convexity adjustment is exactly the same as the one in section 12.10.3.

17.2 Greeks Computation

17.2.1 Introduction

This section shows the Greeks of a generalised Black-Scholes option pricing formula.

17.2.2 Definition

c = price of call option.
p = price of put option.
S = underlying asset price.

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K	=	strike price.
b	=	cost of carry.
r	=	risk free rate.
r_f	=	foreign risk free rate.
q	=	continuous dividend yield.
T	=	time to maturity.
σ	=	price volatility.
$n(\cdot)$	=	standard normal probability density function.
$N(\cdot)$	=	cumulative standard normal distribution function.

17.2.3 Generalised Black-Scholes Formula

The call option price is:

$$c = Se^{(b-r)T}N(d_1) - Ke^{-rT}N(d_2).$$

The put option price is:

$$p = Ke^{-rT}N(-d_2) - Se^{(b-r)T}N(-d_1).$$

We define:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(b + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$ and

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(b - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}.$$

The generalised Black-Scholes formula can be summarised as:

$b = r$	gives the Black-Scholes (1973) stock option model.
$b = r - q$	gives the Merton (1973) stock option model with q.
$b = 0$	gives the Black (1976) futures option model.
$b = r - r_f$	gives the Garman and Kohlhagen (1983) currency option model.

17.2.4 Greeks

Delta

$$\Delta_{call} = \frac{\partial c}{\partial S} = e^{(b-r)T}N(d_1).$$

$$\Delta_{put} = \frac{\partial p}{\partial S} = -e^{(b-r)T}N(-d_1).$$

Gamma

$$\Gamma_{call} = \frac{\partial^2 c}{\partial S^2} = \frac{n(d_1)e^{(b-r)T}}{S\sigma\sqrt{T}}.$$

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$$\Gamma_{put} = \frac{\partial^2 p}{\partial S^2} = \frac{n(d_1)e^{(b-r)T}}{S\sigma\sqrt{T}}.$$

Rho

$$\rho_{call} = \frac{\partial c}{\partial r} = TKe^{-rT}N(d_2).$$

$$\rho_{put} = \frac{\partial p}{\partial r} = -TKe^{-rT}N(-d_2).$$

For option on foreign exchange options, the ρ^f on foreign risk free rate is:

$$\rho^f_{call} = \frac{\partial c}{\partial r_f} = -TSe^{(b-r)T}N(d_1) = -TSe^{-r_fT}N(d_1).$$

$$\rho^f_{put} = \frac{\partial p}{\partial r_f} = TSe^{(b-r)T}N(-d_1) = TSe^{-r_fT}N(-d_1).$$

For option on futures ($b=0$), we have:

$$\rho_{call} = \frac{\partial c}{\partial r} = -Tc.$$

$$\rho_{put} = \frac{\partial p}{\partial r} = -Tp.$$

Vega

$$vega_{call} = \frac{\partial c}{\partial \sigma} = Se^{(b-r)T}n(d_1)\sqrt{T}.$$

$$vega_{put} = \frac{\partial p}{\partial \sigma} = Se^{(b-r)T}n(d_1)\sqrt{T}.$$

Theta

$$\theta_{call} = -\frac{\partial c}{\partial T} = -\frac{Se^{(b-r)T}n(d_1)\sigma}{2\sqrt{T}} - (b-r)Se^{(b-r)T}N(d_1) - rKe^{-rT}N(d_2).$$

$$\theta_{put} = -\frac{\partial p}{\partial T} = -\frac{Se^{(b-r)T}n(d_1)\sigma}{2\sqrt{T}} + (b-r)Se^{(b-r)T}N(-d_1) + rKe^{-rT}N(-d_2).$$

17.3 Interest Rate Risk Analytics

Introduction

The following interest rate risk analytics are defined on any generalized series of cashflows.

The following definitions and derivations will default to use an annual compounding yield to maturity rate r_a. This is to be consistent with the Razor approach which uses an annual compounding yield to maturity for reporting.

Where appropriate the calculations of duration and convexity will use the compounding frequency and coupon periods per year.

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Sometimes these formulae below are expressed from a bond pricing formulae starting point using $k + f/d$ rather than nt_i. In the end the formulae are always consistent and equivalent by substitution, as $t = \frac{1}{n}\left(k + f/d\right)$.

Definitions

t_i - time in years for i'th cashflow, calculated using security days basis or day count convention

df_i - discount factor at time t_i from bootstrapped yield curve

$df_{r,i}$ - discount factor at time t_i derived from yield to maturity rate r of specified yield basis

Annual Compounding Rate r_a

$v_a = \dfrac{1}{1+r_a}$, for all i

$df_{a,i} = (1+r_a)^{-t_i} = (v_a)^{t_i}$

N-Period Compounding Rate r_n

$v_n = \dfrac{1}{1+\frac{r_n}{n}}$, for all i

$df_{n,i} = (1+\frac{r_n}{n})^{-nt_i} = (v_n)^{nt_i}$

Simple Annualised Rate r_s

$$v_{s,i} = \frac{1}{1+r_s t_i}$$

$$df_{s,i} = (1+r_s t_i)^{-1} = v_{s,i}$$

Continuous Compounding Rate r_c

$$v_c = \frac{1}{e^{r_c}}$$

$$df_{c,i} = e^{-r_c t_i} = (v_c)^{t_i}$$

Annualised Discount Rate r_d

$$v_{d,i} = (1 - r_d t_i)$$

$$df_{d,i} = v_{d,i}$$

17.3.1 Valuation for Interest Rate Securities

Market Value

$$P = \sum_i df_i C_i \equiv \sum_i df_{r,i} C_i$$

Accrued Interest

$AI_i = C_i \dfrac{f}{d}$ - cum interest

$AI_i = C_i \left(\dfrac{f}{d} - 1\right)$ - ex interest

where

$d = d_e - d_s$ - number of days of accrued interest, with respect to days basis
$f = d_e - d_v$ - number of days in coupon period, with respect to days basis
d_s - date of previous coupon
d_e - date of next coupon
d_v - value date

Clean Price
CP - clean price

$$P = CP + AI$$

Per $100 Face Value Equivalents
$P100FV = P * 100 / FV$
$CP100FV = CP * 100 / FV$
$AI100FV = AI * 100 / FV$

Ex Interest Date Calculation
If an interest rate security is defined to support ex-interest days (calendar or business days), then the calculation to determine if a coupon is ex-interest is as follows.

Calendar Days
Ex-Interest if (Coupon Date – Calendar Days) >= Value Date, otherwise Cum-Interest

Business Days
Ex-Interest if (Coupon Date – Elapsed Business Days) >= Value Date, otherwise Cum-Interest

17.3.2 Security Interest Rate Risk Analytics

Annual Compounding Rate r_a

Price

$$P = \sum_i (1+r_a)^{-t_i} C_i$$

Macaulay Duration

$$D = \frac{\sum_i (1+r_a)^{-t_i} C_i t_i}{P}$$

, or

$$D = \frac{\sum_i df_i C_i t_i}{P}$$

- Razor uses this form

Modified Duration

$$D' = -\frac{dP}{dr_a}\frac{1}{P}$$

$$\begin{aligned}\frac{dP}{dr_a} &= -\sum_i (1+r_a)^{-t_i-1} C_i t_i \\ &= -(1+r_a)^{-1}\sum_i (1+r_a)^{-t_i} C_i t_i \\ &= -v_a \sum_i (v_a)^{t_i} C_i t_i \\ &= -v_a \sum_i df_i C_i t_i\end{aligned}$$

$$D' = \frac{D}{(1+r_a)} = v_a D$$

- Razor uses this form

Convexity

$$CX = \frac{d^2P}{dr_a^2}\frac{1}{P}$$

$$\frac{d^2P}{dr_a^2} = \frac{d}{dr_a}\left[-\sum_i (1+r_a)^{-t_i-1} C_i t_i\right]$$
$$= \sum_i \left[(1+r_a)^{-t_i-2} C_i t_i^2 + (1+r_a)^{-t_i-2} C_i t_i\right]$$
$$= (1+r_a)^{-2} \sum_i \left[(1+r_a)^{-t_i} C_i t_i^2 + (1+r_a)^{-t_i} C_i t_i\right]$$
$$= v_a^2 \sum_i \left[df_i C_i t_i^2 + df_i C_i t_i\right]$$
$$= v_a^2 \left[\sum_i df_i C_i t_i^2 + \sum_i df_i C_i t_i\right]$$

$CX = v_a\left[D^{''} + D^{'}\right]$, where $D^{''} = v_a \sum_i df_i C_i t_i^2 \frac{1}{P}$

N-Period Compounding Rate r_n

For example, as for bonds

Price

$$P = \sum_i (1+\frac{r_n}{n})^{-nt_i} C_i$$

Macaulay Duration

$$D = \frac{\sum_i (1+\frac{r_n}{n})^{-nt_i} C_i t_i}{P}$$, or

$$D = \frac{\sum_i df_i C_i t_i}{P}$$ - Razor uses this form

Modified Duration

$$D^{'} = -\frac{dP}{dr_n}\frac{1}{P}$$

$$\frac{dP}{dr_n} = -\sum_i (1+\frac{r_n}{n})^{-nt_i-1} C_i t_i$$
$$= -(1+\frac{r_n}{n})^{-1} \sum_i (1+\frac{r_n}{n})^{-nt_i} C_i t_i$$
$$= -v_n \sum_i (v_n)^{nt_i} C_i t_i$$
$$= -v_n \sum_i df_i C_i t_i$$

$$D^{'} = \frac{D}{(1+\frac{r_n}{n})} = v_n D$$

- Razor uses this form

Convexity

$$CX = \frac{d^2P}{dr_n^2}\frac{1}{P}$$

$$\frac{d^2P}{dr_n^2} = \frac{d}{dr_n}\left[-\sum_i (1+\frac{r_n}{n})^{-nt_i-1} C_i t_i\right]$$
$$= \sum_i \left[(1+\frac{r_n}{n})^{-nt_i-2} C_i t_i^2 + \frac{1}{n}(1+\frac{r_n}{n})^{-nt_i-2} C_i t_i\right]$$
$$= (1+\frac{r_n}{n})^{-2} \sum_i \left[(1+\frac{r_n}{n})^{-nt_i} C_i t_i^2 + \frac{1}{n}(1+\frac{r_n}{n})^{-nt_i} C_i t_i\right]$$
$$= v_n^2 \sum_i \left[df_i C_i t_i^2 + \frac{1}{n} df_i C_i t_i\right]$$
$$= v_n^2 \left[\sum_i df_i C_i t_i^2 + \frac{1}{n}\sum_i df_i C_i t_i\right]$$

$CX = v_n\left[D^{''} + \frac{1}{n}D^{'}\right]$, where $D^{''} = v_n \sum_i df_i C_i t_i^2 \frac{1}{P}$

Simple Annualised Rate r_s

This is equivalent to converting to an effective annual compounding rate, and then applying the annual compounding forms of these analytics. For example as used for pricing a Bank Accepted Bill.

Price

$$P = \sum_i (1+r_s t_i)^{-1} C_i$$

Macaulay Duration

$$D = \frac{\sum_i (1 + r_s t_i)^{-1} C_i t_i}{P}$$, or

$$D = \frac{\sum_i df_i C_i t_i}{P}$$ - Razor uses this form

For a BAB or similar single cashflow based instrument, Duration simplifies to term to maturity, ie …

$$D = t_i$$

Modified Duration

$$D' = -\frac{dP}{dr_s}\frac{1}{P}$$

$$\begin{aligned}\frac{dP}{dr_s} &= -\sum_i (1 + r_s t_i)^{-2} C_i t_i \\ &= -\sum_i (v_{s,i})^2 C_i t_i \\ &= -\sum_i (df_i)^2 C_i t_i\end{aligned}$$

$$D' = \sum_i (df_i)^2 C_i t_i \Big/ P$$

For a BAB or similar single cashflow based instrument, Modified Duration simplifies to …

$$D' = v_{s,i} t_i = df_i t_i = v_{s,i} D$$

Convexity

$$CX = \frac{d^2 P}{dr_s^2}\frac{1}{P}$$

$$\begin{aligned}\frac{d^2P}{dr_s^2} &= \frac{d}{dr_s}\left[-\sum_i (1+r_s t_i)^{-2} C_i t_i\right] \\ &= 2\sum_i \left[(1+r_s t_i)^{-3} C_i t_i^2\right] \\ &= 2\sum_i \left(v_{s,i}\right)^3 C_i t_i^2\end{aligned}$$

$$CX = {2\sum_i \left(v_{s,i}\right)^3 C_i t_i^2} \Big/ {P}$$

For a BAB or similar single cashflow based instrument, Convexity simplifies to …

$$CX = {2\left(v_{s,i}\right)^3 C_i t_i^2} \Big/ {\left(v_{s,i}\right) C_i} = 2\left(v_{s,i}\right)^2 t_i^2 = 2\left(D'\right)^2$$

Continuous Compounding Rate r_c

This is equivalent to effective annual compounding rate.

Annualised Discount Rate r_d

Discount rates are not currently supported by Razor.

Price

$$P = \sum_i (1 - r_d t_i) C_i$$

Macaulay Duration

$$\begin{aligned}D &= \frac{\sum_i (1 - r_d t_i) C_i t_i}{P} \\ &= \frac{\sum_i df_i C_i t_i}{P}\end{aligned}$$

, or

For a single cashflow based discount instrument, Duration simplifies to term to maturity, ie …

$$D = t_i$$

Modified Duration

$$D^{'} = -\frac{dP}{dr_d}\frac{1}{P}$$

$$\frac{dP}{dr_d} = -\sum_i C_i t_i$$

$$D^{'} = \frac{\sum_i C_i t_i}{P}$$

Convexity

$$CX = \frac{d^2P}{dr_s^2}\frac{1}{P}$$

$$\frac{d^2P}{dr_s^2} = \frac{d}{dr_s}\left[-\sum_i C_i t_i\right] = 0$$

Duration of Futures, FRA, Forward Bond and Interest Rate Swap

In this section, we discuss on the methods Razor uses to compute the Macaulay and Modified duration of futures, FRA, forward bond and interest rate swaps.

Duration of Futures

Cash Futures

The Macaulay duration of cash futures is the length of the time in years from the valuation date to the futures maturity date using ACT/365 day count basis. Modified duration is calculated from scaling Macaulay by the yield curve forward rate.

$$D_{mac} = \frac{T_m - t}{365} \quad D_{\mathrm{mod}} = \frac{D_{mac}}{(1 + ytm)}$$

Bond Futures

The Macaulay duration of a bond in future in Razor is the Macaulay duration of the underlying bond.

$$D_{mac} = \sum_i \frac{Df_i CF_i t_i}{P} \quad D_{\mathrm{mod}} = \frac{D_{mac}}{(1 + ytm)}$$

Duration of a FRA

The duration of a FRA is the weighted average of the duration of incoming leg and duration of the outgoing leg weighted by the present value of each of the leg. The duration of the incoming and outgoing legs is the length of time period from the forward starting date to the forward ending date.

$$D_{mac} = \frac{\sum_i N_i \cdot Df_i \cdot \tau_i}{\sum_i PV_i} \quad D_{\text{mod}} = \frac{D_{mac}}{(1+ytm)}$$

Duration of Forward Bond

Define

t_0 = valuation date.

T_b = bond starting date.

T_e = bond maturity date.

P = price of the bond at bond starting date.

Currently Razor supports two methods to calculate forward bond durations.
Method 1
The first method is simply to calculate the forward bond duration as for a normal bond but to assume the valuation date is at T_b.

Method 2
The second method is again the same as the duration for a normal bond but to assume the cashflow at the bond starting date, i.e. P, is part of the bond cashflows (with the same + or – sign as other bond cashflows).

Duration of Interest Rate Swap

It is clear that if we hold a pay-fix interest rate swap and a fixed rate bond with the swap tenor dates matching the bond coupon paying date, then we obtain cashflows from the portfolio that replicate the cashflows of a floating rate bond.

We have the relationship

Swap + Fixed Rate Bond = Floating Rate Bond.

Given the cashflows on both side of equations are exactly the same, we can deduce

Swap Duration + Fixed Rate Bond Duration = Floating Rate Bond Duration.

It should be noted that duration of a floating rate bond is zero thus

Swap Duration = - Fixed Rate Bond Duration.

Duration of fixed rate bond can be computed using usual method.

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17.3.3 Portfolio Risk Analytics

The portfolio level approximation for risk analytics such as Macaulay Duration, Modified Duration and Convexity is defined as the net sum of trade level analytic weighted by relative trade market value. The weight by function is configurable.

For any portfolio risk analytic R_P, where R_{t_i}, and V_{t_i}

$$R_P = \frac{\sum_{i=1}^{n} R_{t_i} * V_{t_i}}{\sum_{i=1}^{n} V_{t_i}}$$

Therefore, specifically

Portfolio Macaulay Duration

$$D_P = \frac{\sum_{i=1}^{n} D_{t_i} * V_{t_i}}{\sum_{i=1}^{n} V_{t_i}}$$

Portfolio Modified Duration

$$D_P^{'} = \frac{\sum_{i=1}^{n} D_{t_i}^{'} * V_{t_i}}{\sum_{i=1}^{n} V_{t_i}}$$

Portfolio Convexity

$$CX_P = \frac{\sum_{i=1}^{n} CX_{t_i} * V_{t_i}}{\sum_{i=1}^{n} V_{t_i}}$$

17.4 Yield and Price Volatility

While either yield or price volatility may be used to calculate an interest rate option price, the inputs to the pricing model will need to be adjusted depending on whether yield or price volatility is used.

The implied volatility is the standard deviation of the expected forward price distribution. If the current yield of a forward bond is 5% per annum and has an implied yield volatility of 10%, then the volatility implies that two-thirds of the changes in yield over the next year will be within a range of 4.5% amd 5.5% per annum. If we can calculate the price per US$100 of a bond given the yield, then we can also imply the range of price changes implied by these yields.

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An approximation can be used to calculate price volatility from yield volatility using modified duration:

$$V_{price} = V_{yield} * Modified\ duration * Yield$$

Where modified duration refers to the underlying forward bond.

17.5 Day Count Conventions

From [Per Annex to the 2000 ISDA Definitions (June 2000 Version), Section 4.16. Day Count Fraction]

Defines a scheme of values for specifiying how the number of days between two dates is calculated for purposes of calculation of a fixed or floating payment amount and the basis for how many days are assumed to be in a year.

The full set of conventions is (Invalid, ACT/360, ACT/365, 30/360, 30E/360, ACT/ACT, ACT/365.25, ACT/365.FIXED, ACT/ACT.ISDA, ACT/ACT.ISMA, ACT/ACT.AFB, ACT/365.END, 1/1, ACT/365.ACT).

Currently Invalid defaults to ACT/365.ACT .

30/360

This calculation assumes a year has 360 days comprised of 12 months with 30 days each. In the bond calculation, d is given by the number of months between payment dates multiplied by 30 days. The coupon paid is always the annual coupon rate divided by the payment frequency. Determining $(d-f)$ is more complex as each part month needs to be treated, as if there are 30 days. This type of interest convention is common in Europe and is often referred to as the annual coupon basis.

As part of this calculation if the start day of month is the 31^{st}, then it is reset to the 30^{th}, and if the end date is also the 31^{st} then it is also reset to the 30^{th}.

30E/360

As above for 30/360, except if either the start or end day of month is the 31^{st}, then it will be reset to the 30^{th}.

Actual/Actual (ACT/ACT)

The interest accrual is based on the actual number of days elapsed since the last interest payment date $(d-f)$ in an interest period divided by the actual number of days between the last and next interest payment dates (d). The coupon paid is always the annual coupon rate divided by the payment frequency. This method is commonly used in English-speaking countries and includes instruments such as US Treasury Bonds, A$ and NZ$ government issues and UK Gilts.

ACT/ACT.ISMA

As for Actual/Actual.

Actual/360 (ACT/360)

This method is also referred to as the US Money Market basis because interest is calculated in the same way as for US dollar money market instruments. The interest accrual is based on the actual number of days elapsed since the last

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interest payment date $(d - f)$ in an interest period divided by an assumed 360 days a year (d). In this method, the coupon payment is not always an even amount.

Actual/365 (ACT/365)

As for Actual/360 except d is 365 days per anum.

Actual/365.FIXED (ACT/365.FIXED)

As for Actual/365.

Actual/365.25 (ACT/365.25)

As for Actual/360 except d is 365.25 days per anum.

ACT/365.END

As for Actual/360 except d is 366 if end date is a leap year, else 365.

ACT/365.ACT

As for Actual/360 except d is 366 if start date is a leap year, else 365.

ACT/ACT.ISDA

If the number of whole years between the start and end dates is greater than 0, then the day count fraction is calculated as (start year fraction + end year fraction + number of years – 1). The days in year used for each date will be 366 if a leap year, and otherwise 365.

If the number of whole years is 0, then days in year is by default 365.

ACT/ACT.AFB

If number of whole years between start and end dates is greater than 0, then calculate the days in year according to whether the fractional part of the day count fraction at start end passes over the 29th February.

If the number of whole years is 0, then days in year is by default 365.

BUS/252

Business days / 252. This method is only used for Brazilian Real (BRL). Weekends and public holidays are excluded from the calculation. This requires the use of a calendar. Razor will look for a BUS252 calendar if defined, otherwise only weekends will be excluded.

1/1

This is currently not supported.

17.6 Roll Conventions

The convention for determining the sequence of calculation period end dates. It is used in conjunction with a specified frequency and the regular period start date of a calculation period, e.g. semi-annual IMM roll dates

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EOM, FRN, IMM, IMCAD, SFE, NONE, TBILL, [1, 2 … 29, 30], MON, TUE, WED, THU, FRI, SAT, SUN

Currently FRN is not implemented, so defaults to NONE.

EOM - Rolls on month end dates irrespective of the length of the month and the previous roll day.

FRN - Roll days are determined according to the FRN Convention or Eurodollar Convention as described in ISDA 2000 definitions.

IMM - IMM Settlement Dates. The third Wednesday of the (delivery) month.

IMMCAD - The last trading day/expiration day of the Canadian Derivatives Exchange (Bourse de Montreal Inc) Three-month Canadian Bankers' Acceptance Futures (Ticker Symbol BAX). The second London banking day prior to the third Wednesday of the contract month. If the determined day is a Bourse or bank holiday in Montreal or Toronto, the last trading day shall be the previous bank business day. Per Canadian Derivatives Exchange BAX contract specification.

SFE - Sydney Futures Exchange 90-Day Bank Accepted Bill Futures Settlement Dates. The second Friday of the (delivery) month.

NONE - The roll convention is not required. For example, in the case of a daily calculation frequency.

TBILL - 13-week and 26-week U.S. Treasury Bill Auction Dates. Each Monday except for U.S. (New York) holidays when it will occur on a Tuesday.

[1, 2 … 29, 30] - Rolls on the specific day of the month.

[MON, TUE, WED, THU, FRI, SAT, SUN] - Rolls weekly on the specific day of the week.

17.7 Business Day Conventions

FOLLOWING, MODFOLLOWING, PRECEDING, MODPRECEDING, NONE, FRN

FOLLOWING -The non-business date will be adjusted to the first following day that is a business day.

MODFOLLOWING - The non-business date will be adjusted to the first following day that is a business day unless that day falls in the next calendar month, in which case that date will be the first preceding day that is a business day

PRECEDING - The non-business day will be adjusted to the first preceding day that is a business day

MODPRECEDING - The non-business date will be adjusted to the first preceding day that is a business day unless that day falls in the previous calendar month, in which case that date will be the first following day that us a business day

FRN - Per 2000 ISDA Definitions, Section 4.11. FRN Convention; Eurodollar Convention.

NONE -The date will not be adjusted if it falls on a day that is not a business day.

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NotApplicable - The date adjustments conventions are defined elsewhere, so it is not required to specify them here.

Currently FRN is not implemented, so defaults to NONE.

This enumeration is also referenced as 'Date Roll Convention'.

17.8 Interpolation Methods

17.8.1 Linear Interpolation

Linear interpolation calculates the value of a point lying on a straight line between two other points.

The formula for linear interpolation is:

$$r_i = (r_2 - r_1)\frac{T_i - T_1}{T_2 - T_1} + r_1$$

17.8.2 Log-Linear Interpolation

$$r_i = \left(\frac{r_2}{r_1}\right)^{\left(\frac{T_i - T_1}{T_2 - T_1}\right)} r_1$$

17.8.3 Cubic Interpolation

$$\begin{aligned} r_i = {} & \frac{(T_i - T_2)(T_i - T_3)(T_i - T_4)}{(T_1 - T_2)(T_1 - T_3)(T_1 - T_4)} r_1 + \\ & \frac{(T_i - T_1)(T_i - T_3)(T_i - T_4)}{(T_2 - T_1)(T_2 - T_3)(T_2 - T_4)} r_2 + \\ & \frac{(T_i - T_1)(T_i - T_2)(T_i - T_4)}{(T_3 - T_1)(T_3 - T_2)(T_3 - T_4)} r_3 + \\ & \frac{(T_i - T_1)(T_i - T_2)(T_i - T_3)}{(T_4 - T_1)(T_4 - T_2)(T_4 - T_3)} r_4 \end{aligned}$$

17.9 Cumulative Normal Distribution Function

The standard numerical procedure to evaluate the cumulative normal distribution function adopts a polynomial approximation algorithm[3]. This method provides numerical accuracy to six-decimal-place. Unfortunately, it can be inadequate sometimes. Razor implements a better polynomial approximation utilizing the Hart Algorithm (1968)[4]. This method gives double precision (14-16 decimal places) accuracy. Furthermore, the complexity of this procedure is low, hence its performance is as good as that provided by the standard one.

17.10 Cumulative Bivariate Normal Distribution Function

The two dimensional cumulative normal probability density function with a mean of $\mu = [0,0]$ and standard deviation of $\sigma = [1,1]$ can be described as:

$$f(x,y,\Sigma) = \frac{1}{2\pi\sqrt{|\Sigma|}} e^{-\frac{1}{2}x^T \Sigma^{-1}(x)}$$

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$$\Sigma = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad \sigma_{xy} = (1,1), \quad |\Sigma| \neq 0$$

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$$f(x_1, x_2, \Sigma) = \frac{1}{2\pi\sqrt{|\Sigma|}} e^{-\frac{1}{2}x^T \Sigma^{-1}(x)}$$

Field Code Changed

[3] M.Abramowitz and I. Stegun, *Handbook of Mathematical Functions*. New York, Dover Publications, 1972.

[4] E. G. Haug, The Complete Guide to Option Pricing Formulas. New York, Mcgraw-Hill, 1998.

$$\Sigma = \begin{bmatrix} \sigma_{x_1}^2 & \rho\sigma_{x_1}\sigma_{x_2} \\ \rho\sigma_{x_1}\sigma_{x_2} & \sigma_{x_2}^2 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}, \quad \text{where } \sigma_{x_1} = \sigma_{x_2} = 1, |\Sigma| \neq 0$$

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Here we use the scalar form, in terms of the correlation ρ, by making the following substitutions:

$$|\Sigma| = (1-\rho^2)$$

$$|\Sigma| = \left(1-\rho^2\right)$$

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$$x^T\Sigma^{-1}(x) = \frac{1}{(1-\rho^2)}\begin{bmatrix} 1 & -\rho \\ -\rho & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \text{ for } \sigma_{xy} = (1,1)$$
$$= \frac{x^2 + y^2 - 2\rho xy}{(1-\rho^2)}$$

$\rho = \dfrac{E[XY]}{\sigma_x\sigma_y} = E[XY]$ where $E[XY]$ is the covariance of XY

$$f(x,y,\rho) = \frac{1}{2\pi\sqrt{1-\rho^2}}e^{-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}}, \rho \in [-1,1]$$

The cumulative bivariate normal distribution function is defined as:

$$
\begin{aligned}
\Phi_{xy}(a,b,\rho) &= \int_{-\infty}^{a}\int_{-\infty}^{b} f(x,y,\rho)dxdy \\
&= \frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{a}\int_{-\infty}^{b} e^{-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}}dxdy
\end{aligned}
$$

This integral is solved numerically in Razor using the Drezner (1978)[5] algorithm with the following added special conditions:

17.10.1 Special Conditions

The Drezner approximation does not correctly handle the limiting case of $\rho = \pm 1$. Also in cases where $\rho = 0$ and a=b=0, optimizations can be made to simplify computation. The following additions have been made to the Drezner algorithm:

Case 1: $\rho = 1$

$$\Phi_{xy}(a,b,1) = \Phi(\min(a,b))$$

where Φ is the standard cumulative normal distribution function in one dimension

Case 2: $\rho = -1$

$$
\Phi_{xy}(a,b,-1) = \begin{cases} 0 & b \le -a \\ \Phi(a) + \Phi(b) - 1 & b > -a \end{cases}
$$

Case 3: $|\rho| < \varepsilon, \quad \varepsilon = 1\times 10^{-8}$

$$
\begin{aligned}
\Phi_{xy}(a,b,0) &= \lim_{\rho\to 0}\frac{1}{2\pi\sqrt{1-\rho^2}}\int_{-\infty}^{a}\int_{-\infty}^{b} e^{-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}}dxdy \\
&= \frac{1}{2\pi}\int_{-\infty}^{a}\int_{-\infty}^{b} e^{-\frac{x^2+y^2}{2}}dxdy \\
&= \frac{1}{2\pi}\left[\left(\int_{-\infty}^{a} e^{-\frac{x^2}{2}}dx\right)\cdot\left(\int_{-\infty}^{b} e^{-\frac{y^2}{2}}dy\right)\right] \\
&= \Phi(a)\cdot\Phi(b)
\end{aligned}
$$

[5] John C. Hull, *Options Futures and Other Derivatives*. New York, Mcgraw-Hill, 2002.

Case 4: $|a|,|b| < \varepsilon, \quad \rho \in [-1,1], \quad \varepsilon = 1\times 10^{-8}$

When a,b are very small, the probability density function then reduces to:

$$\lim_{x,y\to 0} \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{x^2+y^2-2\rho xy}{2(1-\rho^2)}} = \frac{1}{2\pi\sqrt{1-\rho^2}}$$

Therefore, the cumulative bivariate normal distribution function can be solved analytically as:

$$\Phi_{xy}(a,b,\rho) = \frac{1}{2\pi}\int \frac{1}{\sqrt{1-\rho^2}} d\rho$$
$$= \frac{\sin^{-1}(\rho)}{2\pi} + \frac{1}{4}$$

17.11 Reiner and Rubinstein Single Barrier Model

For pricing "In" and "Out" barrier options we will use the following notations:

$$x_1 = \frac{\ln(\frac{S}{X})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \qquad x_2 = \frac{\ln(\frac{S}{H})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$y_1 = \frac{\ln(\frac{H^2}{SX})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T} \qquad y_2 = \frac{\ln(\frac{H}{S})}{\sigma\sqrt{T}} + (1+\mu)\sigma\sqrt{T}$$

$$z = \frac{\ln(\frac{H}{S})}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T} \qquad \mu = \frac{b-\frac{\sigma^2}{2}}{\sigma^2} \qquad \lambda = \sqrt{\mu^2 + \frac{2r}{\sigma^2}}$$

$$A = \varphi S e^{(b-r)T} N(\varphi x_1) - \varphi X e^{-rT} N(\varphi x_1 - \varphi\sigma\sqrt{T})$$

$$B = \varphi S e^{(b-r)T} N(\varphi x_2) - \varphi X e^{-rT} N(\varphi x_2 - \varphi\sigma\sqrt{T})$$

$$C = \varphi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_1) - \varphi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_1 - \eta\sigma\sqrt{T})$$

$$D = \varphi S e^{(b-r)T} (\frac{H}{S})^{2(\mu+1)} N(\eta y_2) - \varphi X e^{-rT} (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})$$

$$E = K e^{-rT} [N(\eta x_2 - \eta\sigma\sqrt{T}) - (\frac{H}{S})^{2\mu} N(\eta y_2 - \eta\sigma\sqrt{T})]$$

$$F = K[(\frac{H}{S})^{\mu+\lambda} N(\eta z) + (\frac{H}{S})^{\mu-\lambda} N(\eta z - 2\eta\lambda\sigma\sqrt{T})]$$

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with
S representing the price of underlying asset,
H – option barrier value,
T - option time to maturity,
X - option strike price,
K – pre-specified cash rebate that is paid out at option expiration if the barrier is not hit during option life,
σ - underlying asset volatility of correct maturity
r - continuously compounded risk-free interest rate
b - cost of carry.

“In” Barrier Options.
“In” barrier options come into existence if the underlying asset price process hits the barrier level H before option expiration.
Call option with “down and in” type barrier pays payoff max(S_T -X,0) if $S_t \le H$ at any moment t before maturity T and a (possible) predetermined amount K otherwise.

The price of such option can be expressed as:

$$C_{di(X>H)} = C + E \qquad \eta = 1, \varphi = 1$$
$$C_{di(X<H)} = A - B + D + E \qquad \eta = 1, \varphi = 1$$

Call option with “up and in” type barrier that pays payoff max(S_T -X,0) if $S_t \ge H$ at any moment t before maturity T and a predetermined amount K otherwise priced according to the following equations:

$$C_{ui(X>H)} = A + E \qquad \eta = -1, \varphi = 1$$
$$C_{ui(X<H)} = B - C + D + E \qquad \eta = -1, \varphi = 1$$

For corresponding put options:
Put option with “down and in” barrier, payoff at maturity - max(X- S_T ,0) if $S_t \le H$ at any moment t before maturity T and amount K otherwise priced as:

$$P_{di(X>H)} = B - C + D + E \qquad \eta = 1, \varphi = -1$$
$$P_{di(X<H)} = A + E \qquad \eta = 1, \varphi = -1$$

The value of put “up and in” option is:

$$P_{ui(X>H)} = A - B + D + E \qquad \eta = -1, \varphi = -1$$
$$P_{ui(X<H)} = C + E \qquad \eta = -1, \varphi = -1$$

“Out” Barrier Options.
These type of call and put options remains vanilla type options unless they are knocked out and become worthless at the moment when the

underlying asset process hits barrier level. Similar to “in” type options it is also possible include a predetermined rebate amount K that compensate the knock out event.
“Down and out” calls have payoff at maturity T as max(S_T -X,0) if $S_t > H$ for all $t \le T$ or K otherwise and prices defined:

$C_{do(X>H)} = A - C + F$ $\quad\quad \eta = 1, \phi = 1$

$C_{do(X<H)} = B - D + E$ $\quad\quad \eta = 1, \phi = 1$

“Up and out” calls with payoff at maturity T as max(S_T -X,0) if $S_t < H$ for all $t \le T$ and K compensator priced according to:

$C_{uo(X>H)} = F$ $\quad\quad \eta = -1, \phi = 1$

$C_{uo(X<H)} = A - B + C - D + F$ $\quad\quad \eta = -1, \phi = 1$

Put options values are:
for “Down and out” case -

$P_{do(X>H)} = A - B + C - D + F$ $\quad\quad \eta = 1, \phi = -1$

$P_{do(X<H)} = F$ $\quad\quad \eta = 1, \phi = -1$

and “Up and out” case:

$P_{uo(X>H)} = B - D + F$ $\quad\quad \eta = -1, \phi = -1$

$P_{uo(X<H)} = A - C + F$ $\quad\quad \eta = -1, \phi = -1$

17.12 Finite Difference Methods

17.12.1 Introduction

Though renown Black-Scholes (1973) and Merton (1973) equation and formulae can price the majority of vanilla type financial derivatives many of OTC traded options often include more "exotic" than vanilla features and thus their pricing frequently rely on numerical techniques.
Finite difference methods is one of the most efficient and simple of such numerical procedures that allows calculation of option premium by solving the partial differential equation that represents option price evolution:

$$\frac{df}{dt} + rS\frac{df}{dS} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S} = rf$$

The method requires the construction of a discrete mesh that represents the continuous domain of the state variables of underlying partial differential equation. The time space [0,T] (T – time to option maturity) is discretised into a finite collection of N+1 equally spaced intervals of length $\Delta t = \frac{T}{N}$ and underlying asset space (stock prices) is represented by M+1 equally spaced intervals

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0, $0, \partial S, 2\partial S, 3\partial S, ..., S_{\max}$ of length $\Delta S = \frac{S}{M}$.

The value $S_{\max}$ is chosen sufficiently high that, when it is reached, the put payoff has no value. M is finite enough to get the required accuracy. The values of continues partial derivatives from the above differential equation are approximated then on the pre-built mesh nodes with an explicit and implicit numerical schemes or with their combination.
A numerical scheme commonly used is Crank-Nicholson scheme, which is unconditionally stable and can be considered as the average of the implicit and explicit methods:

$$\frac{u_m^{n+1} - u_m^n}{\Delta t} = \frac{1}{2}\frac{u_{m+1}^{n+1} - 2u_m^{n+1} + u_{m-1}^{n+1}}{\Delta S^2} + \frac{1}{2}\frac{u_{m+1}^n - 2u_m^n + u_{m-1}^n}{\Delta S^2},$$

where $u_m^n \approx f(m\Delta S, n\Delta t)$ - the approximation of option premium f at each node of the mesh.
Using above approximation the option price partial differential equation is converted into a series of ordinary difference equations, and the difference equations are solved iteratively.

Finite difference methods value a derivative by solving the differential equation that the derivative satisfies. The differential equation is converted into a series of difference equations, and the difference equations are solved iteratively.

The differential equation that an American option must satisfy is:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

17.12.2 The Time Axis

Suppose that the life of an option is T. We divide this into N equally spaced intervals of length $\delta t = \frac{T}{N}$. A total of $N+1$ times are therefore considered.

$0, \delta t, 2\delta t, ..., T$

17.12.3 The Price Axis

Suppose the $S_{\max}$ is a stock price sufficiently high that, when it is reached, the put has virtually no value. We define $\delta S = \frac{S_{\max}}{M}$ and consider a total of $M+1$ equally spaced stock prices:

$0, \delta S, 2\delta S, ..., S_{\max}$

The level $S_{\max}$ is chosen so that one of these is the current stock price.

17.12.4 The Grid

The time points and price points define a grid consisting of a total of $(M+1)(N+1)$ points. The (i, j) point on the grid is the point that corresponds to time $i\delta t$ and price $j\delta S$. We will use the variable $f_{i,j}$ to denote the value of the option at the (i, j) point.

17.13 Implicit/Explicit Finite Difference Methods

17.13.1 The Forward Difference Approximation

For an interior point (i, j) on the grid, $\frac{\partial f}{\partial S}$ can be approximated as:

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j}}{\delta S}$$

17.13.2 The Backward Difference Approximation

For an interior point (i, j) on the grid, $\frac{\partial f}{\partial S}$ can also be approximated as:

$$\frac{\partial f}{\partial S} = \frac{f_{i,j} - f_{i,j-1}}{\delta S}$$

17.13.3 Averaging the Forward and Backward Approximations

We can use a more symmetrical approximation by averaging the forward and backward approximations:

$$\frac{\partial f}{\partial S} = \frac{f_{i,j+1} - f_{i,j-1}}{2\delta S}$$

For $\partial f / \partial t$ we will use a forward difference approximation so that the value at time $i\delta t$ is related to the value at time $(i+1)\delta t$:

$$\frac{\partial f}{\partial t} = \frac{f_{i+1,j} - f_{i,j}}{\delta t}$$

The finite difference approximation for $\partial^2 f / \partial S^2$ at the (i, j) point is:

$$\frac{\partial^2 f}{\partial S^2} = \frac{\left(\frac{f_{i,j+1} - f_{i,j}}{\delta S} - \frac{f_{i,j} - f_{i,j-1}}{\delta S}\right)}{\delta S}$$

or

$$\frac{\partial^2 f}{\partial S^2} = \frac{f_{i,j+1} + f_{i,j-1} - 2 f_{i,j}}{\delta S^2}$$

Substituting into the differential equation and noting that $S = j\delta S$ gives

$$\frac{f_{i+1,j} + f_{i,j}}{\delta t} + rj\delta S \frac{f_{i,j+1} + f_{i,j-1}}{2\delta S} + \frac{1}{2}\sigma^2 j^2 \delta S^2 \frac{f_{i,j+1} + f_{i,j-1} - 2 f_{i,j}}{\delta S^2} = rf_{i,j}$$

for $j = 1,2,\ldots,M-1$ and $0,1,\ldots,N-1$. Rearranging terms we obtain:

$$a_j f_{i,j-1} + b_j f_{i,j} + c_j f_{i,j+1} = f_{i+1,j}$$

Where

$$a_j = \frac{1}{2} rj\delta t - \frac{1}{2}\sigma^2 j^2 \delta t$$

$$b_j = 1 + \sigma^2 j^2 \sigma t + r\delta t$$

$$c_j = -\frac{1}{2} rj\delta t - \frac{1}{2}\sigma^2 j^2 \delta t$$

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