Maximum likelihood Estimate (MLE)

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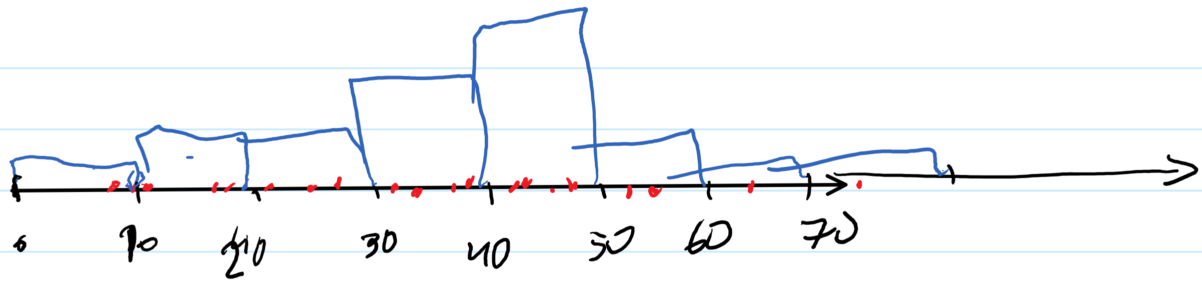
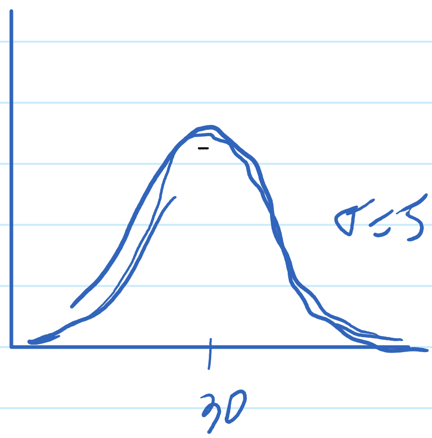


$\mu = 30$

$\sigma = 5$

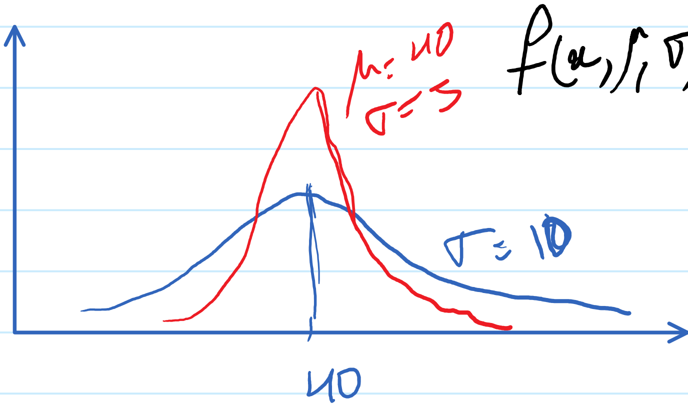


$X \sim N(\mu, \sigma)$

$-\infty < X < \infty$

$$f(x,\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

normal



$$p(x < 10) = \int_{-\infty}^{10} f(x)\,dx$$

$$p(a < x < b) = \int_a^b f(x)\,dx$$

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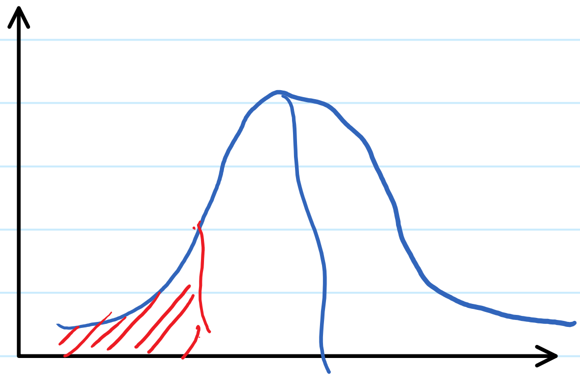
$$p(x<10 \mid \mu=30, \sigma=5) = \int_{-\infty}^{10} f(x,\mu,\sigma)\,dx$$

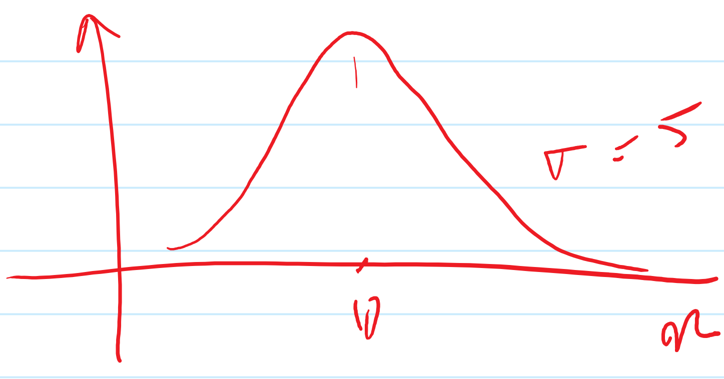


$\mu=30$
$\sigma=5$

$p(x<1000) = 1$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$x \sim N(\mu,\sigma) \Rightarrow \boxed{f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma}}}$$



$\sigma=5$ $\mu=10$
$\sigma=5$

Pdf probability density function

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Gamma Distribution

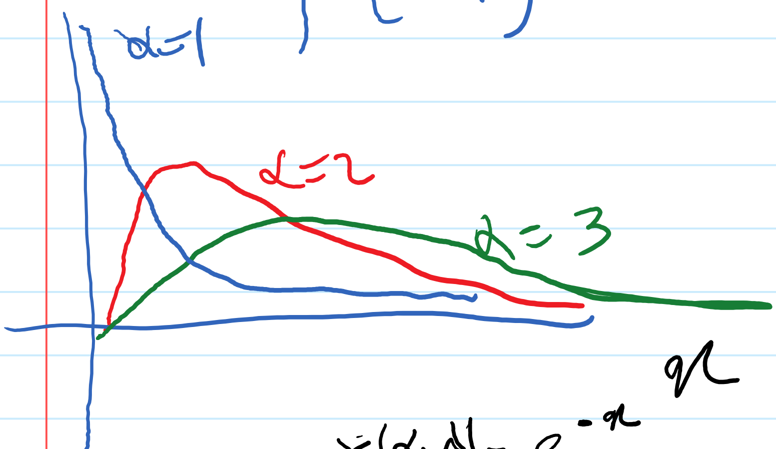
$$\gamma(\alpha, \lambda) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\lambda x}$$

shape factor

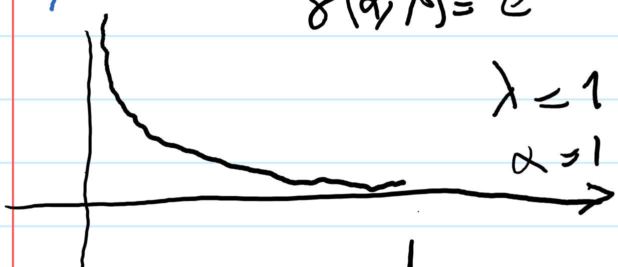
rate

$$\Gamma(\alpha) = \alpha!$$

$x > 0$



$$\gamma(\alpha, \lambda) = e^{-x}$$



$$P(x < 2 \mid \alpha = 1, \lambda = 1) = \int^{2} e^{-x} \, dx$$

$$-e^{-x}\Big|_0^2 = -\left(\frac{1}{e^2} - 1\right) = 1 - \frac{1}{e^2} = 0.86$$

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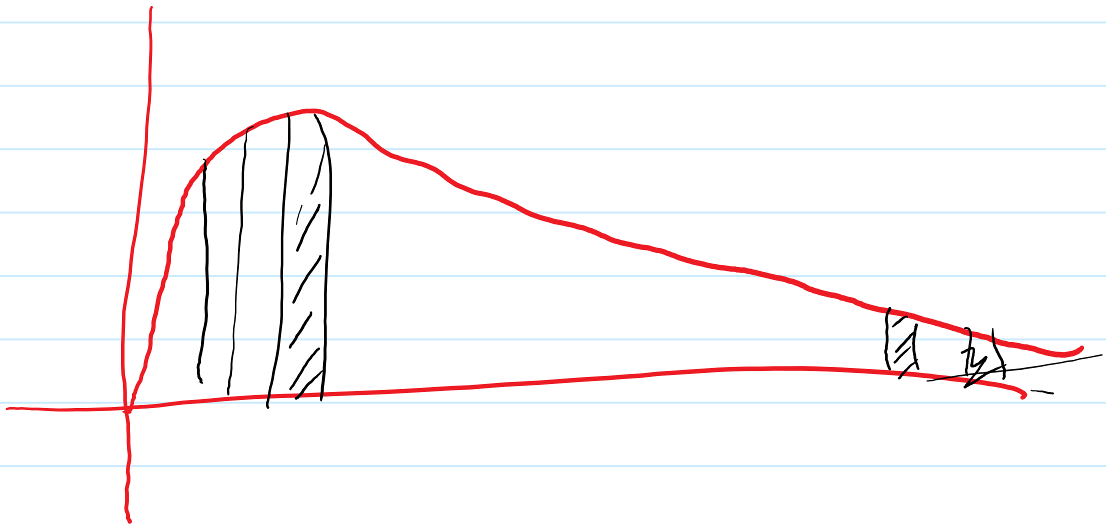
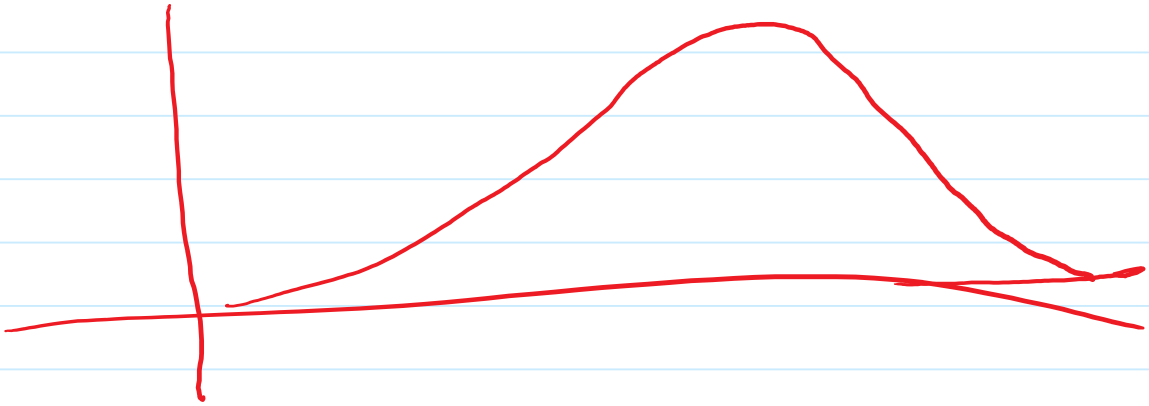
Log Normal dist.

$$\log N(\lambda, \varphi) = \frac{1}{\varphi\sqrt{2\pi}} e^{\frac{-(\ln x - \lambda)}{2\varphi^2}}$$





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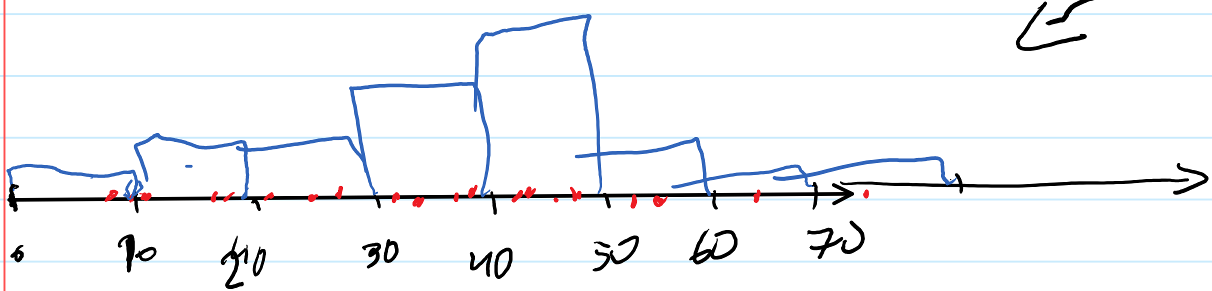
MLE for Normal



$$X \sim N(\mu, \sigma)$$

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

$$P(x < 10 \mid \mu, \sigma)$$

$$L(\mu, \sigma \mid x = 10) = f(x, \mu, \sigma)$$

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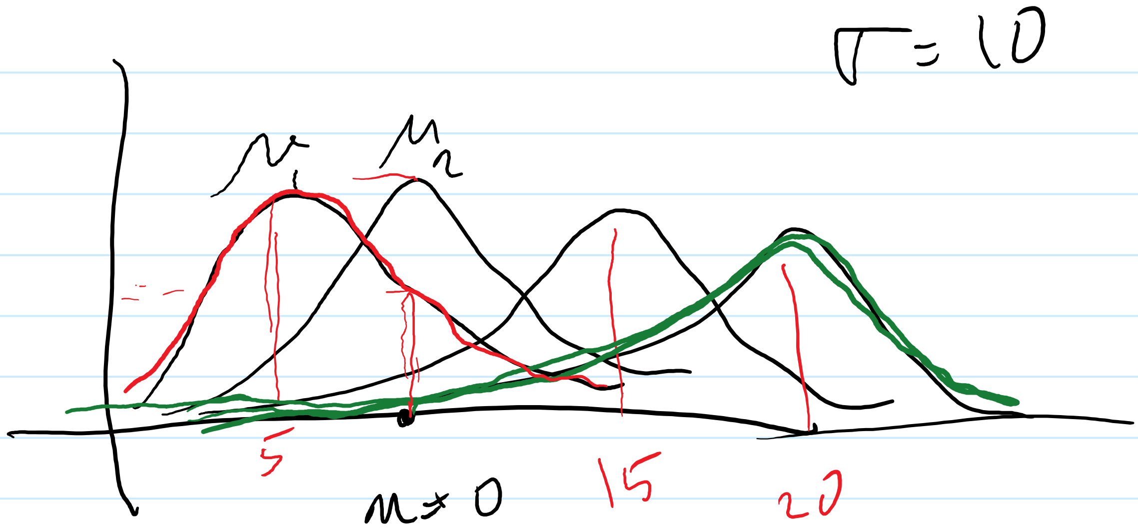

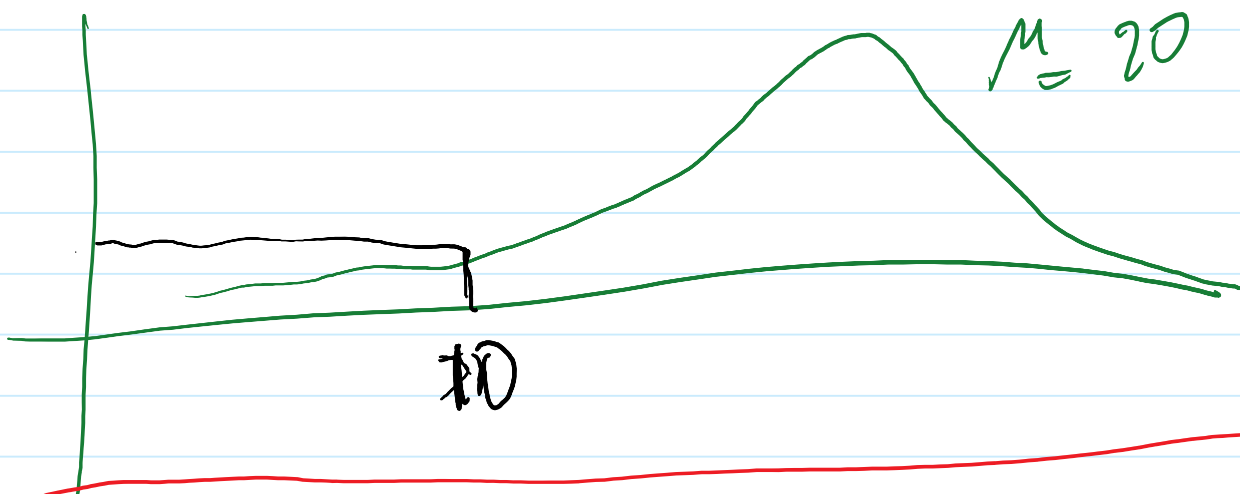

$$\boxed{\hat{\mu}, \hat{\sigma} = \operatorname*{argmax}_{\mu,\sigma} L(\mu, \sigma \mid x)}$$

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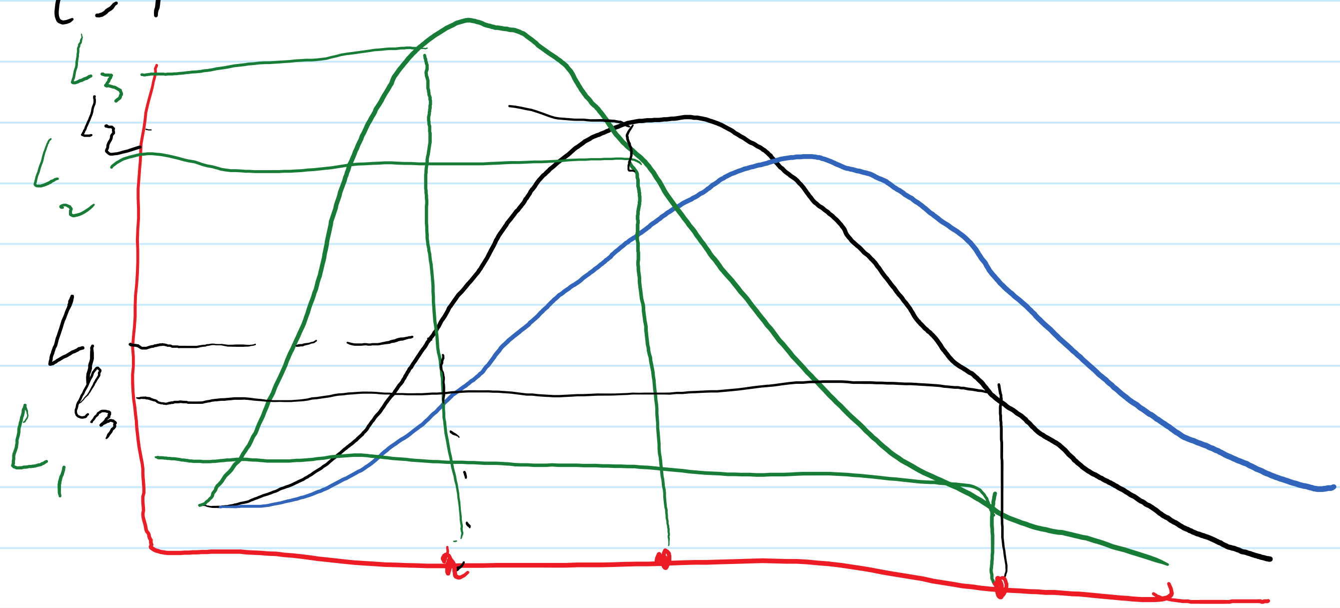
$$L(\mu, \sigma \mid x_1, x_2 \ldots x_n) =$$

$$L(\mu, \sigma \mid x_1) \times L(\mu, \sigma \mid x_2) \ldots$$

$$L(\theta) = \prod_{i=1}^{n} L(\mu, \sigma \mid x_i)$$



$L_1 \times L_2 \times L_3 = L$

$L_1 \times L_2 \times L_3 = L$

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x_i are independent

$$L(\mu,\sigma \mid \text{data}) = \prod_{i=1}^{n} L(\mu,\sigma \mid x_i)$$

$$\text{MLE}: \quad \hat{\mu},\hat{\sigma} = \underset{\mu,\sigma}{\arg\max}\, L(\mu,\sigma \mid \text{data})$$

$$x \sim N(\mu,\sigma)$$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

$$L(\mu,\sigma \mid x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma}}$$

Temperature data

.30	.40		
.20	.15	.15	.10
.10	.5	.15	

dependent as example

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$$L(\mu,\sigma|x) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\log_e(e^x) = x$$

$$\frac{\partial L}{\partial \mu} = 0$$

$$\log(L) = \ell(\mu,\sigma|x)$$

$$\frac{\partial L}{\partial \sigma} = 0$$

$$\log(a.b) = \log(a) + \log(b)$$

$$\log(a^n) = n\log(a)$$

$$\ell(\theta) = \ln(L(\theta)) = \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\right)$$

$$\ell = -\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x-\mu)^2}{2\sigma^2}$$

$$\ell(\mu,\sigma|x_1, x_2 \dots x_n) = \sum(\quad)$$

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$$\frac{d}{dx}\ln(u) = \frac{u'_x}{u}$$

$$\ell(\mu, \sigma \mid x_1, x_2 \dots x_n)$$

$$\ell = \sum_{i=1}^{n} \left\{ -\frac{1}{2}\ln(2\pi\sigma^2) - \frac{(x_i-\mu)^2}{2\sigma^2} \right\}$$

$$\frac{d}{dx}\frac{1}{u} = \frac{-u'}{u^2}$$

$$\frac{\partial \ell}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i - \mu) = 0$$

$$\Rightarrow \sum x_i = n\mu$$

$$\Rightarrow \boxed{\mu = \frac{\sum x_i}{n}}$$

$$\frac{\partial \ell}{\partial \sigma} = \sum \left(-\frac{1}{2}\frac{4\pi\sigma}{(2\pi\sigma^2)} + \frac{(x_i-\mu)^2(4\sigma)}{4\sigma^4} \right)$$

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$$\frac{\partial \ell}{\partial \sigma} = \sum \left(-\frac{\cancel{4}\cancel{\pi}\cancel{\sigma}}{2(\cancel{2}\cancel{\pi}\sigma^{\cancel{2}})} + \frac{(x_i-\mu)^2(\cancel{4}\cancel{\sigma})}{\cancel{4}\sigma^{\cancel{4}\,3}} \right) = 0$$

$$\sum\left(-\frac{1}{\sigma} + \frac{1}{\sigma^3}(x-\mu)^2\right)$$

$$-\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum(x-\mu)^2 = 0$$

$$\boxed{\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2}$$

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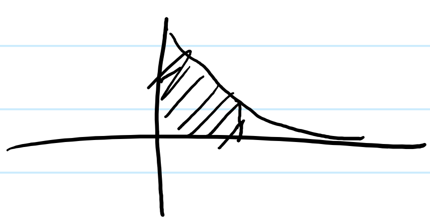
Assignment #1

$\bar{y} = 13.85$ year $\qquad \sigma^2 = 75.9$ $(\text{year})^2$

$\Gamma(\alpha, \lambda)$

$$\begin{cases} P(x \mid \alpha, \lambda) = 0.5 \\ p(x \mid \alpha, \lambda) = 0.25 \end{cases}$$

$$P(x < 10 \mid \alpha, \lambda) = \int_0^{10} M(x, \alpha, \lambda)$$



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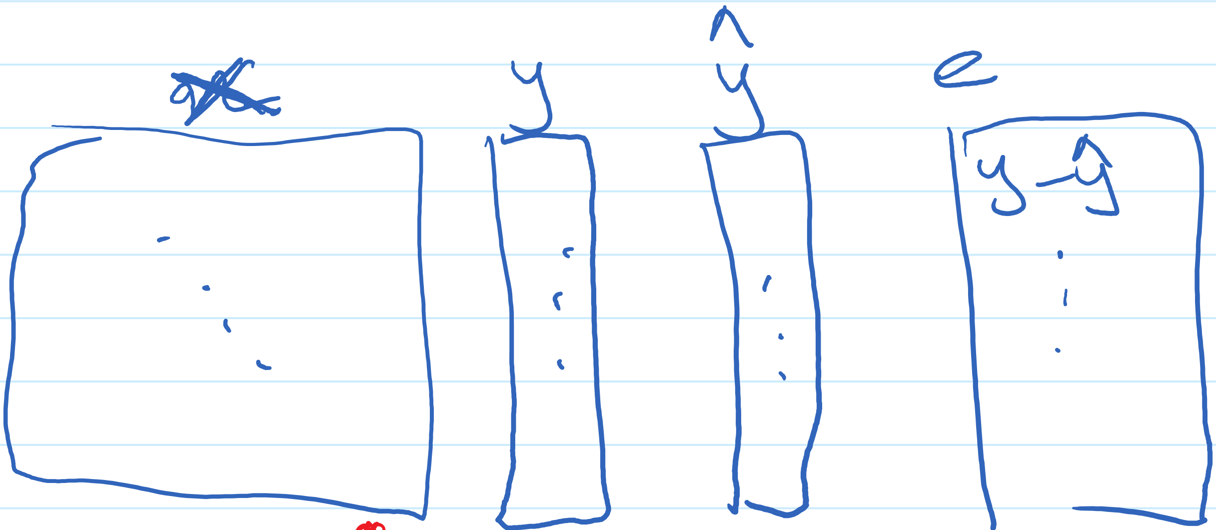
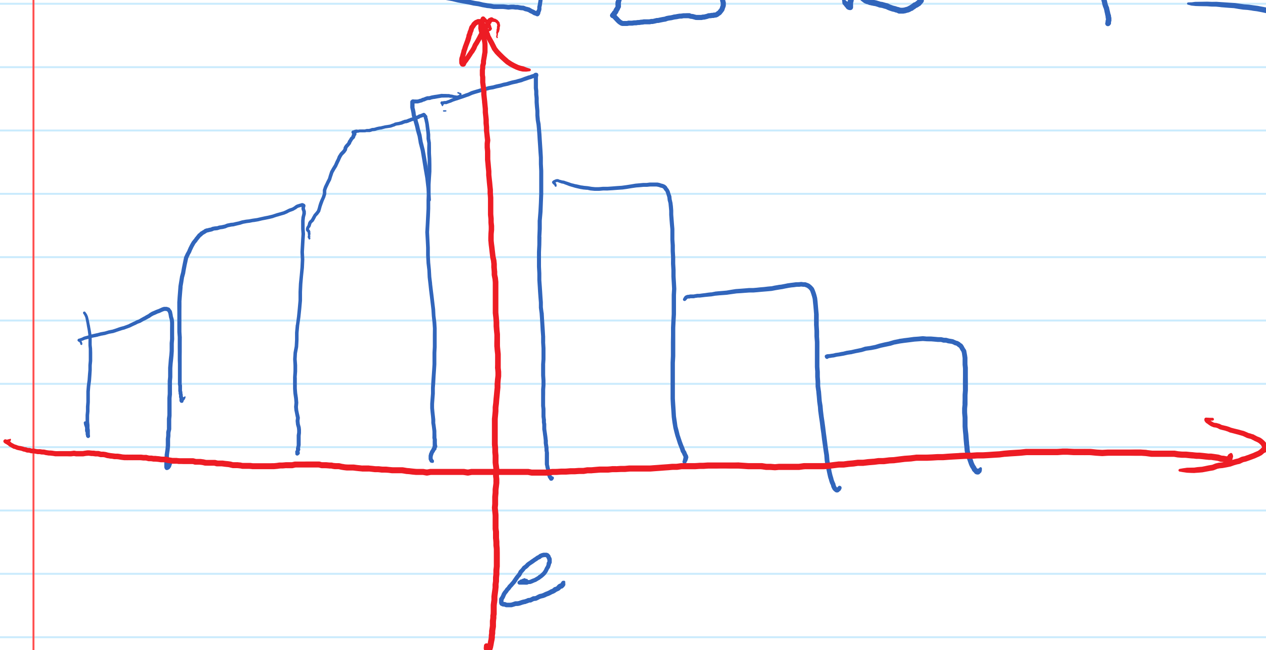
Assignment 2:

Plot histogram of errors for regression





MLE vs lm

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OLS : $\min \sum (y-\hat{y})^2$

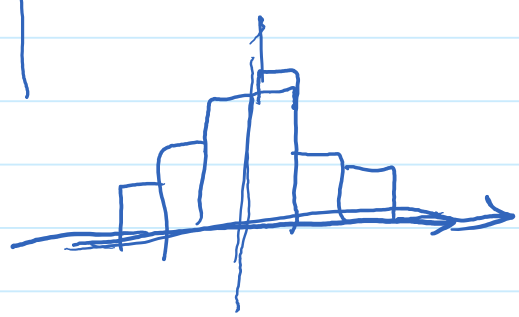
or $\min: \sum |y-\hat{y}|$

why ols is the best linear estimator

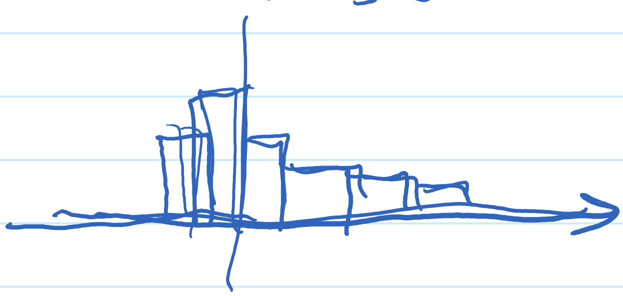
MLE = OLS

$\hat{y} = x\hat{\beta}_1 + \hat{\beta}_0$

$\bar{e} = 0$



$e = y-\hat{y}$



$(y-\hat{y}) \sim N(0, \sigma^2)$

MLE vs Lm

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$\bar{\varepsilon} = 0$

$$L(\beta_0, \beta_1 \mid y_1, \cdots y_n) = \prod \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\varepsilon - \bar{\varepsilon})^2}{2\sigma^2}}$$

$$\varepsilon = y_i - \hat{y}_i = y - \beta_0 - \beta_1 x_i$$

$$\hat{\beta}_0, \hat{\beta}_1 = \underset{\beta_0, \beta_1}{\arg\max}\, L(\beta_0, \beta_1 \mid data)$$

$$\ell(\beta_0, \beta_1 \mid data) = -\boxed{\frac{\sum (y - \beta_0 - \beta_1 x_i)^2}{2\sigma}} - \boxed{\frac{n}{2} \log(2\pi\sigma^2)}$$

MLE $\approx$ OLS

MLE vs lm

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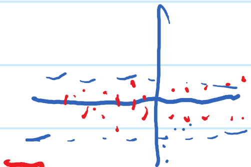
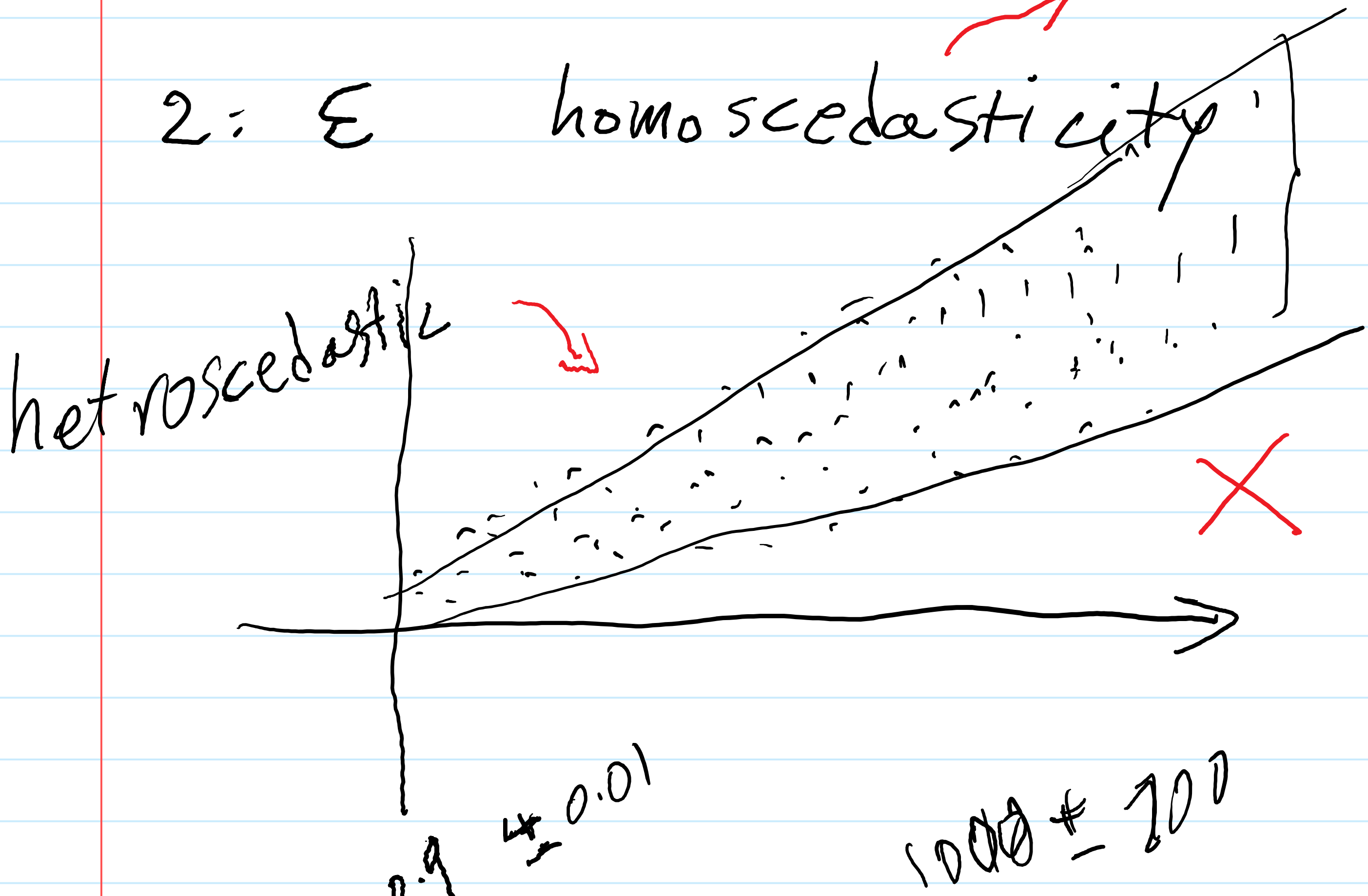
1: $\varepsilon \sim N(0, \sigma^2)$



2: ε homoscedasticity



3. No correlation between X data