Derivation of the Theoretical Framework in Romer and Romer (2010)
(For the Replication Project Macro)

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1 Introduction

This note presents a detailed mathematical derivation of the theoretical framework used in Romer and Romer (2010) to estimate the macroeconomic effects of tax changes on output. The framework leverages a narrative approach to address the endogeneity of tax changes, a pervasive issue in fiscal policy analysis, by isolating exogenous tax changes that are uncorrelated with other factors affecting output.

Key Ideas

- Decompose legislated tax changes into:
  - *Endogenous component*: Tax changes that respond to contemporaneous economic shocks or prospective economic conditions (e.g., countercyclical tax cuts to offset a recession).
  - *Exogenous component*: Tax changes driven by independent policy motives, such as addressing an inherited budget deficit or promoting long-run growth, which are assumed to be uncorrelated with contemporaneous output shocks.
- Use the exogenous component to identify the causal effect of tax changes on output, mitigating omitted variable bias present in regressions using broader tax measures.

2 The Baseline Reduced-Form Model

We start with a simple reduced-form model that relates changes in real output to changes in legislated tax liabilities:

$$\Delta Y_t = \alpha + \beta \Delta T_t + \varepsilon_t, \tag{1}$$

where:

- ΔY_t is the change in the logarithm of real GDP at time t,
- ΔT_t is the change in legislated tax liabilities (expressed as a percentage of GDP) at time t,
- ε_t is a composite error term capturing all other determinants of output growth, such as government spending, monetary policy shocks, and other macroeconomic factors.

The parameter β represents the causal effect of a tax change on output growth. However, estimating β directly using equation (1) is problematic because ΔT_t may be correlated with ε_t, leading to omitted variable bias.

3 Decomposition of the Error Term

The composite error term ε_t in equation (1) consists of multiple distinct shocks that affect output growth. We decompose it as:

$$\varepsilon_t = \sum_{i=1}^{K} \varepsilon_t^{(i)}, \tag{2}$$

where $\varepsilon_t^{(i)}$ represents the i-th shock (e.g., changes in government spending, monetary policy shocks, or supply shocks like oil price changes). These shocks are not necessarily uncorrelated with each other, reflecting the complex interdependencies in the economy.

4 Decomposition of Tax Changes

Legislated tax changes ΔT_t are influenced by both endogenous responses to economic conditions and exogenous policy decisions. Romer and Romer (2010) model this as:

$$\Delta T_t = \underbrace{\sum_{i=1}^{K} b_t^{(i)} \varepsilon_t^{(i)}}_{\text{Endogenous response}} + \underbrace{\sum_{j=1}^{L} \omega_t^{(j)}}_{\text{Exogenous component}}, \tag{3}$$

where:

- $b_t^{(i)}$ are time-varying coefficients capturing the responsiveness of tax changes to the shock $\varepsilon_t^{(i)}$. The time subscript reflects the discrete and episode-specific nature of tax policy responses (e.g., policymakers may respond more aggressively to a spending increase during a boom).
- $\sum_{i=1}^{K} b_t^{(i)} \varepsilon_t^{(i)}$ represents the endogenous component of tax changes, such as countercyclical tax cuts in response to a forecasted recession or tax increases to finance a rise in government spending.
- $\omega_t^{(j)}$ are exogenous tax changes driven by motives unrelated to current or prospective economic conditions, such as addressing an inherited budget deficit or promoting long-run growth (e.g., the Kennedy-Johnson tax cut of 1964).
- The key assumption is that the exogenous components $\omega_t^{(j)}$ are uncorrelated with the shocks $\varepsilon_t^{(i)}$ and the coefficients $b_t^{(i)}$:

$$\mathrm{Cov}(\omega_t^{(j)}, \varepsilon_t^{(i)}) = 0, \quad \mathrm{Cov}(\omega_t^{(j)}, b_t^{(i)}) = 0 \quad \forall i, j.$$

This decomposition is central to the narrative approach, where Romer and Romer use historical records to classify tax changes based on their stated motivations.

5 Substitution into the Baseline Equation

Substitute the tax decomposition (3) into the baseline model (1):

$$\begin{aligned}\Delta Y_t &= \alpha + \beta\left(\sum_{i=1}^{K} b_t^{(i)}\varepsilon_t^{(i)} + \sum_{j=1}^{L}\omega_t^{(j)}\right) + \varepsilon_t \\ &= \alpha + \beta\sum_{i=1}^{K} b_t^{(i)}\varepsilon_t^{(i)} + \beta\sum_{j=1}^{L}\omega_t^{(j)} + \sum_{i=1}^{K}\varepsilon_t^{(i)}. \end{aligned} \tag{4}$$

Group the terms involving $\varepsilon_t^{(i)}$:

$$\Delta Y_t = \alpha + \sum_{i=1}^{K}\left(\beta b_t^{(i)}\varepsilon_t^{(i)} + \varepsilon_t^{(i)}\right) + \beta\sum_{j=1}^{L}\omega_t^{(j)}. \tag{5}$$

Factor out $\varepsilon_t^{(i)}$:

$$\Delta Y_t = \alpha + \sum_{i=1}^{K}\left(1 + \beta b_t^{(i)}\right)\varepsilon_t^{(i)} + \beta\sum_{j=1}^{L}\omega_t^{(j)}. \tag{6}$$

6 Defining the Composite Error Term

Define a new composite error term ν_t as:

$$\nu_t \equiv \sum_{i=1}^{K}\left(1 + \beta b_t^{(i)}\right)\varepsilon_t^{(i)}. \tag{7}$$

Substituting ν_t into equation (6), we obtain:

$$\Delta Y_t = \alpha + \beta\sum_{j=1}^{L}\omega_t^{(j)} + \nu_t. \tag{8}$$

Let:

$$X_t \equiv \sum_{j=1}^{L}\omega_t^{(j)},$$

where X_t is the exogenous component of tax changes identified through the narrative analysis. Equation (8) becomes:

$$\Delta Y_t = \alpha + \beta X_t + \nu_t. \tag{9}$$

7 Identification and Estimation

The key identification assumption is that the exogenous tax shocks X_t are uncorrelated with the composite error term ν_t:

$$\text{Cov}(X_t, \nu_t) = 0.$$

This assumption holds if the $\omega_t^{(j)}$ are indeed exogenous, as assumed in equation (3). Under this condition, an OLS regression of ΔY_t on X_t in equation (9) yields an unbiased estimate of β, the causal effect of an exogenous tax change on output growth.

If instead the full measure ΔT_t is used (including both endogenous and exogenous components), the endogenous component $\sum_{i=1}^{K} b_t^{(i)} \varepsilon_t^{(i)}$ is correlated with ν_t, leading to biased estimates of β. This is the omitted variable bias that Romer and Romer (2010) aim to address. For example, a countercyclical tax cut in response to a recession (where $\varepsilon_t^{(i)}$ is negative) would be negatively correlated with output growth, attenuating the estimated effect of tax changes on output.

Romer and Romer (2010) also highlight that broader measures, such as changes in cyclically adjusted revenues, exacerbate this bias because they include nonpolicy factors (e.g., capital gains from a stock market boom) that are correlated with output growth.

8 Dynamic Specification and Cumulative Effects

The baseline model (9) assumes that tax changes affect output contemporaneously. In practice, the effects of tax changes are dynamic, unfolding over several periods. Romer and Romer (2010) estimate these dynamics using a vector autoregression (VAR) model, but for theoretical purposes, we can specify a distributed lag model:

$$\Delta Y_t = \alpha + \sum_{k=0}^{K} \beta_k X_{t-k} + \sum_{m=1}^{M} \gamma_m \Delta Y_{t-m} + u_t, \tag{10}$$

where:

- X_{t-k} is the exogenous tax shock at lag k,
- β_k is the effect of an exogenous tax shock k periods ago on current output growth,
- ΔY_{t-m} are lagged output growth terms to control for serial correlation in output,
- u_t is an error term.

The cumulative long-run effect of a tax shock is:

$$\beta_{\mathrm{LR}} = \sum_{k=0}^{K} \beta_k, \tag{11}$$

which captures the total impact of a tax change on output over time. Romer and Romer (2010) find that a 1% of GDP exogenous tax increase reduces GDP by approximately 3% over three years, with the effect peaking after 10 quarters.

9 Extensions and Robustness Considerations

9.1 Role of Expectations

The baseline framework dates tax changes at the time of implementation, consistent with evidence that consumers respond to changes in disposable income rather than news about future taxes. However, Romer and Romer (2010) explore the role of expectations by considering a specification that includes both the implementation effect and the news effect:

$$\Delta Y_t = \alpha + \sum_{k=0}^{K} \beta_k X_{t-k} + \sum_{k=0}^{K} \delta_k \mathrm{NEWS}_{t-k} + \sum_{m=1}^{M} \gamma_m \Delta Y_{t-m} + u_t, \tag{12}$$

where NEWS_{t-k} is the present value of future tax changes announced at time $t-k$. They find that output responds more strongly to implementation than to news, suggesting limited expectational effects.

9.2 Time-Varying Effects

Romer and Romer (2010) note that the effects of tax changes may have diminished over time, possibly due to stronger monetary policy responses or changes in fiscal expectations. This can be modeled by allowing β_k to vary across subperiods (e.g., pre- and post-1980).

9.3 Heterogeneity by Motivation

The exogenous tax changes are further categorized into those motivated by long-run goals (e.g., growth) and those addressing inherited deficits. The framework can be extended to estimate separate effects:

$$\Delta Y_t = \alpha + \beta^{\mathrm{LR}} X_t^{\mathrm{LR}} + \beta^{\mathrm{DEF}} X_t^{\mathrm{DEF}} + \nu_t,$$

where X_t^{LR} and X_t^{DEF} are the long-run and deficit-driven exogenous tax changes, respectively. Romer and Romer (2010) find that deficit-driven tax increases have a smaller negative effect on output (potentially even positive) compared to long-run motivated tax changes.

10 Limitations and Potential Biases

- **Misclassification Risk**: The narrative approach relies on correctly identifying the motivation of tax changes. Misclassifying an endogenous tax change as exogenous would violate the identification assumption, leading to bias.

- **Correlated Shocks**: Even exogenous tax changes may be accidentally correlated with other shocks in small samples. Romer and Romer (2010) address this by testing whether X_t is predictable using proxies for other shocks (e.g., government spending, oil prices), finding little evidence of correlation.

- **General Equilibrium Effects**: The framework does not explicitly model feedback effects, such as how tax changes affect interest rates or monetary policy, which may influence the estimated β.