Week 14 - Time Series

Time Series

- A time series is a set of data points which are ordered (indexed) in time.
- Frequent goal: forecasting the future.
- Time Series analysis:
  - Is it stationary?
  - Is there a seasonality component?
  - Is the variable autocorrelated?

Additive Model

$$X_t = T_t + S_t + R_t$$

Decomposition of a time series into three components: the trend (long term direction), the seasonal (calendar movements) and the irregular (unsystematic) components.

Trend Component

Long-term movement in a time series without calendar or irregular effects.

Examples: Economic changes, population growth, inflation etc.

Seasonality Component

Seasonality in a time series can be identified by regularly spaced peaks and troughs which have a consistent direction and approximately the same magnitude every year, relative to the trend.

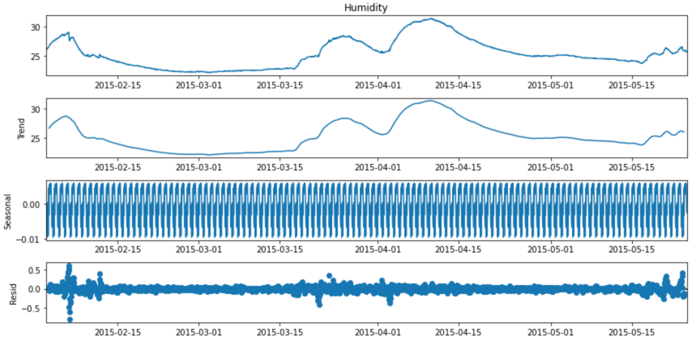
Examples: Weather seasons, administrative effects, social behavior etc.

Residual Component

The residual component is whatever remains after the seasonal and trend components of a time series have been removed. It represents short-time fluctuations which are irregular and cannot be predicted.

Examples: Weather seasons, administrative effects, social behavior etc.

Additive Model



Autocorrelation

Autocorrelation is a type of serial dependence: the correlation of a time series at time t with itself at time t-1. (or other lags)

Autoregressive (AR) Models

AR(1):

$$y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t$$

AR(2):

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \epsilon_t$$

AR(k):

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + ... + \beta_k y_{t-k} + \epsilon_t$$

Stability condition for AR models: $|\beta_1| < 1$

Moving Average (MA) Models

MA(1):

$$y_t = \mu + \theta_1 \epsilon_{t-1} + \epsilon_t$$

AR(2):

$$y_t = \mu + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \epsilon_t$$

AR(k):

$$y_t = \beta_0 + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \ldots + \theta_k \epsilon_{t-k} + \epsilon_t$$

MA models work well if the underlying process varies around the mean, but the mean only drifts slowly in one direction. Invertibility condition for MA models: $|\theta_1| < 1$

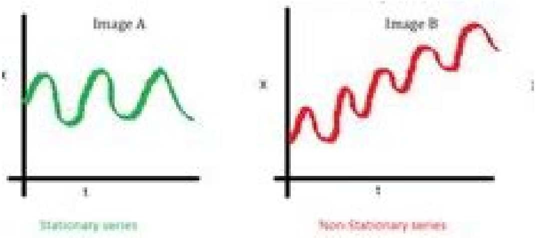
ARMA Models

An ARMA models consists of two processes: an AR (autoregressive) model and an MA (moving average) model. MA(1):

$$y_t = \mu + \beta_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t$$

Conditions for ARMA models: $|\beta_1| < 1$ and $|\theta_1| < 1$.

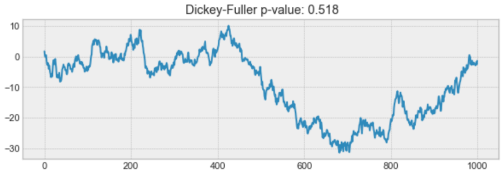
Stationarity



A time series is stationarity, if basic properties of the distribution (such as mean and variance) do not change over time.

Dickey-Fuller Test for Stationarity

- H_0: A unit root is present (so the time series is not stationary).
- If we reject H_0: the series is stationary.



Unit Root Example: an AR(1) process, $y_t = \beta_0 + \beta_1 y_{t-1} + \epsilon_t$ *has a unit root if* $\beta_1 = 1$.

Autocorrelation

Final Remark: Careful when working with autocorrelated variables in a linear regression setting!

One of the assumptions of linear regression applies that the data has no autocorrelation. (we need the errors to be independent from each other!) Therefore, if we include variables with high autocorrelation in a regression then the interpretation can be wrong. (read: coefficients and standard errors are inaccurate)

When working with time series data in a "standard" OLS setting, check for autocorrelation / stationarity and apply first differences whenever necessary!