Hydro Modelling
For Implementation in PGMcpp

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1 Motivation

In order to assess the economic viability of incorporating hydroelectric production into microgrid designs, a general and reliable means of modelling said performance is needed. Fortunately, this is relatively well established, so modelling is fairly straightforward.

2 Ideal Turbine Power

Recall the notion of fluid power

Definition 1. Power of a Fluid Flow: The power of a fluid flow P_{fluid} is given by the product of pressure p and volumetric flow rate Q. That is

$$P_{\text{fluid}} = pQ \tag{2.1}$$

An ideal turbine is then one that would harvest 100% of the available fluid power. Now, in the context of hydroelectric generation, it is more conventional to speak in terms of head rather than pressure. Therefore

Assumption 1. Pressure from Net Head: The pressure at the turbine inlet can be expressed as the product of fluid density ρ, gravity g, and the net head H_{net}. That is

$$p = \rho g H_{\text{net}} \tag{2.2}$$

From Definition 1 and Assumption 1, it then follows that

Definition 2. Ideal Turbine Power: The power output of an ideal turbine P_{ideal} is given by

$$P_{\text{ideal}} = \rho g H_{\text{net}} Q \tag{2.3}$$

3 Accounting for Turbine Efficiency

Of course, no real turbine is ideal. To account for this, one can scale the ideal turbine power by a turbine efficiency factor. That is

Definition 3. Real Turbine Power: The power output of a real turbine P_{turbine} is given by

$$P_{\text{turbine}} = \eta_{\text{turbine}} P_{\text{ideal}} = \eta_{\text{turbine}} \rho g H_{\text{net}} Q \tag{3.1}$$

where η_{turbine} is the turbine efficiency.

From data contained in [1], typical efficiency curves for Francis and Pelton type turbines are as given in Table 3.1 and Figure 3.1.

Table 3.1: Typical hydro turbine efficiencies for Francis, Kaplan, and Pelton type turbines. Power ratio is turbine power P_{turbine} divided by rated power capacity. Data from [1].

Power Ratio [ ]	η_{turbine} **(Francis)** [ ]	η_{turbine} **(Kaplan)** [ ]	η_{turbine} **(Pelton)** [ ]
0.0	0.000	0.000	0.000
0.1	0.400	0.265	0.780
0.2	0.625	0.460	0.855
0.3	0.745	0.550	0.875
0.4	0.810	0.650	0.890
0.5	0.845	0.740	0.900
0.6	0.880	0.805	0.908
0.7	0.900	0.845	0.913
0.8	0.910	0.900	0.918
0.9	0.900	0.880	0.908
1.0	0.850	0.850	0.880

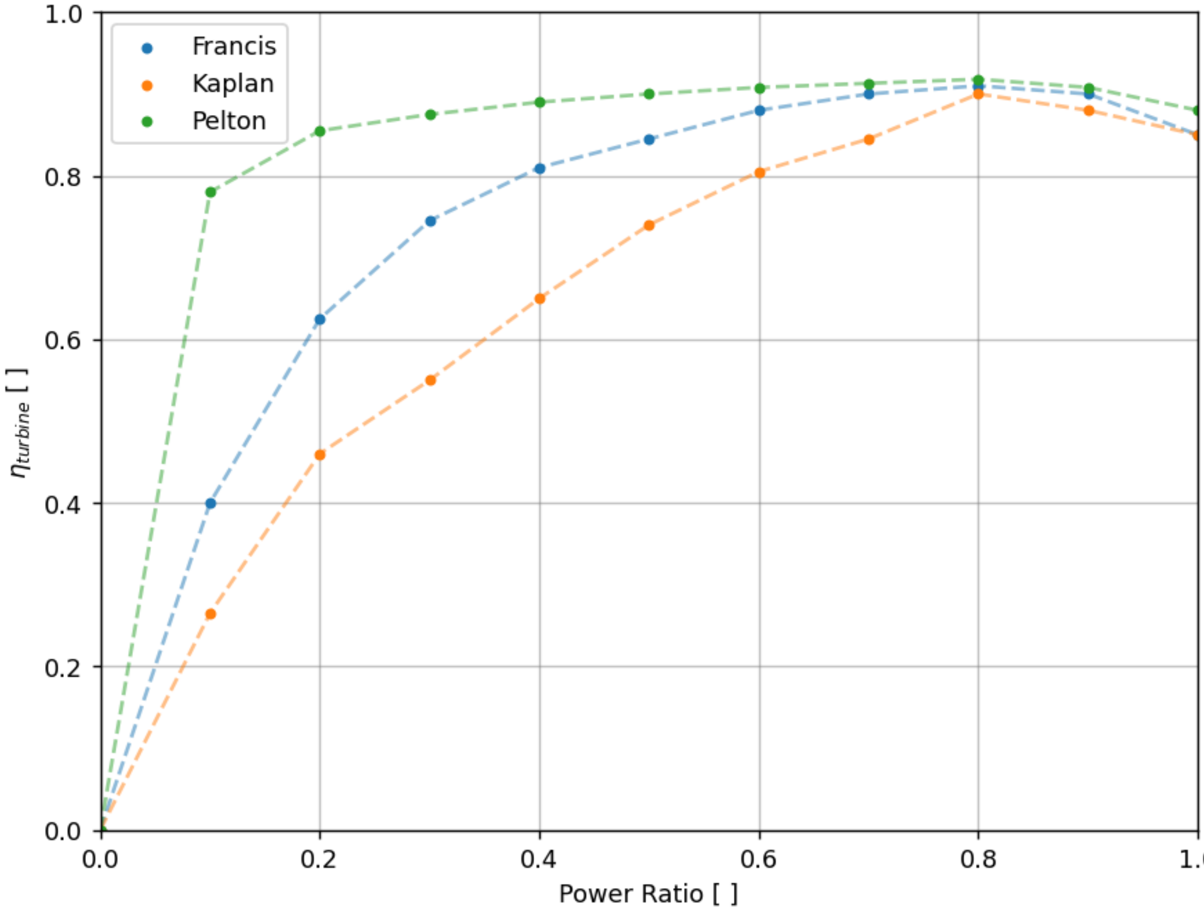

Figure 3.1: Typical hydro turbine efficiency curves for Francis, Kaplan, and Pelton type turbines. Power ratio is turbine power P_{turbine} divided by rated power capacity.

Given the data in Table 3.1, it is possible to interpolate values for η_{turbine} given power ratio $\widehat{P}$. So then

$$P_{\text{turbine}} = \eta_{\text{turbine}} \left(\widehat{P} \right) \rho g H_{\text{net}} Q \tag{3.2}$$

4 Accounting for Generator Efficiency

As a last step, the efficiency of the generator needs to be accounted for. That is

Definition 4. Hydroelectric Power: The power output of a hydroelectric plant P is given by

$$P = \eta_{\text{generator}}\eta_{\text{turbine}}P_{\text{ideal}} = \eta_{\text{generator}}\eta_{\text{turbine}}\rho g H_{\text{net}}Q \tag{4.1}$$

where $\eta_{\text{generator}}$ is the generator efficiency.

Again, data from [1] provides the typical AC generator efficiency curve given in Table 4.1 and Figure 4.1.

Table 4.1: Typical AC generator efficiencies. Power ratio is turbine power P_{turbine} divided by rated power capacity. Data from [1].

Power Ratio [ ]	$\boldsymbol{\eta_{\text{generator}}}$ [ ]
0.0	0.000
0.1	0.800
0.2	0.900
0.3	0.913
0.4	0.925
0.5	0.943
0.6	0.947
0.7	0.950
0.75	0.953
0.8	0.954
0.9	0.956
1.0	0.958

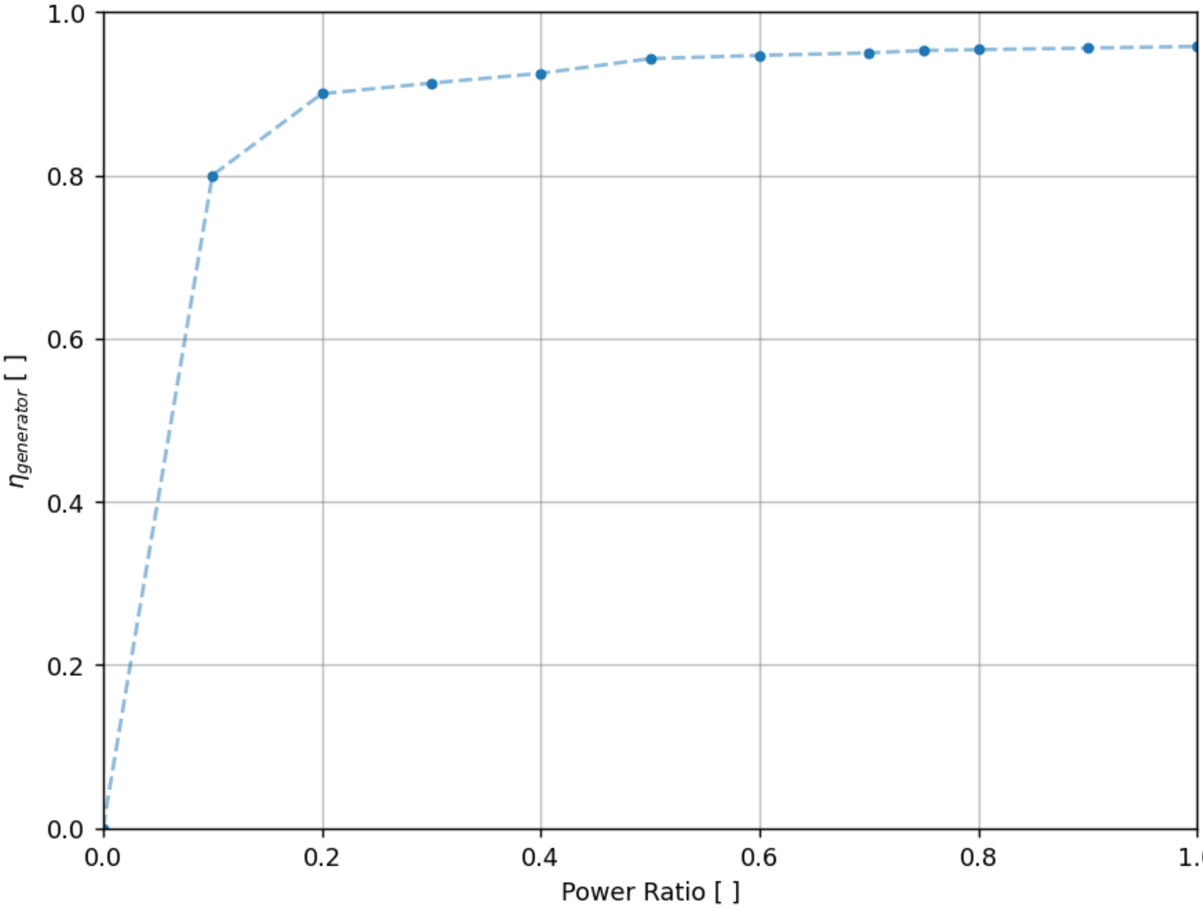

Figure 4.1: Typical AC generator efficiency curve. Power ratio is turbine power P_{turbine} divided by rated power capacity.

Given the data in Table 4.1, it is possible to interpolate values for $\eta_{\text{generator}}$ given power ratio $\widehat{P}$. So then

$$P = \eta_{\text{generator}}\left(\widehat{P}\right)\eta_{\text{turbine}}\left(\widehat{P}\right)\rho g H_{\text{net}} Q \tag{4.2}$$

Combining efficiency terms into a single factor then yields

$$P = \eta\left(\widehat{P}\right)\rho g H_{\text{net}} Q \tag{4.3}$$

From which it follows that

$$Q = \frac{P}{\eta\left(\widehat{P}\right)\rho g H_{\text{net}}} \tag{4.4}$$

and so one can map from flow to power, as well as from power to flow.

References

[1] Marks', *Marks' Standard Handbook for Mechanical Engineers*, 11th ed. McGraw-Hill, iSBN: 978-0-07-142867-5.