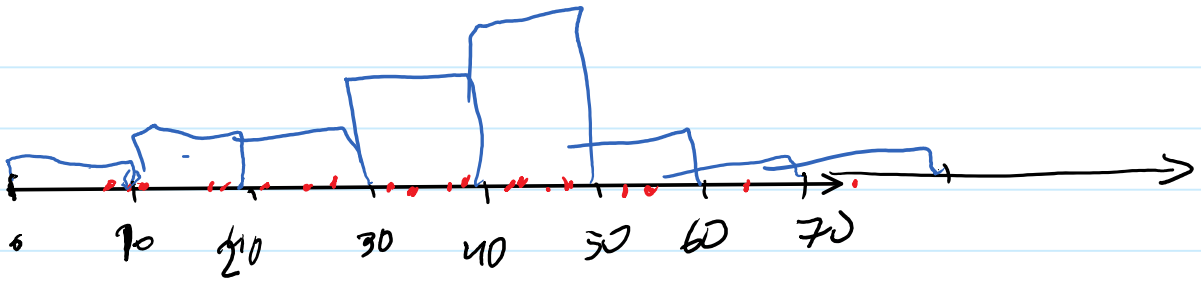
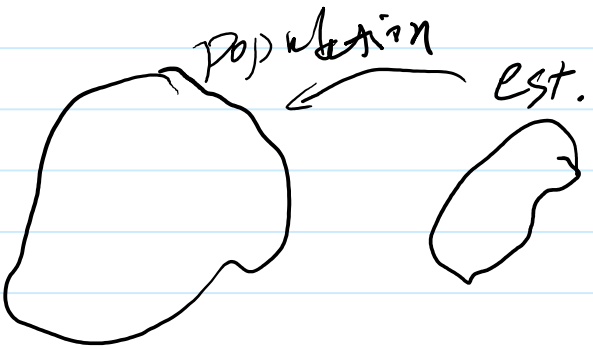
Maximum likelihood Estimate (MLE)

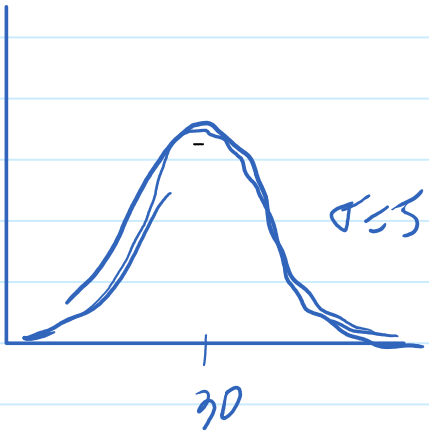
Sunday, May 24, 2020 6:05 PM





$\mu = 30$

$\sigma = 5$

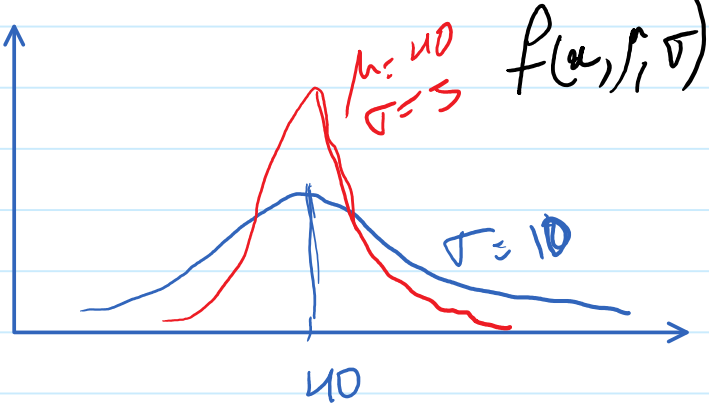


$X \rightsquigarrow N(\mu, \sigma)$

$-\infty < x < \infty$

$$f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

normal



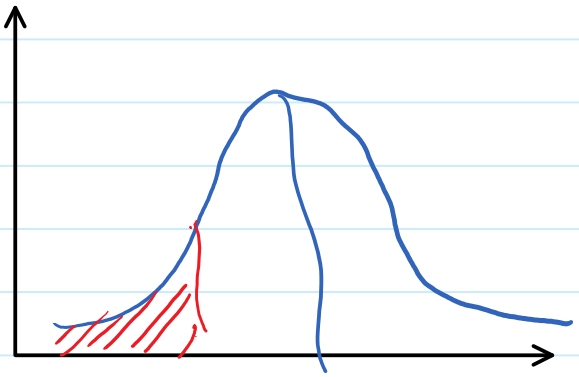
$$p(a < x < b) = \int_a^b f(x)\,dx$$

$$p(x < 10) = \int_{-\infty}^{10} f(x)\,dx$$

MLE

Sunday, May 24, 2020 6:05 PM

$$p(x<10 \mid \mu=30, \sigma=5) = \int_{-\infty}^{10} f(x,\mu,\sigma)\,dx$$



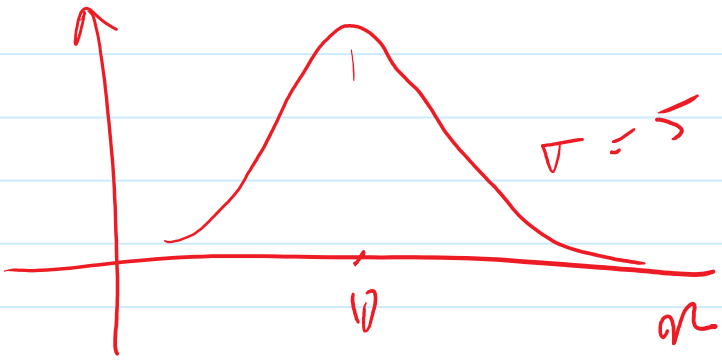
$\mu = 30$

$\sigma = 5$

$$p(x<1000) = 1$$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$x \sim N(\mu, \sigma) \Rightarrow f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$



Pdf probability density function

Sunday, May 24, 2020 6:27 PM

Gamma Distribution

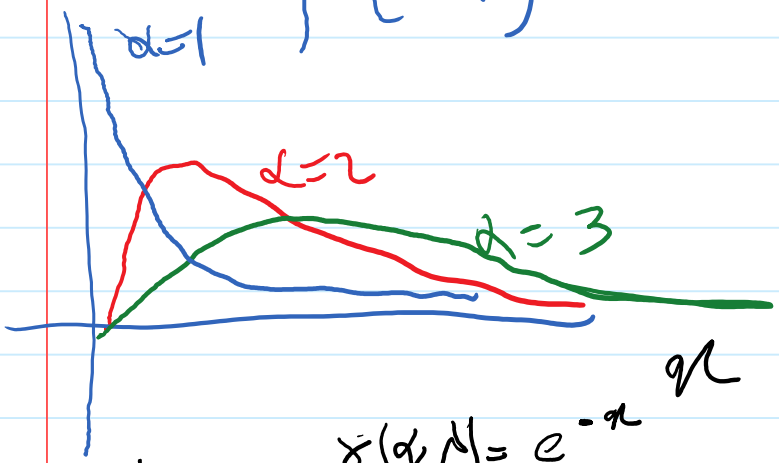
shape factor

$$\gamma(\alpha,\lambda) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$$

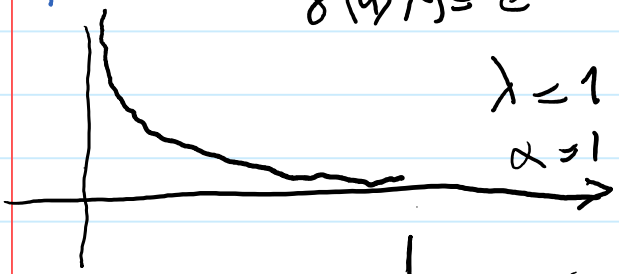
rate

$$\Gamma(\alpha) = \alpha!$$

$x > 0$



$$\gamma(\alpha,\lambda) = e^{-x}$$



$$P(x<2 \mid \alpha=1, \lambda=1) = \int_0^2 e^{-x}\,dx$$

$$-e^{-x}\Big|_0^2 = -\left(\frac{1}{e^2} - 1\right) = 1 - \frac{1}{e^2} = 0.86$$

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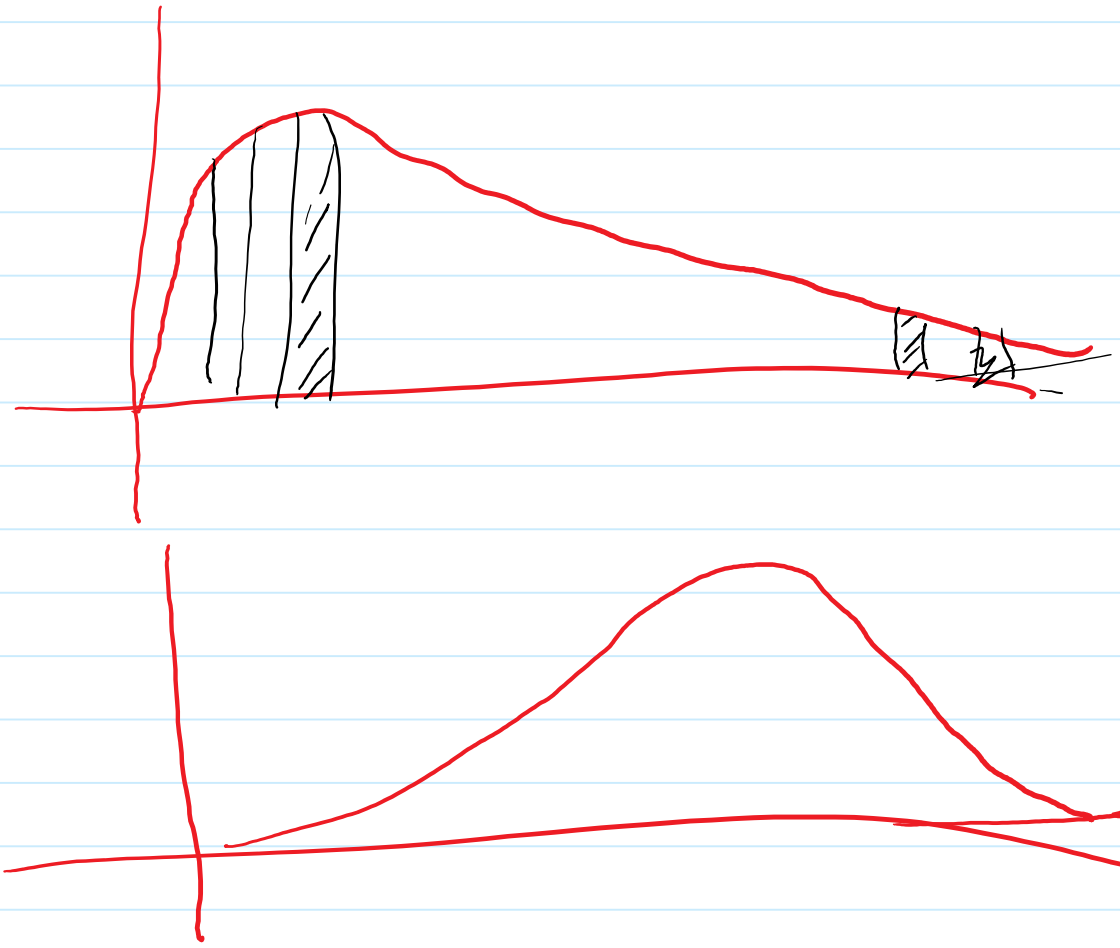
Sunday, May 24, 2020 6:32 PM

Log Normal dist.

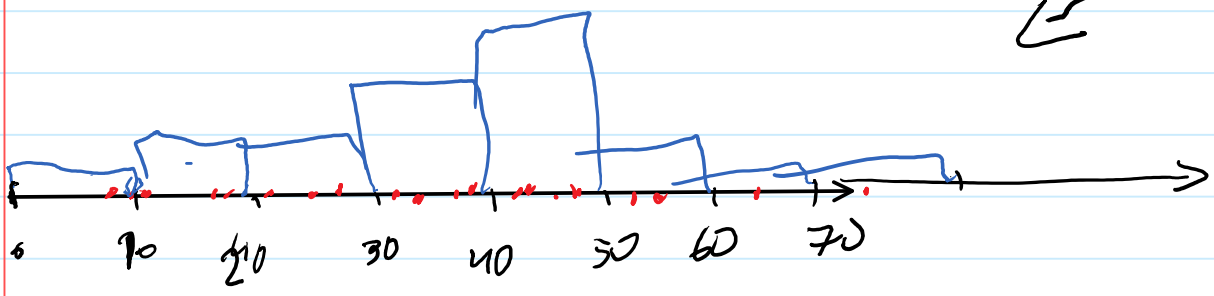
$$\log N(\lambda, \varphi) = \frac{1}{\varphi\sqrt{2\pi}} e^{-\frac{(\ln x - \lambda)}{2\varphi^2}}$$



MLE

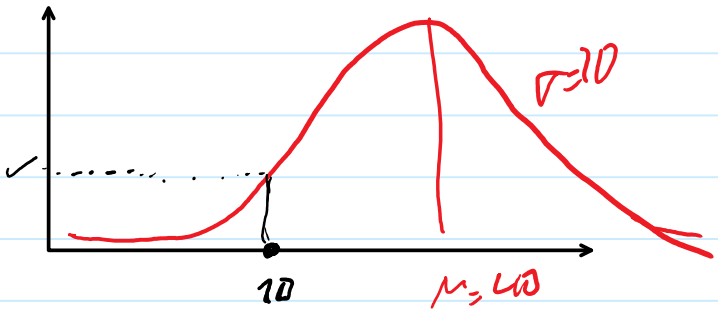
Sunday, May 24, 2020 6:40 PM

MLE for Normal



$$X \sim N(\mu, \sigma)$$

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

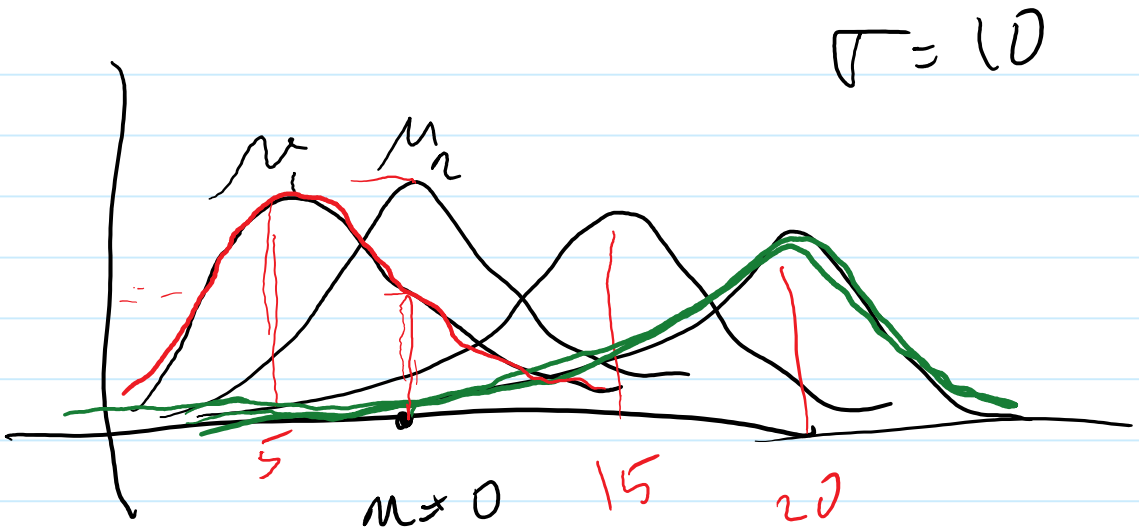


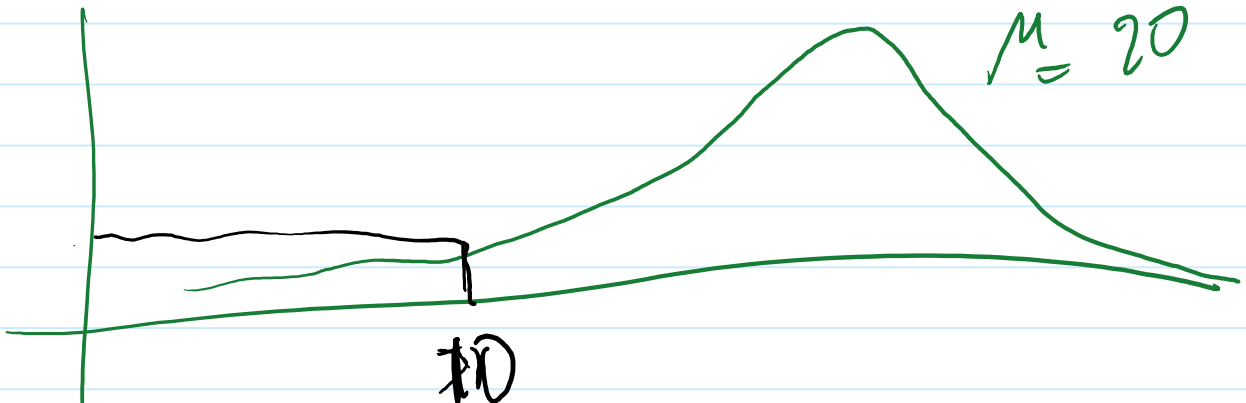
$$P(x < 10 \mid \mu, \sigma)$$

$$L(\mu, \sigma \mid x = 10) = f(x, \mu, \sigma)$$

MLE

Sunday, May 24, 2020 6:48 PM





$$\hat{\mu}, \hat{\sigma} = \underset{\mu,\sigma}{\arg\max} \; L(\mu, \sigma \mid \alpha)$$

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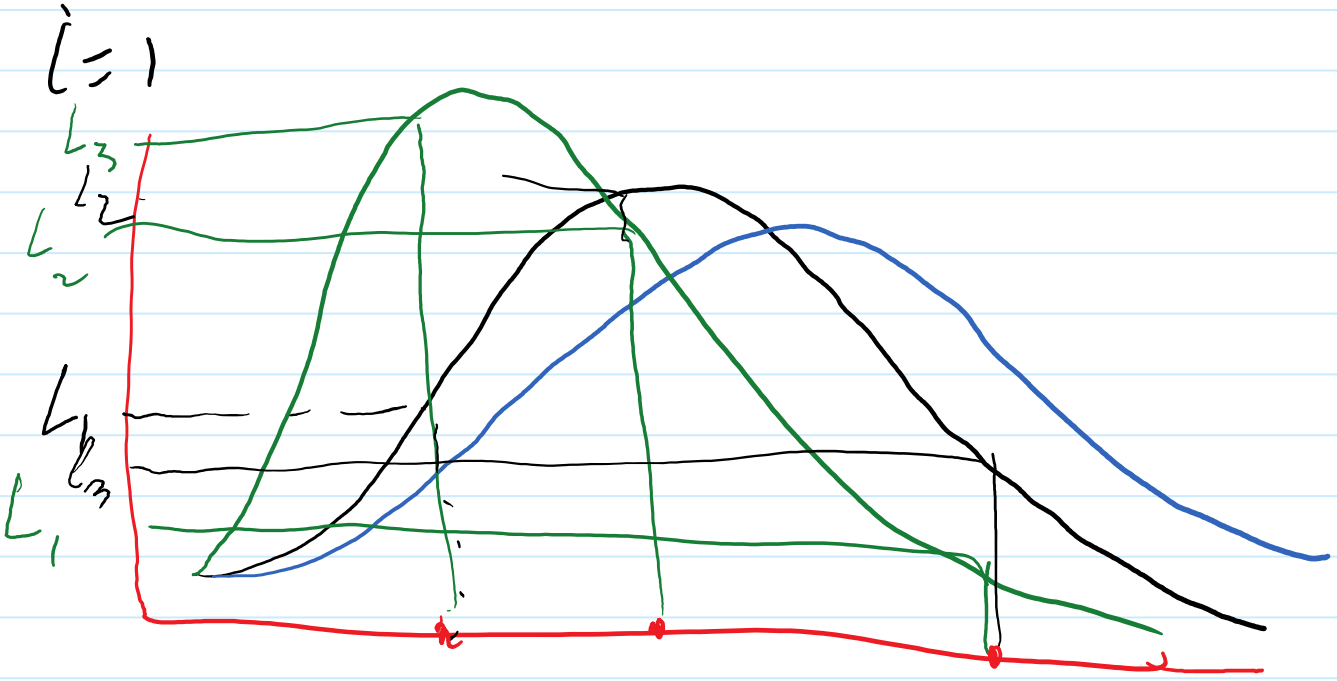
Sunday, May 24, 2020 6:54 PM

$$L(\mu, \sigma \mid x_1, x_2 \dots x_n) =$$

$$L(\mu, \sigma \mid x_1) \propto L(\mu, \sigma \mid x_2) \dots$$

$$L(\theta) = \prod_{i=1}^{n} L(\mu, \sigma \mid x_i)$$



$L_1 \times L_2 \times L_3 = L$

$L_1 \times L_2 \times L_3 = L$

MLE

Sunday, May 24, 2020 6:59 PM

x_i are independent

$$L(\mu, \sigma \mid \text{data}) = \prod_{i=1}^{n} L(\mu, \sigma \mid x_i)$$

$$\text{MLE}: \quad \hat{\mu}, \hat{\sigma} = \underset{\mu, \sigma}{\arg\max}\, L(\mu, \sigma \mid \text{data})$$

$$X \rightsquigarrow N(\mu, \sigma)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

$$L(\mu, \sigma \mid x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

Temperture data

.30	.40		
.20	.15	.15	.10
.10	.5	.15	

dependent rvs example

MLE

Sunday, May 24, 2020 7:07 PM

$$L(\mu,\sigma|x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\log_e(e^x) = x$$

$$\frac{\partial L}{\partial \mu} = 0 \qquad \log(L) = \ell(\mu,\sigma|x)$$

$$\frac{\partial L}{\partial \sigma} = 0 \qquad \log(a\cdot b) = \log(a) + \log(b)$$

$$\log(a^n) = n\log(a)$$

$$\ell(\theta) = \ln(L(\theta)) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)$$

$$\ell = -\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2}$$

$$\ell(\mu,\sigma|x_1, x_2 \dots x_n) = \sum(\quad)$$

MLE

Sunday, May 24, 2020 7:16 PM

$$\frac{d}{dx}\ln(u) = \frac{u'}{u}$$

$$\ell(\mu, \sigma \mid x_1, x_2 \cdots x_n)$$

$$\ell = \sum_{i=1}^{n} \left\{ \frac{-1}{2}\ln\left(2\pi\sigma^2\right) - \frac{(x_i-\mu)^2}{2\sigma^2} \right\}$$

$$\frac{d}{dx}\frac{1}{u} = \frac{-u'}{u^2}$$

$$\frac{\partial \ell}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - \mu) = 0$$

$$\Rightarrow \sum x_i = n\mu$$

$$\Rightarrow \boxed{\mu = \frac{\sum x_i}{n}}$$

$$\frac{\partial \ell}{\partial \sigma} = \sum \left(\frac{4\pi\sigma}{-\frac{1}{2}(2\pi\sigma^2)} + \frac{(x_i-\mu)^2(4\sigma)}{4\sigma^4} \right)$$

Sunday, May 24, 2020 7:21 PM

$$\frac{\partial \ell}{\partial \sigma} = \sum \left(\frac{-1}{2(2\pi\sigma^2)} (4\pi\sigma) + \frac{(x_i-\mu)^2 (4\sigma)}{4\sigma^4} \right) = 0$$

$$\sum \left(\frac{-1}{\sigma} + \frac{1}{\sigma^3} (x-\mu)^2 \right)$$

$$-\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum (x-\mu)^2 = 0$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

Assignment #1

Sunday, May 24, 2020 7:25 PM

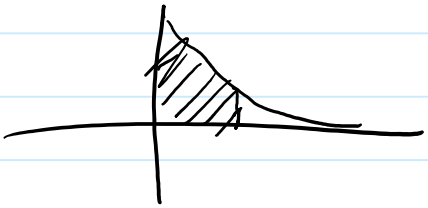
$\bar{y} = 13.85$ year $\quad \sigma^2 = 75.9$ (year)^2

$\Gamma(\alpha, \lambda)$

$$\begin{cases} p(x \mid \alpha, \lambda) = 0.5 \\ p(x \mid \alpha, \lambda) = 0.25 \end{cases}$$

$p(x < 10 \mid \alpha, \lambda) =$

$\int_0^{10} f(x, \alpha, \lambda)$



Sunday, May 24, 2020 8:20 PM

Assignment 2:

Plot histogram of errors for regression