BONUS ASSIGNMENT

Equations

Please make sure your Â“external$_{flow.lib\hat{A}\backslash}$ EES library includes both Â“external$_{flow,sphere\hat{A}\backslash}$ AND Â“external$_{flow,sphere,nd\hat{A}\backslash}$ procedures

Specifications for the ball

$$m = 0.1 \ \text{[kg]} \tag{1}$$

$$D = 0.3 \ \text{[m]} \tag{2}$$

$$A = \pi \cdot \frac{D^2}{4} \tag{3}$$

$$V = \pi \cdot \frac{D^3}{6} \tag{4}$$

Our shell is hollow on the inside, so we cannot obtain its density using EES Thermophysical property functions

$$\rho_{ball} = m/V \tag{5}$$

Temperatures

$$T_{inf} = 17 \ \text{[C]} \tag{6}$$

$$T_{initial} = 250 \ \text{[C]} \tag{7}$$

In previous iterations of this code, I thought It would be wise to consider the density of air and the specific heat of copper to be variable with temperature to improve the accuracy of my final answer. during testing I realized that calculating each of these two in every step of my time integral would only slightly alter my final answer (from 18.98 Celsius to 19.38 Celsius) but would more than triple the processing time. So I went back to using average values for them

$$T_{s,ave} = \frac{(T_{initial} + 18.98 \ \text{[C]})}{2} \qquad \text{I acquired T}_{final} = 18.98 \text{ from previous runs of the code and iterated from there} \tag{8}$$

$$T_{film} = \frac{(T_{inf} + T_{s,ave})}{2} \tag{9}$$

$$\rho_{air} = \rho \left(Air_{ha}, \ \text{T} = T_{film}, \ \text{P} = 101.3 \ \text{[kPa]} \right) \tag{10}$$

$$C_p = \text{c}_\text{p} \left(Copper, \ \text{T} = T_{s,ave} \right) \tag{11}$$

Initial values for remaining height, surface temperature and speed

$$RemainingHeight_1 = 100 \ \text{[m]} + 87 \ \text{[m]} \tag{12}$$

$$T_{s,1} = T_{initial} \tag{13}$$

$$U_1 = 0 \ \text{[m/s]} \tag{14}$$

Our chosen time step for discretization of the time integral

$$\delta_t = 0.04 \ \text{[s]} \tag{15}$$

I set our integrations higher bound (673 * 0.04 seconds   27 seconds) to the exact time it takes for the ball to reach the ground. In other words, RemainingHeight[674] is the first negative value of RemainingHeight[]

$$\text{duplicate} \ \ i = 1, \ 673 \tag{16}$$

Using the EES function for external convective cooling over a smooth sphere, we are only interested in the drag force $F_{D,]andconvectiveheattransfercoefficienth,]}$

$$\text{call } external_{flow,sphere}(\text{'Air\_ha'}, T_{inf}, T_{s,i}, 101.3 \text{ [kPa]}, U_i, D : F_{D,i}, h_i, C_{D,i}, Nusselt_i, Re_i) \tag{17}$$

Finding the Instantaneous acceleration using F_{net} = ma and F_{net} = W - F_B - F_D

$$m \cdot a_i = V \cdot 9.806 \left[\text{m/s}^2\right] \cdot (\rho_{ball} - \rho_{air}) - F_{D,i} \tag{18}$$

$$RemainingHeight_{i+1} = RemainingHeight_i - (U_i \cdot \delta_t) - \left(0.5 \cdot a_i \cdot \delta_t^2\right) \quad \text{delta x = Vt + (1/2)at}^2 \tag{19}$$

$$U_{i+1} = U_i + (a_i \cdot \delta_t) \quad \text{delta V = at} \tag{20}$$

Using the lumped capacitance model. I will show later why it can be used to accurately model this problem

$$T_{s,i+1} - T_{inf} = (T_{s,i} - T_{inf}) \cdot \exp\left(-1 \cdot h_i \cdot 4 \cdot A \cdot \frac{\delta_t}{(m \cdot C_p)}\right) \tag{21}$$

end (22)

Finding the maximum Biot number value to see if using the lumped capacitance model is a good estimation for our problem or not. The Biot number in this problem amounts to only Bi = 0.00000231 which is (much) lower than Bi = 0.1 so lumped capacitance model is accurate
In finding the Biot number, it would have been best to use the thermal resistance definition of the Biot number (Bi = R_{cond} / R_{conv}) as we already know the radial thermal resistance of a hollow shell, but I still decided to use the L_c method

$$V_{real} = \frac{m}{\rho\,(Copper,\ \text{T} = T_{final})} \tag{23}$$

$$L_c = \frac{V_{real}}{(4 \cdot A)} \tag{24}$$

$$Bi = h_{673} \cdot \frac{L_c}{\text{k}\,(Copper,\ \text{T} = T_{final})} \tag{25}$$

Finding out T_s when the ball reaches our hand. T_{final} is equal to 18.98 degrees Celsius So The Ball Is Perfectly Safe to Catch

$$T_{final} = T_{s,673} \tag{26}$$

If we didn't want to use the EES function for external convection, we could have instead used:
A) The equation C_D = [24 / Re] + [(2.6*(Re / 5)) / (1+(Re / 5)$^{1.52}$)] + [(0.411*(Re / 2.63*10^5)$^{-7.94}$) / (1+(Re / 2.63*10^5)$^{-8}$)] + [(0.25*(Re / 10^6)) / (1+(Re / 10^6))]
One advantage of this formula is that it can accurately predict the drag coefficients of flows with bigger than 2 * 10^5 Reynolds numbers (Turbulent flows)
Source: Faith A. Morrison, Â"Data Correlation for Drag Coefficient for Sphere,Â"
Department of Chemical Engineering, Michigan Technological University, Houghton, MI,
www.chem.mtu.edu/ fmorriso/DataCorrelationForSphereDrag2016.pdf
B) the Whitaker correlation to find Nusselt's number as Nusselt = 2 + (0.4*Re$^{0.5}$ + 0.06*Re$^{0.667}$) * Pr$^{0.4}$ * $(\mu/\mu_s)^{0.25}$ C) Re = U * D / kinematicviscosity(Air$_{ha}$,T=T_{film}, P=101.3 [kPa]) for finding Re
D) F_D = (ρ_{air} * U^2 * A * C_D) / 2 to find drag force
And the final segment would have looked something like:
Pr = prandtl(Air$_{ha}$,T=T_{inf}, P=101.3 [kPa])
nu = kinematicviscosity(Air$_{ha}$,T=T_{film}, P=101.3 [kPa])
mu = viscosity(Air$_{ha}$,T=T_{inf}, P=101.3 [kPa])
mu$_s$ = viscosity(Air$_{ha}$,T=T_{film}, P=101.3 [kPa])

k_f = conductivity(Air_{ha},T=T_{film}, P=101.3 [kPa])
Duplicate i=1,673
Re[i] = U[i] * D / ν
$C_{D,i}$ = (24 / Re[i]) + ((2.6*(Re[i] / 5)) / (1+(Re[i] / 5)$^{1.52}$)) + ((0.411*(Re[i] / 2.63*10^5)$^{-7.94}$) / (1+(Re[i] / 2.63*10^5)$^{-8}$)) + ((0.25*(Re[i] / 10^6)) / (1+(Re[i] / 10^6)))
$F_{D,i}$ = ($\rho_{air,i}$ * U[i]^2 * A * $C_{D,i}$) / 2
$F_{net,i}$ = V * 9.806 [m/s^2] * (ρ_{ball} - ρ_{air}) - $F_{D,i}$
m * a[i] = $F_{net,i}$
RemainingHeight[i+1] = RemainingHeight[i] - (U[i] * δ_t) - (0.5 * a[i] * $\delta_t{}^2$)
U[i+1] = U[i] + (a[i] * δ_t)
Nusselt[i] = 2 + (0.4*Re[i]$^{0.5}$ + 0.06*Re[i]$^{0.667}$) * Pr$^{0.4}$ * (μ/μ_s)$^{0.25}$ h[i] = Nusselt[i] * k_f / D
$T_{s,i+1}$ - T_{inf} = ($T_{s,i}$ - T_{inf}) * exp(-1 * h[i] * 4 * A * δ_t / m * C_p)
End