Exercise 1

Perceptrons

The activation function for each perceptron will be the step function

$$\varphi(x) = \begin{cases} 1 & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

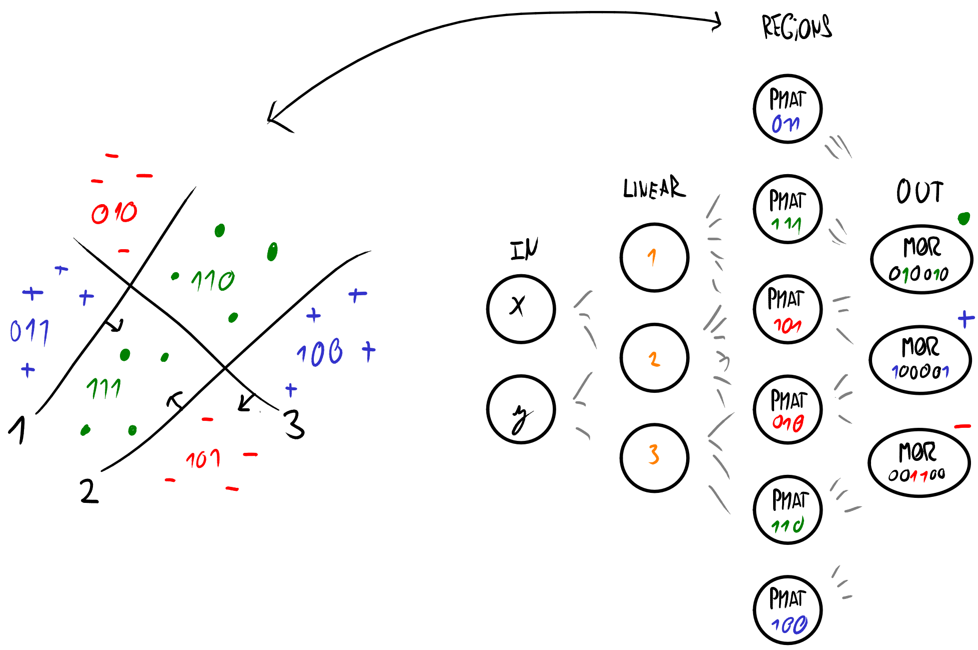
The weights and biases will be the following:

1) OR
   - weights: $\{1\}^D$
   - bias: 0
2) MOR
   - weights: c
   - bias: 0
3) PMAT
   - weights: c with 0s replaced by -1
   - bias: $-\sum c + 1$

The idea for PMAT is that if it perfectly corresponds, βX will sum to $\sum c$ and so it will activate, and if it doesn't it will always be lower due to either $+0$ if it doesn't correspond or -1 .

Network

The network, along with the linear decision planes, is described in the following diagram:



This classifier (especially our version) will likely be very hard to train, since the derivative for the step function will be 0 essentially everywhere.

Exercise 2

We want to prove that we can merge two layers into one by multiplying out the activation:

$$\begin{aligned} y &= \varphi(\varphi(X\beta_1 + b_1)\beta_2 + b_2) \\ &= (X\beta_1 + b_1)\beta_2 + b_2 \qquad \text{identities} \\ &= X\beta_1\beta_2 + b_1\beta_2 + b_2 \qquad \text{multiply out} \\ &= X\underbrace{(\beta_1\beta_2)}_{\beta} + \underbrace{(b_1\beta_2 + b_2)}_{b} \end{aligned}$$

Exercise 3

In its own HTML file.